Field-theoretic pattern of violation of scaling in the timelike and spacelike regions

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We use the Gribov-Lipatov relation to distinguish which class of field theories, conventional type of asymptotically free, can account for the scaling violation observed in ep and e^+e^- interactions. Asymptotically free theories give a better description of these deviations in both timelike and spacelike regions. We also present some predictions for the single-particle inclusive distribution in e^+e^- interactions for PETRA energies.

I. INTRODUCTION

We address ourselves to the problem of scaling breaking effects observed in deep-inelastic lepton-hadron scattering and in electron-positron annihilation processes. Strict Bjorken scaling does not hold in renormalizable field theories.[1] It is therefore of importance to find out whether the data can distinguish which class of field theories (if any at all) are relevant, conventional type (C.T.) or asymptotically free (A.F.) type.

An analysis along these lines has been done by Tung[2] for the case of deep-inelastic electron-proton scattering. Within the limited range of the kinematic variables, he finds that both types of theories can account for the data.

In e^+e^- annihilation, beyond the new particles, the total cross section seems to scale ($R \simeq 5$), while there is a clear deviation from scaling in the inclusive distribution $e^+e^- \to \pi + \chi$. This deviation from scaling follows the same pattern as that observed in deep-inelastic scattering. One might ask whether these scaling-breaking effects can be described by a specific field theory. To that end one is guided by the Gribov-Lipatov relation[3] which relates the structure functions in timelike and spacelike regions. One can then exploit the wider range of energies available when both regions are considered together.

II. BASIC FORMALISM

Let us define the moment integrals in the usual way[4]

$$\int_0^1 dx\, x^{n-2} F(x, q^2) = M(n, q^2)\,. \tag{1}$$

Strict Bjorken scaling would imply that $M(n,q^2)$ does not depend on q^2. Renormalizable field theories on the other hand predict a factorizable q^2 dependence,

$$M(n,q^2) \underset{q^2\to\infty}{\sim} C(n) e^{-\lambda(n)k}\,. \tag{2}$$

Here $C(n)$ is an unknown constant; it contains our basic ignorance of the strong interactions. $\lambda(n)$ is the calculable anomalous dimension of the leading tensor operator in the Wilson expansion of the product of currents in the definition of the structure function. For conventional theories, we have[1]

$$\lambda(n) = A\left[1 - \frac{b}{n(n+1)}\right], \tag{3}$$

while for the nonsinglet part of asymptotically free field[4] theories

$$\lambda^{ns}(n) = G\left[-3 - \frac{2}{n(n+1)} + 4\sum_{m=1}^{n}\frac{1}{m}\right]. \tag{4}$$

A and G are parameters which in the following we shall include in the definition of k. Then

$$k = \begin{cases} A\ln(q^2/q_0^{\,2}) & \text{(C.T.)} \\ G\ln[\ln(q^2/\mu^2)/\ln(q_0^{\,2}/\mu^2)] & \text{(A.F.)}\,, \end{cases} \tag{5}$$

where $q_0^{\,2}$ is some reference value and μ^2 is a scale parameter (which we take to be 1 GeV^2).

We are interested in the isoscalar anomalous dimensions. These have the properties

$$\lambda^s(2) = 0\,,$$

$$\lambda^{ns}(n) - \lambda^s(n) \quad \text{small},$$

and

$$\lambda^{ns}(n) \gtrsim \lambda^s(n)\,.$$

The condition $\lambda^s(2) = 0$ simply expresses the fact that the energy-momentum tensor has canonical dimension 4. For the asymptotic case we shall use the following phenomenological formula which incorporates all the above requirements:

$$\lambda^s(n) = -3 - \frac{18}{n(n+1)} + 4\sum_{m=1}^{n}\frac{1}{m}\,. \tag{6}$$

For the conventional field theories we take[5]

$$\lambda(n) = 1 - \frac{6}{n(n+1)}\,. \tag{7}$$

Different choices from Eqs. (6) and (7) have been considered by Tung.[2] Within his range of q^2 available he found little differences between them; they merely result in a different choice of the normalization constants A or G.

In the now familiar way, given the structure function $F(x, q_0^2)$ at some reference value q_0^2 for $x_0 < x < 1$, Eqs. (1) and (2) can be used to evaluate the structure function $F(x, q^2)$ for all q^2 and $x_0 < x < 1$. We note that to determine $F(x, q^2)$, only $F(\xi, q_0^2)$ with $x < \xi < 1$ is needed.

