PHYSICAL REVIEW D VOLUME 17, NUMBER 6 15 MARCH 1978

Limits from primordial nucleosynthesis on the properties of massive neutral leptons

Duane A. Dicus and Edward W. Kolb
Center for Particle Theory, University of Texas at Austin, Austin, Texas 78712

Vigdor L. Teplitz
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

Robert V. Wagoner
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305
(Received 14 November 1977)

If there exist neutral leptons with masses in the range 50 eV to 5 GeV, they would have been present in thermal equilibrium in the early stages of the hot big bang. In the subsequent evolution of the universe, if their lifetime is sufficiently long, their mass dominated the energy density of the universe. In this paper we consider the effect of their presence on the synthesis of elements in the early universe. Of the observed primordial abundances, we find the helium abundance to be independent of their existence, but we find the deuterium abundance to be sufficiently sensitive to allow bounds to be placed on the mass, lifetime, and decay modes of any heavy neutrinos. In particular, on the basis of present best estimates of astrophysical parameters, we reduce previous radiative lifetime bounds on the order of months to bounds on the order of hours, and expand the range of masses for which no radiatively decaying massive neutral leptons are allowed to 50 eV to 100 keV.

I. INTRODUCTION

Evidence for the existence of a massive charged lepton of mass about 2 GeV has now been confirmed.[1] It is strongly suspected, but not yet proven, that the new charged lepton is accompanied by a new type of neutrino. If the new neutrino exists, its mass can on the basis of present experimental information be as large as several hundred MeV. [Experimentally, we know $m_{\nu_e} < 60$ eV (Ref. 2) and $m_{\nu_\mu} < 0.65$ MeV (Ref. 3).] Indirect evidence from other experiments[4] has as one interpretation extension of the family of leptons to include new, heavy neutrinos possible unrelated to a charged lepton. Direct experimental detection of these particles is exceedingly difficult, and in the near future indirect evidence may provide the only clues to their existence and properties. The object of the present work is to derive such evidence from the synthesis of elements in the primordial big bang. In it we will avoid assuming any particular model for the interactions of the heavy neutrino in order to allow our results to have the widest possible applicability.

Standard cosmological models[5] require neutrinos of masses 50 eV to 5 GeV to be unstable. It was shown by Cowsik and McClelland[6] that a sum of light neutrino masses ($m_{\nu_e} + m_{\nu_\mu} + \cdots$) greater than 50 eV would result in a present energy density greater than the critical density. (Cowsik has recently improved this limit from other evidence.[7]) It was pointed out by Lee and Weinberg[8] and by Dicus, Kolb, and Teplitz[9] that if the neutrino is heavy enough (greater than a few GeV), there will be sufficient annihilation before decoupling to ensure that the present neutrino energy density not exceed the upper bound ($\approx 10^{-29}$ g cm^{-3}) determined by observation of the deceleration of the universe. It was also shown by Dicus *et al.*[9] that for neutrino masses between these two extremes a lifetime shorter than the age of the universe can allow the massless decay products to be red-shifted to produce a present energy density less than the critical density.

In previous work[10] Dicus *et al.* derived restrictions on the mass, lifetime, and decay modes of heavy neutrinos from several cosmological considerations. The principal result was the calculation of an upper bound on the lifetime for radiative decay of massive neutral leptons such that the universe would still be sufficiently dense that the γ's would be degraded by bremsstrahlung and double Compton emission to the temperature of the universe. Restrictions from element formation were only touched on briefly there. The cosmic abundances of light elements are known to be sensitive to most deviations from the standard big-bang cosmology. Recently the helium abundance has been used to limit the number of types of *massless* neutrinos.[11] In this paper we calculate restrictions on the properties of heavy neutrinos (hereafter called ν_H) from their effect on the formation of light elements in the early universe.

In Sec. II we describe briefly the evolution of a universe with heavy neutrinos. We give the present observational limits on astrophysical quantities such as element abundances and the density of the universe; and we describe the calculation of abundances as a function of the density. In Sec. III we calculate the abundances and discuss their dependence on the mass, lifetime, and decay modes of ν_H. In Sec. IV we interpret our bounds and discuss their significance in building models with heavy neutrinos.

Our principal result is that, in the case that the ν_H-decay products include γ's or charged particles, the ν_H lifetime is bounded by a function of the ν_H mass that for most possible masses is less than a few hours (Sec. IV). This is to be compared with the limit of a few months found in Ref. 10 from the requirement that the resulting γ's thermalize. The present limits are, however, dependent on current best measurements for cosmic deuterium abundances. The essential ideas in deriving the lifetime upper bounds in the present paper are as follows: (1) Existence of cosmic deuterium puts bounds on the ratio $R_{B\gamma}$ of baryons to photons at nucleosynthesis; (2) radiative ν_H decay after nucleosynthesis tends to make $R_{B\gamma}$ observed today smaller than $R_{B\gamma}$ at nucleosynthesis.

An observation made in Ref. 10, which should be repeated, is the following: If the decay $\nu_H \to \nu_L \gamma$ proceeds with a coupling constant f, the radiative lifetime τ_R is proportional to $1/f^2$. The same coupling constant enters into the one-photon exchange contribution to the process $\nu_L + X \to \nu_H + Y$ and this cross section is proportional to f^2. We may therefore set model-independent *lower* limits on the lifetime of massive neutrinos by requiring that f^2 be low enough that the ν_H contribution to the total cross section is less than the total observed experimentally. In Ref. 10 Dicus *et al.* found that the most restrictive upper limit on f^2 (lower limit on τ_R) came from the Reines experiment, $\bar{\nu}_e + e \to \bar{\nu}_e + e$. It was found that this lower limit was greater than the thermalization upper limit for $m_{\nu_H} < 50$ keV. The upper limit found in the present work extends the forbidden region to 100 keV.

