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Gauge theory of weak and electromagnetic interactions with an SU(3) × U(1) symmetry

Paul Langacker, Gino Segrè, and Mehdi Golshani*
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19174
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A gauge-theory model of weak and electromagnetic interactions is proposed. The symmetry group is SU(3) × U(1) and the fermions are all placed in triplets (or antitriplets) and singlets. The theory is vectorlike and hence anomaly-free. Discrete symmetries are imposed to avoid unwanted mixings and to appropriately restrict the Higgs potential. The details of fermion and boson mass generation are given. The couplings and masses of the two massive neutral flavor-preserving gauge bosons are determined by fitting the neutral-current data; agreement is excellent. We also analyze the predictions for multimuon production in neutrino scattering as a function of lepton and hadron masses. Finally we examine the limits on such processes as $K_L \to \mu^+\mu^-$, $\mu \to e\gamma$, etc.

I. INTRODUCTION

Unified gauge theories of weak and electromagnetic interactions have enjoyed an enormous success over the past five years and have greatly advanced a possible understanding of the basic forces of nature. The so-called standard[1] model with four quarks and four leptons and gauge group $SU(2)_L \times U(1)$ has appeared to account for most observed phenomena in weak and electromagnetic (em) interactions. There is evidence for new leptons[2] and probably for new quarks[3]: These may be accommodated within the $SU(2)_L \times U(1)$ model by introducing additional multiplets. There are, however, some new phenomena which cannot be accommodated within the standard model. Foremost among these is the absence of parity violation in the atomic-physics experiments[4]: Other difficulties are some of the new phenomena[5-8] seen in very-high-energy neutrino events and possible anomalies in ν and $\bar{\nu}$ inclusive reactions.

Abandoning the $SU(2)_L \times U(1)$ model, one can turn to a vectorlike[9] $SU(2) \times U(1)$ model, but this is in contradiction with recent data from neutrino experiments.[10,11] A hybrid model[12] has some of the same features as $SU(2)_L \times U(1)$, but agrees with the atomic-physics results.[4] Recently much attention has been focused on so-called ambidextrous models[13] with an underlying $SU(2)_L \times SU(2)_R \times U(1)$ gauge group, which we shall not discuss. This paper will deal with the gauge group[14] $SU(3) \times U(1)$ in the context of an underlying vectorlike theory.

There are several reasons for choosing $SU(3) \times U(1)$: Among these are the fact that one can have a vectorlike and hence anomaly-free theory while at the same time having a parity-violating neutral current. The natural classification of fermions in triplets allows for intriguing groupings of light and heavy particles. Finally, we mention that it may be an eventual first step in a grand unification scheme in which the weak and the color gauge groups are combined into a larger gauge group.[15]

In this paper, we shall consider one version of the $SU(3) \times U(1)$ type of theories: It is an elaboration of one proposed by two of us[16] in a published letter. It is far from being unique,[17] but we believe it is useful to display all the details of one particular model.

The paper is organized as follows: In Sec. II we will present the model, that is the classification of the leptons and quarks into left- and right-handed multiplets (with a right-left asymmetry). We will also give the couplings and masses of the nine gauge bosons and specify the couplings of the Higgs mesons needed to generate masses and mixings. The discrete symmetries which the theory needs to insure universality, no $\Delta S = 1$ neutral currents, etc. are also specified.

In Sec. III we discuss some of the phenomenological implications of the model, beginning with charged-current interactions. We then turn to neutral-current phenomenology and fix the masses of our gauge bosons by fitting elastic and inelastic proton[10,11] and neutrino electron scattering.[18,19] We also show that the recent determination of the u and d quark vector and axial-vector coupling constants performed by Sehgal[20] using additional neutrino pion production data is in excellent agreement with the predictions of our model. We then go on to consider dimuon[21] and trimuon[5] data. For this analysis we make use of a charge $-\frac{1}{3}$ b quark with a mass in the 4–5-GeV region. The accompanying high-y anomaly[7,8] appears to be not present: We can get around this by having b_R or u_R mix with heavier quarks. One new feature we discuss is the large B^0-$\bar{B}^0$ oscillations predicted in this model (B^0 is a pseudoscalar bound state of a $\bar{d}$ and a b).

In Sec. IV we discuss in detail the Higgs potential for the model and show how the form of spontaneous symmetry breaking we employed in Sec. II arises naturally. Introduction of additional spin-zero fields with small vacuum expectation values and generation of light-particle (e.g., electron) masses is treated.

In Sec. V we discuss a number of phenomenological questions omitted in Sec. III, such as nonleptonic hyperon decay, the K_L-K_S mass difference and rare K decays, the possibility of $\mu \to e\gamma$, the muon $g-2$, more about B^0-$\bar{B}^0$ mixing, etc.

We then proceed to a few statements about larger unified groups and a conclusion.

II. THE MODEL

A. Yukawa couplings in the model and discrete symmetries

The gauge group is SU(3)×U(1) so that there are nine gauge bosons, of which only one, the photon, remains massless after symmetry breaking. The others will all, as we shall see later, have masses of ~50–100 GeV. We call them $W^{\pm}=(1/\sqrt{2})(W_1 \mp iW_2)$, $W'^{\pm}=(1/\sqrt{2})(W_4 \mp iW_5)$, $W^0=(1/\sqrt{2})(W_6 - iW_7)$, and $\bar{W}^0=(1/\sqrt{2})(W_6+iW_7)$ using SU(3) indices; the two neutral gauge bosons orthogonal to the photon are called Z_1 and Z_2. The charge is the sum of the SU(3) generator $\frac{1}{2}(\lambda_3+\lambda_8/\sqrt{3})$ plus the U(1) generator Y: The gauge-boson charges have already been indicated. The quarks in our model lie in SU(3) triplets with $Y=0$ and charges $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}$, and in SU(3) singlets with charge $\frac{2}{3}$. The leptons will be placed in SU(3) antitriplets with $Y=-\frac{1}{3}$ and charges $-1, 0, 0$ and in SU(3) singlets with charge 0. We can write down separately left- and right-handed triplets (antitriplets). The gauge bosons, of course, couple only left to left and right to right while spin-zero mesons will couple left to right. These models all have a potential left-right asymmetry built into them in that the two particles of charge $-\frac{1}{3}$ (charge 0) in a given multiplet need not assume the same position in the right- and left-handed multiplets: On the other hand, there is no asymmetry in the electron position and hence the electron current coupling to Z_1 and Z_2 is automatically vectorlike. Anticipating for a moment the discussion of Sec. III, this was one important factor in our decision to place leptons in the $\bar{3}$ representation: To ensure vectorlike electron coupling and hence an essentially null effect in the search for parity-violating atomic effects in bismuth.[4]

The spin-zero mesons we need to generate masses lie in octets and triplets: For the latter we shall take both $Y=-\frac{2}{3}$ and $Y=\frac{1}{3}$ representations. The octets may be taken as complex or, equivalently, we may have both scalar and pseudoscalar octets. We will also require the existence of discrete symmetries in order to forbid unwanted couplings. All our masses will be generated via couplings to spin-zero mesons with vacuum expectation values: This is true despite the fact that an SU(3) gauge theory allows for a bare mass term $\sim m_0 \bar{q}_L q_R$ because the gauge group is vectorlike rather than chiral. We forbid such terms, however, by imposing a discrete symmetry such that, e.g., $q_L \to -iq_L$, $q_R \to q_R$, where $q_{L,R}$ are defined by $q_{L,R}=\frac{1}{2}(1\pm\gamma_5)q$.

Let us now be explicit about fermions. We must introduce at least three lepton antitriplets to accommodate, respectively, the e^-, the μ^-, and the heavy lepton first observed at SPEAR,[2] τ^-. We shall label these three $\bar{3}$'s as l_e, l_μ, and l_τ; the two quark triplets will be called q_u and q_c. The subsequent suffix L or R distinguishes between left and right. In Fig. 1 we display the fermion states: Keep in mind that these are not the physical states, but rather the unmixed states. Note the asymmetry between the left and right model triplets. The masses of these states are generated by the Yukawa coupling [here $\varphi=\frac{1}{2}(\vec{\lambda}\cdot\vec{\varphi})$, $\chi=\frac{1}{2}(\vec{\lambda}\cdot\vec{\chi})$]

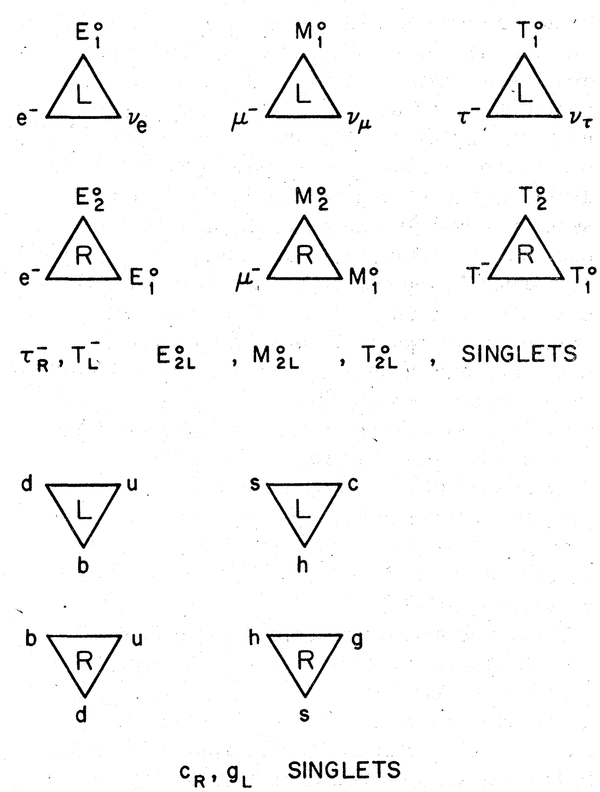

FIG. 1. The fermion multiplets. The horizontal and vertical directions refer to the T^3 and T^8 generators of SU(3), respectively.

