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Lorentz invariance of tachyon theories

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The Lorentz invariance of tachyon theories is investigated. It is found that in the theory of spinless fermion tachyons proposed some time ago, the rates of various physical processes involving tachyons and ordinary particles are properly Lorentz covariant, in the passive sense that the measurements of the same situation by different observers are related by Lorentz transformations. This invariance requires the rates for processes involving various tachyon configurations that can be Lorentz transformed into each other to be added together. Lorentz invariance is not satisfied by spinless boson tachyon theories. The reason for the differences between tachyon theories, and between tachyon theories and ordinary theories is discussed.

I. INTRODUCTION

Some years ago, a quantum field theory was proposed[1] to describe hypothetical, noninteracting particles of imaginary rest mass (tachyons). This proposal stimulated a large number of papers on related theoretical subjects,[2] as well as several experimental searches for such particles,[3] all of which have given negative results. In the original article, a number of questions were left unanswered, and the succeeding literature has not entirely filled this gap. In the present work, I shall reconsider one such question, that of the Lorentz invariance of the *particle theory* associated with the field theory. Specifically, I shall describe a precise criterion for Lorentz invariance of a particle theory, and indicate the extent to which a theory of interacting tachyons can be shown to satisfy this criterion.

Ordinarily, the Lorentz invariance of a field theory leads directly to Lorentz invariance of the particle interpretation of the theory. Suggestions that this might not hold for tachyon theories are connected with the peculiar transformation properties of the particle states in such theories, as opposed to those in ordinary theories. Specifically, in the theory outlined in Ref. 1, the Lorentz transformations are such that a state containing no tachyons changes, under Lorentz boosts, into a state containing antitachyons with each momentum value in a continuous range determined by the velocity of the boost. This state will therefore have a nonzero antitachyon density in each volume of space, and an infinite antitachyon number in an infinite space. Thus two observers, related by Lorentz boosts, will have very different notions about the tachyon content of the "same" configuration of the external world. We shall see below that just this peculiar transformation is required to satisfy Lorentz invariance of the description of physical phenomena. In this connection, it should be recognized that two descriptions may look different, and nevertheless give identical results in all circumstances, in which case the difference is at most one of esthetic preference.

In order to introduce the relevant results, let us consider a situation in which some observer X sees at some time a single charged tachyon present in some region of space, which may be taken arbitrarily large, and no other particles present anywhere. This situation is not stable, because a charged tachyon can radiate photons by Čerenkov radiation. Therefore, at a later time there will also be photons, as well as the charged tachyon. This radiation will continue indefinitely, although the rate of energy loss will decrease rapidly because the tachyon has less energy to lose, and eventually only photons of arbitrarily low energy will be radiated. We disregard the possibility that the tachyons can also decay by other channels. The rate of radiation of photons will depend on the tachyon energy at any given time, and could be computed by the standard methods if we prescribed the matrix element for photon emission by a tachyon. However, we need not consider in this paper the precise form of this matrix element.

The rate of radiation of photons, or equivalently, the rate of appearance of photons, involves only quantities referring to ordinary particles. Hence it must have the simple Lorentz transformation property

$$R_\gamma(\vec{p}_\gamma)=R'_\gamma(\vec{p}'_\gamma)\,, \tag{1.1}$$

where p_γ is the photon momentum and R_γ is the rate at which photons appear in the invariant momentum interval d^3p_γ/E_γ, per unit volume of space, all as measured by one observer, while $\vec{p}'_\gamma, R'_\gamma$ are the corresponding quantities as measured by a different observer. The quantity R'_γ must be calculated from the matrix element evaluated in the transformed tachyon state seen by the second observer, which state, from the above remarks, will be very different from the one-tachyon state seen by the first observer.

In particular, the second observer will see photons appearing as the result of Čerenkov radiation by any of the many tachyons present, or by any of the antitachyons present, or as the result of annihilations of a tachyon with an antitachyon into a single photon, also a kinematically allowed process for tachyons. The total appearance rate of photons will for any observer be the sum of all of these, but for our first observer, who by assumptions detects a single tachyon at t_0, only the first process in the state is measured. In order for the relation (1.1) to be satisfied, two things must happen: There must be a relation between the matrix elements for tachyon Čerenkov radiation and tachyon pair annihilation, which is to be expected from crossing symmetry, since these processes involve the same product of field operators, or the same Feynman graph. Of course, a derivation of such relations would require a quantum *field* theory of interacting tachyons, which we do not have available. In the present work, we shall therefore have to *assume* the required property of the matrix element.

In addition, there must be a relation between the tachyon and antitachyon numbers in the states as seen by the different Lorentz observers. For instance, if the photon appearance rate as measured by the second observer is to get a contribution from tachyon pair annihilation, or from antitachyon Čerenkov radiation, his version of the state must contain antitachyons. This is just a statement of the fact that the laws of quantum mechanics allow only transitions from particles actually present in the initial state to those actually present in the final state.

This required relation between states is exactly of the type that occurs in the theory of Ref. 1. We shall prove below that in this theory, the relation is just such as to ensure the validity of Eq. (1.1) for the case of photon production. We shall then indicate how it can be proved for an arbitrary tachyon process, and for arbitrary initial tachyon states, always provided that the matrix elements satisfy simple "crossing relations" and Lorentz transformation properties. It is not clear that other theories of tachyons that have been proposed have the necessary connection between states in different Lorentz frames to satisfy Eq. (1.1) for arbitrary initial states. Hence it seems likely that such theories fail to satisfy this requirement of Lorentz invariance for quantities referring to ordinary particles, and so fail to meet the minimal requirements of a relativistic quantum field theory.

