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Inertial and gravitational effects in the proper reference frame of an accelerated, rotating observer

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Most experimental laboratories accelerate and rotate relative to inertial frames. This paper derives approximate expressions for the general-relativistic metric and the general-relativistic equations of motion of freely falling particles in such a laboratory. The metric is derived accurate to second order in distance from the origin of coordinates; the equations of motion are derived accurate to first order. The equations of motion contain inertial, Coriolis, and centripetal pseudoforces, electric, magnetic, and magnetic-magnetic type forces due to Riemann curvature (inhomogeneous gravity), "gravitational red-shift" corrections to these forces, and velocity-induced special-relativistic corrections.

Synge[1] defined a natural coordinate system for an accelerated observer, which he called the "Fermi coordinates,"[2] and derived integral expressions for the metric and the inertial (coordinate) accelerations about the observer's world line for these coordinates in spacetime with small curvature. Manasse and Misner[3] obtained the second-order coordinate expansion of the metric in the special case of a freely falling observer. Using a somewhat different coordinate system, and a dyadic formalism, Estabrook and Wahlquist[4] derived an equation for the inertial acceleration near an arbitrary world line. Ni[5] and Mashhoon[6] calculated the second-order expansion of the metric and the first-order expansion of the inertial accelerations in these coordinates for an accelerated observer in special and general relativity, respectively.

A natural extension of the Fermi coordinates of Synge to the case of an accelerated *rotating* observer is the "local coordinates of the observer's proper reference frame" defined by Misner, Thorne, and Wheeler (MTW).[7] Such coordinates are important because they are the ones used by real experimenters in real earth-bound laboratories. MTW calculated the first-order expansion of the metric, and obtained the inertial accelerations on the world line of an arbitrarily accelerating and rotating observer. In this paper, we extend their work to obtain the second-order expansion of the metric and the first-order expansion of the inertial accelerations for the case of an arbitrarily accelerating and rotating observer in general relativity and in other metric theories of gravity. To this order, we include centripetal pseudoforces, second-order red-shifts, relativistic corrections, and electric and magnetic Riemann curvature terms.

Consider an observer moving along the world line $P_0(\tau)$ with four-velocity $u(\tau)$ and four-rotation $\omega(\tau)$ in a gravitational field with Riemann tensor $R^\mu{}_{\nu\alpha\beta}(\tau)$ along the world line. The orthonormal tetrad $\{e_{\hat\alpha}\}$ which the observer carries transports according to[8]

$$\frac{de_{\hat\alpha}}{d\tau} = -\vec{\Omega}\cdot e_{\hat\alpha}\,, \tag{1}$$

where

$$\Omega^{\mu\nu} \equiv a^\mu u^\nu - a^\nu u^\mu + u_\alpha \omega_\beta \epsilon^{\alpha\beta\mu\nu}\,,^{9} \tag{2}$$

$$a(\tau) \equiv \nabla_u u\,, \tag{3}$$

and τ is the proper time along the world line.

Following Sec. 13.6 of MTW, at any event $P_0(\tau)$ we send out geodesics $P(\tau; n; s)$ orthogonal to $u(\tau)$, where n is the unit vector tangent to a particular geodesic at $P_0(\tau)$, and $n\cdot u(\tau) = 0$. An event a distance s out along any geodesic n is then assigned the coordinates $x^{\hat 0} \equiv \tau$, $x^{\hat j} \equiv s n\cdot e_{\hat j}$. These coordinates are called local coordinates.

This coordinate system is good for events near the world line, i.e., for

$$s \ll \min\left\{\frac{1}{|a|}, \frac{1}{|\omega|}, \frac{1}{|R^{\hat\alpha}{}_{\hat\nu\hat\alpha\hat\beta}|^{1/2}}, \frac{|R^{\hat\mu}{}_{\hat\nu\hat\alpha\hat\beta}|}{|R^{\hat\mu}{}_{\hat\nu\hat\alpha\hat\beta,\hat\gamma}|}\right\},$$

since within this distance the geodesics coming out of the world line do not cross ($s \ll 1/|a|$), the "light-cylinder" has not been reached ($s \ll 1/|\omega|$), curvature has not yet caused geodesics to cross ($s \ll 1/|R^{\hat\mu}{}_{\hat\nu\hat\alpha\hat\beta}|^{1/2}$), and the Riemann tensor has not yet changed much from its value on the world line ($s \ll |R^{\hat\mu}{}_{\hat\nu\hat\alpha\hat\beta}|/|R^{\hat\mu}{}_{\hat\nu\hat\alpha\hat\beta,\hat\gamma}|$). This last condition is usually the most severe restriction when using this coordinate system in an earth-bound laboratory.