$$
\begin{aligned}
\mathcal{L}^{(\text{no mixing})}_{\text{Yukawa}} = {} & f_u \bar{q}_u(\varphi - i\gamma_5\chi)q_u + f_c\bar{q}_c(\varphi - i\gamma_5\chi)q_c + h_c\bar{q}_{cL}c_R\Omega + h_g\Omega^{+}\bar{g}_L q_{cR} \\
& + f_e\bar{l}_e(\varphi^T + i\gamma_5\chi^T)l_e + f_\mu \bar{l}_\mu(\varphi^T + i\gamma_5\chi^T)l_\mu + k_e\Phi^T\bar{E}^0_{2L}l_{eR} + k_\mu\Phi^T\bar{M}^0_{2L}l_{\mu R} \\
& + f_\tau\bar{l}_\tau(\varphi^T + i\gamma_5\chi^T)l_\tau + k_\tau\Phi^T\bar{T}^0_{2L}l_{\tau R} + h_\tau\bar{l}_{\tau L}\tau^-_R\Omega^{T\dagger} + h_T\Omega^T\bar{T}^-_L l_{\tau R} + \text{H.c.}, \qquad (2.1)
\end{aligned}
$$

where φ and χ are scalar and pseudoscalar octets, Φ and Ω are $y=\frac{1}{3}$ and $y=-\frac{2}{3}$ triplets. As we shall see later, we can impose discrete symmetries such that their vacuum expectation values take the form

$$
\begin{aligned}
&\langle\varphi\rangle_0 = \left\langle\frac{\vec{\lambda}\cdot\vec{\varphi}}{2}\right\rangle_0 = \frac{1}{\sqrt{2}}\, v_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \frac{v_1\lambda_6}{\sqrt{2}}\,, \quad \langle\chi\rangle_0 = \left\langle\frac{\vec{\lambda}\cdot\vec{\chi}}{2}\right\rangle_0 = \frac{1}{\sqrt{2}}\, v_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} = \frac{v_2\lambda_7}{\sqrt{2}}\,, \\
&\langle\Phi\rangle_0 = v_\Phi \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \langle\Omega\rangle_0 = v_\Omega \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
\qquad (2.2)
$$

and hence, with $v_1 = -v_2 = v$ the mass matrix is of the form

$$
\begin{aligned}
&\sqrt{2} f_u v\bar{b}_L b_R + \sqrt{2} f_c v\bar{h}_L h_R + h_c v_\Omega \bar{c}_L c_R + h_g v_\Omega \bar{g}_L g_R + \sqrt{2} f_e v\bar{E}^0_{1L}E^0_{1R} + \sqrt{2} f_\mu v\bar{M}^0_{1L}M^0_{1R} + \sqrt{2} f_\tau v\bar{T}^0_{1L}T^0_{1R} \\
&\quad + k_e v_\Phi \bar{E}^0_{2L}E^0_{2R} + k_\mu v_\Phi \bar{M}^0_{2L}M^0_{2R} + k_\tau v_\Phi \bar{T}^0_{2L}T^0_{2R} + h_\tau v_\Omega \bar{\tau}^-_L\tau^-_R + h_T v_\Omega \bar{T}^-_L T^-_R + \text{H.c.}
\end{aligned}
$$

so b, h, c, g, E^0_1, M^0_1, T^0_1, E^0_2, M^0_2, T^0_2, τ^-, and T^- all have arbitrary masses which may be adjusted to agree with experimental data, while u, d, s, e^-, and μ^- all have zero mass. We shall subsequently work out schemes for them to acquire small masses. The three neutrinos ν_e, ν_μ, and ν_τ will, however, never acquire a mass as they are only left-handed. Let us recapitulate for a moment: φ and χ couplings lead to masses for the fermion triplet fields, Φ couplings lead to masses for E^0_2, M^0_2, and T^0_2 and Ω couplings to masses for c, g, τ^-, and T^-: Separate spin-zero fields, as we shall see, lead to mixing.

We require that the Lagrangian be invariant under the discrete symmetry V under which

$$
\begin{aligned}
& q_L \to -iq_L, \quad g_L \to ig_L, \\
& q_R \to q_R, \quad c_R \to c_R, \\
& l_L \to il_L, \quad E^0_{2L} \to iE^0_{2L}, \quad -T_L \to iT_L, \\
& l_R \to l_R, \quad \tau_R \to \tau_R, \\
& \varphi \to \chi, \quad \chi \to -\varphi, \\
& \Phi \to i\Phi, \quad \Omega \to -i\Omega,
\end{aligned}
\qquad (2.4)
$$

where we have not differentiated for the time between particles belonging to different multiplets; i.e., we assume q_{uL} and q_{cL} have the same transformation properties under V. We shall show later that when the most general fourth-order Higgs potential involving the above spin-zero fields and satisfying the discrete symmetry V is written down, there is a range of parameters for which the fields have vacuum expectation values as indicated in Eq. (2.2). We also emphasize that the discrete symmetry is already implicit in Eq. (1) by our choice of equal Yukawa coupling of the fields φ and χ to q; this is forced upon us by V symmetry. The change in sign between couplings and transformation properties of q_L and l_L follows from q_L lying in the 3 representation, whereas l_L is in the $\bar{3}$. Note also that since φ, χ couple only L to R fields the γ_5 may be dropped as $\gamma_5\psi_R = -\psi_R$. Finally, we mention that the discrete symmetry as expressed in (4) would allow for off-diagonal couplings, such as, e.g., $\bar{q}_u \times(\varphi - i\gamma_5\chi)q_c$, $\Phi^\dagger\bar{E}^0_{2L}l_{\mu R}$, etc. These may all be forbidden by associating under V different phases θ_u, θ_c, θ_e, θ_l, θ_τ with each set of fields, e.g., $q_{uL} \to -ie^{i\theta_u}q_{uL}$, $q_{uR} \to e^{i\theta_u}q_{uR}$, etc., giving the singlets the appropriate phases as well.

B. Secondary features of the model

So far we have presented the gross structure of the model. We must now introduce additional scalar fields, of lesser importance in that they will have smaller products of Yukawa couplings times field vacuum expectation values. We will choose our Higgs potential parameters such that these smaller products are due to the fields and not to the couplings. These additional fields will therefore not play an important role in the gauge-boson mass matrix. They will be of importance, however, in giving initially massless fermions a mass. Let us give two examples: Assuming CP invariance for the moment (see Sec. IV) $\langle\lambda\chi\rangle_0$ must lie along the seven direction in SU(3) while $\langle\lambda\varphi\rangle_0$ can lie along the three, six, or eight direction. In terms of U-spin generators $U_1=\lambda_6$, $U_2=\lambda_7$, $U_3=\frac{1}{2}(\lambda_3-\sqrt{3}\,\lambda_8)$, and $U_0=\frac{3}{2}(\lambda_3+\lambda_8/\sqrt{3})$

$$\begin{aligned}\langle\chi\rangle_0&=\frac{1}{\sqrt{2}}v_2U_2\,,\\ \langle\varphi\rangle_0&=\frac{1}{\sqrt{2}}v_1(\alpha U_1+\beta U_3+\gamma U_0)\end{aligned} \tag{2.5}$$

normalized to $\alpha^2+\beta^2+3\gamma^2=1$. Previously we took $\beta=\gamma=0$, $\alpha=1$, and $v_1=-v_2$. We may, however, choose the parameters of our potential such that the minimum is at $v_2=0$ instead of $v_1^2=v_2^2$ and independently $\alpha=0$ rather than $\beta=\gamma=0$. We generate masses for the e, μ, u, d, and s if we allow for the existence of a pair of fields σ,δ analogous to φ,χ but with the above properties, i.e., $\alpha=0$ rather than $\beta=\gamma=0$. These couplings of σ and δ will also generate small mixings, e.g., between the d and b quarks and between the s and h quarks, but these do not cause any phenomenological difficulties. In Sec. IV we will discuss these schemes at some length, emphasizing the possible types of development.

Before completing this examination of the model, we must comment on how to generate the Cabibbo angle; we wish to do this by having mixing between u_L and c_L. The simplest way is to introduce a second field Ω' which couples q_{uL} to c_R: As an example let the transformation properties under V be

$$\begin{aligned}&q_{uL}\to iq_{uL},\quad q_{cL}\to -iq_{cL},\\ &q_{uR}\to -q_{uR},\quad q_{cR}\to q_{cR},\\ &c_R\to c_R,\\ &\Omega\to -i\Omega,\quad \Omega'\to i\Omega'.\end{aligned} \tag{2.6}$$

The couplings of c_R are then

$$h_c\bar{q}_{cL}c_R\Omega+h_c'\bar{q}_{uL}c_R\Omega' \tag{2.7}$$

and the u-c mass matrix is

$$\begin{aligned}&h_cv_\Omega\bar{c}_Lc_R+h_c'v_\Omega'\bar{u}_Lc_R\\ &\quad=h_cv_\Omega(\bar{c}_Lc_R+(h_c'v_\Omega'/h_cv_\Omega)\bar{u}_Lc_R)\end{aligned} \tag{2.8}$$

so the left-handed Cabibbo angle is defined by

$$\tan\theta_L=h_c'v_\Omega'/h_cv_\Omega\,. \tag{2.9}$$

One could also introduce a right-handed angle θ_R by allowing Ω' to couple g_L to q_{uR} so that u_R and g_R would mix. There are no strangeness-or charm-changing neutral currents or other undesired effects introduced by this mixing. We have, of course, not calculated the Cabibbo angle, but rather introduced a new field Ω' with arbitrary adjustable coupling, allowing us to parametrize the mixing.

C. The gauge-boson mass matrix

In this section we will specify the form of the couplings of gauge bosons to fermions and the form of the gauge-boson mass matrix. We shall neglect the effect of the Higgs mesons σ,δ,Ω' described at the end of the previous section. Their only effect is to shift the masses slightly. They do not lead to any new mixing between the gauge bosons. We write the total Lagrangian for spin-one-half fields $\psi(x)$, Hermitian scalar fields $\phi_i(x)$, and gauge fields $W_\mu^\alpha(x)$, $B_\mu(x)$ as

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu}-\tfrac{1}{4}F_{i\mu\nu}F_i^{\mu\nu}\\ &-\tfrac{1}{2}(D_\mu\phi)_i(D_\mu\phi)^i+i\bar{\psi}\gamma_\mu D^\mu\psi\\ &+\bar{\psi}\Gamma_i\psi\phi^i-P(\phi)\end{aligned} \tag{2.10}$$

with

$$\begin{aligned}G_{\mu\nu}&=\partial_\mu B_\nu-\partial_\nu B_\mu\,,\\ F_{i\mu\nu}&=\partial_\mu W_{i\nu}-\partial_\nu W_{i\mu}+gf_{ijk}W_{j\mu}W_{k\nu}\,,\\ iD_\mu&=i\partial_\mu+gT_iW_{i\mu}+g'YB_\mu\,,\end{aligned} \tag{2.11}$$

where D_μ is the covariant derivative. T_i and Y take on the appropriate values determined by the representation of the multiplet to which D_μ is being applied. In the above $\bar{\psi}\Gamma_i\psi\phi^i$ represents the Yukawa potential written in Eq. (1) and we have omitted mass terms because of the discrete symmetry. A few examples, to specify our normalization conventions are

$$\begin{aligned}i(D_\mu\varphi)_i&=(i\partial_\mu\delta_{ik}+igf_{ijk}W_{j\mu})\varphi_k\,,\\ i(D_\mu\Phi)&=(i\partial_\mu+g\tfrac{1}{2}(\vec{\lambda}\cdot\vec{W}_\mu)+\tfrac{1}{3}g'B_\mu)\Phi\,.\end{aligned} \tag{2.12}$$