The theory of Ref. 1 therefore is Lorentz invariant insofar as quantities referring to ordinary particles are concerned. This is true in the passive sense that observers moving at different velocities will have measurements on the same system that are related by Lorentz transformations. Measurements by a single observer on two systems, say with different energy, are not simply related in general, so that "active" Lorentz invariance may not be satisfied, although it may hold for special tachyon distributions.

Since all measurements ultimately are made on ordinary particles, one may be satisfied with a Lorentz invariance that refers only to their properties. Note that the tachyons that interact with the ordinary particles are not virtual particles, in that they have been taken to satisfy the usual mass-shell conditions. A statement of Lorentz invariance for quantities directly involving tachyons will require a better understanding of tachyon states which, unlike the few particle states considered here, transform in a simple way under Lorentz transformations. This problem will be treated elsewhere.

II. ČERENKOV RADIATION BY TACHYONS

We calculate here the photon production rate through various tachyon processes related by Lorentz transformations. Actually, the formulas that I shall use are valid for the production rate of any combination of ordinary particles by tachyons, either by emission or by pair annihilation, as I shall not use anything about the properties of photons. With the minor changes $N \to 1-N$, $k \to -k$, they would apply to the decay of some combination of ordinary particles into tachyons, and by superposing these two cases, to any reaction in which tachyons scatter or convert into ordinary particles through Fig. 1.

Let the rate per unit volume of photon production in an invariant momentum interval be denoted $R(\vec{k})$, i.e.,

$$dN(k) = R(\vec{k})\frac{d^3k}{\omega_k} \tag{2.1}$$

is the number of photons produced with this momentum, per unit volume of space. $R(k)$ is the result of the related processes of Čerenkov radiation by tachyons, by antitachyons, and of tachyon pair annihilation into a single photon. We neglect other photon-producing processes. We imagine that all of the processes we consider are described by a single Feynman graph (Fig. 1) and a single matrix element, evaluated for various possible positive and negative four-momenta of the tachyon lines. We hasten to add that the use of negative energies here does not involve the existence of negative-energy states any more than when it is done to express crossing symmetry for ordinary particles.

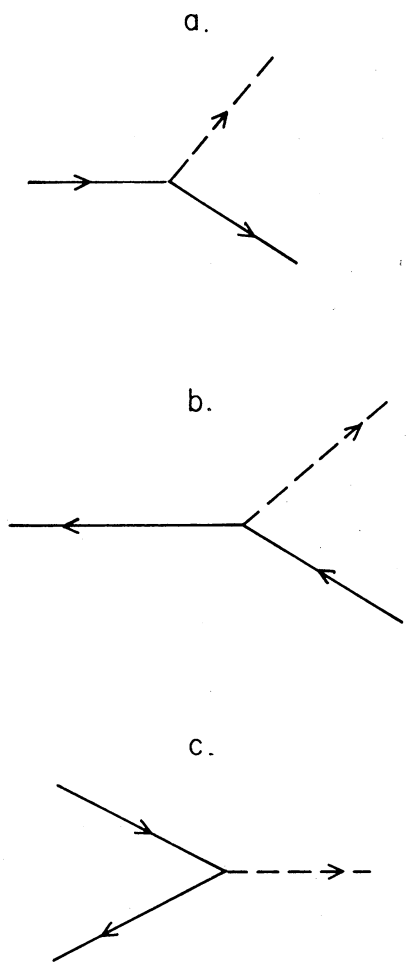

FIG. 1. Three processes which are related by Lorentz transformations: (a) Emission of a photon by a tachyon, (b) emission of a photon by an antitachyron, (c) annihilation of a tachyon and antitachyon into a photon.

We adopt the notational convention that a four-vector p_μ always has a positive energy, so that $-p_\mu$ has negative energy. Let $M(p_1,p_2,k)$ be the matrix element corresponding to Fig. 1. Then $M(p_1,p_2,k)$ is the matrix element for Čerenkov radiation by a tachyon, $M(-p_2,-p_1,k)$ is the matrix element for Čerenkov radiation by an antitachyon, $M(p_1,-p_2,k)$ and $M(-p_1,p_2,k)$ are the matrix elements for tachyon-antitachyon pair annihilation into a photon. We *assume* here that a single analytic function describes all these processes.

We further define $N(p)$ to be the tachyon number density per unit volume, in a momentum interval, and $\overline{N}(p)$ to be the corresponding antitachyon density, both as measured by some observer. That is, $N(p)d^3p$ is the number of tachyons with momentum p, per unit volume of space. Strictly speaking, we should consider separately the quantities N_i, N_f that characterize the initial and final states. However, for a given process such as Čerenkov radiation, N_f is determined by N_i and so we need not consider it as an independent variable. Stated differently, the density matrix for the final state is determined by that for the initial state together with the S matrix. For the fermion tachyon of Ref. 1, we have $0 \leq N(p) \leq 1$, but for the moment we consider the general case. By allowing $N(p)$ to differ from an integer, we are considering a statistical mixture of occupation numbers, rather than a pure state, a generalization that will be of some value to us.

