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High-energy deep-inelastic scattering and analytic model of the virtual Compton amplitude

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Recent experimental data from Fermilab on the large-ω behavior and scale-invariance breaking of the nucleon structure function $\nu W_2(\omega,Q^2)$ measured over a wide range of ω and Q^2 covering $1 \lesssim \omega \leq 1000$, $0.2 \leq Q^2 \leq 50$ GeV^2 are in surprisingly good agreement with the *absolute prediction* of an analytic model of the virtual (forward) Compton amplitude proposed recently by the authors. On the basis of this agreement we conclude that (i) the observed decrease of $\nu W_2(\omega,Q^2)$ at large ω *is* consistent with diffractive behavior of the nucleon scaling function $F_2(\omega)$, (ii) the decrease results most probably from the kinematic constraint, $\lim_{Q^2\to 0}\nu W_2(\omega,Q^2)=0$, valid for all $\omega > \omega_{\text{threshold}}$, (iii) the observed pattern of scaling deviation is consistent with "precocious scaling" around $Q^2 \simeq 2$ GeV^2, and (iv) there is a $1/Q^2$ approach to scale invariance. The predicted results of the model for the Callan-Gross and the Gottfried sum rules are in very good agreement with the predictions of the Kuti-Weisskopf model. We also present and discuss the moments of the structure function.

I. INTRODUCTION

Recently, the nucleon structure function $\nu W_2(\omega, Q^2)$ has been measured[1] at Fermilab over a wide range of ω and Q^2 ($1 \lesssim \omega \lesssim 1000$, $0.2 \leqslant Q^2 \leqslant 50$ GeV^2) by scattering high-energy (147-GeV) muons off deuterium. These experimental data are of much current interest presumably because they offer the possibility of a much more quantitative comparison of theory with experiment by virtue of their extension over a considerably wider range in ω and Q^2 than earlier electroproduction data[2] (which were confined to $\omega \lesssim 12$, $Q^2 \lesssim 15$ GeV^2). Further useful applications of the recent data are expected to include (i) possible determination of the relative size of the contribution of diffractive vs nondiffractive exchanges at large ω or equivalently[3] (ii) the resolution of the issue of asymptotic dominance of the contribution of the "$q\bar{q}$ sea" vs that of the "valence quarks," (iii) a more reliable estimation of the sum rules[4,5] which test the predictions of the quark-parton models, (iv) a direct study of the pattern of scale-invariance breaking, and (v) a reliable test of the hypothesis of "precocious scaling" observed in the earlier data.

The purpose of the present paper is to show that the *absolute predictions* of a recently proposed[6] analytic model of the virtual (forward) Compton amplitude are in surprisingly good agreement with the recent data over the entire range of ω and Q^2. The model, which is based on the general principles of S-matrix theory, incorporates (a) manifest analyticity in the ν plane, (b) s-u crossing symmetry, (c) asymptotic Regge behavior, (d) scale invariance in the Bjorken limit, (e) correct behavior in the real-photon ($Q^2 \to 0$) limit, and (f) a low-energy (Thomson) limit. It describes[6] earlier electroproduction and photoabsorption data excellently and yields the correct value of the p-n mass difference.

The remarkable agreement of the predictions of the model with the present data enables us to study in some detail the applications mentioned above as well as to compare with and comment on the other current models[7] describing deep-inelastic scattering. On the basis of the analysis of the recent data in the framework of the present model, we arrive at a number of important conclusions regarding the high-energy behavior, scale-invariance breaking of the nucleon structure function and various sum rules involving the latter.

In Sec. II, we briefly recapitulate the salient features of the model relevant to the present analysis and compare its prediction with the observed large-ω behavior of the nucleon structure function. We also discuss in Sec. II the definite predictions of other current models in the kinematical range spanned by the recent data and note our main conclusions regarding the high-energy behavior of the structure function. In Sec. III we evaluate the Callan-Gross[4] and the Gottfried[5] sum rules and compare the results with the predictions of the conventional quark-parton models.[8] Section IV deals with the direct study of scale-invariance breaking by confronting the model with the data on $\nu W_2(\omega, Q^2)$ plotted as a function of Q^2 for various fixed ranges of ω. We also present and discuss the predictions for the structure function moments.