Model-dependent calculations of ν_H lifetimes related to the work of Refs. 9 and 10 have also been done by Goldman and Stephenson[12] and by Sato and Kobayashi.[13] Sato and Sato have also considered analogous limits on Higgs meson lifetimes.[14] They have, in addition, informed us that they have in progress nucleosynthesis calculations similar to those of the present paper but using a different computer code. Finally, a careful study of the implications for galaxy and cluster formation and structure of stable neutral massive leptons has been performed by Gunn *et al.*[15]

We may summarize the status of information on massive-neutral-lepton lifetimes as follows: (1) If ν_H has a mass between 50 eV and 5 GeV it must decay;[6,8,9] (2) astrophysical considerations of Cowsik[7] place severe restrictions on the existence of neutrinos of masses lower than 50 eV; (3) knowledge of an upper bound on the total mass of the universe puts a limit (dependent on the neutrino mass) on the order of a fraction of the lifetime of the universe on the ν_H lifetime for 50 eV $< m_{\nu_H} < 5$ GeV; (Ref. 9) (4) if ν_H decays into charged particles or γ's, thermalization of the resulting γ's puts a mass-independent upper limit on the order of months on the ν_H lifetime; (5) if ν_H decay produces γ's, the present work shows that primordial nucleosynthesis puts a mass-dependent upper limit on the order of hours on the ν_H lifetime; (6) if the ν_H decay produces ν_e's, the Reines experiment puts a mass-dependent *lower* limit on the ν_H lifetime.[10]

II. HEAVY NEUTRINOS AND COSMOLOGY

A. Evolution of a universe with heavy neutrinos

The evolution of a universe with heavy neutrinos has been described elsewhere.[10] Here we review briefly the important features. In the very early stages the heavy neutrinos ν_H are kept in thermal equilibrium by their interactions with ν_e, ν_μ, etc. ν_e and ν_μ are in turn kept in thermal equilibrium by their interactions with electrons and muons. As the universe expands, the number densities and temperature of all particles drop. We can define a decoupling temperature T_D for the heavy neutrinos by equating their average interaction time as a function of the temperature $\tau_H(T)$ to the age of the universe at that temperature[16].

$$\tau_H = \frac{\langle n_H \rangle}{\langle n_H n_H \sigma |v| \rangle}, \tag{1a}$$

$$t_{\text{universe}} \cong \frac{1.09 \times 10^{20}}{T^2(°\text{K})} \text{ sec}. \tag{1b}$$

$n_H(T)$ is their equilibrium number density at temperature T, $|v|$ is their relative flux, and σ is the sum of the cross sections for

$$\begin{aligned} \nu_H + \bar{\nu}_H &\to \nu_e + \bar{\nu}_e, \\ &\to \nu_\mu + \bar{\nu}_\mu, \\ &\to e^+ + e^-. \end{aligned} \tag{2}$$

The use of other interactions to keep the neutrinos in thermal equilibrium has been considered, but the use of (2) will be sufficient for our needs.

In Sec. III we will evaluate the temperature at decoupling T_D, and the number density at decoupling n_D as a function of the mass of the heavy neutrino by equating (1a) and (1b) for reasonable values of the cross section of (2). After decoupling, the neutrinos are in free expansion and their energy density decreases in the expansion of the universe as $\rho_{\nu_H} \sim R^{-3} \sim T^3$. The energy density of relativistic particles decreases as $\rho_\gamma \sim R^{-4} \sim T^4$. Therefore, if the lifetime of ν_H is large enough (calculated in Sec. III), domination of the mass of the universe by relativistic particles gives way to domination of the mass of the universe by the mass of the heavy neutrinos. For neutrino masses from 0.5 to 25 MeV this will occur either immediately before, or during, nucleosynthesis. The universe remains dominated by the massive neutrinos until they decay (as they must to allow $\rho_{\text{now}} < \rho_c$).

We consider two complementary scenarios for ν_H decay: (1) ν_H decay creates photons, either prompt ($\nu_H \to \nu_L \gamma$) or delayed ($\nu_H \to e^+e^-\nu_L$, $\pi^0\nu_L, \ldots$); or (2) ν_H decay does not create photons ($\nu_H \to \nu_L \nu_L \bar{\nu}_L$). ν_L is a massless neutrino. In the event photons are created by ν_H decay, and the lifetime of ν_H is sufficient to allow the universe to have been dominated by the ν_H, the photons from the decay deposit a large amount of entropy in the universe which increases the photon temperature appreciably. If only massless neutrinos result from ν_H decay, the photon temperature of the universe remains unchanged.

B. Limits from present-day observables

The major product of big-bang nucleosynthesis is ^{4}He. From various observations we may *conservatively* place the primordial mass fraction of ^{4}He in the range 22–29%, and *probably* require the mass fraction to be closer to 29%.[17] The standard big-bang calculation[18] predicts a helium abundance between 17% and 27%, in excellent agreement with observation. The fact that the helium abundance is correctly predicted lends credence both to the assumptions in the standard model, and the method of the abundance calculations. Most modifications of the standard big-bang model are in gross disagreement with the observations.[17] These observations also indicate the He is of primordial origin.[17]

^{2}H is the next most likely element of primordial abundance that may be determined by present observations. The current abundance observations suggest a ^{2}H primordial mass fraction of $2 \times 10^{-5} \le X(^2\text{H}) \le 10^{-4}$. The deuterium abundance is very sensitive to the one input parameter in the calculation; this fact will be exploited below to place bounds on the properties of the heavy neutrino.

Determination of the primordial abundances of other nuclei is obscured by the fact that even if precise present-day abundances were known, it is at present difficult to separate the percent of each that was made in the big bang from the percent made later in stars. We will determine the abundances of other elements produced in the big bang, but their interpretation is unclear at this time.

The single input parameter in the standard calculation depends (solely in the case $\nu_H \to \bar{\nu}_L + 2\nu_L$) on the present-day baryon density. The present baryon density may be expressed as (using $H_0 = 55$ km sec^{-1}Mpc^{-1}),

$$\rho_B = 5.7 \times 10^{-30}\Omega \text{ g cm}^{-3}. \tag{3}$$

The galactic contribution to Ω is known to be in the range from ≈ 1 to 0.01 and the latest determination gives as its best value $\Omega = 0.06$, and an upper bound of 0.3.[17]

The assumptions and details of the abundance calculations have been described elsewhere.[17,18] Here we briefly discuss the input parameters. The calculation of primordial abundances depends only on: (1) total energy density and temperature and (2) baryon density before and during nucleosynthesis. The dependence on the baryon density is expressed through a parameter h defined as

$$\rho_B = hT_9^{\,3}, \tag{4}$$

where T_9 is the temperature in units of 10^9 °K. Conservation of baryons requires h to be constant, except when entropy is generated in the universe. Assuming all ν_H decay at the same time,

$$h_{\text{after}} \approx h_{\text{before}} \left(\frac{T_b}{T_a}\right)^3, \tag{5}$$

if T_a is the temperature after release of entropy and T_b is the temperature before. Annihilation of electron pairs serves to increase the temperature of the universe by a factor of 1.4 (see Ref. 19):

$$h_0 = h(1.4)^3 = 2.75h. \tag{6}$$

h_0, the value of h before pair annihilation, will be used as the input parameter.