For the observer in question, the photon production rate is then the sum of three terms,

$$R(\vec{k}) = R_T(\vec{k}) + R_{\overline{T}}(\vec{k}) + R_{T\overline{T}}(\vec{k}) , \tag{2.2}$$

where

$$R_T(\vec{k}) = \int N(p_1)[1-N(p_2)]\,|M(p_1,p_2,k)|^2 \delta(p_1^2-\mu^2)\delta(p_2^2-\mu^2)\theta(E_1)\theta(E_2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2 , \tag{2.3}$$

$$R_{\overline{T}}(\vec{k}) = \int \overline{N}(p_1)[1-\overline{N}(p_2)]\,|M(-p_2,-p_1,k)|^2 \delta(p_1^2-\mu^2)\delta(p_2^2-\mu^2)\theta(E_1)\theta(E_2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2 , \tag{2.4}$$

$$R_{T\overline{T}}(\vec{k}) = \int N(p_1)\overline{N}(p_2)\,|M(p_1,-p_2,k)|^2 \delta(p_1^2-\mu^2)\delta(p_2^2-\mu^2)\theta(E_1)\theta(E_2)\delta^4(p_1+p_2-k)d^4p_1d^4p_2 . \tag{2.5}$$

These three terms represent respectively the rates for Čerenkov radiation by tachyons, Čerenkov radiation by antitachyons, and tachyon-antitachyon pair annihilation into a photon. Note that the rate corresponding to spontaneous appearance of a tachyon-antitachyon pair together with a photon, all other particles being undisturbed, vanishes by energy conservation, since no negative-energy states occur in this theory. The presence of the factors $(1-N)$ and $(1-\overline{N})$ in R_T and $R_{\overline{T}}$ requires some comment. Since our tachyons are fermions, a given process cannot occur if it results in a tachyon being emitted into a final state already containing a tachyon, whereas it is permitted if no tachyon is present. On the other hand, the rate for any of the processes will vanish if $N=0$, if there are no tachyons present in the initial state with the relevant momenta. This explains the presence of the factors N, $\overline{N}$ in $R_T, R_{\overline{T}}, R_{T\overline{T}}$. A general derivation of (2.2), (2.3) and (2.4) will be given in Appendix A. The use of the same matrix element M in R_T, $R_{\overline{T}}$ and $R_{T\overline{T}}$ is the crossing relation referred to earlier.

The rate R, or the partial rates $R_T, R_{\overline{T}}, R_{T\overline{T}}$, cannot be calculated, even if M is known, unless we specify $N, \bar{N}$. We could of course calculate them for specific N or $\bar{N}$, such as

$$N = \frac{\delta^3(\vec{p} - \vec{p}_0)}{V}, \quad \bar{N} = 0$$

where V is the volume of space and $\vec{p}_0$ is some specific three-momentum. This would correspond to the photon production rate by a single tachyon in all space. However, the rates obtained for this case have little direct significance, because the distribution assumed is a very special one, valid in at most one Lorentz frame.

We instead ask how N and $\bar{N}$, and hence R, transform from one Lorentz observer to another. The quantities $N, \bar{N}$ can be expressed in terms of matrix elements of the tachyon or antitachyon number operators $c^\dagger(p)c(p)$, $d^\dagger(p)d(p)$.

The Lorentz transformation of such operators is determined by the transformation properties of the tachyon field $\phi(x)$, as described in Ref. 1. Therefore, we can relate the values of $N, \bar{N}$ as seen by different observers. In line with our comment above, we need only consider the transformation of particles in the initial state. In the theory of Ref. 1, Lorentz transformations do not take particles from initial to final state. It is straightforward to show (see Appendix B) that

$$N'(p') = N(p)\theta(p_0) + [1 - \bar{N}(-p)]\theta(-p_0), \qquad (2.6)$$

$$\bar{N}'(p') = \bar{N}(p)\theta(p_0) + [1 - N(-p)]\theta(-p_0). \qquad (2.7)$$

Here N and N' are the tachyon densities as measured by two observers, related by a Lorentz transformation $a_{\mu\nu}$, and p' is the four-vector Lorentz transform of p, i.e., $p'_\mu = a_{\mu\nu}p_\nu$ or $p_\mu = a_{\nu\mu}p'_\nu$. When a given Lorentz transformation $a_{\mu\nu}$ is applied, p_0 and p'_0 will have the same sign for some p and opposite sign for other p. Then (2.6) and (2.7) say that for those p, where sign $p'_0/p_0 = +1$, the tachyon densities at corresponding momenta are equal. On the other hand, for those p where sign $p'_0/p_0 = -1$, the tachyon density measured by one observer is proportional to the antitachyon deficit $(1 - \bar{N})$ seen by the other observer at the corresponding momentum. In writing (2.6) and (2.7), we have taken the final energies p'_0 to be positive, as this is the application we shall make below. These rules are a straightforward extension of those given in Ref. 1 for states with definite occupation numbers. They obviously make sense only for fermion tachyons, where $0 \leq N \leq 1$. We shall see that for boson tachyons, there is no simple variant of (2.6) and (2.7) which can be made to give Lorentz-invariant results.

The second observer will detect photons with different momenta $k'_\mu = a_{\mu\nu}k_\nu$ and with different rates, which we denote as R'. These can be expressed in terms of $N', \bar{N}'$, and the Lorentz-transformed matrix element M', by formulas analogous to (2.2), (2.3), and (2.4):

$$R'_T(k') = \int N'(p'_1)[1 - N'(p'_2)]\,|M'(p'_1, p'_2, k')|^2 \delta(p_1'^2 - \mu^2)\delta(p_2'^2 - \mu^2)\theta(E'_1)\theta(E'_2)d^4p'_1 d^4p'_2 \delta^4(p'_1 - p'_2 - k'),$$

$$R'_{\bar{T}}(k') = \int \bar{N}'(p'_1)[1 - N'(p'_2)]\,|M'(-p'_2, -p'_1, k')|^2 \delta(p_1'^2 - \mu^2)\delta(p_2'^2 - \mu)\theta(E'_1)\theta(E'_2)\delta^4(p'_1 - p'_2 - k')d^4p'_1 d^4p'_2,$$

$$R'_{T\bar{T}}(k') = \int N'(p'_1)\bar{N}'(p'_2)\,|M'(p'_1, -p'_2, k)|^2 \delta(p_1'^2 - \mu^2)\delta(p_2'^2 - \mu^2)\theta(E'_1)\theta(E'_2)\delta^4(p'_1 + p'_2 - k')\,d^4p'_1 d^4p'_2.$$

In order to compare the rates as measured by the two observers, we must substitute the formulas (2.6) and (2.7) for the transformed densities $N', \bar{N}'$, into the expressions for R', and make some assumptions about the relation between M' and M. We can also use the identities

$$d^4p'_1 = d^4p_1,$$

$$\delta^4(p'_1 + p'_2 - k') = \delta^4(p_1 + p_2 - k),$$

$$\delta(p_1'^2 - \mu^2) = \delta(p_1^2 - \mu^2),$$

etc.