In conclusion, we summarize the main results of the present analysis in Sec. V.

II. THE MODEL AND THE HIGH-ENERGY BEHAVIOR OF THE NUCLEON STRUCTURE FUNCTION

By unitarity, the nucleon structure function $\nu W_2^N(\nu, Q^2)$ is obtained at any fixed (spacelike) Q^2, from the discontinuity of the virtual (forward) Compton amplitude across the ν-plane cuts and is given by

$$\nu W_2^N(\nu, Q^2) = \left(\frac{1}{\alpha}\right)\nu Q^2 \text{disc } t_2^N(\nu, Q^2) \ . \tag{1}$$

The kinematic-singularity-free invariant amplitude $t_2(\nu, Q^2)$ is given in the model[6] as a sum of contributions from the Born pole and the inelastic cut by

$$t_2^N(\nu, Q^2) \simeq \frac{4m}{Q^4 - 4m^2\nu^2}\left[Q^2 f_2^N(Q^2) + \left|\frac{z_p - z}{z_p + 1}\right|^{2l_N} \sum_j \beta_j \frac{(1+z)^{-\alpha_j}}{\sin(\alpha_j \pi/2)}\right] . \tag{2}$$

The structure function νW_2 is obtained from (2) using (1) and is given by

$$\nu W_2(\nu, Q^2) \underset{\omega > \omega_c}{=} \left(\frac{1}{\alpha}\right)\left(\frac{2\omega}{1-\omega^2}\right)\left|\frac{z_p - z}{z_p + 1}\right|^{2l_N} \times \left(\frac{\omega}{2\omega_c}\right)^{\alpha_j} \frac{\sin(-\alpha_j\theta)}{\sin(\alpha_j \pi/2)} \ , \tag{3}$$

where $\omega \equiv 2m\nu/Q^2$ is the scaling variable, $\omega_c = 1 + m_0^2/Q^2$ ($m_0^2 = 2mm_\pi + m_\pi^2$) is the threshold[6] for ω in electron (muon) scattering. $\alpha_j \equiv \alpha_j(t=0)$ are the $t=0$ intercepts of the relevant t-channel Regge exchanges,[9] i.e., $j = P$, P', and A_2. Other quantities appearing in (2) and (3) have been defined in Ref. 6. There are *five parameters* of the model for νW_2, i.e., the two integers l_p and l_n and the three Regge-residue parameters

$$\beta_P (\equiv \beta_P^p = \beta_P^n),$$

$$\beta_{P'} (\equiv \beta_{P'}^p = \beta_{P'}^n) \ ,$$

and

$$\beta_{A_2} (\equiv \beta_{A_2}^p = -\beta_{A_2}^n) \ .$$

All these parameters have been previously determined from fits to earlier electroproduction data.[2]

In Fig. 1 we plot the *absolute prediction* of the model (3) together with the recent data on $\nu W_2(\omega, Q^2)$ per nucleon, i.e. $(\nu W_2^p + \nu W_2^n)/2$ as a function of ω. It is to be noted that the value of Q^2 corresponding to each data point is different at different values of ω (varying between $Q^2 \sim 50$ GeV^2 at $\omega \sim 3$ and $Q^2 \sim 0.2$ GeV^2 at $\omega \sim 1000$; see Ref. 1). These widely different values of Q^2 at different values of ω are of crucial importance (as shown below) in the comparison of the data with the detailed prediction of any model. The solid curve in Fig. 1 is the prediction of the model (3) and corresponds to the actual value of Q^2 at each data point. We have also plotted for comparison and clarity the prediction of the model (dashed curve of Fig. 1) for the scaling function $F_2(\omega)$ obtained from $\nu W_2(\omega, Q^2)$ in the strict Bjorken limit given by

$$F_2(\omega) = \lim_{\substack{Q^2 \to \infty \\ \omega \text{ fixed} > 1}} \nu W_2(\omega, Q^2) = \left(\frac{1}{\alpha}\right)\left(1 - \frac{1}{\omega^2}\right)^{l-1} \sum_j \beta_j \left(\frac{\omega}{2}\right)^{\alpha_j - 1} \frac{\sin[\alpha_j \theta(\omega)]}{\sin[\alpha_j \pi/2]} \ . \tag{4}$$