In the local coordinate system we decompose a four-vector V as $V=(V^{\hat 0};V^{\hat i})\equiv(V^{\hat 0};\vec V)$. Now defining $b\equiv\nabla_u a$, $\eta\equiv\nabla_u\omega$ and using Eqs. (1) and (2) we have

$$b^{\hat 0}=\vec a\cdot\vec a\,,\quad \vec b=\frac{d\vec a}{d\tau}+\vec\omega\times\vec a\,,$$
$$\eta^{\hat 0}=\vec\omega\cdot\vec a\,,\quad \vec\eta=\frac{d\vec\omega}{d\tau}\,. \tag{4}$$

Along $P_0(\tau)$, MTW derived the connection coefficients and the first-order partial derivatives to be

$$\left.\begin{aligned}&\Gamma^{\hat 0}{}_{\hat 0\hat 0}=\Gamma^{\hat\alpha}{}_{\hat j\hat k}=0\\&\Gamma^{\hat 0}{}_{\hat j\hat 0}=a^{\hat j}\\&\Gamma^{\hat j}{}_{\hat k\hat 0}=-\omega^{\hat i}\epsilon^{\hat i\hat j\hat k}\end{aligned}\right\}\ \text{all along } P_0(\tau)\,, \tag{5}$$

$$\left.\begin{aligned}&g_{\hat\alpha\hat\beta,\hat 0}=g_{\hat j\hat k,\hat i}=0\\&g_{\hat 0\hat 0,\hat j}=-2a_{\hat j}\\&g_{\hat 0\hat j,\hat k}=-\epsilon^{\hat j\hat k\hat i}\omega^{\hat i}\end{aligned}\right\}\ \text{all along } P_0(\tau)\,. \tag{6}$$

Differentiating Eqs. (5) along the trajectory with respect to τ and using Eqs. (4), we have

$$\left.\begin{aligned}&\Gamma^{\hat 0}{}_{\hat 0\hat 0,\hat 0}=\Gamma^{\hat\alpha}{}_{\hat j\hat k,\hat 0}=0\\&\Gamma^{\hat 0}{}_{\hat j\hat 0,\hat 0}=b^{\hat j}(\tau)+\epsilon^{\hat j\hat k\hat i}a^{\hat k}(\tau)\omega^{\hat i}(\tau)\\&\Gamma^{\hat i}{}_{\hat j\hat 0,\hat 0}=-\eta^{\hat k}(\tau)\epsilon^{\hat i\hat j\hat k}\end{aligned}\right\}\ \text{all along } P_0(\tau)\,. \tag{7}$$

From the definition of the Riemann tensor,

$$\Gamma^{\hat\alpha}{}_{\hat\mu\hat 0,\hat\nu}=R^{\hat\alpha}{}_{\hat\mu\hat\nu\hat 0}+\Gamma^{\hat\alpha}{}_{\hat\mu\hat\nu,\hat 0}+(\Gamma^{\hat\sigma}{}_{\hat\mu\hat\nu}\Gamma^{\hat\alpha}{}_{\hat\sigma\hat 0}-\Gamma^{\hat\sigma}{}_{\hat\mu\hat 0}\Gamma^{\hat\alpha}{}_{\hat\sigma\hat\nu})\,. \tag{8}$$

Combining this equation with Eqs. (5), we find

$$\left.\begin{aligned}&\Gamma^{\hat 0}{}_{\hat 0\hat 0,\hat i}=b^{\hat i}(\tau)+2a^{\hat j}(\tau)\omega^{\hat k}(\tau)\epsilon^{\hat i\hat j\hat k}\\&\Gamma^{\hat j}{}_{\hat 0\hat 0,\hat i}=R_{\hat 0\hat j\hat 0\hat i}-\eta^{\hat k}\epsilon^{\hat j\hat i\hat k}+a^{\hat i}a^{\hat j}\\&\qquad+\omega^{\hat i}\omega^{\hat j}-\delta_{\hat i\hat j}(\omega^{\hat l})^2\\&\Gamma^{\hat 0}{}_{\hat j\hat 0,\hat i}=R_{\hat 0\hat j\hat 0\hat i}-a^{\hat i}a^{\hat j}\\&\Gamma^{\hat j}{}_{\hat k\hat 0,\hat i}=R_{\hat j\hat k\hat i\hat 0}+a^{\hat k}\omega^{\hat l}\epsilon_{\hat l\hat j\hat i}\end{aligned}\right\}\ \text{all along } P_0(\tau)\,. \tag{9}$$