We obtain the expressions

$$\begin{aligned}R'_T(k') &= \int N(p_1)[1 - N(p_2)]\,|M'(p'_1, p'_2, k')|^2 \delta(p_1^2 - \mu^2)\delta(p_2^2 - \mu^2)\theta(E'_1)\theta(E'_2)\theta(E_1)\theta(E_2)\delta^4(p_1 - p_2 - k)d^4p_1 d^4p_2 \\ &+ \int N(p_1)\bar{N}(-p_2)\,|M'(p'_1, p'_2, k')|^2 \delta(p_1^2 - \mu^2)\delta(p_2^2 - \mu^2)\theta(E'_1)\theta(E'_2)\theta(E_1)\theta(-E_2)\delta^4(p_1 - p_2 - k)d^4p_1 d^4p_2 \\ &+ \int [1 - \bar{N}(-p_1)][1 - N(p_2)]\,|M'(p'_1, p'_2, k')|^2 \delta(p_1^2 - \mu^2)\delta(p_2^2 - \mu^2)\theta(E'_1)\theta(E'_2)\theta(-E_1)\theta(E_2) \\ &\quad \times \delta^4(p_1 - p_2 - k)d^4p_1 d^4p_2\end{aligned}$$

$$+\int [1-\bar{N}(-p_1)]\bar{N}(-p_2)\delta(p_1{}^2-\mu^2)|M'(p_1',p_2',k')|^2\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(-E_1)\theta(-E_2)$$
$$\times\,\delta^4(p_1-p_2-k)d^4p_1d^4p_2\,, \tag{2.8}$$

$$R_{\bar{T}}'(k')=\int N(p_1)[1-\bar{N}(p_2)]|M'(-p_2',-p_1',k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(E_1)\theta(E_2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2$$
$$+\int \bar{N}(p_1)N(-p_2)|M'(-p_2',-p_1',k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(E_1)\theta(-E_2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2$$
$$+\int [1-N(-p_1)][1-\bar{N}(p_2)]|M'(-p_2',-p_1,k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(-E_1)\theta(E_2)$$
$$\times\,\delta^4(p_1-p_2-k)d^4p_1d^4p_2$$
$$+\int [1-N(-p_1)]N(-p_2)|M'(-p_2,-p_1',k')\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(-E_1)\theta(-E_2)$$
$$\times\,\delta^4(p_1-p_2-k)d^4p_1d^4p_2\,, \tag{2.9}$$

$$R_{T\bar{T}}'(k')=\int N(p_1)\bar{N}(p_2)|M'(p_1',-p_2',k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(E_1)\theta(E_2)\delta^4(p_1+p_2-k)d^4p_1d^4p_2$$
$$+\int N(p_1)[1-N(-p_2)]|M'(p_1',-p_2',k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(+E_1)\theta(-E_2)$$
$$\times\,\delta^4(p_1+p_2-k)d^4p_1d^4p_2$$
$$+\int [1-\bar{N}(-p_1)]\bar{N}(p_2)|M'(p_1',-p_2',k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\theta(E_1')\theta(E_2')\theta(-E_1)\theta(E_2)$$
$$\times\,\delta^4(p_1+p_2-k)d^4p_1d^4p_2$$
$$+\int [1-\bar{N}(-p_1)][1-N(-p_2)]|M'(p_1',-p_2',k')|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu)\theta(E_1')\theta(E_2')\theta(-E_1)\theta(-E_2)$$
$$\times\,\delta^4(p_1+p_2-k)d^4p_1d^4p_2\,, \tag{2.10}$$

$R'=R_T'+R_{\bar{T}}'+R_{T\bar{T}}'$.

It may be seen from Eqs. (2.8), (2.9), and (2.10) that the individual rates $R_{\bar{T}}', R_{T\bar{T}}'$ cannot be equal to the corresponding unprimed rates, for arbitrary $N, \bar{N}$, because of the extra terms that occur in each of them. This is just because the change of the tachyon and antitachyon densities from observer to observer implies that some individual processes that were possible for one observer are impossible for another, and conversely. However, we may take the attitude that there is no need to require the invariance of those individual rates, since they refer to particles that are unobserved. On the other hand, the total production rate of photons can be determined just from measurements on known particles, and so should be invariant.

We now assume that the matrix element $|M|^2$ satisfies the invariance property $|M'(p_1',p_2',k')|^2 = |M(p_1,p_2,k)|^2$, as it would in a theory of ordinary particles. This amounts to the assumption that the S matrix in a field theory of interacting tachyons could be written as a Lorentz-invariant functional of the tachyon and photon fields. In the absence of such a theory, we will simply assume the required property. It is simple to obtain examples of M, satisfying this condition by analogy with ordinary charges.