The quantitative agreement of the *absolute prediction* of the model with the recent data is surprisingly good as revealed by a χ^2 test (we obtain total $\chi^2 = 15$ for 19 data points). A very remarkable feature of the model is that although the scaling function is diffractive (as can be seen from Fig. 1), the observed decreasing behavior of νW_2 at large ω is reproduced excellently presumably as a result of the finite-Q^2 effect resulting from the kinematic constraint[6]

$$\lim_{Q^2 \to 0} \nu W_2(\omega, Q^2) = 0 \text{ for all } \omega > \omega_c \ , \tag{5}$$

since Q^2 is small <2 GeV^2 for $\omega > 80$. We therefore arrive at the following important conclusions regarding the high-energy behavior of $\nu W_2(\omega, Q^2)$:

(i) The observed decrease of $\nu W_2(\omega, Q^2)$ for $\omega \gg 10$ is completely consistent with diffractive dominance (Pomeron exchange or[3] excitation of the $q\bar{q}$ sea

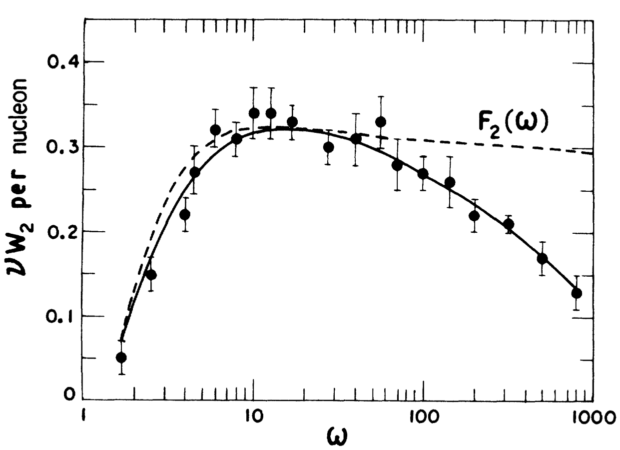

FIG. 1. νW_2 per nucleon vs $\omega = 2m\nu/Q^2$. The data are from Ref. 1. The solid curve is the prediction of the model for $\nu W_2(\omega, Q^2)$ per nucleon and the dashed curve corresponds to the scaling function $F_2(\omega)$ of the model, plotted for comparison.

constituting the core) of $F_2(\omega)$ at large ω.

(ii) This decrease arises most probably from the kinematic constraint $\lim_{Q^2\to 0}\nu W_2(\omega, Q^2)=0$, valid for all $\omega>\omega_c$.

Further strong support in favor of the second conclusion comes from the excellent description[6] in the model of the data on total photoabsorption cross sections for both proton and neutron given by

$$\sigma_T^{\gamma N}(\nu)=\lim_{Q^2\to 0}(4\pi^2\alpha/Q^2)\nu W_2^N(\omega, Q^2)\ ,$$

as well as of the data on the approach to scaling of $\nu W_2(\omega, Q^2)$ for small $Q^2<1$ GeV^2 at different ranges of ω.[10]

An alternative explanation for the decrease of $\nu W_2(\omega, Q^2)$ for $\omega>10$ is provided by a number of models which ascribe the decrease to the asymptotic dominance of nonleading Regge exchanges[11] or, equivalently, to the dominance of the valence quarks.[12] A common feature of these models is the identification of the decrease of $\nu W_2(\omega, Q^2)$ for $\omega\gg 10$ with that of the scaling function $F_2(\omega)$ *assuming scaling at large* $\omega\gg 10$. It must, however, be emphasized that the data on $\nu W_2(\omega, Q^2)$ for large $\omega\gg 10$ have small $Q^2<1$ GeV^2 (Q^2 is, in fact, as small as 0.2 GeV^2 for $\omega\sim 1000$), and there is at present little theoretical or experimental justification for scaling to persist to such low values of Q^2.

