Narrow resonance in an open channel

D. Horn*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

Daniel E. Novoseller†
Stanford Linear Accelerator Center, Stanford, California 94305
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We discuss the possibility that a very narrow resonance will exist in a multiresonance system above the threshold of an open decay channel. We show that its width varies as the square of the commutator of the mass and width matrices near the vanishing point of this commutator. A hypothetical example of such an effect, relevant to narrow-resonance searches in e^+e^- annihilation, is discussed.

Very narrow resonances can exist in strongly interacting systems even in the presence of open decay channels. This fact was observed by Fonda and Newton in their formalism of resonance reactions[1] and by Coulter and Shaw in an N/D approach.[2] We analyze this phenomenon within the Weisskopf-Wigner approach[3] where it becomes a simple quantum-mechanical effect. We present the criterion for its existence and discuss the stability of this phonomenon. We point out that in the realm of particle physics it could be observed in e^+e^- annihilation, i.e., a narrow bound state of heavy quarks may be found above the production threshold of pairs of states which have the new quantum numbers of these heavy quarks.

This phenomenon may occur in a system of overlapping resonances in which the number of open decay channels is smaller than the number of resonance states. In particular, we will consider a system of two overlapping resonance states which couple to only one open decay channel. Using for the resonance states a basis $|1\rangle$, $|2\rangle$ which is invariant under time reversal, and denoting the coupling of these states to the open channel by α and β, respectively, it is clear that the combination $\beta|1\rangle-\alpha|2\rangle$ is a state which decouples from the open channel, i.e., has zero width. However, in general such a decoupling does not occur.[4] The system of resonance states, which can be described by the effective Hamiltonian[5]

$$\underline{H}=\underline{M}-\frac{i}{2}\underline{\Gamma}, \tag{1}$$

where $\underline{M}$ and $\underline{\Gamma}$ are real symmetric matrices, cannot be diagonalized by the orthogonal transformation that diagonalizes the matrix $\underline{\Gamma}$. The reason is that in general

$$\underline{C}=[\underline{M},\underline{\Gamma}]\neq 0. \tag{2}$$

Hence both complex eigenvalues of Eq. 1 will have nonvanishing imaginary parts.

The investigation and understanding of resonance systems dates back to the original paper of Weisskopf and Wigner.[3] In recent years there appeared in the literature several studies of overlapping resonance systems with the intent of application to particle physics,[5,6] in particular relating the Hamiltonian (1) to an S matrix which is proportional to $(E\underline{I}-\underline{M}+\frac{1}{2}i\underline{\Gamma})^{-1}$. In narrow-resonance systems one often regards $\underline{M}$ and $\underline{\Gamma}$ as constant matrices. In general, they may have an energy dependence which is regulated by a dispersion relation. The S matrix may then be described in terms of two fixed poles and an energy-dependent background or, alternatively, as having two effective poles whose location and decay structure vary with energy. The rich hadronic spectrum and structure observed in e^+e^- annihilation around and above 4 GeV is an example of a physical system which should be explored in terms of overlapping resonances.[7] Here the resonances are produced by a virtual photon. Very narrow states, whose nonzero width comes from electromagnetic decays and strongly suppressed hadronic decays, are observed below 3.72 GeV. Above this energy allowed hadronic decay channels open up and broad resonance structures are obtained. Our purpose is to show that even in the presence of such an open channel one may find a narrow resonance.

The phenomenon that we discuss here occurs when the commutator $\underline{C}$ is small. Let us parametrize our 2×2 mass and width matrices using as a basis the Pauli spin matrices $\underline{\sigma}$:

$$\begin{aligned}\underline{M}&=\overline{M}\underline{I}+a\underline{\sigma}_z+b\underline{\sigma}_x,\\ \underline{\Gamma}&=(c^2+d^2)^{1/2}\underline{I}+c\underline{\sigma}_z+d\underline{\sigma}_x,\end{aligned} \tag{3}$$

where $\overline{M}$ is the average mass of the two-resonance system and $\underline{\Gamma}$ is a matrix of rank one, i.e., $\det\underline{\Gamma}=0$, as required in a single-channel problem. The commutator is then

$$\underline{C}=i\underline{\sigma}_y\Delta,\quad \Delta=2(ad-bc). \tag{4}$$

When Δ vanishes $\underline{H}$ can be diagonalized by the orthogonal transformation that diagonalizes $\underline{\Gamma}$. The condition for obtaining a zero-width resonance is therefore

$$\Gamma_1(E)=0 \text{ if } \Delta(E)=0\,, \tag{5}$$

where we denote by Γ_1 the width of our narrowest resonance. Satisfying this condition may be quite an accident. We will, however, show that it suffices to have Δ small in order to obtain a very small Γ_1. Intuitively, this is clear if one considers the variation of Γ_1 as a function of Δ. At the point $\Delta=0$, Γ_1 vanishes. However, since zero is also the minimal value that Γ_1 can obtain,

$$\left.\frac{\partial \Gamma_1}{\partial \Delta}\right|_{\Delta=0}=0 \tag{6}$$

and Γ_1 will vary as Δ^2 for small Δ.