We can then group the terms in R' that contain a factor such as $N\bar{N}$, with the following results (after various interchanges of p_1 with $-p_1$, or p_1 with p_2):

$$R'(k')=\int N(p_1)[1-N(p_2)]|M(p_1,p_2,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2[\theta(E_1)\theta(E_2)]$$
$$\times[\theta(E_1')\theta(E_2')+\theta(-E_1')\theta(-E_2')+\theta(E_1')\theta(-E_2')]$$
$$+\int N(p_1)[1-\bar{N}(p_2)]|M(-p_2,-p_1,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2[\theta(E_1)\theta(E_2)]$$
$$\times[\theta(E_1')\theta(E_2')+\theta(-E_1')\theta(-E_2')+\theta(+E_1')\theta(-E_2')]$$
$$+\int N(p_1)\bar{N}(p_2)|M(p_1,-p_2,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(p_1+p_2-k)d^4p_1d^4p_2[\theta(E_1)\theta(E_2)]$$
$$\times[\theta(E_1')\theta(E_2')+\theta(-E_1')\theta(E_2')+\theta(E_1')\theta(-E_2')]$$

$$+\int [1-N(p_1)][1-\bar{N}(p_2)]\,|M(-p_2,p_1,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(-p_1-p_2-k)[\theta(E_1)\theta(E_2)]$$
$$\times\,[\theta(E_1')\theta(-E_2')+\theta(-E_1')\theta(E_2')+\theta(-E_1')\theta(-E_2')]\,. \qquad (2.11)$$

Since k_0 is positive the last term vanishes, because the δ^4 factor requires $p_{1,0}+p_{2,0}$ to be negative, while the $\theta(E_1)\theta(E_2)$ factor forbids this. The other terms can be written as

$$R'(k')=\int N(p_1)[1-N(p_2)]\,|M(p_1,p_2,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(p_1-p_2-k)d^4p_1d^4p_2\theta(E_1)\theta(E_2)\,[1-\theta(-E_1')\theta(E_2')]$$
$$+\int \bar{N}(p_1)[1-\bar{N}(p_2)]\,|M(-p_2,-p_1,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(p_1-p_2-k)$$
$$\times\, d^4p_1d^4p_2\theta(E_1)\theta(E_2)[1-\theta(-E_1')\theta(+E_2')]$$
$$+\int N(p_1)\bar{N}(p_2)\,|M(p_1,-p_2,k)|^2\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\delta^4(p_1+p_2-k)$$
$$\times\, d^4p_1d^4p_2\theta(E_1)\theta(E_2)[1-\theta(-E_1')\theta(-E_2')]\,. \qquad (2.12)$$

It may now be seen that the θ functions in the last set of brackets in each of the terms in R' vanish, when multiplied by the corresponding δ^4 function, by the argument given above. When these terms are dropped, we get immediately the desired result that, for any $N,\bar{N}$,

$$R'(k')=R(k)$$

by comparing Eq. (2.12) with Eqs. (2.3), (2.4), and (2.5).

The two observers will therefore agree on the photon production rate at corresponding momenta, although they will ascribe the rate of origin of the photons to different linear combinations of the three rates we have considered.

We note that because of this last circumstance, if we try to make an "active" interpretation of the Lorentz transformation, and use it to compare the photon production rate under two different circumstances for one observer, we obtain rather uninteresting results. These would say that the photon production rates are equal for two different tachyon distributions, related by (2.6), and (2.7), where now N,N' represent two densities as measured by the same observer for two *different* states of the world, in which all particles have different momenta, related by $a_{\mu\nu}$. Because of the $\theta(-p_0)$ terms in (2.6) and (2.7), these two states involve very different numbers of tachyons and so the relations are not very enlightening. This is to be contrasted with the case of ordinary particles, where the corresponding relations are between states with equal number of particles, and so give useful information about how the transition rates depend on the momenta of the initial particles.

The difference between these two cases is not, however, an indication of non-Lorentz invariance of the tachyon theory. It is instead a consequence of the fact that when tachyons are involved, the states we deal with always have complicated Lorentz transformation properties. An instructive comparison can be made with processes involving only ordinary particles, but in which there is a background distribution of matter involved in the process. For example, one may consider the annihilation of positrons with the electrons in a solid body, whose average momentum is zero in some Lorentz frame. In this case, because of the momentum distribution of the individual electrons in the solid, the pair annihilation rate into one photon, say, will depend in a complicated way on the positron momentum. In other words, the annihilation rate as measured by any one observer for two different positrons will not be simply related. The reason is of course that Lorentz invariance will instead relate processes in which the solid's momentum is changed as well as the positron's. The latter relations will be of the same type that we have derived here for tachyons, that is, they will relate the photon production rates measured by two different observers in the same external state, e.g., the state in which a single positron of some momentum is incident on a solid in a solid's rest system. The only difference between this case and the tachyon case is that here it is obvious that something other than the positron must be Lorentz transformed, whereas for the tachyons it is the formal theory that indicates the need to transform the tachyon numbers as indicated in (2.6) and (2.7).

The lack of an active form of Lorentz invariance for processes involving physical tachyons will probably spill over into a similar lack of active Lorentz invariance for processes in which only virtual tachyons are involved. For example, it is possible that the self-mass of an electron will receive a contribution from the emission and absorption of tachyons, which is not the same for electrons of different velocity as seen by a single observer. I say this is possible, rather than certain, because this conclusion depends on being able to do calculations with virtual tachyons, by way of quantum field theory, and it is not yet clear (to me) how this can be done.

If this type of noninvariance does occur for ordinary particles, its significance for the existence of tachyons will depend heavily on the strength of their interaction with ordinary particles. Other considerations indicate that the tachyon–ordinary-particle interaction must in any case be extremely small, in which case there may be no practical consequences of the lack of active invariance. I shall return to this question in another palce.

