Color-octet weak current induced by quark mixing

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We generalize a previous study, in which we have discussed the possible existence of a color-octet weak charged current by assuming tacitly the integral-charge quark model, so that the formulation may also be applicable to the fractional-charge quark model. In spite of this generalization, the conclusions stated in the previous paper are not revised.

In a previous paper[1] we have discussed the possible existence of a color-octet weak charged current induced by quark mixing on the basis of a four-flavor model with unconfined color in the framework of the $SU(2)_L \times U(1)$ gauge theory.

Recently, evidence for the existence of fractional charge on matter has been reported by LaRue, Fairbank, and Hebard.[2] This suggests that quarks have fractional charges. In the fractional-charge quark model, the color-octet weak current can be induced only by mixing among the quark fields.[3] Although a model which provides color-octet charged and neutral currents induced by mixing between the flavor and color gauge bosons has penetratingly been proposed by Pati and Salam,[4] such a mechanism is not applicable to the fractional-charge quark model.

The purpose of the present paper is to extend the previous studies based on the integer-charge quark model. We assign our quark fields to the doublets of the left-handed gauge group $SU(2)_L$ as follows:

$$\begin{pmatrix} u_i \\ d_i' \end{pmatrix}_L, \quad \begin{pmatrix} c_i \\ s_i' \end{pmatrix}_L, \quad i=1,2,3. \tag{1}$$

Here the fields d_i' and s_i' are given by a unitary transformation U as

$$\begin{pmatrix} d' \\ s' \end{pmatrix}_L = U\begin{pmatrix} d \\ s \end{pmatrix}_L \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix}_L, \tag{2}$$

where $d=(d_1,d_2,d_3)$ and $s=(s_1,s_2,s_3)$. In order that our theory may lead to results compatible with the semileptonic interactions of ordinary hadrons, we must impose

$$\tfrac{1}{3}\mathrm{Tr}(A)=e^{i\alpha}\kappa\cos\theta_C\,, \tag{3}$$

$$\tfrac{1}{3}\mathrm{Tr}(B)=e^{i\beta}\kappa\sin\theta_C\,, \tag{4}$$

where θ_C is the Cabibbo angle and κ is a real (positive) parameter. Hereafter, we choose $\alpha=\beta=0$ because the phases α and β can be absorbed into the quark fields d and s.

In general, the relation

$$|a+b+c|^2 \leq 3(|a|^2+|b|^2+|c|^2) \tag{5}$$

holds for arbitrary numbers a, b, and c, where the equality holds only when $a=b=c$. Applying the relation (5) to Eqs. (3) and (4), we get

$$\kappa^2 \leq \tfrac{1}{3}\sum_i (|A_{ii}|^2+|B_{ii}|^2). \tag{6}$$

Since $|A_{ii}|^2+|B_{ii}|^2 \leq 1$ $(i=1,2,3)$, we can obtain

$$\kappa^2 \leq 1\,, \tag{7}$$

where $\kappa^2=1$ holds only when $A_{11}=A_{22}=A_{33}=\cos\theta_C$ and $B_{11}=B_{22}=B_{33}=\sin\theta_C$, in which case there are no color-octet components.

We next investigate a condition of the $\Delta I=\frac{3}{2}$ suppression in the $\Delta C=0$, $\Delta S=1$ part of the color-singlet nonleptonic interaction. A piece of the current product concerned with nonleptonic decays of ordinary hadrons is

$$\tfrac{1}{24}f_{84}\sum_{i,j=1}^{3}[\bar{u}_i\gamma_\mu(1-\gamma_5)d_i\bar{s}_j\gamma^\mu(1-\gamma_5)u_j+\bar{u}_i\gamma_\mu(1-\gamma_5)d_j\bar{s}_j\gamma^\mu(1-\gamma_5)u_i]$$
$$+\tfrac{1}{12}f_{20}\sum_{i,j=1}^{3}[\bar{u}_i\gamma_\mu(1-\gamma_5)d_i\bar{s}_j\gamma^\mu(1-\gamma_5)u_j-\bar{u}_i\gamma_\mu(1-\gamma_5)d_j\bar{s}_j\gamma^\mu(1-\gamma_5)u_i]\,, \tag{8}$$

where the coefficients are given by

$$f_{84(20)}=\mathrm{Tr}(A)\,\mathrm{Tr}(B^\dagger)+(-)\,\mathrm{Tr}(AB^\dagger)\,. \tag{9}$$