Let us solve for Γ_1 in terms of the five free parameters of the problem which are given in Eq. (3). Diagonalization of the algebraic system of equations is a straightforward exercise. Solving the problem in the neighborhood of $\Delta=0$ we find

$$\frac{\Gamma_1}{\Gamma_2}\approx\frac{\Delta^2}{16(ac+bd)^2+4(c^2+d^2)^2}\,. \tag{7}$$

We conclude that as long as $|\Delta|$ is considerably smaller than the other bilinear combinations of parameters which appear in the denominator of Eq. (7), a narrow resonance should exist.

An example of this effect is given by the figures. Figure 1 is a trial fit to the e^+e^- annihilation cross-section ratio R in the neighborhood of 4 GeV.[8] This fit is obtained by using a constant background plus two overlapping resonances in a two-channel problem. One of the two channels is assumed to open up only at 4.0114 GeV. Below this energy we have then a system which can be

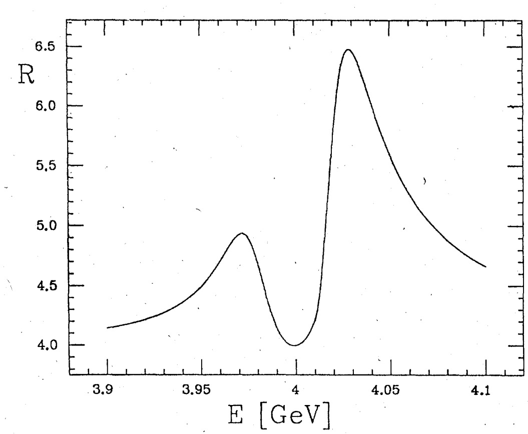

FIG. 1. Pattern of two interfering resonances produced by a trial fit to e^+e^- annihilation data under the assumptions specified in the text.

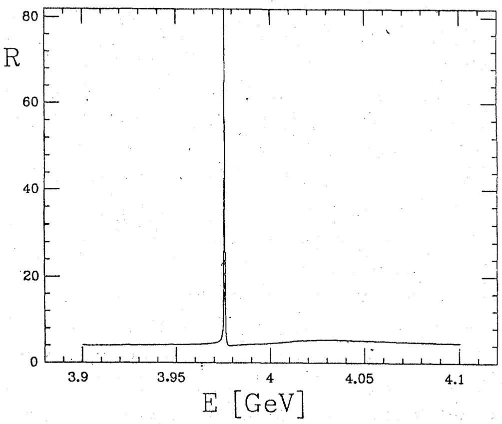

FIG. 2. The result obtained by changing only the sign of the parameter d in the effective Hamiltonian of the system that is shown in Fig. 1.

described by Eq. (3).[9] By reversing the sign of the parameter d in this fit we obtained the structure shown in Fig. 2. The latter is dominated by a very strong narrow resonance since the condition for Eq. (7) is met. This resonance occurs on top of a seemingly slowly varying "background" which consists of the wide resonance. Clearly both signs of d are physically allowed and, within this model, Fig. 2 is *a priori* as probable and plausible a result as Fig. 1. Figure 2 is by no means exceptional. The parameters a, b, c and d are of the same order of magnitude, and whenever their relative signs are such as to make $|\Delta|$ small, a narrow resonance will appear.

The location of the narrow resonance depends on $\overline{M}$. By increasing $\overline{M}$ we can shift the peak to the region in which the two channels are open in the example discussed above. We find that, depending on the values of the other parameters, sometimes the resonance stays narrow and sometimes it broadens considerably. It is clear that in general a narrow resonance will occur if both the conditions

$$\underline{C}\approx 0\,,\quad \det\underline{\Gamma}\approx 0 \tag{8}$$

are met. Whenever the number of open channels is smaller than the number of overlapping resonances, $\det\underline{\Gamma}$ vanishes, and only one condition is left. However, even if the number of channels is large, the possibility exists that two resonances will have very similar eigenchannels, in which case a narrow resonance will appear if the conditions for Eq. (7) are met.

The actual physical situation in the region of our example is somewhat different from the assumptions that we have made in order to illustrate our point. We know that there exist two open channels ($D\bar{D}$ and $D\bar{D}^* + D^*\bar{D}$) below 4 GeV. This makes the existence of a very narrow resonance less likely. Such an effect as we have discussed could have existed in the region between 3.72 to 3.87 GeV, where only the $D\bar{D}$ channel is open, if there were overlapping 1^{--} resonances in this region. A more favorable situation may be found in systems of higher quark masses such as the recently discovered Υ states,[10] where one expects a denser set of levels to appear in the region of the first continuum channel.[11] The narrow-resonance effect may of course occur in many other systems. It can, however be quite elusive when searched for in normal scattering channels. The clear advantage of a process such as e^+e^- annihilation is that the production of the resonance is determined by its coupling to the photon, a parameter which is independent of the details of its hadronic decay modes.

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*On leave of absence from Tel-Aviv University.
†Present address: TRW Systems, Inc., Redondo Beach, Calif. 90278.

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