On the other hand, the decrease of $\nu W_2(\omega, Q^2)$ obtained in the present model is based on the purely kinematic constraint (5) following from gauge invariance, which has been supported by the earlier data. In this respect, therefore, the present result is clearly distinguished from the results of models mentioned above since it demonstrates unambiguously the simultaneous realization of a diffractive scaling function $F_2(\omega)$ and a decreasing structure function $\nu W_2(\omega, Q^2)$ at large ω.

Secondly, the excellent agreement of the present high-energy data supplemented by the earlier successful description[6] of the data on $\nu W_2^p - \nu W_2^n$, $\nu W_2^n/\nu W_2^p$ and the high-energy photoabsorption cross sections $\sigma_T^{\gamma p}$, $\sigma_n^{\gamma n}$, $\sigma_T^{\gamma p}-\sigma_T^{\gamma n}$ constitutes, in our opinion, strong evidence in favor of diffractive dominance in real and virtual photoabsorption cross sections. Further evidence for diffractive dominance comes from the evaluation of the electroproduction sum rules[4,5] discussed in the next section.

As a final remark on the large-ω behavior of the nucleon structure function, it is worthwhile mentioning the lack of any evidence in the present data of an increase of νW_2 with ω at large ω, which has been conjectured[13] to arise from the threshold excitation of a new heavy quark. Similar behavior is also predicted to result from the triple-Pomeranchukon coupling.[14]

III. SUM RULES

The behavior of the nucleon scaling function $F_2(\omega)$ for large ω plays an important role in the evaluation of sum rules which test the prediction of quark-parton models. These are the Callan-Gross[4] sum rule given by

$$I_1=\int_1^\infty \frac{d\omega}{\omega^2}F_2(\omega)=\sum_N P(N)\Big\langle\sum_{i=1}^N Q_i^2\Big\rangle\Big/N \qquad (6)$$

and the Gottfried[5] of sum rule given by

$$I_2=\int_1^\infty \frac{d\omega}{\omega}F_2(\omega)=\sum_N P(N)\Big\langle\sum_{i=1}^N Q_i^2\Big\rangle\ . \qquad (7)$$

I_1 has the significance of denoting the weighted mean square charge per parton and I_2 denotes the weighted sum of the square of parton charges. As has been remarked above, the integral I_2 depends crucially on the behavior of $F_2(\omega)$ as $\omega\to\infty$. In particular, I_2 diverges in models which include a diffractive contribution or contribution from an infinite sea of $q\bar{q}$ pairs, implying

$$F_2(\omega)\xrightarrow[\omega\to\infty]{}\text{constant}\ .$$

On the other hand, the difference

$$I_2^p-I_2^n=\int_1^\infty \frac{d\omega}{\omega}\left[F_2^p(\omega)-F_2^n(\omega)\right] \qquad (8)$$

is expected to converge on account of the vanishing of the quantity $F_2^p(\omega)-F_2^n(\omega)$ with $\omega\to\infty$ as suggested by the data[2] as well as theoretical considerations[3] assuming the absence of an isoscalar contribution to the p-n difference in this limit. However, unlike I_2, the integral I_1 is expected to be less sensitive to the $\omega\to\infty$ limit because of the damping factor ω^{-2}. In Table I, we summarize the comparison of the experimental values of the sum rules I_i evaluated using the earlier data with the predictions of the conventional quark-parton models[8] and the present model.