To express $\Gamma^{\hat\mu}{}_{\hat j\hat k,\hat i}$ in terms of $R_{\hat\alpha\hat\beta\hat\gamma\hat\delta}$, $\vec a$, $\vec b$, $\vec\omega$, and $\vec\eta$, we follow the method of Manasse and Misner[3] and use the geodesic deviation equation

$$\frac{d^2N^\mu}{ds^2}+2\frac{dN^\sigma}{ds}\Gamma^\mu{}_{\sigma\alpha}U^\alpha+N^\sigma U^\alpha U^\beta R^\mu{}_{\alpha\sigma\beta}$$
$$+N^\sigma U^\alpha U^\beta(\Gamma^\mu{}_{\sigma\alpha,\beta}+\Gamma^\tau{}_{\sigma\alpha}\Gamma^\mu{}_{\tau\beta}-\Gamma^\mu{}_{\sigma\tau}\Gamma^\tau{}_{\alpha\beta})=0\,, \tag{10}$$

where $N=\partial/\partial N$ and $U=\partial/\partial s$ of a one-parameter family of geodesics $\mathcal{R}(N,s)$, and where s is an affine parameter along the geodesic $\mathcal{R}(N,s)$ for N fixed. The family of geodesics we want to consider is $P(\tau;\alpha^{\hat i};s)\equiv P(\tau;n;s)$ where $n=\alpha^{\hat i}e_{\hat i}$. The case $N=\partial/\partial\tau$ merely leads to part of Eqs. (18). The case $N=\partial/\partial\alpha^{\hat i}$ leads to the desired results. In this case $N=\partial/\partial\alpha^{\hat i}=s\partial/\partial x^{\hat i}$, hence $N^{\hat i}=s\delta_{\hat i}{}^{\hat\mu}$. Expanding the second term in the geodesic deviation equation in powers of s, we have

$$2\delta_{\hat i}{}^{\hat\sigma}\Gamma^{\hat\mu}{}_{\hat\sigma\hat j}\alpha^{\hat j}=2\lambda\Gamma^{\hat\mu}{}_{\hat i\hat j,\hat k}|_{P_0(\tau)}\alpha^{\hat j}\alpha^{\hat k}+O(\lambda^2)\,. \tag{11}$$

Substituting (5) and (11) into (10), dividing (10) by s, and then setting $s=0$, we obtain

$$(3\Gamma^{\hat\mu}{}_{\hat i\hat j,\hat k}+R^{\hat\mu}{}_{\hat j\hat i\hat k})|_{P_0(\tau)}\alpha^{\hat j}\alpha^{\hat k}=0 \tag{12}$$

Since $\alpha^{\hat j}$ can be arbitrary, (12) leads to

$$(\Gamma^{\hat\mu}{}_{\hat i\hat j,\hat k}+\Gamma^{\hat\mu}{}_{\hat i\hat k,\hat j})|_{P_0(\tau)}=-\tfrac{1}{3}(R^{\hat\mu}{}_{\hat j\hat i\hat k}+R^{\hat\mu}{}_{\hat k\hat i\hat j})|_{P_0(\tau)}\,. \tag{13}$$

This equation can be solved for $\Gamma^{\hat\mu}{}_{\hat i\hat j,\hat k}|_{P_0(\tau)}$ by adding to it one cyclic permutation and subtracting another:

$$\Gamma^{\hat\mu}{}_{\hat i\hat j,\hat k}|_{P_0(\tau)}=-\tfrac{1}{3}(R^{\hat\mu}{}_{\hat i\hat j\hat k}+R^{\hat\mu}{}_{\hat j\hat i\hat k})|_{P_0(\tau)}\,. \tag{14}$$