A further insight into the need for these transformations can be obtained by examination of the expression for R_T, $R_{\overline{T}}$, and $R_{T\overline{T}}$. As defined, these quantities represent the photon production rates per unit volume of space. If we multiply both sides of the equation by the volume of space, we obtain the total rate at which tachyons are producing photons in all space. This latter rate is what is usually calculated in particle physics by using Fermi's golden rule. One generally does this by taking $N, \overline{N}$ to represent one particle in all of space, so that $N, \overline{N}$ will be inversely proportional to V. In that case, we see that $VR_T, VR_{\overline{T}}$ will be independent of the volume for large V, and represent finite rates for the amount of Čerenkov radiation in all space. On the other hand, $R_{T\overline{T}}$ will be proportional to V^{-2}, so that $VR_{T\overline{T}}$ will vanish in the limit of infinite volume. This is just an expression of the well-known fact that for one particle in the initial state, the transition rate is finite in the infinite volume limit, while for two particles in the initial state, it is the cross section that is finite, and the cross section has an extra power of volume compared to the transition rate.

These results imply that $VR_{T\overline{T}}$ would make a vanishing contribution to VR if $N, \overline{N}$ represent a finite number of tachyons in all space. But we know that $VR_{T\overline{T}}$ must make a finite contribution in some Lorentz systems, since tachyon Čerenkov radiation in one Lorentz frame may kinematically become pair annihilation in another Lorentz frame. This implies that even if N represents one particle in all space in some Lorentz frame, so that $N \propto 1/V$ in this frame, in another Lorentz frame, $\overline{N}$ must have a part that is independent of V, in order that $VR_{T\overline{T}}$ not vanish in that Lorentz frame. This is exactly what is accomplished by the factors $(1-\overline{N})\theta(-p_0)$ appearing in (2.6), since if $\overline{N} \propto 1/V$ the term 1 will give something independent of V. Therefore, the finite tachyon densities that appear under Lorentz transformations in the theory of Ref. 1, far from indicating a lack of Lorentz invariance of the theory, are essential to the demonstration of Lorentz invariance of the particle theory.

We have also investigated whether a similar demonstration of Lorentz invariance, in the sense of Eq. (1.1), can be given for spinless boson tachyons. In this case, the factors $(1-N)$ and $(1-\overline{N})$ appearing in (2.2) and (2.3) would be replaced by $1+N, 1+\overline{N}$, by the standard properties of bosons. We can then ask whether any transformation analogous to (2.6) and (2.7) can be written, which would lead to Eq. (1.1). The answer is no, at least if one requires this equation to hold for arbitrary $N, \overline{N}$. This is shown in Appendix C. This is expected, because the factor $1+N$ is never negative, and there is no possibility of suppressing the emission of bosons, if such emission is kinematically possible. Therefore, while a single boson tachyon cannot produce photons if it has zero energy, a Lorentz transform of this situation, in which nonzero-energy tachyons and antitachyons are present, will in general lead to photon production, and the two Lorentz observers will not agree on the photon production rate. Also, theories of boson tachyons do not have proper behavior under Lorentz transformations of the commutation relations among creation and annihilation operators,[1] and so it is not surprising that the results are not Lorentz invariant.

The result for fermions indicates that several previous discussions[4] of Čerenkov radiation by tachyons are not relevant. These discussions try to relate by Lorentz transformations the rate of Čerenkov radiation as seen by different observers for the *same* tachyon distribution, which is not what the theory actually relates. Specifically, there is no contradiction with having a single tachyon emit Čerenkov radiation as seen by one observer, and the fact that if this tachyon is transformed to zero energy, it cannot emit any radiation. The radiation that will be observed in the latter case will come from the extra tachyons and antitachyons that are introduced by the transformation, which do not have zero energy. Similarly, an electron emitting Čerenkov radiation in a medium can be Lorentz transformed so that it is at rest. The radiation that is still detected will now be attributed to the moving particles of the medium, interacting with an electron at rest.

The actual calculation of the rate of Čerenkov radiation by tachyons depends on the assumption of some matrix element for the process. This will be discussed in a future paper in which various tachyon searches are analyzed.

III. DISCUSSION

It is not difficult to extend the discussion of Sec.

II to other tachyon processes. What is necessary is to consider all processes that can be transformed into one another by crossing symmetry of the external tachyon lines. The standard integral in the rate for such processes must be multiplied by factors $N(p), \bar{N}(p)$ for tachyons or antitachyons in the initial state, and by $(1-N), (1-\bar{N})$ for tachyons or antitachyons in the final state. The assumption of Lorentz invariance of the matrix element, together with the formulas for Lorentz transformation of the densities (2.6) and (2.7), can then be used to relate the rates in different coordinate systems for the production or absorption of any combination of ordinary particles. No new problems arise in doing this, and the result is essentially the same as in Sec. II. That is, the appearance or disappearance rates of any combination of ordinary particles are invariant, just as they would be for a theory in which no tachyons appeared. Therefore, always under the assumption of Lorentz-invariant matrix elements, the results of observations on ordinary particles are no less Lorentz invariant in the theory of Ref. 1 than in standard quantum field theories. Since all attempts to detect tachyons have and will continue to rely on the observation of ordinary particles, this result is important for the interpretation of such observations (or their lack). However, I do not believe that the result completely answers the question of Lorentz invariance of tachyon theories. There is no example yet available of a quantum field theory of interacting tachyons, and so no way of deriving the invariance properties of the transition matrix element that we have assumed. Until this is accomplished, some question of Lorentz invariance remains.

Nevertheless, the results of this paper are of interest in two connections. One is that a clear distinction arises between fermion tachyons and boson tachyons, with the latter not leading to a Lorentz-invariant theory. The other item of interest is that because of the peculiar Lorentz transformation property of the tachyon densities, it is likely that in any Lorentz frame, there will be a finite number density of tachyons over some region of momentum space. Because of this, the production rate or decay rate of ordinary particles through tachyon mediated processes is likely to get contributions both from scattering by tachyons and from tachyon pair creation or annihilation. Any experimental search for tachyons should allow for both types of contributions, and also allow for the possible suppression of effects because of the tachyon exclusion principle. The analyses given previously have not done this, and I shall reanalyze various tachyon searches in this way in a subsequent paper.