If the decay of the heavy neutrino creates photons, then there is an additional increase in h_0,

$$h_0 = 2.75\left(\frac{T_a}{T_b}\right)^3 h. \tag{7}$$

Finally we may use (4) to express ρ_B, the present baryon energy density in terms of h_0, and the present temperature T_0 as

$$\rho_B\left(\frac{2.7}{T_0}\right)^3 = \frac{h_0}{2.75}\left(\frac{T_b}{T_a}\right)^3 (2.7\times 10^{-9})^3$$
$$= 7.15\times 10^{-27}\left(\frac{T_b}{T_a}\right)^3 h_0. \tag{8}$$

The results of the calculations in the next section will be expressed in terms of h_0 which is related to the present baryon density through (8).

III. CALCULATIONS

A. Decoupling temperature and density

ν_H remains in thermal equilibrium with the rest of the particles in the early universe through the process of (2), shown in Fig. 1. Our present task is to calculate the decoupling temperature and the ν_H number density at decoupling. We do not know the coupling of ν_H to Z. Where the massive neutrinos have the largest effect on nucleosynthesis (0.5–25 MeV) we will consider several possibilities. The first coupling uncertainty involves the space-time structure of the ν_H-Z vertex. We consider two extremes, pure vector (also equivalent to pure axial vector) and the combination V-A. The second uncertainty is in the overall strength of the coupling. The presence of the ν_H causes the largest deviation in abundances from the results of the standard big-bang calculation for the 5 MeV case. Here, in addition to the usual coupling strength expected in gauge theories, we arbitrarily multiply and divide the cross section by a factor of ten. For light ($\leq 10^{-1}$ MeV) and heavy ($\geq 10^2$ MeV) ν_H, where the effect on the final abundance is smallest and insensitive to the above uncertainties, we consider only V-A coupling with

$$\nu_H + \bar{\nu}_H \to \nu_e + \bar{\nu}_e$$

as the reaction keeping ν_H in thermal equilibrium. Using only this part of (2) allows us to consider the possibility of ν_H as ν_μ in the case of $m < 0.1$ MeV. A summary of the cases considered may be found in Table I.

Following the prescription given in Sec. II and discussed in detail in Ref. 10, we calculate the decoupling temperature and the number density at decoupling for the several cases mentioned above. The results are given in Table I.

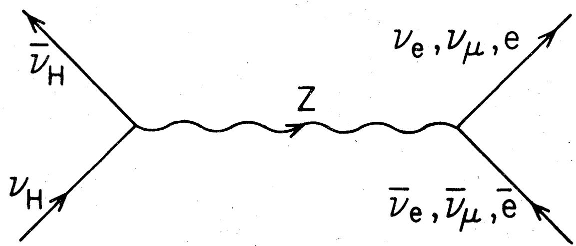

FIG. 1. The reaction that keeps ν_H in thermal equilibrium in the early universe.

Also included in Table I is the temperature T_1 at which the universe becomes matter dominated by the heavy neutrinos. T_1 may be found by equating the energy density of heavy neutrinos to the energy density of relativistic particles

$$2mn_H(T_1) = 1.45aT_1^4, \tag{9}$$

where $a = 4.72 \times 10^{-9}$ MeV °K^{-4} cm^{-3}.

The numerical factor of 2 takes into account the presence of both ν_H and $\bar{\nu}_H$; the factor of 1.45 takes into account the energy densities of γ, ν_e, $\bar{\nu}_e$, ν_μ, and $\bar{\nu}_\mu$, as well as the fact that the neutrino temperatures are less than the photon temperature after e^+e^- annihilation ($T_\gamma = 1.4T_\nu$).

Since after decoupling the number of neutrinos is conserved, the *number* density n_H decreases as T_ν^3, therefore we have

TABLE I. The temperature (T_D) and number density (n_D) at decoupling, and the temperature (T_1) at which ν_H dominates the mass of the universe for different values of its mass and its coupling to Z.

Mass (MeV)	Coupling form	Coupling strength α	n_D (cm^{-3})	T_D (°K)	T_1 (°K)
10^{-2}	$V-A$	1	3.02×10^{32}	3.41×10^{10}	8.06×10^{6}
10^{-1}	$V-A$	1	3.02×10^{32}	3.41×10^{10}	8.06×10^{7}
0.5	$V-A$	1	2.25×10^{32}	3.10×10^{10}	4.00×10^{8}
0.5	V	1	4.50×10^{32}	3.10×10^{10}	8.00×10^{8}
5	$V-A$	1	1.15×10^{32}	3.00×10^{10}	2.26×10^{9}
5	$V-A$	0.1	1.90×10^{33}	6.60×10^{10}	3.50×10^{9}
5	$V-A$	10.0	4.70×10^{30}	1.52×10^{10}	7.09×10^{8}
5	V	1	1.60×10^{32}	2.70×10^{10}	4.30×10^{9}
5	V	0.1	3.40×10^{33}	6.40×10^{10}	6.87×10^{9}
5	V	10.0	4.70×10^{30}	1.35×10^{10}	1.01×10^{9}
10	$V-A$	1	5.19×10^{31}	3.22×10^{10}	1.65×10^{9}
10	V	1	5.25×10^{31}	2.86×10^{10}	2.38×10^{9}
25	$V-A$	1	1.80×10^{31}	4.55×10^{10}	5.06×10^{8}
25	V	1	1.65×10^{31}	4.15×10^{10}	6.11×10^{8}
10^{2}	$V-A$	1	1.19×10^{31}	1.14×10^{11}	8.51×10^{7}
10^{3}	$V-A$	1	4.32×10^{30}	6.74×10^{11}	1.49×10^{6}