In the evaluation of the integrals I_i we have used the model for $F_2(\omega)$ given by (4). The errors indicated with the obtained results reflect the errors in determining the Regge parameters β_j. It is clear from Table I that the experimental results for I_1^p and I_1^n are in good agreement with the results from the present model as well as the Kuti-Weisskopf model[8] which includes neutral gluons in addition to the valence quarks and the $q\bar{q}$ sea. On the other hand, the simple three-quark model[15] with or without the $q\bar{q}$ sea predicts values for $I_1^{p,n}$ which are considerably higher than the experimental values.

The results for $I_2^{p,n}$ are divergent in the present

TABLE I. Comparison of the prediction of the quark-parton models (Ref. 8.) and the present model for the sum rules (see text) with the existing experimental result.

Sum rule	Model predictions: 3 quark [a]	3 quark + $q\bar{q}$ sea [a]	3 quark + $q\bar{q}$ sea + gluons (Kuti-Weisskopf) [b]	Results from present model	Experimental measurement [c]	ω_m (cutoff)	Q^2 (GeV^2)
I_1^p	$\frac{1}{3}$	$\frac{2}{9}+\frac{1}{3\langle N\rangle}$	$\frac{1}{9}+(\frac{2}{9})^2 \simeq 0.16$	0.18 ± 0.018	0.159 ± 0.005	20	1.0
					0.165 ± 0.005	20	1.0
					0.172 ± 0.009	20	1.5
					0.154 ± 0.005	12	2.00
I_1^n	$\frac{2}{9}$	$\frac{2}{9}$	$(\frac{2}{3}\times\frac{1}{9})+(\frac{2}{9})^2 \simeq 0.12$	0.13 ± 0.013	0.120 ± 0.008	20	1.0
					0.115 ± 0.008	20	1.5
					0.107 ± 0.009	12	2.0
I_2^p	1	$\frac{1}{3}+\frac{2}{9}\langle N\rangle$	∞	∞	0.739 ± 0.029	20	1.0
					0.761 ± 0.027	20	1.5
					0.780 ± 0.04	20	1.5
					0.607 ± 0.021	12	2.0
I_2^n	$\frac{2}{3}$	$\frac{2}{9}\langle N\rangle$	∞	∞	0.592 ± 0.051	20	1.0
					0.584 ± 0.050	20	1.5
					0.429 ± 0.036	12	2.0
$I_2^p - I_2^n$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0.28 ± 0.04	0.147 ± 0.059	20	1.0
					0.177 ± 0.057	20	1.5
					0.178 ± 0.042	12	2.0

[a] J. D. Bjorken and E. Paschos (Ref. 8). Here $\langle N\rangle$ is the expectation value of number of quarks N.
[b] J. Kuti and V. F. Weisskopf (Ref. 8).
[c] See Ref. 7 and references contained therein.

model which is diffractive and hence[3] consistent with the dominance of the $q\bar{q}$ sea contribution. *The important result of the present model is the value obtained for* $I_2^p - I_2^n$ which is predicted to be $\frac{1}{3}$ in all the conventional quark-parton models.[8] Table I shows that the earlier discrepancy between theoretical predictions and experimental evaluations[16] is considerably reduced by the present model which successfully accounts for the high-energy data.

The overall agreement of the prediction of the Kuti-Weisskopf model for the sum rules with the results of the present model is perhaps significant and interesting since the former takes into account the constituents of the nucleon, whereas the latter model is based on general principles of the S-matrix approach. The agreement also presumably lends further credence to the formal correspondence[3] between the Regge-pole description and the quark-parton approach formulated in terms of the identification: Pomeron $\to q\bar{q}$ sea and Reggeon $\to$ valence quarks.

IV. SCALE-INVARIANCE BREAKING

Recently, the study of deviation from strict Bjorken scaling in deep-inelastic electron (muon) scattering has received considerable theoretical and experimental attention. Previous measurements at SLAC[17] and Fermilab[18] have shown conspicuous deviation from scaling. Theoretical investigations into the cause of scale-invariance breaking have emphasized the choice of new scaling variables,[19] the idea of the substructure of the hadronic constituents,[20] the role of anomalous dimensions,[21] and asymptotically free field theories.[22] It is now believed that the qualitative features of the observed pattern of scale-breaking are fairly well understood in the framework of field-theoretic models.[23] As a result of these new developments, there is now some confusion and doubt regarding the validity of the earlier belief of precocious scaling in deep-inelastic scattering.