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APPENDIX A

In this appendix, we shall indicate the derivation of Eqs. (2.3), (2.4), and (2.5). In these equations, the factors of $|M|^2$, the δ functions, and the θ functions are the standard terms and will not be further explained. The novelty comes from the factors of N and $1-N$ for fermions, and $N, 1+N$ for bosons in the corresponding formulas of Appendix C. To see how these factors emerge, we consider a system described by a statistical mixture, in which the number of particles of any momentum is not well defined. For simplicity, we fix on a scattering from a given initial momentum to a given final momentum of the tachyon. Consider first the fermion case. The probability of scattering is certainly proportional to the probability P_1 that there is one, rather than zero tachyon in the initial state. Furthermore, assuming that the initial and final momenta are different, which is almost always the case, the probability of scattering into the final state is proportional to P_0', the probability that the final momentum had no tachyon present before the scattering. If there were already a tachyon with the final momentum, then by the exclusion principle a second tachyon could not be scattered into this momentum.

The average number N of tachyons with the initial momentum is clearly $P_1 = 1 - P_0$. The average number N' of tachyons with the final momentum is $P_1' = 1 - P_0'$. Therefore

$$N = 1 - P_0 = P_1 ,$$

$$N' = 1 - P_0' = P_1' .$$

The transition probability for the scattering is proportional to $P_0' P_1 = N(1-N')$ as assumed in (2.3). A similar derivation indicates that the tachyon-antitachyon annihilation term is proportional to $N\bar{N}$ as in (2.4).

For the boson case, assume instead that P_n is the probability that n particles are found in the initial state and that P_n' is the same probability for the final state. We have

$$\sum_n P_n = 1 = \sum_n P_n' .$$

In this case, the transition probability is proportional to

$$(P_1 + 2P_2 + 3P_3 + \cdots) \times (P_0' + 2P_1' + 3P_2' + \cdots) .$$

The average particle number is given by

$$N = P_1 + 2P_2 + 3P_3 + \cdots .$$

It is easy to see that the factor $P_0' + 2P_1' + 3P_2' + \cdots = 1 + N'$ so that the transition probability in the boson case is proportional to $N(1+N')$ as assumed in Appendix C. A more formal derivation of these formulas can be given along the lines used by Tolman.[5]

APPENDIX B

In this appendix, we derive the transformation of the number densities $N(p)$ that have been taken to describe the tachyons. The system of tachyons and antitachyons is taken to be a mixture, described by a statistical matrix ρ.

It is convenient to use a formalism introduced by Van Weert and de Boer[6] to describe relativistic transport theory. In this formalism, the number density per unit volume and per momentum interval is

$$N(p) = \int d^4u \,\mathrm{tr}[\rho c^\dagger(p - \tfrac{1}{2}u)c(p + \tfrac{1}{2}u)]\delta(p\cdot u)\,, \tag{B1}$$

$$\bar{N}(p) = \int d^4u \,\mathrm{tr}[\rho d^\dagger(p - \tfrac{1}{2}u)d(p + \tfrac{1}{2}u)]\delta(p\cdot u)\,. \tag{B2}$$

Here $c, c^\dagger$ and $d, d^\dagger$ are annihilation and creation operators for tachyons and antitachyons, differing in normalization from those used in Ref. 1 by a factor $\sqrt{p_0}$. Thus

$$\{c^\dagger(p - \tfrac{1}{2}u), c(p + \tfrac{1}{2}u)\} = \delta^3(u)p_0\,,$$

and similarly for $d, d^\dagger$. The formalism of Ref. 6 is valid provided that the system is sufficiently homogeneous that one can neglect variations of densities over the De Broglie wavelengths of the particles involved.[6] Under this assumption, it is valid to neglect terms such as u_0/p_0 in (B1) and (B2) and these formulas reduce to more conventional expressions for number densities.

Consider now a Lorentz-transformed observer, for whom $p, \omega_p \to p', \omega_{p'}$ where the primed quantities are the geometric transforms of the unprimed. This observer will measure number densities

$$N'(p') = \int d^4u' \,\mathrm{tr}[L\rho L^{-1} c^\dagger(p' - \tfrac{1}{2}u')c(p' + \tfrac{1}{2}u')] \times \delta(p'\cdot u') \tag{B3}$$

and similarly for $\bar{N}'(p')$.

From Eq. (D13) of Ref. 1, and the above definition of c, we have

$$Lc(p)L^{-1} = c(p)\theta(p_0) + c^\dagger(p)\theta(-p_0)\,.$$

By the assumptions made in Ref. 6, we must take the energy variables in $c^\dagger$ and in c above to be the same, i.e., we must neglect u_0 compared to p_0 in the argument of $c, c^\dagger$. Then

$$\begin{aligned} N'(p') &= \int d^4u' \,\mathrm{tr}[\rho\theta(+p_0)c^\dagger(p - \tfrac{1}{2}u)c(p + \tfrac{1}{2}u) \\ &\qquad + \rho\theta(-p_0)d(-p - \tfrac{1}{2}u)d^\dagger(-p + \tfrac{1}{2}u)] \\ &\qquad \times \delta(p'\cdot u') \end{aligned} \tag{B4}$$

$$\begin{aligned} &= \int d^4u\{\theta(+p_0)\,\mathrm{tr}[\rho c^\dagger(p - \tfrac{1}{2}u)c(p + \tfrac{1}{2}u)] \\ &\qquad \times \theta(-p_0)[\delta^3(u)p_0 - d^\dagger(-p + \tfrac{1}{2}u) \\ &\qquad\qquad \times d(-p - \tfrac{1}{2}u)]\}\delta(p\cdot u) \\ &= \theta(+p_0)N(p) + \theta(-p_0)[1 - \bar{N}(-p)]\,. \end{aligned} \tag{B5}$$

Similarly

$$\bar{N}'(p') = \theta(+p_0)\bar{N}(p) + \theta(-p_0)[1 - N(-p)]\,. \tag{B6}$$

Note that if the total number of tachyons in some Lorentz system is a finite number N_0, then in the infinite-volume limit, $N(p) \propto N_0/V$ approaches zero. However, in another Lorentz system, $\bar{N}(p)$ will contain a finite density for some range of p, because of the 1 in the second term of (B6). This finite density is what is needed according to the discussion in the text.