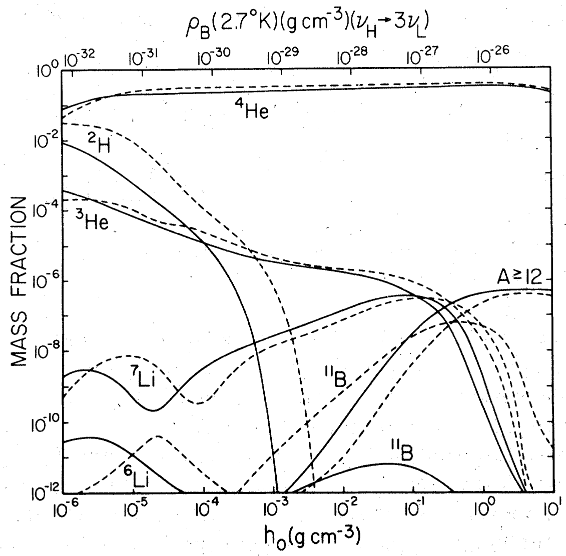

FIG. 2. A comparison of the canonical abundances (solid line) and the abundances predicted for the case of a 5-MeV neutrino with vector coupling and $\sigma/10$ (dashed lines) (assuming the neutrino has not decayed before nucleosynthesis). All other cases in Table I lie between these two extremes.

$$n_H(T)=\frac{n_D}{2.75}\left(\frac{T}{T_D}\right)^3, \tag{10}$$

and (9) becomes

$$T_1=\frac{mn_D}{2aT_D^{\,3}}. \tag{11}$$

Well before the time of nucleosynthesis the heavy neutrinos have decoupled from the rest of the universe and subsequently interact only gravitationally. Therefore they affect the final abundances only through increasing the expansion rate of the universe during element formation. The general solution of the Einstein field equations with a Robertson-Walker metric for the expansion rate in the early universe is

$$V^{-1}\frac{dV}{dt}\approx(24\pi G\rho)^{1/2}. \tag{12}$$

Nucleosynthesis occurs when $T_9\sim 1$, and Table I shows that for a mass of from 0.5 to 25 MeV the universe will be matter dominated by heavy neutrinos before or during this time. The effect of matter domination is to increase the expansion rate. For masses outside this range, the energy density of the heavy neutrinos is much less than the energy density of radiation during nucleosynthesis and may be ignored in (12). Thus the dependence of the abundances on h_0 outside the 0.5 to 25 MeV mass range remains the same as in the canonical model.

After decoupling, the ν_H density as a function of temperature may be approximated as

$$\rho_{\nu_H+\bar\nu_H}(T)=\frac{8n_\lambda\pi}{h^3}\int_0^\infty\left[m^2+p^2\left(\frac{T_\nu}{T_D}\right)^2\right]^{1/2}\times\frac{p^2dp}{1+\exp[(p^2+m^2)^{1/2}/kT_D]}, \tag{13}$$

where $T_\gamma=1.4T_\nu$ and n_λ is the number of spin projections allowed. We assume $n_\lambda=1$ for V-A interactions and $n_\lambda=2$ for V.

We have incorporated (12) into the calculation of element abundances by adding (13) to the old density in the calculation of the expansion rate. The rest of the standard program remains the same as described In Ref. 18.

The result of nucleosynthesis with heavy neutrinos is given in graphical form in Figs. 2 and 3 as a function of the input parameter h_0. In Fig. 2 the case of 5 MeV heavy neutrinos with a vector coupling and the cross section divided by ten has been compared to the canonical results. This case gives the maximum deviation from the standard model; i.e., all other cases lie between these two. Figure 3 gives only the deuterium abundance. Figure 3(a) is for the V-A ($\nu_H Z$) interaction, and Fig. 3(b) is for the V coupling.

The general nature of the deuterium result is easily understood: The increase in the expansion rate due to the ν_H contribution to the energy density implies less time to convert d's into heavier elements therefore increasing, for given h_0, the d abundance.

h_0 is the input parameter of the calculation, but we need to know the abundances in terms of the observable present-day baryon density. The baryon density is related to h_0 by (8). If no entropy is generated in ν_H decay ($\nu_H\to\nu_L\nu_L\bar\nu_L$) then

$$\rho_B\left(\frac{2.7}{T_0}\right)^3=7.15\times10^{-27}h_0. \tag{14}$$

These values of ρ_B are shown on Figs. 2 and 3 but are correct only if ν_H decay does not create photons, or if the ν_H decay before the universe is dominated by their mass.

Now, we calculate the relation of h_0 and ρ_B, assuming the ν_H decays do create photons. The energy density of the nonrelativistic neutrinos before decay is