Furthermore, in spite of the qualitative understanding of the pattern of scaling deviation, a further quantitative comparison of theory with experiment has not been successful primarily because of the ambiguity in the choice of the "proper" scaling variable,[19] since all the quantitative results are sensitive to such choice in the kinematic range explored by the earlier experiments. This problem presumably becomes insignificant for the present measurement of high-energy muon scattering at Fermilab since any reasonable mass term appearing in the new choice of the scaling variable is much less than $2m\nu$. The present data are, therefore, expected to provide a much more quantitative test of theoretical predictions on scale-invariance breaking.

In Fig. 2 we have plotted the recent data on νW_2 per nucleon as a function of Q^2 for various fixed ranges of ω together with the *prediction* (solid curve) of the model. A remarkable feature of the data is the ω dependence of the slope of Q^2 variation of νW_2: No gross violation of scaling is observed for $3<\omega<80$, but for $\omega>80$, νW_2 rises with Q^2, whereas it shows a mild decrease with Q^2 in the range $\omega<3$. Such a pattern of scaling deviation, also observed in the earlier measurements,[18] is conjectured by field-theoretic models,[23] which predict logarithmic violation of scaling[22] or violations characterized by anomalous dimensions.[21] The present model predicts a $1/Q^2$ approach to scaling which has been conjectured earlier by West[24] on consideration of the naive extension of Bjorken's original arguments for the current commutators. The pattern of scaling deviation predicted by the present model is given by

$$\lim_{\substack{Q^2 \gg m_0^2 \\ \omega \text{ fixed} > 1}} \nu W_2(\omega, Q^2) = F_2(\omega)\left[1+\frac{m_0^2}{Q^2}G(\omega)\right], \qquad (9)$$

where $m_0^2 = 2mm_\pi + m_\pi^2 \simeq 0.28\ \mathrm{GeV}^2$ sets the scale for the approach to scaling and explains, by virtue of its smallness, the rapid onset of scale invariance around 2 GeV^2. The function $G(\omega)$ accounts for the ω dependence of the slope of Q^2 variation and has the behavior

$$G(\omega) \underset{\omega\to\infty}{\sim} \omega$$

and

$$G(\omega) \underset{\omega\to 1}{\sim} \text{constant} .$$

Thus, all the prominent features of the data are explained in the framework of the present model and the absolute predictions of the model are found to be in remarkable agreement with the data over the entire range of ω and Q^2 (we obtain a total $\chi^2=37$ for 52 data points).

It should be emphasized that the parameter m_0^2, which controls the rapid onset of scaling around $Q^2 \simeq 2\ \mathrm{GeV}^2$, arises naturally in the present model from considerations of analyticity[25] of the virtual Compton amplitude in the ν plane, unlike other models[26] where it is introduced as an extra parameter determined only by fits to the experimental data. The excellent agreement of the prediction for scale-breaking given by (9) with recent data enables us to conclude that *the observed pattern of scale-invariance breaking is consistent with precocious scaling*. The sharp deviations from scaling observed for $\omega>80$ are presumably due partly to the behavior of $G(\omega)$ which rises with ω