The vacuum expectation values of the fields φ,χ,Φ,Ω were already specified in Eq. (2). We may now substitute this into (10) to determine the gauge-boson mass matrix remembering $W^\pm=(1/\sqrt{2})(W_1\mp iW_2)$, $W'^\pm=(1/\sqrt{2})(W_4\mp iW_5)$ and, for simplicity, dropping Lorentz indices

$$\begin{aligned}\mathcal{L}_{\text{mass}}(W,B)=&\,g^2(\tfrac{1}{2}v_1^2+\tfrac{1}{2}v_2^2+\tfrac{1}{2}v_\Phi^2)W^+W^-+g^2(\tfrac{1}{2}v_1^2+\tfrac{1}{2}v_2^2+\tfrac{1}{2}v_\Phi^2+\tfrac{1}{2}v_\Omega^2)W'^+W'^-\\ &+\tfrac{1}{2}g^2v_\Phi^2\bar{W}^0W^0+g^2v_2^2W_6W_6+g^2v_1^2W_7W_7\\ &+g^2\tfrac{1}{2}(v_1^2+v_2^2)\tfrac{1}{2}(W_3-\sqrt{3}\,W_8)^2+\tfrac{1}{4}v_\Phi^2[-\tfrac{2}{3}g'B+(2g/\sqrt{3})W_8]^2+\tfrac{1}{4}v_\Omega^2[-\tfrac{4}{3}g'B+g(W_3+W_8/\sqrt{3})]^2\,,\end{aligned} \tag{2.13}$$

where, when we write v^2 we always mean absolute value squared. Since $v_1{}^2 = v_2{}^2 = v^2$ the expression simplifies and we may write, remembering also that $W^{0,\bar{0}} = (1/\sqrt{2})(W_6 \mp iW_7)$ and labeling by M_+, M'_+, M_0, M_1, and M_2 the masses of W, W', W^0, Z_1, and Z_2

$$\begin{aligned} M_+{}^2 &= g^2(v^2 + \tfrac{1}{2}v_\Omega{}^2)\,, \\ M'^2_+ &= g^2(v^2 + \tfrac{1}{2}v_\Omega{}^2 + \tfrac{1}{2}v_\Phi{}^2)\,, \\ M_0{}^2 &= g^2(2v^2 + \tfrac{1}{2}v_\Phi{}^2)\,. \end{aligned} \tag{2.14}$$

To find eigenstates Z_1, Z_2, and γ of the mass matrix, we must diagonalize the submatrix involving W_3, W_8, and B. This has the form,

$$\begin{aligned} \mathcal{L}_{\text{mass}}(W_3, W_8, B) &= \tfrac{1}{4}v_\Omega{}^2[-\tfrac{4}{3}g'B + g(W_3 + W_8/\sqrt{3})]^2 \\ &\quad + \frac{v_\Phi{}^2}{4}[-\tfrac{2}{3}g'B + (2g/\sqrt{3})W_8]^2 \\ &\quad + \tfrac{1}{2}g^2v^2(W_3 - \sqrt{3}\,W_8)^2\,. \end{aligned} \tag{2.15}$$

To diagonalize this matrix we proceed in two steps. The first is to introduce fields A, N_1, N_2 and an angle θ such that

$$\begin{aligned} A &= \cos\theta B + \sin\theta\,\tfrac{1}{2}(\sqrt{3}\,W_3 + W_8)\,, \\ N_1 &= -\sin\theta B + \cos\theta\,\tfrac{1}{2}(\sqrt{3}\,W_3 + W_8)\,, \\ N_2 &= \tfrac{1}{2}(-W_3 + \sqrt{3}\,W_8)\,, \end{aligned} \tag{2.16}$$

where $0 \le \theta \le \pi/2$ and

$$\cos\theta = \frac{g}{(g^2 + \frac{4}{3}g'^2)^{1/2}}\,. \tag{2.17}$$

The mass matrix now involves only N_1 and N_2

$$\begin{aligned} \mathcal{L}_{\text{mass}}(N_1, N_2) &= \frac{g^2 v_\Omega{}^2 N_1{}^2}{3\cos^2\theta} + 2g^2v^2N_2{}^2 \\ &\quad + \frac{g^2 v_\Phi{}^2}{12\cos^2\theta}(N_1 + \sqrt{3}\cos\theta N_2)^2 \end{aligned} \tag{2.18}$$

so the field A, which remains massless, is the photon. $\mathcal{L}_{\text{mass}}(N_1, N_2)$ is then diagonalized by an orthogonal transformation to obtain the mass-matrix eigenstates Z_1 and Z_2

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} \tag{2.19}$$

with masses $M_1{}^2$ and $M_2{}^2$

$$\begin{aligned} M_{1,2}{}^2 &= a + b \mp [(b-a)^2 + c^2]^{1/2}\,, \\ a &= \frac{g^2}{3\cos^2\theta}(v_\Omega{}^2 + \tfrac{1}{4}v_\Phi{}^2)\,, \\ b &= g^2(2v^2 + \tfrac{1}{4}v_\Phi{}^2)\,, \\ c &= g^2 v_\Phi{}^2\sqrt{3}/6\cos\theta\,. \end{aligned} \tag{2.20}$$

Also $0 < \beta < \pi/2$ where β is given by

$$\cos\beta = \frac{1}{\sqrt{2}}\left[\left(1 + \frac{b-a}{[(b-a)^2 + c^2]^{1/2}}\right)\right]^{1/2}. \tag{2.21}$$

If the octets σ, δ are present with small vacuum expectation values relative to Φ, Ω, φ, no substantial change in the gauge-boson mass matrix occurs. It would, however, be hard to tolerate any appreciable contribution to the mass matrix of a single octet field which had vacuum expectation value along the U_1 *and* the U_3 or U_0 directions. This would introduce W^+-W'^+ mixing and other highly undesirable effects. Fortunately it does not occur in our model.

D. The gauge-boson couplings

The gauge-boson coupling to fermions is

$$\begin{aligned} \mathcal{L}_{\text{coupling}} &= \tfrac{1}{2}g\bar{q}\gamma^\mu\lambda_i W_i{}^\mu q \\ &\quad - \tfrac{1}{2}g\bar{l}\gamma^\mu\lambda_i^T W_i^\mu l - \tfrac{1}{3}g'\bar{l}\gamma^\mu B_\mu l \\ &\quad + \sum_a g' Y_a \bar{F}_a \gamma^\mu B_\mu F_a\,, \end{aligned} \tag{2.22}$$

where we have written q and l, but intend this to mean separate entries for q_u, q_c, l_e, l_μ, and l_τ. The F_a are the fermion singlet fields which couple to B and have weak hypercharge Y_a. The gauge fields of course couple only L to L and R to R. Finally we point out that if we denote by λ_i the SU(3) coupling matrix for the $\underline{3}$ representation, the $\underline{3}^*$ couples by $-\lambda_i^T$ as indicated. From the above Lagrangian we obtain the usual effective current-current Hamiltonian with

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_+{}^2} \tag{2.23}$$

and two other terms corresponding to $W'^\pm$ and $W^0\overline{W}^0$, exchange with, respectively,

$$\frac{G'_F}{\sqrt{2}} = \frac{g^2}{8M'^2_+}\,, \quad \frac{G_F^0}{\sqrt{2}} = \frac{g^2}{8M_0{}^2}\,, \tag{2.24}$$

M_+, M'_+, and M_0 being given in (2.14). As a representative sample, the left-handed quark couplings to W^+, W'^+, and W^0 are

$$g/\sqrt{2}\,(\bar{u}_L\gamma^\mu W_\mu^+ d_L + \bar{u}_L\gamma^\mu W'^+_\mu b_L + \bar{d}_L\gamma^\mu W_\mu^0 b_L)\,. \tag{2.25}$$

To obtain the couplings of the fermions to $Z_{1,2}$ and A we must invert the matrix (2.16). Upon doing this we find

$$eA^\mu J_\mu^{\text{em}} + gZ_1^\mu J_{1\mu} + gZ_2^\mu J_{2\mu} \tag{2.26}$$

with J_μ^{em} being the usual electromagnetic current (u, d, b are taken to have charges $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}$),

$$\begin{aligned} e &= \tfrac{1}{2}\sqrt{3}\,g\sin\theta \\ &= \frac{gg'}{(g^2 + \frac{4}{3}g'^2)^{1/2}}\,, \end{aligned} \tag{2.27}$$

and using the U-spin notation that U_3 is the matrix $\frac{1}{2}(\lambda_3 - \sqrt{3}\,\lambda_8)$:

$$J_{1\mu} = \tfrac{1}{2}\sqrt{3}\,\cos\theta\,\cos\beta\,J_\mu^{\rm em} + \tfrac{1}{2}\sin\beta(\bar{q}\gamma_\mu U_3 q - \bar{l}\gamma_\mu U_3 l) - \tfrac{1}{2}\sqrt{3}\;\frac{\cos\beta}{\cos\theta}\left(-\tfrac{1}{3}\bar{l}\gamma_\mu l + \sum_a Y_a \bar{F}_a\gamma_\mu F_a\right) \tag{2.28}$$

and $J_{2\mu}$ is obtained from $J_{1\mu}$ by letting $\cos\beta \to \sin\beta$ and $-\sin\beta \to \cos\beta$.

We close this section by noting that in the limit $v_\Omega = 0$, $v_\Phi \to \infty$, the present model reduces to an SU(2)×U(1) model, with gauge fields $W^\pm$, Z_1, and A (the other gauge bosons have infinite mass in this limit). The SU(2) doublets in this limit are simply the left- and right-handed $T_3 = \pm\frac{1}{2}$ components of the SU(3) triplets and antitriplets. The other Fermion fields are SU(2) singlets. The spontaneous-symmetry-breaking term $\langle\varphi\rangle_0 + i\langle\chi\rangle_0$ plays the role of a complex doublet.

III. PHENOMENOLOGY OF THE MODEL

In this section we will discuss the phenomenology of the model. In Sec. III A we record and describe the flavor-changing interactions. Section III B is concerned with the neutral (i.e., flavor conserving) interactions. The effective Hamiltonian for neutral-current interactions will be seen to depend on three unknown parameters ($\cos^2\theta$ and two ratios of vacuum expectation values of Higgs fields). We will describe our fit to the existing neutral-current data and show that all three parameters are uniquely determined within a narrow range. Using these results from the neutral-current data, the masses of all of the intermediate vector bosons can be calculated. The results are given in Sec. III C. Section III D is devoted to a discussion of dimuon, trimuon, and tetramuon events associated with heavy leptons. The relevant production cross sections and branching ratios are estimated using the results in Sec. III C. Further features of the heavy-lepton phenomenology are discussed in Sec. III E.