$$\rho_{\nu_H}(T_b)=2mn_H(T_b). \tag{15}$$

Conservation of neutrinos requires that

$$n_H(T_b)=n_D\left(\frac{T_b}{T_D}\right)^3\frac{1}{2.75}. \tag{16}$$

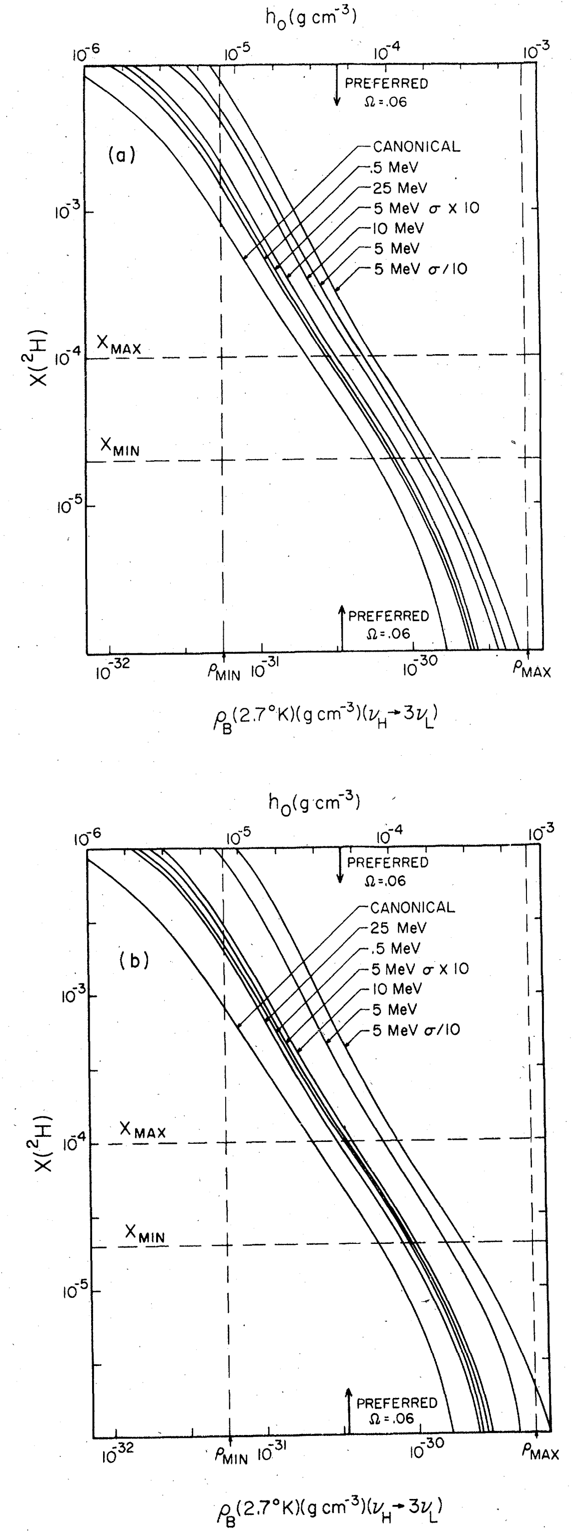

FIG. 3. The predicted ^{2}H abundances as a function of h_0 for all cases of Table I. (a) is for $V-A$ coupling; (b) for V coupling. If $m \geq 10^2$ or $m < 0.5$ MeV, the canonical result accurately describes these cases.

For $T_b < 10^9$ °K it is necessary to include the factor of 2.75 from the heating of photons by e^+e^-. We assume that the decay proceeds via $\nu_H \to \nu_L + \gamma$ so that one-half of the energy density of $\nu_H + \bar{\nu}_H$ before decay is deposited in the photon sea. (Other assumptions for the dominant decay, e.g., $\nu_H \to e^+e^-\nu_L$, $\nu_L + 2\gamma$, . . . could lead to different predictions.) Therefore, we have

$$\rho_{\gamma_H}(T_a) \approx m n_H(T_b) = \frac{m n_D}{2.75}\left(\frac{T_b}{T_D}\right)^3 . \tag{17}$$

If we assume the photons γ_H quickly thermalize,[20] then

$$\rho_{\gamma_H}(T_a) = aT_a^{\,4} . \tag{18}$$

Equating (17) and (18) and using (11) we obtain

$$\left(\frac{T_b}{T_a}\right)^3 = \left(\frac{2.75 a T_D^{\,3}}{m n_D}\right) T_a = \frac{2.75}{2}\left(\frac{T_a}{T_1}\right). \tag{19}$$

It will be convenient to express T_a in terms of a red-shift z:

$$T_a = 2.7(1+z)\ \text{°K} = 2.7z\ \text{°K}, \quad z \gg 1 . \tag{20}$$

Further we expect $z \approx 10^6$ so, for convenience, we define x as

$$x = \frac{10^6}{z} , \tag{21}$$

i.e., $T_a = 2.7 \times 10^6 / x$. Then we have

$$\left(\frac{T_b}{T_a}\right)^3 = \frac{3.71 \times 10^6}{T_1 x} \quad (T_1 \text{in °K}) , \tag{22}$$

and (8) becomes

$$\rho_B\left(\frac{2.7}{T_0}\right)^3 = 2.65 \times 10^{-20} \frac{h_0}{T_1 x} . \tag{23}$$

The present baryon density as a function of h_0 is shown in Fig. 4 for several values of the parameters x and m. The variable of interest in (23) is

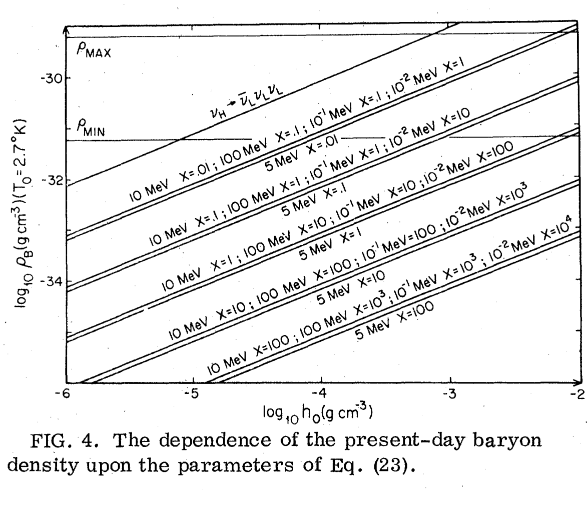

FIG. 4. The dependence of the present-day baryon density upon the parameters of Eq. (23).

x, which below will be related to the lifetime. Assuming here and below that $T_0 = 2.7\,°\mathrm{K}$,

$$x = 4.65 \times 10^9 \frac{h_0}{\Omega T_1}\,. \tag{24}$$

The observational limits on Ω were discussed in the previous section, and from Fig. 3 limits may be placed on h_0 by the deuterium abundance. The final step in this section will be to relate x to the lifetime of the heavy neutrino.

At the time of ν_H decoupling the age of the universe is less than one second. Therefore the lifetime of ν_H may be well approximated by the age of the universe when ν_H decays. The age of the universe when it becomes matter dominated is

$$t_1 = \frac{1.92 \times 10^{20}}{T_1^{\,2}}\,. \tag{25}$$

During matter domination (from T_1 until ν_H decays at T_b) the age of the universe scales as $T^{-3/2}$, so we may express the lifetime of ν_H as

$$\begin{aligned}\tau &= \frac{1.92 \times 10^{20}}{T_1^{\,2}}\left(\frac{T_1}{T_b}\right)^{3/2} \\ &= \frac{1.92 \times 10^{20}}{T_1^{\,1/2}}\,\frac{T_1^{\,1/2}}{T_a^{\,2}}\left(\frac{2}{2.75}\right)^{1/2} \\ &= 2.25 \times 10^7 x^2\,, \end{aligned} \tag{26}$$

where use has been made of Eqs. (25), (19), and (21).

In the following section we use the above results to obtain our lifetime bounds; we conclude this section by noting the following:

The helium abundance for the case given in Fig. 2 (the case of maximum deviation from the standard result) is larger than the canonical result and may slightly exceed the maximum value observationally allowed. The mass fraction of ^{4}He is nearly independent of h_0, and therefore insensitive to the decay mode of ν_H. A limit is not warranted from the helium abundance, but it should be noted that if many such heavy neutrinos exist their effect on the ^{4}He abundance may become important.