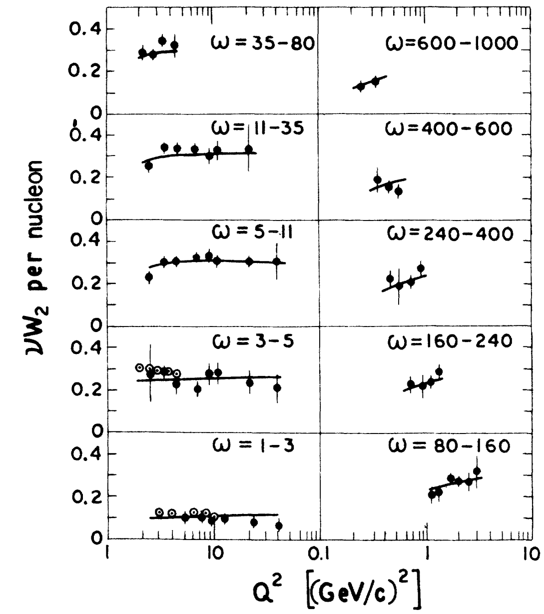

FIG. 2. νW_2 per nucleon as a function of Q^2 for various fixed ω bins. The open circles indicate the data measured at SLAC by Riordan *et al.* (Ref. 17). The solid curves are the predictions of the model.

for large ω and partly again to the kinematical constraint (5) which becomes operative because of the small values of Q^2 involved in this range of ω.

We point out here that the present model which is based on analyticity in the ν plane for fixed Q^2 is applied to the variable-Q^2 data describing scaling deviation. The justification comes from the knowledge[27] that for all spacelike $Q^2 \equiv -q^2 > 0$, neither the virtual Compton amplitude nor the structure functions W_i are expected to possess any singularity in Q^2.

Before closing this section, we briefly discuss the moment integrals of the structure function $\nu W_2(\omega, Q^2)$ defined[28] as

$$M_n(Q^2) = \int_{\omega_c}^{\infty} d\omega\, \omega^{-n} F(\omega, Q^2) , \qquad (10)$$

where $F(\omega,Q^2)=\nu W_2(\omega,Q^2)$, $n=1,2,3,\ldots$ and ω_c is the threshold of electron (muon) scattering for fixed Q^2, defined earlier. These moments are of some current interest in the context of the asymptotically free field theories and theories with anomalous dimensions, presumably because these theories predict[23] definite asymptotic behavior of the moments with Q^2. However, since the situation regarding the direct experimental evaluation of these moments is not quite satisfactory[29] at present, the theoretical predictions on their asymptotic behavior with Q^2 can be utilized only indirectly in the study of scale-invariance breaking by translating these to testable large-Q^2 behavior of $\nu W_2(\omega, Q^2)$ by inversion[30] of relation (10). These moments are therefore of less significance in models, such as the present one, whose predictions can be directly confronted with the existing data on νW_2 plotted as a function of Q^2 for fixed ω, to test scale invariance.

We have nevertheless shown in Fig. 3 the prediction[31] for these moments in our model, in anticipation of their more satisfactory determination from experimental data in the future and for other possible useful applications[32] in the context of asymptotically free gauge theories. It is to be noted that the approach to scaling as expressed by relation (9) implies the following asymptotic behavior of $M_n(Q^2)$ in the model.

$$\lim_{Q^2 \gg m_0^2} M_n(Q^2) = A(n) + B(n)(m_0^2/Q^2) \; . \qquad (11)$$

As expected, $M_n(Q^2)$ becomes approximately independent of Q^2 for $Q^2 \gtrsim 4$ GeV^2 reflecting the early onset of scaling.

V. SUMMARY AND CONCLUSION

We have shown in this paper that a simple model of the virtual (forward) Compton amplitude, consisting of the Born and inelastic contributions and exhibiting the usual analytic and invariant properties, is in remarkable quantitative agreement with the existing data on the structure functions over the entire range of the kinematic region explored experimentally. It is to be emphasized that we have not tried to fit the recent data[1] by varying the (few) parameters of the theory, but have rather compared these with the *absolute predictions* of the model, with parameters predetermined from fits to the earlier data (which were confined to a much narrower kinematic region than the recent data). Nevertheless, the surprising agreement achieved by this comparison is perhaps indicative of the stability and soundness of the underlying principles in the present description of deep-inelastic scattering.

It is important to further recognize that the present model does not employ any explicit assumption regarding the constituent nature of the nucleon, unlike the conventional quark-parton models,[8] but is based on rather general principles such as analyticity, crossing symmetry, Regge, behavior, low-energy theorems, and gauge-invariance constraints in the real photon limit. It is therefore interesting and perhaps significant to find the results for the Callan-Gross[4] and Gottfried sum rules[5] obtained in the quark-parton model of Kuti and Weisskopf[8] to be in satisfactory agreement with the prediction of the present model in spite of the basic difference of the underlying concepts.