A. Flavor-changing interactions

From the Lagrangian in (2.22) one can easily derive the effective four-fermion Hamiltonian valid for momentum transfers small compared to the gauge-boson masses. It is

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\,J_{+\mu}J_+^{\mu\dagger} + \frac{G_F'}{\sqrt{2}}\,J'_{+\mu}J_+^{\prime\mu\dagger} + \frac{G_F^0}{\sqrt{2}}\,J_{0\mu}J_0^{\mu\dagger}\,, \tag{3.1}$$

where G_F, G_F', and G_F^0 are given in (2.24). In (3.1), the current J_+, which is responsible for the ordinary charged-current weak interactions, is

$$\begin{aligned}J_{+\mu} &= 2\bar{q}_{uL}\tfrac{1}{2}(\lambda_1 + i\lambda_2)\gamma_\mu q_{uL} + 2\bar{q}_{uR}\tfrac{1}{2}(\lambda_1+i\lambda_2)\gamma_\mu q_{uR} - 2\bar{l}_{eL}\tfrac{1}{2}(\lambda_1^T + i\lambda_2^T)\gamma_\mu l_{eL} - 2\bar{l}_{eR}\tfrac{1}{2}(\lambda_1^T+i\lambda_2^T)\gamma_\mu l_{eR} + q_c, l_\mu, l_\tau \text{ terms}\\ &= \bar{u}(\theta_L)\gamma_\mu(1+\gamma_5)d + \bar{u}(\theta_R)\gamma_\mu(1-\gamma_5)b + \bar{c}(\theta_L)\gamma_\mu(1+\gamma_5)s + \bar{g}(\theta_R)\gamma_\mu(1-\gamma_5)h - \bar{\nu}_e\gamma_\mu(1+\gamma_5)e - \bar{E}_1^0\gamma_\mu(1-\gamma_5)e\\ &\quad - \bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu - \bar{M}_1^0\gamma_\mu(1-\gamma_5)\mu - \bar{\nu}_\tau\gamma_\mu(1+\gamma_5)\tau - \bar{T}_1^0\gamma_\mu(1-\gamma_5)T\,.\end{aligned} \tag{3.2}$$

In (3.2), θ_L and θ_R are the left- and right-handed Cabibbo angles, and the transitions involving light particles (u, d, s, ν, e, μ) are purely $V-A$. The conjugate of the $\bar{u}_R\gamma_\mu b_R$ term is responsible for the high-y anomaly.[7,8]

The currents J'_+ and J_0 are similar to (3.2), except $\lambda_1 + i\lambda_2$ is replaced by $\lambda_4 + i\lambda_5$ and $\lambda_6 + i\lambda_7$, respectively. Hence

$$\begin{aligned}J'_{+\mu} &= \bar{u}(\theta_L)\gamma_\mu(1+\gamma_5)b + \bar{u}(\theta_R)\gamma_\mu(1-\gamma_5)d\\ &\quad - \bar{M}_1^0\gamma_\mu(1+\gamma_5)\mu - \bar{M}_2^0\gamma_\mu(1-\gamma_5)\mu\\ &\quad + q_c, l_e, l_\tau \text{ terms}\,.\end{aligned} \tag{3.3}$$

The last three terms displayed in (3.3) are important in the decays of the M_1^0 and M_2^0 leptons.

Similarly, the electrically neutral but flavor-changing current $J_{0\mu}$ is

$$\begin{aligned}J_{0\mu} &= \bar{d}\gamma_\mu(1+\gamma_5)b + \bar{b}\gamma_\mu(1-\gamma_5)d\\ &\quad - \bar{M}_1^0\gamma_\mu(1+\gamma_5)\nu_\mu - \bar{M}_2^0\gamma_\mu(1-\gamma_5)M_1^0\\ &\quad + q_c, l_e, l_\tau \text{ terms}\,.\end{aligned} \tag{3.4}$$

The first and third terms are responsible for the reaction $\nu_\mu d_L \to M_1^0 b_L$, which can lead to trimuons,[5] while the first two can lead to transitions between the neutral mesons that are bound states of $\bar{b}d$ and $\bar{d}b$.

B. Neutral currents

The neutral-current phenomena in this model will be seen to depend on three unknown parameters: the analog of the Weinberg angle $\cos^2\theta$, and two positive numbers η and η' defined by

$$\eta = \left(\frac{v}{v_\Phi}\right)^2, \qquad \eta' = \left(\frac{v_\Omega}{v_\Phi}\right)^2. \tag{3.5}$$

Fortunately, all three parameters are reasonably well determined by the existing neutral-current data,[10,11,18,19] and can therefore be used to predict the masses of the various gauge bosons.

Before proceeding, we reemphasize that the electron component of the neutral current is purely vector, so there is no enhanced parity violation in bismuth[4] (there is parity violation in light atoms, involving the hadronic axial-vector current).[22] This also implies that the cross sections for $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ are equal. In the hadron sector the parity violation enters entirely through the charge $-\frac{1}{3}$ quarks. For the Higgs scheme described in Sec. II the neutral currents are entirely diagonal. The additional Higgs multiplets that will be introduced in Sec. IV to give mass to the light Fermions (u, d, s, e, μ) will introduce small off-diagonal terms such as $\bar{b}d$, $\bar{h}s$, $\bar{M}_1\nu_\mu$, $\bar{M}_2\nu_\mu$, and possibly $\bar{c}u$ and $\bar{g}u$, but these present no phenomenological difficulty.

The effective four-Fermion Hamiltonian for low momentum transfer neutral-current interactions is

$$H_{\text{eff}} = \frac{g^2}{2}\left(\frac{J_{1\mu}J_1^\mu}{M_1^2} + \frac{J_{2\mu}J_2^\mu}{M_2^2}\right), \tag{3.6}$$

where the currents $J_{i\mu}$ and Z-boson masses M_i are given in (2.31) and (2.20). This expression can be simplified by a straightforward but tedious calculation.

The term relevant to parity violation in light atoms such as hydrogen is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\lambda\bar{e}\gamma_\mu e\bar{d}\gamma^\mu\gamma_5 d, \tag{3.7}$$

where

$$\lambda = \tfrac{1}{2}(3\cos^2\theta - 1)\frac{\eta' + 2\eta}{\eta' + 2\eta + 8\eta\eta'} \tag{3.8}$$

and η' and η are defined in (3.5).

The part relevant to neutrino scattering from electrons and ordinary hadrons is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\sum_i \bar{\psi}^i\gamma_\mu(g_V^i + g_A^i\gamma_5)\psi^i, \tag{3.9}$$

where the sum is over the various Fermion fields. The expressions for g_A and g_V for the u and d quarks and the electron are given in Table I. They depend on $\cos^2\theta$ and two positive parameters r and r' defined as

$$\begin{aligned} r &= \frac{\eta' + 2\eta + 4\eta\eta' + 8\eta^2}{\eta' + 2\eta + 8\eta\eta'}, \\ r' &= \frac{\eta' + 2\eta + 4\eta\eta' + 2\eta'^2}{\eta' + 2\eta + 8\eta\eta'}. \end{aligned} \tag{3.10}$$

The hadronic terms depend only on r' and the combination $r'' = r\cos^2\theta$, while the lepton terms depend on r and $\cos^2\theta$.

The fit to the data is performed using the parameters r, r', and $\cos^2\theta$. From the resulting values for r and r' the parameters η and η' can be found from the inverse equations

$$\begin{aligned} \sigma &= -\frac{r' - 1}{r - 1}, \\ \eta &= \frac{(1+\sigma)(r-1)}{4(1-\sigma) - 8\sigma(r-1)}, \\ \eta' &= 2\sigma\eta. \end{aligned} \tag{3.11}$$

However, some care must be exercised in that not all values of r and r' can be obtained for positive values of η and η'. The allowed region, obtained as η and η' vary from zero to infinity, is shown in Fig. 2.

TABLE I. The parameters g_V^i and g_A^i which appear in the effective neutral-current Hamiltonian defined in Eq. (3.9).

i (fermion)	g_V^i	g_A^i
u	$r'' = r\cos^2\theta$	0
d	$-\frac{1}{2}r''$	$-\frac{1}{2}r'$
e	$\frac{1}{2}r(1 - 3\cos^2\theta)$	0

1. *Inclusive scattering*

Consider the quantitities

$$\begin{aligned} R^{\nu N} &= \frac{\sigma(\nu N \to \nu + X)}{\sigma(\nu N \to \mu^- + X)}, \\ R^{\bar{\nu} N} &= \frac{\sigma(\bar{\nu} N \to \bar{\nu} + X)}{\sigma(\bar{\nu} N \to \mu^+ + X)}, \\ R &= \frac{\sigma(\bar{\nu} N \to \bar{\nu} + X)}{\sigma(\nu N \to \nu + X)}. \end{aligned} \tag{3.12}$$

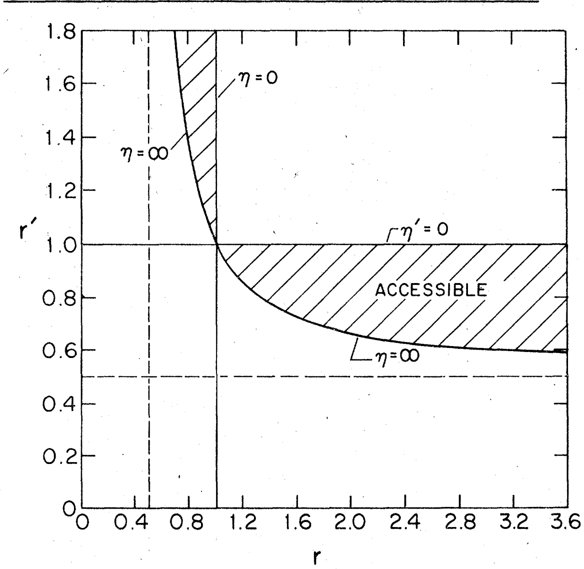

FIG. 2. The region of the r-r' plane corresponding to positive values of η and η' [see Eq. (3.10)].

TABLE II. Inclusive scattering data (Ref. 11). The data are from Gargamelle (GGM), Harvard-Pennsylvania-Wisconsin-Fermilab (HPWF), Caltech-Fermilab (CITF), and Brookhaven National Laboratory (BNL) experiments. The predictions of the present model are given for $r'=0.95$, $r''=0.50$.

	GGM	HPWF	CITF	BNL	Present model
$R^{\nu N}$	0.25 ± 0.04	0.29 ± 0.04	0.24 ± 0.02	0.25 ± 0.05	0.22
$R^{\bar\nu N}$	0.39 ± 0.6	0.39 ± 0.10	0.34 ± 0.09	···	0.42
R	0.59 ± 0.14	$\leqslant 0.61\pm0.25$	···	···	0.64

The experimental[11] values for these quantities are summarized in Table II and Fig. 3.

The parton-model expressions for $R^{\nu N}$, $R^{\bar\nu N}$, and R are

$$R^{\nu N}=\sum_{i=u,d}\tfrac{1}{4}(g_V^i+g_A^i)^2+\tfrac{1}{12}(g_V^i-g_A^i)^2A\,,$$

$$R^{\bar\nu N}=\sum_{i=u,d}\tfrac{1}{4}(g_V^i+g_A^i)^2+\tfrac{3}{4}(g_V^i-g_A^i)^2(1/A)\,,$$

$$R=\frac{1}{3}\frac{R^{\bar\nu N}}{R^{\nu N}}A\,, \tag{3.13}$$

where

$$A=\frac{3\sigma(\bar\nu N\to\mu^+X)}{\sigma(\nu N\to\mu^-X)}=\frac{1+3C}{1+\frac{1}{3}C} \tag{3.14}$$

and where

$$C=\int_0^1\bar q(x)dx\Big/\int_0^1 q(x)dx \tag{3.15}$$

is a measure of the antiquark content of the nucleon. Equations (3.13) through (3.15) are valid for isoscalar targets.

In the valence quark approximation, which we adopt, $\bar q(x)=0$ so that $A=1$. Then, from (3.13) and Table I, we have

$$R^{\nu N}=\frac{5r''^2}{12}+\frac{r'^2}{12}+\frac{r'r''}{12}\,,$$

$$R^{\bar\nu N}=\frac{5r''^2}{4}+\frac{r'^2}{4}-\frac{r'r''}{4}\,, \tag{3.16}$$

$$R=\frac{1}{3}\frac{R^{\bar\nu N}}{R^{\nu N}}\,,$$

where $r''=r\cos^2\theta$.