The general trend of scaling breaking derives from the condition $\lambda(2) = 0$ and from the fact that $\lambda(n)$ is an increasing function of n. The requirement $\lambda(2) = 0$, which, as we have seen follows on general grounds, implies that the area under $F(x, q^2)$ scales, i.e., $\int_0^1 dx\, F(x, q^2)$ is independent of q^2.

It is well known that because the higher moments obtain their support mainly from the region $x \approx 1$ the structure function $F(x, q^2)$ for large x will be a decreasing function of q^2. Since the area under $F(x, q^2)$ must be conserved, this decrease can only be offset by an increase at small x. As noted by Tung[2] this is indeed the pattern observed in deep-inelastic *ep* scattering. There, both conventional and asymptotically free field theories give similar results for $x \gtrsim 0.2$ where most of the experimental data are available.[6] Only at smaller x ($x \lesssim 0.1$) can one hope to disentangle the two classes of field theories for a reasonable range of q^2.

The Gribov-Lipatov relation[3] gives a connection between the structure functions in the timelike and the spacelike region; it reads

$$x\frac{d\sigma}{dx}(e^+e^- \to h + \chi) = \frac{2\pi\alpha^2}{3s}F^{eh}(x)\,. \tag{8}$$

This relation has a simple intuitive basis in the parton model.[7] It is supported by the few available proton data.[8] We assume it to be valid in nature.[9]

Combining Eq. (8) with Eq. (1) we see that everything we have said about $F(x, q^2)$ can also be said about $x d\sigma/dx(e^+e^- \to \bar{h} + \chi)$. In particular, we expect to see scaling-breaking effects, and these should be governed by the same anomalous dimensions as for the deep-inelastic case. (That the ratio R is roughly constant above the region of the new-particle production gives us some confidence that we have reached the "scaling" region.) Most one-particle inclusive data in electron-positron annihilation are for pions. From Eq. (2) we know that the q^2 dependence of the moments can be factored out and should not depend on the specific type of hadron, so that we can write

$$\frac{3s}{4\pi\alpha^2}\int_0^1 dx\, x^{n-2} x\frac{d\sigma}{dx}(e^+e^- \to \pi + \chi) = C^\pi(n) e^{-\lambda(n)k}\,. \tag{9}$$

Hence once the inclusive distribution $x d\sigma/dx(e^+e^- \to \pi + \chi)$ is known for some reference value q_0^2, it is possible, using the same technique as before, to evaluate this quantity for all q^2. The q^2 dependence should be governed by the same anomalous dimensions $\lambda(n)$. The region in x and q^2 available is now much extended. In particular, data are available at small x ($x < 0.1$), where the difference between the two types of field theories is expected to be strongest, and for $q^2 = 23-55$ GeV^2 (SPEAR-II regime), which is hopefully large enough to work out the uncertainty in μ^2. In short, we may hope to distinguish between the two types of field theory.

Finally we note that the field theories can also be compared for their respective merits in timelike and spacelike regions separately.[10]

III. RESULTS

The broad trends in scale breaking discussed in Sec. II are indeed observed in the latest one-particle inclusive spectra $d\sigma/dx(e^+e^- \to \pi + \chi)$ from SPEAR.[11] (In Fig. 1 we have adjusted the overall normalizations of the data such that the areas under the curves are the same for all q^2. This amounts at most to a 10% correction.) The slow decrease with q^2 for $x > 0.3$ and the rapid increase for $x < 0.2$ are striking. There is also a shift of the peak toward smaller x.

We used the $\sqrt{s} = 4.8$ GeV data as input to calculate the distribution at $\sqrt{s} = 6.2$ GeV and $\sqrt{s} = 7.4$ GeV. These are compared with the data in Fig. 1 for both types of field theories. The best values for the parameters are $A = 0.4$ and $G = 0.35$. It is seen that the A.F. theories fit the data much better than the C.T. ones. C.T. theories give the wrong q^2 behavior: for $x > 0.2$ there is a decrease with q^2, but the peak broadens and flattens. The area under the curve is conserved only because of a substantial increase in the small-x region. Parameter A cannot be taken too large since this would enhance the very-small-x region too much, to such an extent that the forward dip even disappears. On the other hand, A.F. field theories yield a q^2 behavior qualitatively very much like that of the data. The peak does not grow fast enough with q^2, but it increases with q^2 and it shifts toward smaller x.