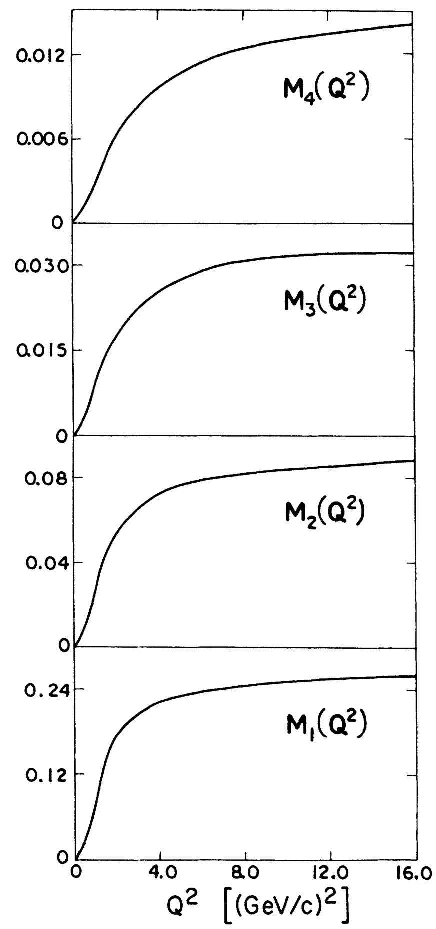

FIG. 3. The prediction of the model for the moments of structure function $M_n(Q^2)$ as a function of Q^2 for $n = 1, 2, 3,$ and 4.

In conclusion, we summarize the main results obtained by the present analysis of the high-energy deep-inelastic scattering data:

(i) The observed decrease of the nucleon structure function $\nu W_2(\omega, Q^2)$ at large $\omega \gg 10$ is consistent with diffractive behavior of the scaling function $F_2(\omega)$.

(ii) The decrease arises most probably from the kinematic constraint, $\lim_{Q^2 \to 0} \nu W_2(\omega, Q^2) = 0$, which is valid for all $\omega > \omega_{\text{threshold}}$.

(iii) The predictions of the present model for the Callan-Gross[4] and Gottfried[5] sum rules are in satisfactory agreement with the corresponding results of the Kuti-Weisskopf[8] model.

(iv) The observed scaling deviation is consistent with precocious scaling around $Q^2 \simeq 2$ GeV^2.

(v) The pattern of scale-invariance breaking is also satisfactorily described by a $1/Q^2$ approach[24] to scaling.

Note added in proof. After this paper was submitted for publication, high-energy data on the proton structure function $\nu W_2^p(\omega, Q^2)$ was reported [H. L. Anderson *et al.* Phys. Rev. Lett. 38, 1450 (1977)] which was used together with the deuterium data (Ref. 1) in the evaluation of the Callan-Gross sum rules for the proton and the neutron at different Q^2. The results are shown to be almost independent of Q^2. The average values obtained, $I_1^p = 0.171 \pm 0.006$ and $I_1^p - I_1^n = 0.034 \pm 0.015$, are in very good agreement with the predictions of the present model (Table I), thus providing further experimental evidence in its support.

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[29] To the authors' knowledge, the only experimental data on the moments have been reported by Bloom (Ref. 16). This experimental determination is, in our opinion, not very reliable since it uses a cutoff to the integral (10) (corresponding to $\omega' = 5$) thus excluding the high-energy contributions. Besides, the free use of a "scaling fit" (to the earlier data, Ref. 2) where experimental data on the structure function are not available is another drawback of this determination.

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[31] The predictions for the moments in the present model are in qualitative agreement with the existing experimental data (see Ref. 29) and can be made to agree more quantitatively by using scaling over a portion of the kinematic region as has been done (Ref. 29) in obtaining these experimental data.

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