IV. RESULTS AND CONCLUSIONS

A. The bounds

Our most interesting result involves bounds on the radiative lifetime τ_R for $\nu_H \to \nu_L \gamma$. We can express the lifetime in terms of the following parameters: h_0 which defines the baryon density at the time of nucleosynthesis, Ω, the ratio of the present baryon density to the critical density $5.7 \times 10^{-30}\ \mathrm{g\,cm^{-3}}$, and T_1, the temperature at which the mass of the universe is dominated by the contribution from heavy neutrinos. Using (24) and (26), we have

$$\tau_R = 4.865 \times 10^8 \frac{(h_0/\Omega)^2}{(T_1/10^9)^2}\,. \tag{27}$$

For each case considered, we read T_1 from Table I, find the upper and lower limits on h_0 necessary to give the observed ^{2}H abundance from Fig. 3, and use (27) to calculate upper and lower bounds on τ_R. These bounds are given in Fig. 5 as a function of Ω for representative cases, and given for all cases in Table II. If the ν_H decays before the universe is matter dominated, our use of (19) is invalid and the time at matter domination is a bound below which we can say nothing, i.e., if $\tau_R < t_1(T_1)$, our statements for $\nu_H \to 3\nu_L$ will apply also to $\nu_H \to \nu_L + \gamma$ since there will be no significant increase in the photon temperature.

In Fig. 5 the solid lines are the upper bounds on the radiative lifetime for the masses indicated. The dotted lines may be interpreted as a lower bound *only* if the ν_H exist long enough to dominate the universe with their mass. The dashed line marked "previous result" arises from the condition that the photons thermalize, and was calculated in Ref. 10; it applies to all masses. Values of τ_R less than the dashed line are allowed; values above are forbidden.

Only in the case of a low-density universe (one in which cosmic deuterium is produced) do the results of Table II and Fig. 5 apply; however, that is exactly the result of the recent best measurements. For a mass range of 10^{-1} MeV to 10.0 MeV, and a value of $\Omega = 0.06$, the (independent) restrictions obtained above from the deuterium abundance on the maximum radiative lifetime exceed previous limits by as much as 2.5 orders of magnitude.

Allowed bounds on τ_R as a function of mass for a particular value of Ω, $\Omega = 0.06$, are shown in Fig. 6. For the sake of simplicity we will only consider V-A and $\alpha = 1$ cases from Table II. From

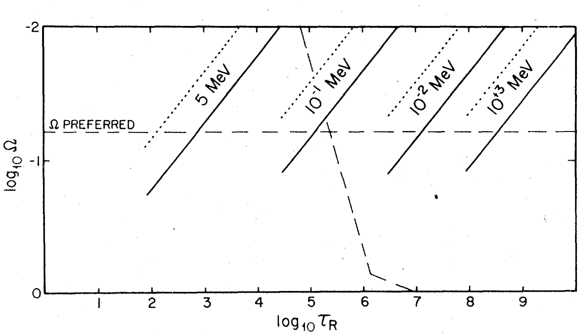

FIG. 5. Upper and lower limits on the radiative lifetime as a function of Ω for representative values of Table II. All cases are $V-A$ with coupling strength of 1.

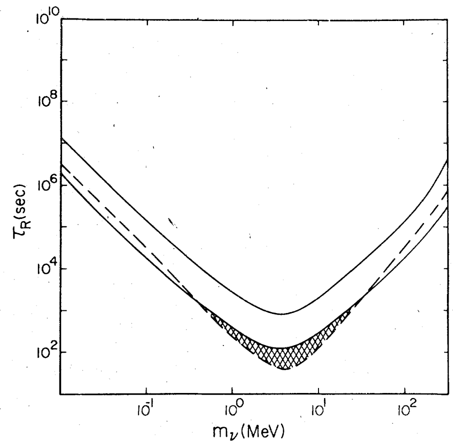

FIG. 6. Upper and lower bounds (solid line) for $\Omega = 0.06$ on the radiative lifetime of ν_H as a function of the mass. The allowed region is between the solid line or beneath the dashed line (the time when the universe becomes dominated by ν_H). The cross-hatched region is disallowed.

Table II it is obvious that deviations from the results for these choices are minor. The regions in Fig. 6 that are allowed to lie between the solid lines (results from Table II from ^{2}H bounds) *or* beneath the dashed line (ν_H decays before matter domination—Table I). Thus all the values below the upper solid line are allowed except for a small forbidden region that is cross-hatched.

B. Weak-interaction models

Now we address the question of whether requiring lifetimes to be in the allowed regions of Fig. 6 on τ_R can select among weak-interaction models. Toward this end, consider the matrix element for $\nu_H \rightarrow \nu_L \gamma$. The only gauge-invariant form is

$$M = f\bar{u}(p')\sigma^{\mu\nu}q_\nu(1 \pm \gamma_5)u(p)\epsilon_\mu , \tag{28}$$

where $p = p' + q$; ϵ_μ is the polarization vector for the photon, and f is an arbitrary coupling constant of dimension mass^{-1}. The width is

$$\Gamma = \frac{f^2}{4\pi} m^3 , \tag{29}$$

where m is the mass of ν_H. Below, we write all masses in MeV and the time in seconds. Corresponding to (29) we have

$$\tau_R = \frac{4\pi\hbar}{f^2 m^3} = \frac{8.27 \times 10^{-21}}{f^2 m^3} \text{ sec.} \tag{30}$$

Following Goldman and Stephenson[12] we now consider three reasonable choices for f: (A) the result of a first-order weak coupling with neutrino mixing, (B) first-order weak with heavy charged leptons, and (C) "second"-order weak.

For first-order weak, the matrix element is

$$M_1^\mu = \frac{G_F e}{\sqrt{2}8\pi^2}(Am + BM)\bar{u}(p')\sigma^{\mu\nu}q_\nu(1 \pm \gamma_5)u(p) , \tag{31}$$

with G_F as the Fermi constant, and the numerical factors coming from, for example, the graph of Fig. 7; M is the mass of a heavy charged lepton. There are more graphs involving Higgs particles, etc., so we consider (31) as only an order-of-magnitude estimate. We have also suppressed

TABLE II. Limits on the radiative lifetime of heavy neutrinos for the cases of Table I to correctly predict the ^{2}H abundance.