From the definition of the Christoffel symbols,

$$g_{\mu\nu,\alpha}=g_{\mu\sigma}\Gamma^\sigma{}_{\nu\alpha}+g_{\sigma\nu}\Gamma^\sigma{}_{\mu\alpha}\,, \tag{15}$$

we find by differentiation that

$$g_{\hat\mu\hat\nu,\hat\alpha\hat\beta}|_{P_0(\tau)}=\eta_{\hat\mu\hat\sigma}\Gamma^{\hat\sigma}{}_{\hat\nu\hat\alpha,\hat\beta}|_{P_0(\tau)}+\eta_{\hat\sigma\hat\nu}\Gamma^{\hat\sigma}{}_{\hat\mu\hat\alpha,\hat\beta}|_{P_0(\tau)}$$
$$+g_{\hat\mu\hat\sigma,\hat\beta}|_{P_0(\tau)}\Gamma^{\hat\sigma}{}_{\hat\nu\hat\alpha}|_{P_0(\tau)}$$
$$+g_{\hat\sigma\hat\nu,\hat\beta}|_{P_0(\tau)}\Gamma^{\hat\sigma}{}_{\hat\mu\hat\alpha}\,. \tag{16}$$

Combining Eqs. (5), (6), (7), (9), (14), and (16) we have

$$\left.\begin{aligned}
&g_{\hat\alpha\hat\beta,\hat0\hat0}=0\\
&g_{\hat j\hat k,\hat l\hat0}=0\\
&g_{\hat0\hat0,\hat j\hat0}=-2(b^{\hat j}+\epsilon^{\hat j\hat k\hat l}a^{\hat k}\omega^{\hat l})\\
&g_{\hat0\hat j,\hat k\hat0}=-\epsilon_{\hat j\hat k\hat l}\eta^{\hat l}\\
&g_{\hat0\hat0,\hat j\hat k}=-2R^{\hat0}{}_{\hat j\hat k\hat0}-2a^{\hat j}a^{\hat k}+2\delta^{\hat j}{}_{\hat k}(\omega^{\hat l})^2-2\omega^{\hat j}\omega^{\hat k}\\
&g_{\hat0\hat i,\hat j\hat k}=-\tfrac{2}{3}(R_{\hat0\hat k\hat i\hat j}+R_{\hat0\hat j\hat i\hat k})\\
&g_{\hat l\hat m,\hat i\hat j}=-\tfrac{1}{3}(R_{\hat i\hat l\hat j\hat m}+R_{\hat i\hat m\hat j\hat l})
\end{aligned}\right\}\ \text{all along } P_0(\tau)\,. \tag{17}$$

From Eqs. (17), we obtain the second-order expansion of the metric at the point $P(x^{\hat0},x^{\hat j})$ as

$$\begin{aligned}
ds^2=&-(dx^{\hat0})^2[1+2a_{\hat j}x^{\hat j}+(a^{\hat l}x^{\hat l})^2+(\omega^{\hat l}x^{\hat l})^2\\
&\qquad-(\omega)^2x^{\hat l}x^{\hat l}+R_{\hat0\hat l\hat0\hat m}x^{\hat l}x^{\hat m}]\\
&+2dx^{\hat0}dx^{\hat i}(\epsilon_{\hat i\hat j\hat k}\omega^{\hat j}x^{\hat k}-\tfrac{2}{3}R_{\hat0\hat l\hat i\hat m}x^{\hat l}x^{\hat m})\\
&+dx^{\hat i}dx^{\hat j}(\delta_{\hat i\hat j}-\tfrac{1}{3}R_{\hat i\hat l\hat j\hat m}x^{\hat l}x^{\hat m})\\
&+O(dx^{\hat\mu}dx^{\hat\nu}x^{\hat l}x^{\hat m}x^{\hat k})\,,
\end{aligned}\tag{18}$$

where $a_{\hat j}$, $\omega^{\hat l}$, and $R_{\hat\alpha\hat\beta\hat\mu\hat\nu}$ are evaluated on the world line at time $x^{\hat0}$.