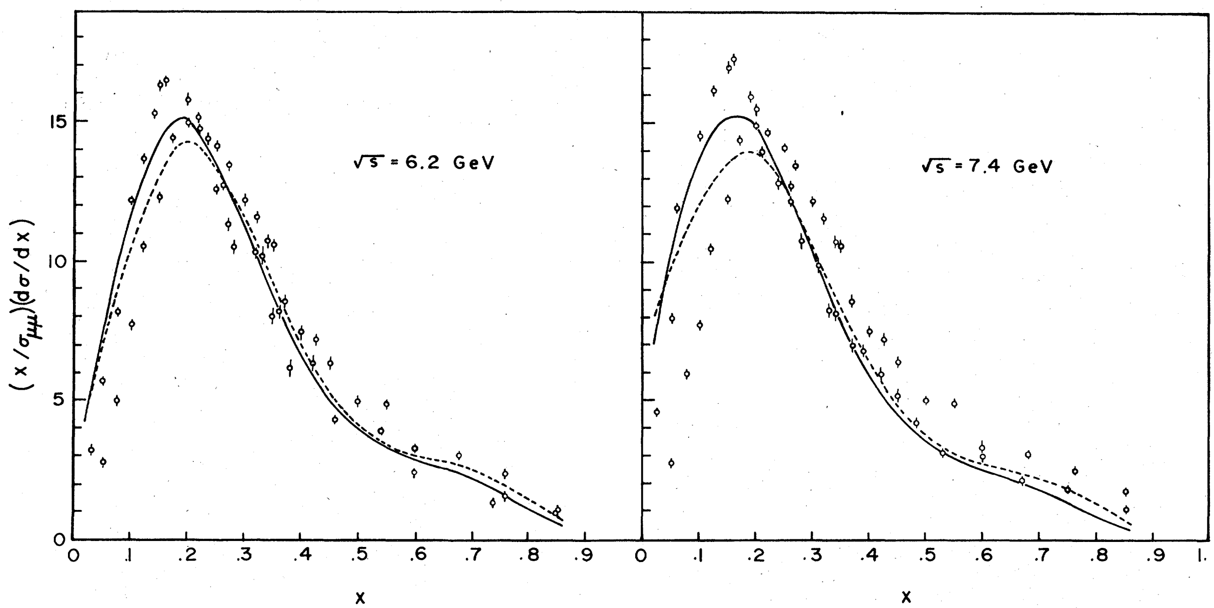

FIG. 1. Comparison of the conventional theory and the asymptotically free theories at the SPEAR-II energies. Solid line represents asymptotically free theory and dashed line represents conventional theory.

The region $x>0.25$ is well described. As for the C.T. case the small-x region puts an upper limit on the value of G. The best value of G, $G=0.35$, compares favorably with $G=0.16$ as expected from the four-color-triplet model. Different choices of the anomalous dimensions, of the value of parameter μ^2 and/or use of a nonasymptotic expression for k in Eq. (5) can all lead to changes in the value of G. The value of $G=0.35$ differs from the best value $G=0.085$ found by Tung[2] in deep-inelastic scattering. For C.T. theories there is no theoretical value for A. The value $A=0.4$ compares with $A=0.25$ found by Tung.

Finally, since PETRA will be in operation within some years, we also make predictions up to its energy regime. These are shown in Fig. 2.

IV. CONCLUSIONS

The scaling-breaking effects seen in the single-particle inclusive distributions in e^+e^- are similar to those observed in deep-inelastic ep scattering, and one may argue that they have a similar field-theoretic interpretation. In particular the Gribov-Lipatov relation, which gives a direct relation between the timelike and spacelike structure functions, implies that both regimes are governed by the same field theory. In a comparison with the data, especially in the smaller-x region ($x<0.2$), where one expects a very different behavior for the two classes of field theories, it turns out that the asymptotically free theories fare much better. Although neither field theory gives a perfect fit to the observed q^2 behavior, only the A.F. theories give qualitatively the features seen in the data.

