Comments and Addenda

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Electromagnetic mass splitting of charmed mesons

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The nonrelativistic Schrödinger equation is used with a one-gluon–perturbed linear potential to calculate the electromagnetic mass splitting of charmed mesons in the quark model. It is estimated that the D^+-D^0 and D^{*+}-D^{*0} mass differences are about 7.4 MeV and 5.9 MeV respectively.

There have been many papers estimating the electromagnetic mass splitting (ems) of charmed mesons. Itoh *et al.*[1] and Lichtenberg[2,3] have considered the ems of hadrons in a quark model, assuming the mass splittings to arise from intrinsic quark mass differences and Coulomb and magnetic-moment interactions between quarks. Assuming the same causes for the ems of charmed mesons, Ono[4] has used the harmonic-oscillator charmed-quark model to calculate the splittings. Chan[5] has also treated the ems of charmed mesons in an SU(8) model, assuming the mass differences to arise from intrinsic quark mass differences and two-body spin-spin interactions together with Coulomb and magnetic interactions.

Considering only intrinsic quark-mass differences and Coulomb interactions, De Rújula, Georgi, and Glashow[6] obtained the ems of charmed mesons. To evaluate the Coulomb term they calculated $\langle 1/r\rangle$ from the pion mass splitting and assumed this would be roughly the same for charmed mesons. Lane and Weinberg[7] considered the same two mass-splitting terms but evaluated $\langle 1/r\rangle$ from the K^+-K^0 splitting, using the π^+-π^0 mass difference and Dashen's theorem to separate the quark mass difference from the Coulomb part. Both of these papers considered the D and K mesons in a nonrelativistic atomic model, but Lane and Weinberg thought that it was unlikely that this model was applicable to pions as was assumed by De Rújula *et al.* Celmaster[8] has argued that more electromagnetic terms should be considered in estimating the ems. He has treated the splittings in a one-gluon–perturbed harmonic potential, keeping terms in the Fermi-Breit potential up to first order in the fine-structure constant α and the strong-interaction constant α_s. Peaslee[9] has also used the Fermi-Breit potential in a phenomenological quark model to calculate the ems.

In this paper, I consider a linear potential with one-gluon–exchange corrections. The $1/r$ part of the one-gluon exchange is treated correctly and the remaining terms of the Fermi-Breit potential in perturbation theory. Following De Rújula *et al.*,[10] I assume that the Hamiltonian for a quark-antiquark system in the center-of-mass system is

$$H = L(r) + m_1 + m_2 + \frac{p^2}{2\mu} + (\alpha Q_1 Q_2 - \tfrac{4}{3}\alpha_s) S_{12} + \cdots ,$$

where

$$S_{12} = \frac{1}{r} + \frac{1}{2m_1 m_2}\left\{\frac{1}{r}p^2 + \frac{1}{r^3}[\vec{r}\cdot(\vec{r}\cdot\vec{p})\vec{p}]\right\} - \frac{\pi}{2}\delta^3(\vec{r})\left(\frac{1}{m_1{}^2} + \frac{1}{m_2{}^2} + \frac{16\vec{S}_1\cdot\vec{S}_2}{3m_1 m_2}\right)$$

and $L(r)$ is the universal interaction responsible for binding, which is here assumed to be linear, i.e., $L = \beta(r - r_0)$. I have omitted terms that are zero in S-wave states and terms that are of higher order than $(v/c)^2$ or are higher than first order in α or α_s. Following De Rújula *et al.*, I have assumed that the hadrons are governed by nonrelativistic dynamics and that the long-range binding depends only on the spatial separation of the quarks. The perturbation was treated in the following way: Let $H = H_0 + H_{\text{pert}}$, where

$$H_0 = \beta(r - r_0) - \frac{4}{3}\frac{\alpha_s}{r} + \frac{p^2}{2\mu} + m_1 + m_2 ,$$

and H_{pert} is everything else. Then the nonrelativistic Schrödinger equation was solved numerically to obtain the energy eigenvalue E and the wave function $\psi(r)$.

The masses of the quarks (u, d, s, c) were taken to be those of De Rújula *et al.*,[10] i.e., $m_u = 336$ MeV, $m_d = m_u + \epsilon$, $m_s = 540$ MeV, and $m_c = 1660$ MeV, where ϵ is a parameter to be determined. Values of α_s and β were used that were the same as those used by Barbieri *et al.*,[11] who did a similar perturbation treatment to calculate the meson-mass spectrum, but they did not consider the electromagnetic terms or the mass difference of the u and d quarks. The values for $\frac{4}{3}\alpha_s$ were linearly interpolated from the values, 0.27 for a $c\bar{c}$ bound state, 0.36 for an $s\bar{s}$, and 0.42 for a $u\bar{u}$. The value for β was 0.25 GeV^2. Using the calculated wave function $\psi(r)$, the terms in H_{pert} can then be evaluated in first-order perturbation theory. These matrix elements will depend on β, α_s, and the quark masses, but not on r_0.