It is easy to verify from (3.16) that for all real values of r' and r'',

$$R\geqslant(2\sqrt{5}-1)/(2\sqrt{5}+1)\,,$$

$$R^{\bar\nu N}\geqslant 1.90R^{\nu N}\,. \tag{3.17}$$

This region of values compatible with the model is shown in Fig. 3 along with the data. It is seen that the model predicts a some what high value for R. However, there is a narrow range of values for r' and r'' which come within one standard deviation of the data.[11] This range of values of r' and r'', for which $R^{\bar\nu N}\lesssim 0.43$ and $R^{\nu N}\gtrsim 0.22$, is shown in Fig. 4.

Since the completion of these fits, new results for $R^{\nu N}$ and $R^{\bar\nu N}$ have been reported by the CERN-Dortmund-Heidelberg-Saclay (CDHS) collaboration.[23] Their values, which are included in Fig. 3,

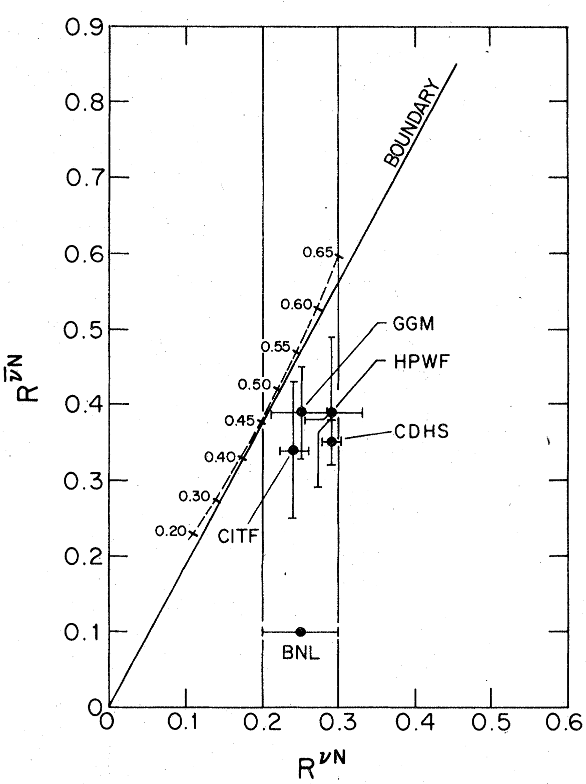

FIG. 3. The inclusive neutral-current parameters $R^{\nu N}$ and $R^{\bar\nu N}$. The data points are from Ref. 11. Only the region to the left of the straight line is accessible in the present model. The dashed line represents the predictions for $r'=0.95$ and various r''. Curves for other r' in the range $0.8\leqslant r'\leqslant1.2$ are similar. The CDHS point [Ref. (23)] should not be compared directly with the theoretical curve because of the large sea contribution suggested by the CDHS charged-current data. See text.

should not be compared directly with the theoretical curve because the CDHS charged-current cross sections, unlike earlier experiments, requires a substantial sea contribution. If the formulas in (3.16) are modified by adding a sea contribution, the theoretical curve in Fig. 3 moves downward. For $C=0.10$ the discrepancy with the CDHS point is 3 standard deviations; for $C=0.15$, it is 2 standard deviations. The CDHS results may therefore require an eventual modification of the model (such as by placing the u_R in a singlet or invoking the existence of substantial scaling violation). At present, however, there is considerable disagreement between experiments on the magnitudes and y distributions of the charged-current cross sections. A serious study of the effects of a sea or of scaling violations must be deferred until these experimental questions are resolved.

2. *Elastic scattering*

Define the quantities

$$R^{\nu p} = \frac{\sigma(\nu p \to \nu p)}{\sigma(\nu n \to \mu^- p)}, \tag{3.18}$$

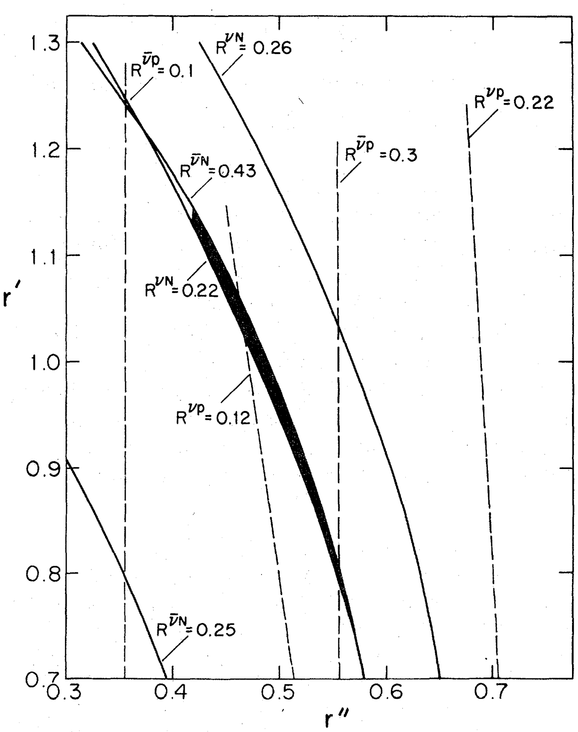

FIG. 4. The solid lines are contours of constant $R^{\bar\nu N}$ or $R^{\nu N}$. The region $R^{\nu N} \geq 0.22$ and $R^{\bar\nu N} \leq 0.43$ fits the inclusive data (Ref. 11). The dashed lines are contours of constant $R^{\nu p}$ or $R^{\bar\nu p}$ for $M_A = 1.15$ GeV. The region $R^{\nu p} > 0.12$ and $R^{\bar\nu p} < 0.30$ fits the elastic data (Ref. 10). For $M_A = 0.84$, the elastic scattering data requires $0.42 < r'' < 0.48$. The region that correctly describes all hadronic data is shaded.

TABLE III. ν_μ-$\bar\nu_\mu$ proton elastic scattering (Ref. 10). The data are from Harvard-Pennsylvania-Wisconsin (HPW) and Caltech-Rockefeller (CTR) experiments. Predictions of the model are given for $r' = 0.95$, $r'' = 0.50$, and two values of the axial-vector mass M_A.

	HPW	CTR	Model $M_A = 0.84$	Model $M_A = 1.15$
$R^{\nu p}$	0.17 ± 0.05	0.23 ± 0.09	0.17	0.13
$R^{\bar\nu p}$	0.2 ± 0.1	$\cdots$	0.32	0.24
$R_{\rm el}$	0.4 ± 0.2	$\cdots$	0.65	0.55

$$R^{\bar\nu p} = \frac{\sigma(\bar\nu p \to \bar\nu p)}{\sigma(\bar\nu p \to \mu^+ n)}, \tag{3.19}$$

$$R_{\rm el} = \frac{\sigma(\bar\nu p \to \bar\nu p)}{\sigma(\nu p \to \nu p)}. \tag{3.20}$$

The available data for these parameters,[10] as well as the Q^2 dependence of the differential cross sections, are given in Table III and in Figs. 5–7.

The elastic scattering cross sections are predicted in this model as follows. The vector part of the hadronic neutral current is decomposed into isovector and isoscalar components, the matrix elements of which can be parametrized[24] in terms of known electromagnetic form factors. (We use dipole form factors with a mass parameter of

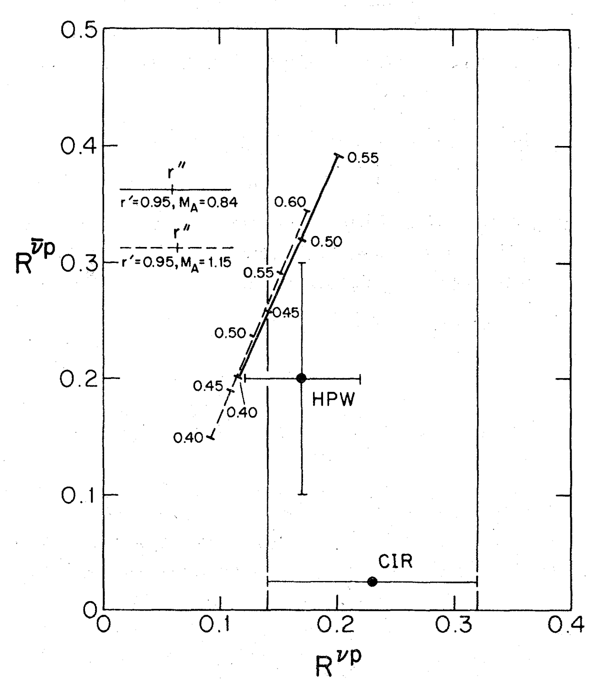

FIG. 5. The ν_μ-$\bar\nu_\mu$ proton elastic scattering data (Ref. 10). The solid and dashed lines are the predictions of the model are $M_A = 0.84$ and 1.15 GeV, respectively. Both curves are for $r' = 0.95$ and various r''.

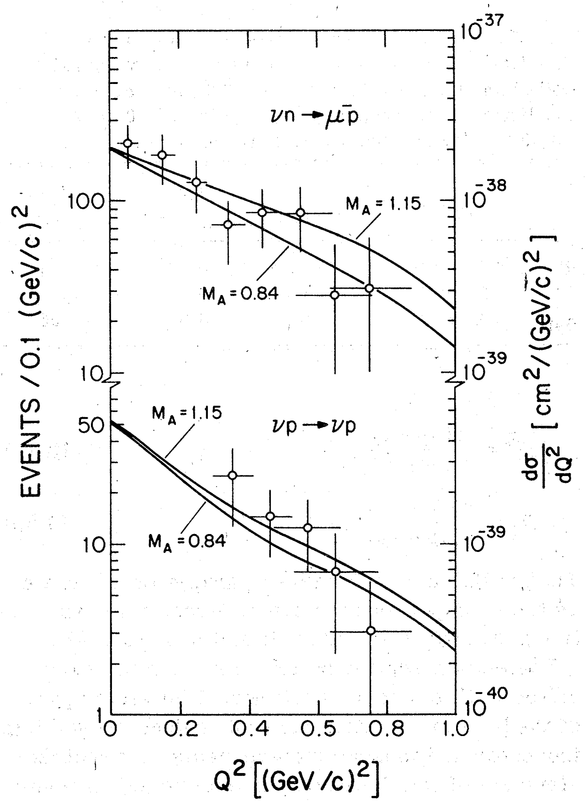

FIG. 6. The Q^2 distributions for $\nu_\mu p \to \nu_\mu p$ and $\nu_\mu n \to \mu^- p$ (Ref. 10). The theoretical curves are the predictions of the model for $r' = 0.95$, $r'' = 0.50$.