To calculate the coordinate acceleration of a freely falling body, we use the geodesic equation in the form

$$\frac{d^2x^{\hat i}}{d(x^{\hat0})^2}+\left(\Gamma^{\hat i}{}_{\hat\mu\hat\nu}-\Gamma^{\hat0}{}_{\hat\mu\hat\nu}\frac{dx^{\hat i}}{dx^{\hat0}}\right)\frac{dx^{\hat\mu}}{dx^{\hat0}}\frac{dx^{\hat\nu}}{dx^{\hat0}}=0 \tag{19}$$

and substitute into it the first-order expansion of the Γ's. Defining $w^i\equiv dx^{\hat i}/dx^{\hat0}$, the velocity measured by the accelerated rotating observer, the resulting coordinate acceleration is

$$\begin{aligned}
\frac{d^2x^{\hat i}}{d(x^{\hat0})^2}=&-(1+\vec a\cdot\vec x)a^{\hat i}-(\vec\omega\times(\vec\omega\times\vec x))^{\hat i}-(\vec\eta\times\vec x)^{\hat i}\\
&-2(\vec\omega\times\vec w)^i+2(\vec a\cdot\vec w)(\vec\omega\times\vec x)^i\\
&+w^i[2\vec a\cdot(\vec\omega\times\vec x)+2\vec a\cdot\vec w(1-\vec a\cdot\vec x)+\vec b\cdot\vec x]\\
&-x^{\hat l}R_{\hat0\hat i\hat0\hat l}-2x^{\hat l}w^jR_{\hat i\hat j\hat l\hat0}\\
&+\tfrac{2}{3}x^{\hat l}w^jw^kR_{\hat i\hat j\hat k\hat l}+2x^{\hat l}w^iw^jR_{\hat0\hat j\hat0\hat l}\\
&+\tfrac{2}{3}x^{\hat l}w^iw^jw^kR_{\hat0\hat j\hat k\hat l}+O((x^{\hat l})^2)\,.
\end{aligned}\tag{20}$$

To express $d^2x^{\hat i}/d(x^{\hat0})^2$ in terms of the velocity v^i observed in the local coordinates of an unaccelerated nonrotating observer, we use the relation

$$\vec w=\vec v(1+\vec a\cdot\vec x)-\vec\omega\times\vec x+O((x^{\hat l})^2)\,, \tag{21}$$

which is obtained by integrating Eq. (20).

Substituting Eq. (21) into (20) we obtain

$$\begin{aligned}
\frac{d^2x^{\hat i}}{d(x^{\hat0})^2}=&-(1+\vec a\cdot\vec x)a^{\hat i}+2(\vec a\cdot\vec v)(1+\vec a\cdot\vec x)v^i\\
&+(\vec b\cdot\vec x)v^i-2(1+\vec a\cdot\vec x)(\vec\omega\times\vec v)^{\hat i}+(\vec\omega\times(\vec\omega\times\vec x))^{\hat i}\\
&-(\vec\eta\times\vec x)^i-R_{\hat0\hat i\hat0\hat l}x^{\hat l}-2R_{\hat i\hat j\hat l\hat0}x^{\hat l}v^j\\
&+\tfrac{2}{3}R_{\hat i\hat j\hat k\hat l}x^{\hat l}v^jv^k+2R_{\hat0\hat j\hat0\hat l}x^{\hat l}v^iv^j+\tfrac{2}{3}R_{\hat0\hat j\hat k\hat l}x^{\hat l}v^iv^jv^k\\
&+O((x^{\hat l})^2)\,.
\end{aligned}\tag{22}$$

The various terms in this equation are interpreted in Table I. Notice that to the order calculated there could be no coupling between the Riemannian terms and the a, ω, b, and η terms. Therefore, we can also derive the above results by combining a simpler special-relativistic derivation with the results for a freely falling observer in curved spacetime.

The results presented in this paper may be useful in analysis of tidal deformation of objects due to various types of close encounters, or in analysis of gravitational wave detectors and laboratory experiments where the size of the apparatus is small compared with inhomogeneities in the gravitational fields being observed. A Newtonian physcist can think about the terms in Eq. (22) or Table I as simply Newtonian forces, as described in box 37.1 of MTW. Moreover, a Newtonian physicist can use the equation of motion (20) or (22) to analyze mechanical apparatus in an experimental laboratory. All he needs to do is multiply this equation by the mass of a mass element in his apparatus, and add it linearly onto the forces that would be present if the apparatus were at rest in an inertial reference frame (see, e.g., box 37.1 of MTW).

TABLE I. Various inertial and gravitational effects in coordinate acceleration.