The expression for the mass of a meson is

$$M = E + m_1 + m_2 + \alpha Q_1 Q_2 \left\langle \frac{1}{r} \right\rangle + (\alpha Q_1 Q_2 - \tfrac{4}{3}\alpha_s)\left[\frac{1}{2m_1 m_2} \left\langle \frac{1}{r} p^2 + \frac{1}{r^3}[\vec{r}\cdot(\vec{r}\cdot\vec{p})\vec{p}] \right\rangle - \frac{\pi}{2}|\psi(0)|^2 \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16\vec{S}_1\cdot\vec{S}_2}{3m_1 m_2} \right) \right],$$

where E is the energy eigenvalue of H_0 and $\psi(0)$ is the wave function at the origin. For the eigenvalue E, I used the value that, together with the other terms, would give the mass of the meson (in the particular doublet under consideration) containing the u quark. This amounts to using a different r_0 for each meson. Barbieri *et al.*[11] were unable to fit the meson-mass spectrum very well with one value of r_0 due, perhaps, to relativistic effects. With different values of r_0 for different mesons, the contribution to the ems from the ground-state energy term is not predicted from this model. Therefore I use the contribution from the ground-state energy term as a parameter and denote $\Delta = E_{dq} - E_{uq}$, where E_{dq} is the value of E for the d-q quark system. I also approximate Δ to be the same for the meson doublets considered here. This seems to be the simplest assumption needed to make some prediction on the charmed-meson ems, other than just neglecting Δ altogether as has been done by others.[1-7]

I also calculated $\psi(r)$ for the meson containing the u quark. Then, since the linear part of H_0 should dominate these values, the dependence of $\psi(0)$ and the matrix elements on the reduced mass μ was approximated to be the same as that for a strictly linear potential. When taking mass differences of the doublet, terms of order α and ϵ/m_u were kept.

It is then straightforward to obtain mass differences for the meson doublets in terms of the two parameters ϵ and Δ. These parameters can be evaluated from the experimental values of the K^0-K^+ and K^{*0}-K^{*+} mass splittings. Some authors dispute[5] the experimental value of $K^{*0} - K^{*+}$; however, I used the value 4.1 ± 0.6 MeV given by the Particle Data Group.[12] Then I obtain the value of ϵ,

$$\epsilon = 5.0 \pm 3 \text{ MeV}$$

and the value of Δ,

$$\Delta = 6.26 \text{ MeV} - 1.107\epsilon = 0.73 \pm 3.3 \text{ MeV}.$$

My values for the ems of charmed mesons are given in Table I along with the results of other calculations and the experimental value.[13] My splittings were not too sensitive to the values of α_s and β. For example, if I had taken $\frac{4}{3}\alpha_s = 0.4$ for $c\bar{d}$ and $\frac{4}{3}\alpha_s = 0.8$ for $u\bar{s}$ and $\beta = 0.2$ GeV^2, the results would differ by 5% to 10%.

There is quite a large spread in the possible values for ϵ due to the experimental errors for the $K^{*0} - K^{*+}$ ems, but fortunately the values for the predicted ems of the charmed mesons vary

TABLE I. Calculated and experimental values of charmed-meson electromagnetic mass splitting in MeV.

	$D^+ - D^0$	$D^{*+} - D^{*0}$
Experiment[13]	5.1 ± 0.8	2.6 ± 1.8
This calculation	7.4	5.9
Itoh *et al.*[1]	13.2	9.0
Lichtenberg[3]	>4.0	>0
Ono[4]	15 ± 5	15 ± 5
Chan[5]	5.2	3.7
De Rújula *et al.*[6]	15	15
Lane and Weinberg[7]	6.7	6.7
Celmaster[8]	1.37	−0.57
Peaslee[11]	5.4	4.7

only by about ± 1 MeV. For example, with $\epsilon = 8$ MeV, $\Delta = -2.6$ MeV we obtain

$$D^{+} - D^{0} = 6.4 \text{ MeV} \quad D^{*+} - D^{*0} = 4.9 \text{ MeV}$$

and with $\epsilon = 2$ MeV, $\Delta = 4.0$ MeV

$$D^{+} - D^{0} = 8.4 \text{ MeV} \quad D^{*+} - D^{*0} = 7.0 \text{ MeV}.$$

I have used a linear potential with one-gluon-exchange corrections rather than a one-gluon-perturbed harmonic potential as Celmaster[8] did, and I have included more terms in the mass expressions than other authors[1-7] have. It can be seen that Chan's results are closest to the experimental values. He introduced a phenomenological spin-dependent term to obtain agreement with the strong hadron mass splitting. He then used this same spin-dependent term to fit some of the observed ems in the baryons and was then able to predict the others along with the ems of the mesons. His method was quite different from mine since I did a dynamical calculation based on the specific model of De Rújula *et al.*[10] Yet my results agree with those of Chan and also with those of Peaslee, who used a phenomenological quark model with terms suggested by the Fermi-Breit potential, to within about 2 MeV. This agreement is very good considering the approximations made in my calculation. In my perturbation expansion, the perturbing mass terms were sometimes as large as the ground-state energy term. It is, however, plausible that part of the error from this improper perturbation expansion cancels out in the mass differences. Also, relativistic effects were not completely negligible, and I had to parametrize the contribution to the ems from the ground-state energy term. To be able to predict the ems of the charmed mesons, I also had to assume this contribution from the ground-state energy term to be the same for the kaons and the charmed mesons. If one looks at the value of

$$(D^{*+} - D^{*0}) - (D^{+} - D^{0}),$$

the value of Δ for the charmed mesons should cancel out, and it can be seen that my value for this difference agrees well with that of Chan. Whether or not this is a more reliable prediction is not clear since it is the difference of two terms. The similarity between my results and Peaslee's results may actually be closer than that shown in Table I. To obtain the values shown in Table I, Peaslee used a value for ϵ that was the average of what he found for the mesons and baryons. Using the value of ϵ he obtained from the mesons, his results are

$$D^{+} - D^{0} = 7.4 \text{ MeV} \quad \text{and} \quad D^{*+} - D^{*0} = 6.7 \text{ MeV},$$

which are in better agreement with mine.

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