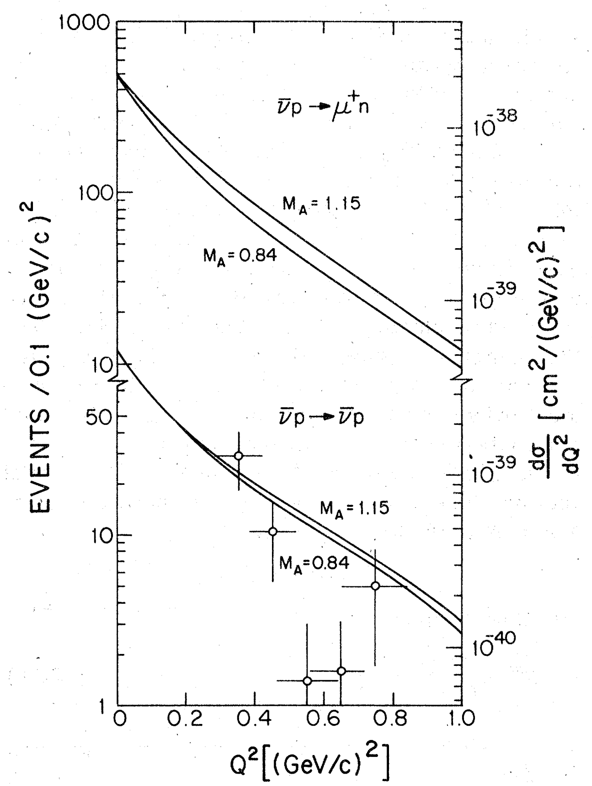

FIG. 7. The Q^2 distributions (Ref. 10) for $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$ and $\bar{\nu}_\mu p \to \mu^+ n$.

$M_V = 0.84$ GeV). The matrix element of the axial-vector current $\bar{d}\gamma_\mu \gamma^5 d$ is

$$\langle p|\bar{d}\gamma_\mu\gamma_5 d|p\rangle = -\langle p|A_\mu^3|p\rangle + \sqrt{3}\langle p|A_\mu^8|p\rangle + \langle p|\bar{s}\gamma_\mu\gamma_5 s|p\rangle, \tag{3.21}$$

where $A_\mu^i = \frac{1}{2}\bar{q}\gamma_\mu\gamma^5\lambda^i q$. We assume that the $\bar{s}s$ term is negligible and use ordinary SU(3) to evaluate the other terms. This implies

$$\langle p|\bar{d}\gamma_\mu\gamma_5 d|p\rangle = \bar{u}_F\gamma_\mu\gamma_5 u_i\, G_A(Q^2), \tag{3.22}$$

$$G_A(Q^2) = \left[(F+D)\left(1 - \frac{2D}{F+D}\right)\right]\left(1 + \frac{Q^2}{M_A^2}\right)^{-2},$$

where $F + D = 1.24$ and $D/(F+D) \approx 0.66$ from β decay and hyperon decay,[25] yielding $G_A(0) \approx -0.40$. We have assumed that the form factors associated with A^3 and A^8 have the same Q^2 dependence. For the axial-vector mass M_A we take the two values $M_A = 0.84$ and 1.15 GeV.

The differential and integrated charged- and neutral-current cross sections, averaged over the BNL ν and $\bar{\nu}$ spectra,[10] are then computed.

The results are relatively insensitive to r', but are strongly dependent on r''. For r' in the vicinity of one (as required by the inclusive data), the predicted values for $R^{\nu p}$ and $R^{\bar{\nu} p}$ are within one standard deviation of the experimental points for $0.42 \lesssim r'' \lesssim 0.48$ with $M_A = 0.84$ and for $0.47 \lesssim r'' \lesssim 0.55$ with $M_A = 1.15$, as can be seen in Fig. 5. [The fit would be improved if $|G_A(0)|$ were allowed to be somewhat larger than the SU(3) value of 0.40]. The predicted Q^2 distributions are shown in Figs. 6 and 7 for the representative values $r' = 0.95$ and $r'' = 0.50$. It is seen that they agree reasonably well with the data and are only moder-

TABLE IV. Comparison of the present model for $r' = 0.95$, $r'' = 0.50$ with Sehgal's (Ref. 20) analysis of the constraints from inclusive pion production. Also listed are the predictions of the Weinberg-Salam model for $\sin^2\theta_W = 0.25$. g_L^i and g_R^i are defined as $g_{L,R}^i = \frac{1}{2}(g_V^i \pm g_A^i)$.

	SU(3) × U(1) (present model)	Weinberg-Salam	Sehgal
$\lvert g_L^u\rvert^2$	0.063	0.11	0.082 ± 0.035
$\lvert g_R^u\rvert^2$	0.063	0.028	0.055 ± 0.025
$\lvert g_L^d\rvert^2$	0.131	0.174	0.158 ± 0.035
$\lvert g_R^d\rvert^2$	0.013	0.007	0.001 ± 0.025

TABLE V. ν_μ-$\bar{\nu}_\mu$ electron (Ref. 18) scattering. The Aachen-Padova data include only events with electron energy greater than 0.4 GeV. The theoretical values are for $g_A^e=0$. All energies are in GeV.

	GGM	Aachen-Padova	$g_V^e=0.4$	0.5	0.6
$\sigma(\nu_\mu e)$ (10^{-42} cm^2)	$<2.6E_\nu$	$(2.1\pm1.2)E_\nu$	$0.9E_\nu$	$1.4E_\nu$	$2.0E_\nu$
$\sigma(\bar{\nu}_\mu e)$ (10^{-42} cm^2)	$(1.0^{+2.1}_{-0.9})\times E_{\bar\nu}$	$(2.4\pm1.3)E_{\bar\nu}$	$0.9E_{\bar\nu}$	$1.4E_{\bar\nu}$	$2.0E_{\bar\nu}$

ately sensitive to M_A.

The region in the r''-r' plane for which the predicted inclusive and elastic data all agree with the data is shown in Fig. 4.

Sehgal has recently shown[20] that inclusive pion production can be used to determine the isospin structure of the neutral current. We compare the results of our fit to the inclusive and elastic cross sections with Sehgal's constraints in Table IV. The agreement is excellent. Hung and Sakurai[26] have subsequently argued that the various neutral-current experiments restrict the neutral-current parameters to two possible regions. Our fit corresponds very well with Hung and Sakurai's region B.

3. Neutrino-electron scattering

The cross sections for $\nu_\mu e$ and $\bar{\nu}_\mu e$ elastic scattering are

$$\sigma(\nu_\mu,\bar{\nu}_\mu)=\frac{G_F{}^2}{2\pi}m_eE[(g_V^e\pm g_A^e)^2+\tfrac{1}{3}(g_V^e\mp g_A^e)^2]$$

$$\xrightarrow[g_A^e=0]{}\frac{4}{3}\;\frac{G_F{}^2m_e}{2\pi}\,E(g_V^e)^2$$

$$=5.5\times10^{-42}E\text{ cm}^2\times(g_v^e)^2\,, \quad (3.23)$$

where E is the neutrino (antineutrino) energy in GeV and the last two lines are specialized to the present model, for which $g_A^e=0$. The experimental results[18] are listed in Table V, and the corresponding allowed values of g_V^e and g_A^e are shown in Fig. 8.

Figure 8 also shows the regions of g_V^e and g_A^e that are compatible with the cross section[19] for $\bar{\nu}_e e\to\bar{\nu}_e e$. From Fig. 8 one sees that all of the neutrino-electron data are in agreement with $-0.6\lesssim g_V^e\lesssim-0.4$. Similarly, all of the data are compatible with $0.4\lesssim g_V^e\lesssim0.6$ except for the low electron energy $\bar{\nu}_e e$ cross section, for which the discrepancy is two to three standard deviations.

The value of g_V^e in the present model is

$$g_V^e=\tfrac{1}{2}r(1-3\cos^2\theta)$$

$$=\tfrac{1}{2}r-\tfrac{3}{2}r''\,. \quad (3.24)$$

In Fig. 4 we have displayed the region in the r''-r' plane that fits all hadronic neutral-current data. For a fixed value of g_V^e, this region can be mapped onto an allowed region in the r-r' plane using $r=2g_V^e+3r''$. Such a mapping is presented in Fig. 9. The two roughly rectangular regions are bounded by the lines of $g_V^e=-0.4$ and -0.6 and by

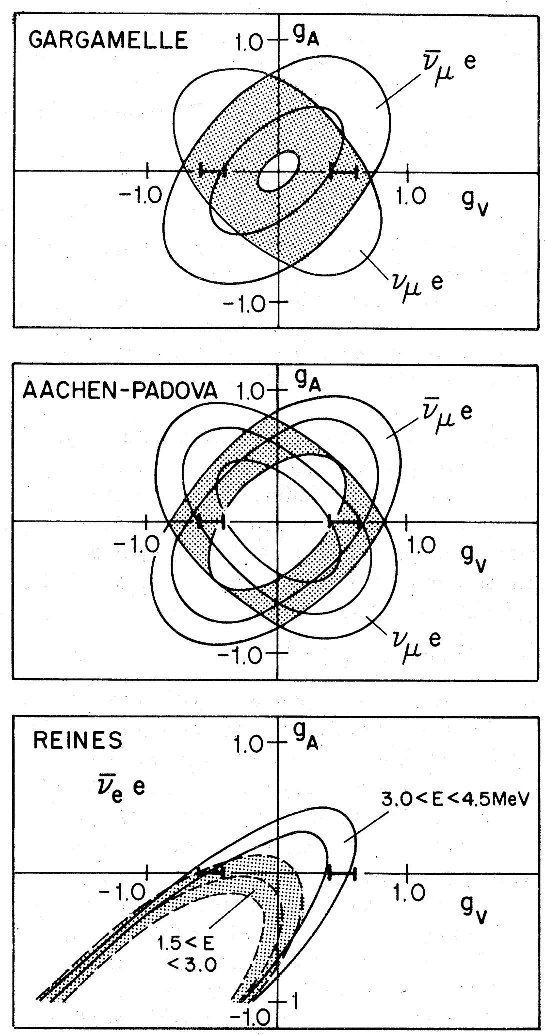

FIG. 8. Allowed regions in g_A^e and g_V^e for $\nu_\mu e$, $\bar{\nu}_\mu e$ elastic scattering (Ref. 18) and for $\bar{\nu}_e e$ elastic scattering (Ref. 19). The regions $-0.6<g_V^e<-0.4$ and $0.4<g_V^e<0.6$ (with $g_A^e=0$) are indicated.

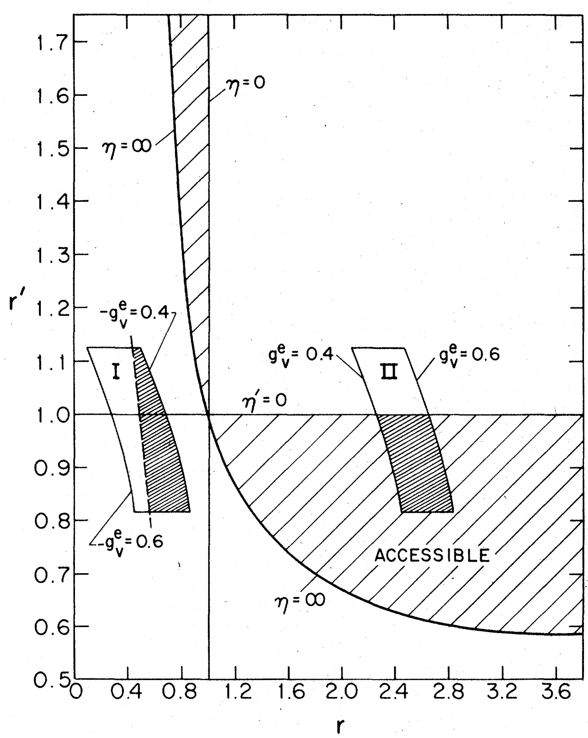

FIG. 9. The regions in r and r' compatible with all neutral-current data. Region I corresponds to $\cos^2\theta \approx 1$ (only the shaded portion has $\cos^2\theta \leqslant 1$). Region I cannot be reached by positive values of η' and η. The shaded part of region II ($\cos^2\theta \approx 0.2$) fits all data and is compatible with positive η' and η.