Effect	Term in coordinate acceleration $d^2x^{\hat{i}}/d(x^{\hat{0}})^2$
1. Usual inertial acceleration	$-a^{\hat{i}}$
2. Usual Doppler ("gravitational") red-shift correction to term 1: physical processes "overhead" run fast compared to observer's proper time	$-(\vec{a}\cdot\vec{x})a^{\hat{i}}$
3. Special-relativistic (SR) correction to acceleration [due to $\gamma=1/(1-v^2)^{1/2}$]	$+2(\vec{a}\cdot\vec{v})v^{i}$
4. Red-shift correction to term 3	$+2(\vec{a}\cdot\vec{v})(\vec{a}\cdot\vec{x})v^{i}$
5. ∂(red-shift)/$\partial\tau$ correction to acceleration	$+(\vec{b}\cdot\vec{x})v^{i}$
6. Coriolis acceleration	$-2(\vec{\omega}\times\vec{v})^{i}$
7. Red-shift correction to term 6	$-2(\vec{a}\cdot\vec{x})(\vec{\omega}\times\vec{v})^{i}$
8. Centripetal acceleration (Ref. 10)	$+[\vec{\omega}\times(\vec{\omega}\times\vec{x})]^{i}$
9. Coordinate acceleration if ω changes	$-(\vec{\eta}\times\vec{x})^{i}$
10. "Electric-type" (usual) gravitational effect	$-R_{\hat{0}\hat{i}\hat{0}\hat{l}}$
11. SR correction to term 10	$+2R_{\hat{0}\hat{j}\hat{0}\hat{l}}x^{\hat{l}}v^{\hat{i}}v^{\hat{j}}$
12. "Magnetic-type" gravitational effect	$-2R_{\hat{i}\hat{j}\hat{l}\hat{0}}x^{\hat{l}}v^{\hat{j}}$
13. SR correction to term 12	$+\frac{2}{3}R_{\hat{0}\hat{j}\hat{k}\hat{l}}x^{\hat{l}}v^{i}v^{j}v^{k}$
14. "Double-magnetic" gravitational effect	$+\frac{2}{3}R_{\hat{i}\hat{j}\hat{k}\hat{l}}x^{\hat{l}}v^{\hat{j}}v^{\hat{k}}$

In actual experiments, while the second-order inertial effects are small, so are the Riemann forces which are being observed. Terms 2, 8, and 9 in Table I, for example, have a dependence on the coordinates similar to the usual $R_{\hat{0}\hat{j}\hat{0}\hat{l}}$ acceleration, term 10; likewise, terms 5 and 7 resemble the "magnetic" Riemann effect, term 12, and term 4 resembles terms 11 and 14. In typical resonant-device experiments, for instance, one might be concerned about noise fluctuations in the acceleration of gravity: If $g=g_0(1+\epsilon\cos\omega t)$, then the second term (red-shift) gives an acceleration which simulates an $R_{\hat{0}\hat{z}\hat{0}\hat{z}}$ of magnitude $2g_0{}^2\epsilon$. Thus, one might ask that the dimensionless amplitude (metric perturbation) of the wave h ($h\omega^2 \sim |R_{\hat{0}\hat{i}\hat{0}\hat{l}}|$) be greater than

$$h_{\min}\sim\left(\frac{g_0}{\omega}\right)^2\epsilon\sim 10^{-17}\,\frac{\epsilon}{(\nu/1\text{ Hz})^2}\,,$$

where ν is the frequency of the wave. For the Crab pulsar, which is estimated[11] to produce $h\sim 10^{-27}$ at 60 Hz, one thus would want to reduce ϵ below 10^{-7} (or orient the apparatus horizontally). Second-order accelerations due to angular motions may be more serious; there are no good measurements at present of angular seismic noise.[12]

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[2]The term "Fermi coordinates" is usually used in a different sense, namely, to describe coordinates such that the affine connections vanish on some curve or other subspace; cf. E. Fermi, Atti R. Accad. Lincei Rend. Cl. Sci. Fis. Mat. Nat. 31, 21 (1922); 31, 51 (1922), and L. O'Raifeartaigh, Proc. R. Irish Acad. 59A, 15 (1958).

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[8] Throughout this paper we use MTW notations and conventions.

[9]$\epsilon^{\alpha\beta\mu\nu}$ is the completely antisymmetric tensor with $\epsilon^{0123} = -1$ in an inertial frame.

[10]If we use $\vec{w}$ instead of $\vec{v}$ in the definition of Coriolis acceleration, then the sign of this term flips and we have centrifugal acceleration.

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