APPENDIX C

We consider here the possibility of Lorentz invariance of boson theories of spinless tachyons. We illustrate the result for the case of Čerenkov and annihilation radiation.

In analogy to Eqs. (2.3), (2.4), and (2.5), we now assume

$$\begin{aligned} R_T^{(B)}(k) &= \int N(p_1)[1 + N(p_2)]\,|M(p_1, p_2, k)|^2\delta(p_1{}^2 - \mu^2)\delta(p_2{}^2 - \mu^2)\theta(E_1)\theta(E_2)\delta^4(p_1 - p_2 - k)d^4p_1d^4p_2\,, \\ R_{\bar{T}}^{(B)}(k) &= \int \bar{N}(p_1)[1 + \bar{N}(p_2)]\,|M(-p_2, -p_1, k)|^2\delta(p_1{}^2 - \mu^2)\delta(p_2{}^2 - \mu^2)\theta(E_1)\theta(E_2)\delta^4(p_1 - p_2 - k)d^4p_1d^4p_2\,, \\ R_{T\bar{T}}^{(B)}(k) &= \int N(p_1)\bar{N}(p_2)\,|M(p_1, -p_2, k)|^2\delta(p_1{}^2 - \mu^2)\delta(p_2 - \mu^2)\theta(E_1)\theta(E_2)\delta^4(p_1 + p_2 - k)d^4p_1d^4p_2\,. \end{aligned} \tag{C1}$$

Here the superscript B refers to boson tachyons, and we have changed the factors $1 - N$ of the fermion case to $1 + N$.

We assume that a Lorentz-transformed observer will measure tachyon and antitachyon densities linearly related to the original densities

$$N'(p')=N(p)\theta(p_0)+[A+B\bar{N}(-p)]\theta(-p_0)\,,$$
$$\bar{N}'(p')=\bar{N}(p)\theta(p_0)+|A+BN(-p)|\theta(-p_0)\,, \tag{C2}$$

where A, B are constants, which we try to choose to satisfy Lorentz invariance. This linear transformation of $N(p)$ is a consequence of a linear transformation of the fields $\phi(x)$ under Lorentz transformation, together with the bilinear expression for $N(p)$ in terms of $c, c^\dagger$, given in Eq. (B1), or its equivalent for bosons.

By manipulations such as those of Sec. II, we can rewrite R' so that it depends on $N, \bar{N}$. Upon doing so, we find that we must choose $B=1$ in order to get the term proportional to $N\bar{N}$ to come out right. The term proportional to N, however, takes the form

$$\begin{aligned}\int d^4p_1 d^4p_2 N(p_1)[&A\theta(-E_1')\theta(E_2')\delta^4(+p_1-p_2-k)M(+p_1,p_2,k)\\ &+\theta(E_1')\theta(E_2')\delta^4(p_1-p_2-k)M(p_1,p_2,k)+(1+A)\theta(E_1')\theta(-E_2')\delta(p_1+p_2-k)M(p_1,-p_2,k)\\ &+(1+2A)\theta(-E_1')\theta(-E_2')\delta(p_1-p_2-k)M(p_1,p_2,k)]\theta(E_1)\theta(E_2)\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\,.\end{aligned} \tag{C3}$$

Also, the boson case leads to a term independent of N and of $\bar{N}$, proportional to $A(1+A)$. In order to make this term vanish, it is necessary to take $A=0$, since $A=-1$ would imply negative tachyon densities—an absurdity. If we choose $A=0$ (or indeed any positive A) it is impossible to transform the term (C3) into the form it should have from Eq. (C1) above. The bad term is

$$(1+A)\int d^4p_1 d^4p_2 N(p_1)\theta(E_1')\theta(-E_2')\delta(p_1+p_2-k)M(p_1,-p_2,k)\theta(E_1)\theta(E_2)\delta(p_1{}^2-\mu^2)\delta(p_2{}^2-\mu^2)\,. \tag{C4}$$

This term has the kinematics of part of a tachyon-antitachyon annihilation term, rather than of a tachyon Čerenkov radiation term, as it should have from the factor $N(p_1)$. In the fermion case, the first factor is $1-A$, which vanishes for $A=1$, and so the term disappears.

Strictly speaking I have only shown that boson theories of tachyons are not Lorentz invariant if they describe processes such as the emission of ordinary particles by tachyons and pair annihilation or creation of tachyons. I do not know if other boson tachyon theories could be Lorentz invariant.

[1]G. Feinberg, Phys. Rev. 159, 1089 (1967).

[2]See the bibliography in E. Recami and R. Mignani, Riv. Nuovo Cimento 4, 209 (1974) for partial list of such papers.

[3]For example, T. Alvager and M. Kreisler, Phys. Rev. 171, 1357 (1958); C. Baltay *et al.*, Phys. Rev. D 1, 759 (1970); J. S. Danburg *et al.*, *ibid.* 4, 53 (1971).

[4]For example, that of H. K. Wimmel, Lett. Nuovo Cimento 1, 645 (1971).

[5]R. C. Tolman, *The Principles of Statistical Mechanics* (Oxford Univ. Press, London, 1938), p. 437 ff.

[6]C. Van Weert and W. de Boer, Physica 81A, 597 (1975).