$g_V^e = +0.4$ and $+0.6$. Hence, any point within either region describes a reasonable fit to all neutral-current data (with the possible exception of the low-electron-energy $\bar{\nu}_e e$ cross section). However, not every value of g_V^e, r, and r' corresponds to possible values of $\cos^2\theta$, η, and η'. For a definite r and g_V^e one can determine

$$\cos^2\theta = r''/r = (r - 2g_V^e)/3r\,. \tag{3.25}$$

Region I in Fig. 9 corresponds to $\cos^2\theta$ near unity. In fact, only the shaded (roughly) triangular wedge in Region I has $\cos^2\theta \lesssim 1$. Region II corresponds to $\cos^2\theta \approx 0.2$. Finally, only a portion of the r-r' plane corresponds to real and positive values of η and η'. This region is disjoint with the region I but includes the shaded portion of region II.

Hence, a reasonable fit to all neutral-current data is obtained for $0.4 \lesssim g_V^e \lesssim 0.6$, $2.3 \lesssim r \lesssim 2.8$, and $0.81 \lesssim r' \lesssim 1.0$. Table VI lists the values of r'', $\cos^2\theta$, η, and η' in this region. (We concentrate on $r' \gtrsim 0.95$, which corresponds to smaller values of M_A.) Tables III–VI and the figures illustrate the neutral-current cross sections obtained for the representative point $r' = 0.95$, $r'' = 0.50$, $\cos^2\theta = 0.20$.

C. Masses of the gauge bosons

In the previous section we have shown that the neutral-current data determine $\cos^2\theta$, η, and η' to lie within a narrow range. From the results in Sec. II, we can determine all the gauge-boson masses, the effective four-Fermion coupling constants, and the parameter λ introduced in (3.7) to describe parity violation in light atoms. In addition to (3.8) and (2.14), the relevant formulas are

$$\begin{aligned}
M_+ &= \frac{g}{(8G_F/\sqrt{2})^{1/2}} = \left(\frac{2\sqrt{2}\pi\alpha}{3G_F\sin^2\theta}\right)^{1/2} = \frac{43.4\ \text{GeV}}{\sin\theta}\,,\\
\frac{G_F'}{G_F} &= \frac{M_+^2}{M_+'^2} = \frac{2\eta+\eta'}{2\eta+\eta'+1}\,,\\
\frac{G_F^0}{G_F} &= \frac{M_+^2}{M_0^2} = \frac{2\eta+\eta'}{4\eta+1}\,,\\
\frac{M_+^2}{M_{1,2}^2} &= \frac{6\cos^2\theta(2\eta+\eta')}{3\cos^2\theta(8\eta+1)+4\eta'+1\mp\sqrt{P}}\,,
\end{aligned} \tag{3.26}$$

where

$$P = [3\cos^2\theta(8\eta+1) - (4\eta'+1)]^2 + 12\cos^2\theta \tag{3.27}$$

and where $M_{1,2}$ are the masses of the neutral bosons $Z_{1,2}$. The results are given in Table VII for the parameters of Table VI.

D. Multimuon events

1. Motivations

Recently, two groups[5] have reported the existence of neutrino-induced trimuon ($\mu^-\mu^-\mu^+$) events. In particular, several of the Fermilab-Harvard-Pennsylvania-Rutgers-Wisconsin (FHPRW) events are characterized by high muon energies, which suggests[6] that they may be due to the production and cascade decay of a new heavy lepton. (However, purely hadronic origins, at least for some of the events, cannot be entirely ruled out.[27])

The following sequence has been suggested[6] for

TABLE VI. Values of parameters for the range of r, r', g_V^e which fits the neutral-current data.

Point	r	r'	g_V^e	$\cos^2\theta$	r''	η	η'
A	2.3	0.95	0.4	0.22	0.50	0.39	0.030
B	2.5	0.95	0.5	0.20	0.50	0.45	0.030
C	2.7	0.95	0.6	0.19	0.50	0.50	0.029
D	2.25	1.0	0.4	0.21	0.48	0.31	0
E	2.45	1.0	0.5	0.20	0.48	0.36	0
F	2.65	1.0	0.6	0.18	0.48	0.41	0
G	2.64	0.85	0.5	0.21	0.54	0.74	0.13

trimuon production

$$\nu_\mu N \to L^- + X \searrow L^0 \mu^- \bar{\nu}_\mu \searrow \mu^- \mu^+ \nu_\mu \;, \tag{3.28}$$

where L^- and L^0 are new heavy leptons. The kinematic distributions for this sequence are in agreement with the data if the masses of L^- and L^0 are around[28] 7–8 and 3–4 GeV, respectively.[29] However, if all six events reported by the FHPRW group[5] are due to this sequence, the large rate (~5×10^{-4} relative to single-muon production) is difficult to accommodate in SU(2)×U(1) models. In SU(2)×U(1) the relative rate is

$$R(\mu^-\mu^-\mu^+) \approx K \sin^2\alpha B_1 B_2 \;, \tag{3.29}$$

where K is a phase-space factor ($K \approx 0.14$ when weighted against the FHPRW neutrino spectrum[5]), $\sin\alpha$ is the mixing angle describing the $\nu_\mu L^-$ coupling, and B_1 and B_2 are the relevant branching ratios for L^- and L^0 decay. For $B_1 \approx B_2 \approx \frac{1}{5}$ and $\sin\alpha \approx 0.1$ (compatible with approximate universality), $R(\mu^-\mu^-\mu^+) \approx 6 \times 10^{-5}$.

The rate can be increased within SU(2)×U(1) by extending the concept of universality[30] or by taking L^0 relatively light (1–2 GeV) so that the alternate sequence

$$L^- \to L^0 \mu^- \bar{L}^0, \quad L^0 \to \mu^- + X, \quad \bar{L}^0 \to \mu^+ + X \tag{3.30}$$

becomes important (as two of us have discussed[6]). Another possibility is to go to a larger gauge group so that the $\bar{L}\nu$ current can couple with full strength to a new gauge boson. One must then consider what happens at the hadron vertex. If the new gauge boson couples light quarks to light quarks (such as with a $\bar{u}_R d_R$ current), then the competing decay mode $L^- \to \nu_\mu \bar{u} d$ causes B_1 to be small. If the new boson only couples light quarks to heavy quarks, this decay mode is eliminated. However, the production cross section is reduced due to the need to produce a heavy quark in addition to the heavy lepton.

The present model is an example of the last possibility, except that the heavy lepton is neutral rather than charged. Large trimuon rates can occur from the production of the M_1^0 along with the b quark.

2. *Production of the M_1^0*

In this model the neutrino-induced multimuon events (other than those associated with charm) are due to the reaction $\nu_\mu d_L \to M_{1L}^0 b_L$, which proceeds via the exchange of the W^0. The production cross section is

$$\frac{\sigma(\nu_\mu d_L \to M_{1L}^0 b_L)}{\sigma(\nu_\mu N \to \mu^- + X)} = K \left(\frac{G_F^0}{G_F}\right)^2 \tag{3.31}$$

The ratio $(G_F^0/G_F)^2$ is ≈0.11 from Table VII. K includes the phase-space suppression associated with the two heavy particles as well as the threshold factors in the hadronic structure functions (since both vertices are $V-A$ there is no factor of $\frac{1}{3}$ from the y distribution). K depends sensitively on m_1, M_b, and m_b, which are the mass of the M_1^0, the mass of the lightest hadron containing the b quark, and the effective mass of the b quark, respectively. Following Albright, Smith, and Vermaseren,[28] we estimate K assuming[31] ξ scaling.

In Fig. 10(a) we present K as a function of m_1 for $M_b = 5$ GeV and $m_b = 4.0$ and 4.9 GeV. (The cross sections have been averaged over the FHPRW[5] neutrino spectrum.)

3. *Decay modes*

The possible decay modes of the M_1^0, with relative weights, are

$$\begin{aligned} &M_{1L}^0 \to \mu_L^- + X_1 \quad (5G_F'^2)\;, \\ &M_{1L}^0 \to \mu_L^- \mu^+ M_{2R}^0, \; M_{2R}^0 \to \mu^- + X_2 \quad (G_F'^2 \Delta(m_2/m_1))\;, \\ &M_{1R}^0 \to \mu_R^- + X_3 \quad (5G_F{}^2)\;, \\ &M_{1R}^0 \to \mu_R^- \mu^+ \nu_L \quad (G_F{}^2)\;. \end{aligned} \tag{3.32}$$

X_1 represents $u_R \bar{d}_R$, weighted by three for color, $E_2 \bar{e}$, and possibly $g_R \bar{s}$. X_3 represents $u_L \bar{d}_L$, $e^+ \nu_e$, and possibly $c_L \bar{s}_L$. $\Delta(m_2/m_1)$ is the phase-space suppression for the decay into a heavy M_2^0. Δ is plotted in Fig. 10(b), and its functional form is given in Ref. 32. We have assumed that M_b is sufficiently large ($\geq 0.7 m_1$) that the decays $M_1^0 \to \nu b \bar{d}$ and $M_1^0 \to \nu \bar{b} d$ are forbidden or strongly suppressed.

The M_2^0, once produced, always decays into a μ^-, along with $X_2 = u_R \bar{d}_R$ or $E_2 e^+$.

Similarly, we estimate the branching ratios for

TABLE VII. Values of the gauge-boson masses and effective four-Fermion couplings for the seven sets of parameters in Table VI. λ, defined in Eq. (3.7) is relevant to parity violation in hydrogen. All masses are in GeV.