We have also made predictions for the one-particle inclusive spectra in the PETRA regime. We hope to report on a similar analysis for $ep \to ehX$ and for the two-particle inclusive cross sections in e^+e^- interactions. A similar analysis of the scaling-breaking pattern for massive-lepton-pair production in pp collisions at Fermilab and CERN-ISR energies has been reported elsewhere.[12]

Note added. After this work was completed Glück and Reya[13] have independently shown that

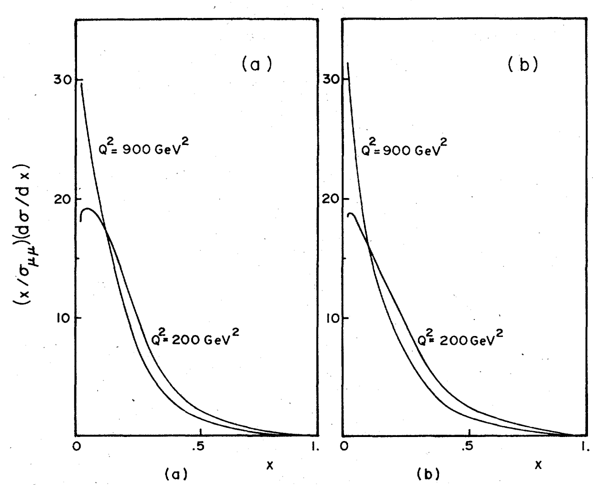

FIG. 2. Predictions at the PETRA energies. (a) Asymptotically free theory (b) conventional theory.

only asymptotically free theories are compatible with experiment and not other theories whose fixed points are not at the origin of the coupling-constant space. Thus, their result agrees completely with the present analysis. However, we had to overcome the difficult task of continuing from the spacelike to the timelike region. Besides, since most of the data for $e^+e^- \to h+\chi$ come from $x \leq 0.3$, there might be some contamination coming from the nonleptonic decays of charmed particles, which we could not take into account properly. However, the interesting task of distinguishing the two classes of field theories is too tempting to warrant a strict view on the field-theoretic validity of the Gribov-Lipatov relation or some contamination from the nonleptonic decays of charmed particles. With such a *raison d'etre* in mind, we have addressed ourselves to the pattern of scale breaking, whose analysis agrees completely with the more theoretically oriented approach of Glück and Reya.

ACKNOWLEDGMENT

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[1]G. Parisi, Phys. Lett. 42B, 114 (1972); C. Callan and D. Gross, Phys. Rev. D 8, 4383 (1972).

[2]W.-K. Tung, Phys. Rev. Lett. 35, 490 (1975).

[3]V. N. Gribov and L. N. Lipatov, Phys. Lett. 37B, 78 (1971); Yad. Fiz. 15, 781 (1972) [Sov. J. Nucl. Phys. 15, 438 (1972).

[4]For review on asymptotically free theory, see H. D. Politzer, Phys. Rep. 14, 129 (1974).

[5]This choice is typical of any nongauge theory in perturbation theory. If strong interactions are governed by such a field theory, the fixed point will not be at the origin of the coupling-constant space. Hence the anomalous dimensions, which are determined at the fixed point, cannot be reliably obtained by perturbation theory. As pointed out by Tung (Ref. 2), an obstacle to a systematic study of distinguishing gauge theory from nongauge theory lies in the lack of precise knowledge of the anomalous dimensions in the latter kind of field theory. Such obstacles can be overcome only by adopting a phenomenological parametrization of the anomalous dimension Eq. (7) consistent with the known general constraints of the function derived some time back [O. Nachtmann, Nucl. Phys. B63, 237 (1973)].

[6]The latest SLAC data [E. M. Riordan *et al.*, SLAC Report No. SLAC-PUB-1634 (unpublished)] go down to $x=0.1$, but for this x value the q^2 range is very small. We have redone Tung's analysis on these data without substantial change in conclusions.

[7]For a review, see J. D. Sullivan, talk presented at the SLAC Summer Institute of Physics, 1974 [SLAC Report No. 167, Vol. 1 (unpublished)].

[8]See F. J. Gilman, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire England, 1974).

[9]The field-theoretic status of the Gribov-Lipatov relation is still in its infancy and so is the principle of the continuation from the spacelike region to the timelike region. Following the pioneering work of A. H. Mueller [Phys. Rev. D 9, 963 (1974)], C. G. Callan and M. L. Goldberger [Phys. Rev. D 11, 1542 (1975)] have shown that the annihilation structure functions have a behavior completely analogous to electroproduction structure functions. Their moments scale for large virtual-photon mass and this scaling is described by anomalous dimensions very similar to the usual anomalous dimensions.

[10]Scale breaking in e^+e^- has previously been studied by V. Rittenberg and D. H. Schiller [Nucl. Phys. B87, 480 (1975)]. However, they used only conventional field theories.

[11]C. Morehouse, report presented at the SLAC Summer School, 1975 (unpublished).

[12]D. K. Choudhury, Tata Institute Report No. TIFR/76-45 (unpublished).

[13]M. Glück and E. Reya, Phys. Rev. D 16, 3242 (1977).