Mass (MeV)	Coupling form	Coupling strength α	Limits on h_0 (Fig. 5)	Limits on τ_R [Eq. (27)]
10^{-2}	$V-A$	1	$2.88\times10^{-5} \le h_0 \le 7.94\times10^{-5}$	$6.21\times10^{3}/\Omega^2 \le \tau_R \le 4.72\times10^{4}/\Omega^2$
10^{-1}	$V-A$	1	$2.88\times10^{-5} \le h_0 \le 7.94\times10^{-5}$	$6.21\times10^{1}/\Omega^2 \le \tau_R \le 4.72\times10^{2}/\Omega^2$
0.5	$V-A$	1	$3.80\times10^{-5} \le h_0 \le 1.05\times10^{-4}$	$4.39\times10^{0}/\Omega^2 \le \tau_R \le 3.35\times10^{1}/\Omega^2$
0.5	V	1	$4.78\times10^{-5} \le h_0 \le 1.27\times10^{-4}$	$1.74\times10^{0}/\Omega^2 \le \tau_R \le 1.23\times10^{1}/\Omega^2$
5	$V-A$	1	$6.92\times10^{-5} \le h_0 \le 1.78\times10^{-4}$	$4.56\times10^{-1}/\Omega^2 \le \tau_R \le 3.02\times10^{0}/\Omega^0$
5	$V-A$	0.1	$7.94\times10^{-5} \le h_0 \le 2.04\times10^{-4}$	$2.50\times10^{-1}/\Omega^2 \le \tau_R \le 1.65\times10^{0}/\Omega^2$
5	$V-A$	10	$4.47\times10^{-5} \le h_0 \le 1.20\times10^{-4}$	$1.93\times10^{0}/\Omega^2 \le \tau_R \le 1.39\times10^{1}/\Omega^2$
5	V	1	$8.51\times10^{-5} \le h_0 \le 2.19\times10^{-4}$	$1.91\times10^{-1}/\Omega^2 \le \tau_R \le 1.26\times10^{0}/\Omega^2$
5	V	0.1	$1.10\times10^{-4} \le h_0 \le 2.88\times10^{-4}$	$1.25\times10^{-1}/\Omega^2 \le \tau_R \le 8.55\times10^{-1}/\Omega^2$
5	V	10	$5.01\times10^{-5} \le h_0 \le 1.32\times10^{-4}$	$1.20\times10^{0}/\Omega^2 \le \tau_R \le 8.31\times10^{0}/\Omega^2$
10	$V-A$	1	$6.03\times10^{-5} \le h_0 \le 1.58\times10^{-4}$	$6.50\times10^{-1}/\Omega^2 \le \tau_R \le 4.46\times10^{0}/\Omega^2$
10	V	1	$5.37\times10^{-5} \le h_0 \le 1.41\times10^{-4}$	$2.48\times10^{-1}/\Omega^2 \le \tau_R \le 1.71\times10^{0}/\Omega^2$
25	$V-A$	1	$4.07\times10^{-5} \le h_0 \le 1.10\times10^{-4}$	$3.05\times10^{0}/\Omega^2 \le \tau_R \le 2.30\times10^{1}/\Omega^2$
25	V	1	$4.27\times10^{-5} \le h_0 \le 1.14\times10^{-4}$	$2.38\times10^{0}/\Omega^2 \le \tau_R \le 1.69\times10^{1}/\Omega^2$
10^{2}	$V-A$	1	$2.88\times10^{-5} \le h_0 \le 7.94\times10^{-5}$	$5.57\times10^{1}/\Omega^2 \le \tau_R \le 4.24\times10^{2}/\Omega^2$
10^{3}	$V-A$	1	$2.88\times10^{-5} \le h_0 \le 7.94\times10^{-5}$	$1.82\times10^{5}/\Omega^2 \le \tau_R \le 1.38\times10^{6}/\Omega^2$

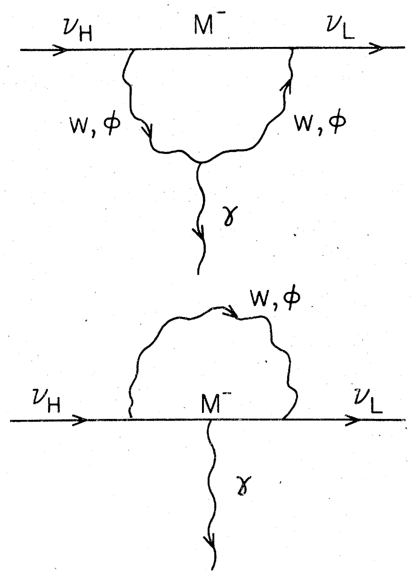

FIG. 7. An example of diagrams involved in the calculation of the radiative ν_H lifetime. W is the charged intermediate vector boson; ϕ is the Higgs field; and M is a heavy lepton.

Cabibbo-type leptonic angles. Because of these uncertainties we will consider A and B as parameters and consider values in the range

$$0.1 < B^2, A^2 < 10 \tag{32}$$

reasonable.

It would not be unreasonable in gauge theories for the first-order weak diagrams to cancel. The "second"-order weak (actually order $G/m_W{}^2$) amplitude is

$$M_2^\mu = \frac{CG_F e}{\sqrt{2}8\pi^2}\frac{\Delta M^2}{m_W{}^2}\, m\bar{u}(p')\sigma^{\mu\nu}q_\nu(1\pm\gamma_5)u(p)\,, \tag{33}$$

where ΔM^2 is the difference of the square of two heavy leptons and m_W^2 is the mass of the W boson. C^2, like A^2 and B^2, may be assumed to vary in the range 0.1 to 10. Using (28), (31), and (33), the possibilities for f are

$$f^2 = 9.51\times 10^{-28}\times\begin{cases} A^2m^2 & [\text{case (A)}]\\ B^2M^2 & [\text{case (B)}]\\ C^2\left(\dfrac{\Delta M^2}{m_W{}^2}\right)^2 m^2 & [\text{case (C)}]\end{cases}$$

Using (30) for τ_R we compare in Fig. 8 the three possibilities for τ_R from (34) using $(\Delta M^2)^{1/2} = 4$ GeV, $m_W = 40$ GeV, $M = 2$ GeV; and A^2, B^2, and C^2 in the range 0.1 to 10. The solid line in Fig. 8 is the upper curve in Fig. 6. We also show in Fig. 8, for comparison, the upper limits on τ from $\rho < \rho_c$ and from the requirement of thermalization as well as the lower limit on τ from laboratory experiments.