We confine our studies to the CP-conserving weak interactions. Then we can choose the coefficients

f_{84} and f_{20} real.[5] From the inequality

$$\begin{aligned}\mathrm{Re}\,[\mathrm{Tr}(AB^\dagger)] &= \mathrm{Re}\Big[\sum_{i,j}(1-\delta_{ij})A_{ij}B^*_{ij}\Big] + \mathrm{Re}\Big(\sum_i A_{ii}B^*_{ii}\Big)\\ &\geq -\tfrac{1}{2}\sum_{i,j}(1-\delta_{ij})(|A_{ij}|^2+|B_{ij}|^2) + \mathrm{Re}\Big(\sum_i A_{ii}B^*_{ii}\Big) = -\tfrac{3}{2}+\tfrac{1}{2}\sum_i |A_{ii}+B_{ii}|^2\\ &\geq -\tfrac{3}{2}+\tfrac{1}{6}\,|\mathrm{Tr}(A)+\mathrm{Tr}(B)|^2\,, \end{aligned} \tag{10}$$

we obtain

$$\frac{f_{84}-f_{20}}{f_{84}+f_{20}} \geq \frac{-1+\kappa^2(\cos\theta_C+\sin\theta_C)^2}{6\kappa^2\cos\theta_C\sin\theta_C}\,, \tag{11}$$

or

$$\kappa^2 \leq \frac{1+\omega}{1+4\sin 2\theta_C+\omega(1-2\sin 2\theta_C)}\,, \tag{12}$$

where

$$\omega \equiv f_{84}/f_{20}\,. \tag{13}$$

When we put[6] $\omega \simeq \frac{1}{20}$, which comes from the analysis of $K \to 2\pi$ decays, we get a rigid upper bound,

$$\kappa^2 \lesssim 0.388\,, \tag{14}$$

for $\sin\theta_C = 0.22$. If we put $\omega = 0$, we get $\kappa^2 \leq 0.368$ which agrees with the value obtained by a Monte Carlo search in the previous paper.[1] Thus the restriction for κ^2 is not loosened even if we extend the color 2-3 mixing to the most general mixing.

The small value of κ^2 predicts[1] the large ratio of the slopes of the linearly rising cross sections, $\sigma(\nu_\mu(\bar\nu_\mu)N \to \mu^-(\mu^+)X)$, well above both charm and color thresholds to that below both thresholds,

$$\frac{(\alpha_{\nu(\bar\nu)})_{\text{"above"}}}{(\alpha_{\nu(\bar\nu)})_{\text{"below"}}} \simeq \frac{1}{\kappa^2}\,, \tag{15}$$

and the small weak-boson mass

$$\begin{aligned} m_{W^\pm} &= \left(\frac{\pi\alpha}{\sqrt{2}\,G_F}\right)^{1/2}\frac{\kappa}{\sin\theta_W}\\ &\simeq \kappa \times \frac{38}{\sin\theta_W}\,\mathrm{GeV}\,. \end{aligned} \tag{16}$$

If we attempt to adopt the suppression mechanism[7] of the $\Delta I = \frac{3}{2}$ part, we are forced to accept that the slope must vary more than twice at energies high enough to produce the color-octet states and the weak-boson mass must be smaller than about 43 GeV ($\sin^2\theta_W \simeq 0.3$).

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[1]M. Katuya and Y. Koide, Phys. Rev. D 16, 165 (1977).

[2]G. S. LaRue, W. M. Fairbank, and A. F. Hebard, Phys. Rev. Lett. 38, 1011 (1977).

[3]A model of a color-octet weak current induced by quark mixing was first proposed by G. Feldman and P. T. Matthews, Phys. Lett. 63B, 68 (1976).

[4]J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); 10, 275 (1974).

[5]Even if we choose the coefficients f_{84} and f_{20} to be real, we can introduce *CP*-violating effects into our model through an effective $(\bar d s)(\bar d s)$ interaction.

[6]In the framework of the four-flavor model with $V-A$ interactions, it is further necessary that the $\Delta I = \frac{3}{2}$ part is suppressed by a factor of ~4 relative to the $\Delta I = \frac{1}{2}$ part, even if we take into account the correction in a short-distance expansion. [M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974).] The value $\omega = \frac{1}{4}$ leads to the restriction $\kappa^2 \leq 0.466$.

[7]The idea of the cancellation has been proposed by Kingsley and Fujii *et al.* on the basis of the three-triplet model: R. L. Kingsley, Phys. Lett. 40B, 387 (1972); K. Fujii, M. Katuya, S. Okubo, and S. Tamura, Prog. Theor. Phys. 49, 995 (1973).