From Fig. 8 we see that for a heavy neutrino mass less than $\approx$20 MeV, second-order weak transitions [case (C)] will have too long a lifetime, the universe will be matter dominated by ν_H for an appreciable amount of time resulting in the deuterium abundance being incorrectly predicted. For masses less than $\approx$5 MeV, the same problem arises in the neutrino mixing case [case (A)]. For $m \lesssim 0.8$ MeV case (B) will result in $\bar{\nu}_e e$ scattering larger than allowed by the Reines experiment (line 5).

The limits given here cannot be used with complete confidence until more definite values of the primordial ^{2}H abundance and the present baryon density are available. Our limits on the radiative lifetime and the coupling involved may best be interpreted as suggestive of reasonable values.

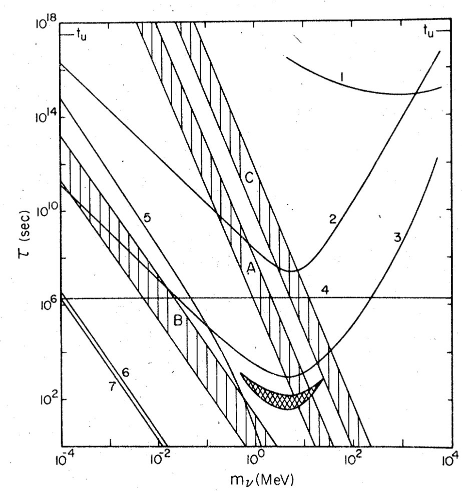

FIG. 8. Comparison of the radiative ν_H lifetime (as a function of the ν_H mass) for the three cases discussed in the text with astrophysical upper and experimental lower limits. Lines 1 through 4 are *upper* bounds on τ from astrophysics: 1 is from requiring the decay neutrinos not be detected in the Davis experiment (Ref. 10), 2 is from requiring the present energy density of ν_H decay products not exceed the maximum allowed (Refs. 9, 10), 3 is from nucleosynthesis (Fig. 6), 4 is from requiring photons from ν_H decay thermalized (Ref. 10). The cross-hatched region is the disallowed region from Figure 2. Lines 5 through 7 are *lower* bounds on the radiative ν_H lifetime from laboratory experiments (Ref. 10): 5 is from $\nu_e e$ scattering, 6 is from $\nu_\mu e$ scattering. Possible values of the radiative lifetimes lie below 1–4 and above 5–7. Therefore, radiative decays for neutrinos of mass less than 0.1 MeV are ruled out. The shaded regions are cases (A), (B), and (C) from Sec. IVB.

However, because of the difficulty of detecting such neutrinos, indirect evidence such as presented here may for some time provide the best available evidence of their properties.

It is also possible to apply the results of this paper to particles with other spins such as scalar or pseudoscalar mesons (Higgs, "axions," etc.). The only condition necessary is that their lifetime be long enough that they are present after nucleosynthesis.

In conclusion, we draw the reader's attention once again to Fig. 8 which summarizes the information currently available from cosmology on the lifetimes of massive neutral weakly interacting leptons as a function of their mass. It is in many respects remarkable that such detailed information may be inferred from such a remote event.

ACKNOWLEDGMENTS

One of us (EWK) would like to thank S. Brodsky for making available the SLAC computing facilities where much of the numerical work was done, and B. Baaquie for his help. We have benefited from conversations with B. W. Lee, R. Bowers, G. Steigman, D. N. Schramm, and S. Weinberg, as well as from the members of the VPI Astronomy group, J. Broderick, J. Condon, B. Denison, and J. Ledden. The work of D.A.D. and E.W.K. was supported in part by the Energy Research and Development Administration under Contract No. E (40-1)3992. The work of V.L.T. was supported in part by the National Science Foundation. The work of R.V.W. was also supported in part by the National Science Foundation under Contract No. PHY 76-21454.

---

[1]For a review of the experimental evidence for the existence of τ see G. Feldman and M. Perl, Phys. Rep. 33C, 285 (1977).

[2]K. Bergkvis *et al.*, Nucl. Phys. B39, 317 (1972).

[3]A. Clark *et al.*, Phys. Rev. D 9, 533 (1974).

[4]Examples of experiments for which heavy neutrinos have been used to explain unexpected results are: parity violation in atomic physics, L. Lewis *et al.*, Phys. Rev. Lett. 39, 795 (1977); and P. Baird *et al.*, *ibid.* 39, 798 (1977); and trimuon events in ν_μ inelastic scattering, A. Benvenuti *et al.*, *ibid.* 38, 1110 (1977).

[5]For the details of what is referred to as the "standard" big bang see, for example, C. Misner, K. Thorne, and J. Wheeler, *Gravitation* (Freeman, San Francisco, 1973); or S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[6]R. Cowsik and J. McClelland, Phys. Rev. Lett. 29, 669 (1972).

[7]R. Cowsik, Phys. Rev. Lett. 39, 784 (1977).

[8]B. Lee and S. Weinberg, Phys. Rev. Lett. 39, 165 (1977).

[9]D. Dicus, E. Kolb, and V. Teplitz, Phys. Rev. Lett. 39, 168 (1977); 39, 973(E) (1977).

[10]D. Dicus, E. Kolb, and V. Teplitz, Astrophys. J. 221, 327 (1978).

[11]G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. 66B, 202 (1977).

[12]T. Goldman and G. J. Stephenson, Jr., Phys. Rev. D 16, 2256 (1977).

[13]K. Sato and M. Kobayashi, Prog. Theor. Phys. 58, 1775 (1977).

[14]K. Sato and H. Sato, Prog. Theor. Phys. 54, 912 (1975).

[15]J. Gunn *et al.*, Astrophys. J. (to be published). We are grateful to Professor G. Steigman for informing us of the existence of this work.

[16]At very early times the particle separation R is proportional to the square root of the time. See Misner, Thorne, and Wheeler, or Weinberg, Ref. 5. Equation (1b) is valid in the temperature range (3×10^9)–10^{12} °K.

[17]For a review of pertinent abundances, see D. Schramm and R. Wagoner, in Annu. Rev. Nucl. Sci. 27, 37 (1977).

[18]P. J. E. Peebles, Astrophys. J. 146, 542 (1966); R. Wagoner, W. Fowler, and F. Hoyle, *ibid.* 148, 3 (1967); and R. Wagoner, *ibid.* 179, 343 (1973). For a review of these calculations see Ref. 17.

[19]See Weinberg, Ref. 5, Chap. 16.

[20]P. J. E. Peebles, in *Physical Cosmology* (Princeton Univ. Press, Princeton, 1971), Chap. VII.