Point	M_+	M_+'	M_0	M_1	M_2	$\frac{G_F'}{G_F}$	$\frac{G_F^0}{G_F}$	λ
A	49.0	73	87	60	117	0.45	0.32	−0.16
B	48.5	70	84	59	114	0.48	0.33	−0.18
C	48.1	68	82	58	111	0.51	0.34	−0.20
D	48.9	79	93	61	126	0.38	0.28	−0.18
E	48.4	75	89	60	121	0.42	0.30	−0.21
F	48.0	72	86	59	117	0.45	0.31	−0.23
G	48.9	62	77	56	104	0.62	0.41	−0.13

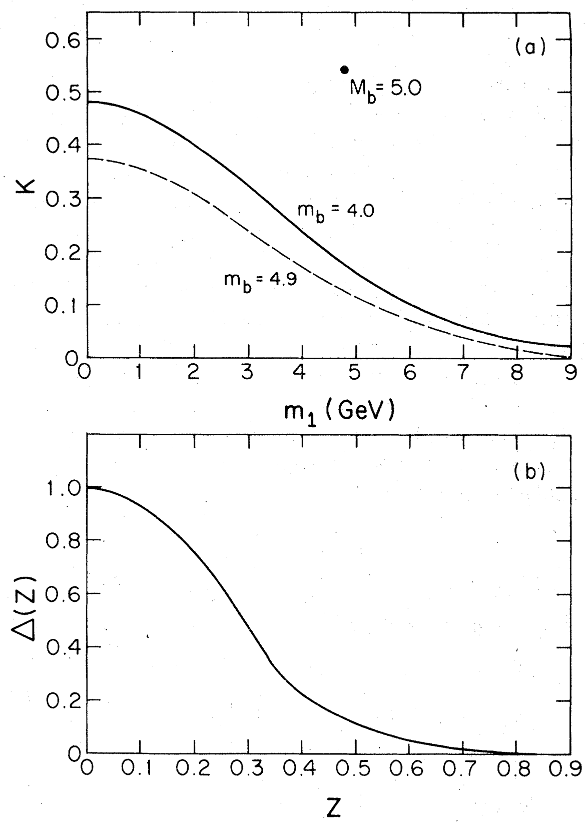

FIG. 10. (a) The kinematic-suppression factor K averaged over the FHPRW neutrino spectrum for $M_1^0 + b$ production, as a function of the lepton mass m_1. The hadron mass M_b is taken as 5 GeV and the effective quark mass m_b is 4.0 GeV (solid line) and 4.9 GeV (dashed line). We are grateful to W. Su for generating this graph. (b) The phase-space suppression factor $\Delta(Z)$ (Ref. 32) for three-body weak decays in the case that one final particle is massive (m_F) and the other two are massless. The curve is valid when each vertex in the amplitude has a definite helicity.

the b quark by counting decay modes with relative weights (the procedure is less reliable than for M_1^0 decay because the nonleptonic modes may be enhanced by the strong interactions):

$$\begin{aligned}
&b_R \to u_R + X \quad (5G_F^2)\,,\\
&b_R \to u_R \mu^- \bar{\nu}_\mu \quad (G_F^2)\,,\\
&b_L \to u_L X \quad (G_F'^2)\ (5G_F'^2)\,,\\
&b_L \to u_L \mu^- \bar{M}_2^0 \qquad (G_F'^2 \Delta(m_2/m_b))\,.\\
&\qquad\qquad\searrow \mu^+ + X
\end{aligned} \tag{3.33}$$

An interesting feature of this model is that the oscillations between the mesons B^0 and $\bar{B}^0$, which are bound states of $b\bar{d}$ and $\bar{d}b$, are very rapid compared to their lifetime (see Sec. V). Hence, half of the b quarks appearing in B^0 mesons will have oscillated into $\bar{b}$ quarks at the time of their decay. Let f be the fraction of events in which the b is in a B^0 meson state. Then, the additional modes

$$\begin{aligned}
&b \to \bar{b} \to \bar{u} + X \quad (\tfrac{5}{2} f (G_F^2 + G_F'^2))\,,\\
&b \to \bar{b} \to \bar{u}\mu^+\nu_\mu \quad (\tfrac{1}{2} f G_F^2)\,,\\
&b \to \bar{b} \to \bar{u}\mu^+ M_2^0\\
&\qquad\qquad\searrow \mu^- + X \quad (\tfrac{1}{2} f G_F'^2 \Delta(m_2/m_b))
\end{aligned} \tag{3.34}$$

are possible. We assume that f is fairly small (in the range 0.1 to 0.25). It is also possible to produce $\bar{b}$ quarks directly from $\bar{d}$ quarks in the sea.

The branching ratios found from (3.32) through (3.34) are summarized in Table VIII, using $(G_F'/G_F)^2 \approx 0.23$ (from Table VII).

4. Multimuons

The various multimuon event rates depend very sensitively on the masses m_1, m_2, M_b, and m_b. There are in fact three decay sequences that lead to trimuon events. Hence, estimates of the heavy-lepton masses that have been made by considering the single sequence in (3.27) are not applicable here. The kinematics of the data suggest that m_1 is in the range 5–8 GeV. We require that m_2 be reasonably small compared to m_1 so that the purely leptonic trimuon source $M_1 \to \mu^-\mu^+ M_2 \to \mu^-\mu^+\mu^- + X$ is important.

We will illustrate the various event rates for the following typical cases:

$$\begin{aligned}
&A:\ m_1 = 7\text{ GeV},\quad m_2 = 2\text{ GeV},\\
&B:\ m_1 = 5\text{ GeV}\,,\quad m_2 = 2\text{ GeV}\,,\\
&C:\ m_1 = 5\text{ GeV}\,,\quad m_2 = 1\text{ GeV}\,.
\end{aligned} \tag{3.35}$$

We always take $M_b = 5$ GeV, $m_b = 4$ GeV.

The important decays are as follows:

(a) *Dimuons*. The rate $R(\mu^-\mu^+)$ for $\mu^-\mu^+$ events

TABLE VIII. Branching ratios for M_1^0, M_2^0, and b, using the coupling constants from Table VII. Δ and f are defined in the text.

Quantity	Value
$B(M_1^0 \to \mu^- X)$	0.85
$B(M_1^0 \to \mu^-\mu^+\nu)$	0.14
$B(M_1^0 \to \mu^-\mu^+ M_2^0 \to \mu^-\mu^+\mu^- + X)$	$0.03\Delta(m_2/m_1)$
$B(M_2 \to \mu^- + X)$	1
$B(b \to X)$	0.85
$B(b \to \mu^- + X)$	0.14
$B(b \to \mu^- \bar{M}_2^0 + X \to \mu^-\mu^+ + X)$	$0.03\Delta(m_2/m_b)$
$B(b \to \bar{b} \to \mu^+ + X)$	$0.07f$
$B(b \to \bar{b} \to \mu^+ M_2^0 + X \to \mu^+\mu^- + X)$	$0.015f\Delta(m_2/m_b)$

TABLE IX. Rates for neutrino-induced multimuon events for three combinations of lepton masses. In each case, $M_b=5$ GeV and $m_b=4$ GeV. The rates are relative to single-muon production and are averaged over the FHPRW spectrum.

Process	Relative rate A $m_1=7$ GeV $m_2=2$ GeV	B $m_1=5$ GeV $m_2=2$ GeV	C $m_1=5$ GeV $m_2=1$ GeV
$\mu^-\mu^+$ (noncharm)	$(0.9+0.4f)\times 10^{-3}$	$(2.1+1.1f)\times 10^{-3}$	$(2.1+1.1f)\times 10^{-3}$
$\mu^-\mu^-$	0.9×10^{-3}	2.1×10^{-3}	2.1×10^{-3}
$\mu^-\mu^-\mu^+$ (total)	2.8×10^{-4}	6.2×10^{-4}	9.6×10^{-4}
First sequence	0.9×10^{-4}	1.2×10^{-4}	2.9×10^{-4}
Second sequence	1.4×10^{-4}	3.4×10^{-4}	3.4×10^{-4}
Third sequence	0.5×10^{-4}	1.6×10^{-4}	3.3×10^{-4}
$\mu^-\mu^+\mu^+$	$0.7\times 10^{-4}f$	$1.7\times 10^{-4}f$	$1.7\times 10^{-4}f$
$\mu^-\mu^-\mu^-\mu^+$	1.6×10^{-5}	2.2×10^{-5}	5.4×10^{-5}
$\mu^-\mu^+\mu^-\mu^+$	$(9.0+8.0f)\times 10^{-6}$	$(2.2+1.2f)\times 10^{-5}$	$(5.4\times 2.8f)\times 10^{-5}$
$\mu^-\mu^-\mu^+\mu^-\mu^+$	1.0×10^{-6}	1.5×10^{-6}	8.6×10^{-6}

from $\nu d\to M_1^0 b$ relative to ordinary single-muon production is (B are branching ratios)

$$R(\mu^-\mu^+)=K\left(\frac{G_F^0}{G_F}\right)^2 \times[B(M_1^0\to\mu^-\mu^+\nu_\mu)B(b\to X) + B(M_1^0\to\mu^-+X)B(b\to\bar{b}\to\mu^++X)]. \quad (3.36)$$

The value of R is given for the three cases in Table IX.

The dimuons from the first sequence, though only $\frac{1}{5}$ to $\frac{1}{10}$ as numerious as those from charm decay, are kinematically quite different: The two muons are both from a heavy-lepton decay. They should be relatively symmetric, with their average energies lying within the Pais-Treiman[29] bound.

The dimuons from $b\to\bar{b}\to\mu^++X$ are kinematically undistinguished.

The rate for same-sign dimuons is

$$R(\mu^-\mu^-)=K\left(\frac{G_F^0}{G_F}\right)^2 B(M_1^0\to\mu^-+X)B(b\to\mu^-+X)\ . \quad (3.37)$$

The values are given in Table IX. Notice that the two muons will be unsymmetric. The muon from the hadronic vertex will tend to be the least energetic. There is no purely leptonic source of same sign dimuons in this model.

(*b*) *Trimuons*. There are three sources of $\mu^-\mu^-\mu^+$ events. The relative rate is

$$R(\mu^-\mu^-\mu^+)=K\left(\frac{G_F^0}{G_F}\right)^2[B(M_1^0\to\mu^-\mu^+M_2^0\to\mu^-\mu^+\mu^-+X)B(b\to X)+B(M_1^0\to\mu^-\mu^+\nu)B(b\to\mu^-X) + B(M_1^0\to\mu^-X)B(b\to\mu^-\bar{M}_2^0X\to\mu^-\mu^+X)]\ . \quad (3.38)$$

Table IX lists the relative rates for each sequence. The total rate can easily be 5×10^{-4} or more. The first sequence is purely leptonic. The three muons should be energetic with invariant mass less than m_1. The other sequences can produce three-muon invariant masses larger than m_1. The muons from b decay will tend to have low energy on the average. Hence, many of these events, especially from the third sequence, would be missed in present experiments. Note that no neutrinos are produced in the first and third sequences (when X consists of hadrons). Hence, the visible energy is the same as the total energy in these cases.

Trimuons of the type $\mu^-\mu^+\mu^+$ can be produced through B^0-$\bar{B}^0$ oscillations (or by $\bar{b}$ production from the sea):

$$R(\mu^-\mu^+\mu^+)=K\left(\frac{G_F^0}{G_F}\right)^2 B(M_1^0\to\mu^-\mu^+\nu_\mu)B(b\to\bar{b}\to\mu^+X)\ . \quad (3.39)$$

(*c*) *Tetramuons and pentamuons*. The model also predicts small numbers of four- and five-muon events:

$$R(\mu^-\mu^-\mu^-\mu^+)=K\left(\frac{G_F^0}{G_F}\right)^2 B(M_1^0\to\mu^-\mu^+M_2^0\to\mu^-\mu^+\mu^-+X)B(b\to\mu^-+X)\ ,$$

$$R(\mu^-\mu^+\mu^-\mu^+)=K\left(\frac{G_F^0}{G_F}\right)^2 [B(M_1^0\to\mu^-\mu^+\nu_\mu)B(b\to\mu^-\overline{M}_2^0+X\to\mu^-\mu^++X)$$
$$+B(M_1^0\to\mu^-\mu^+M_2^0\to\mu^-\mu^+\mu^-+X)B(b\to\overline{b}\to\mu^++X)]\ ,$$

$$R(\mu^-\mu^-\mu^+\mu^-\mu^+)=K\left(\frac{G_F^0}{G_F}\right)^2 B(M_1^0\to\mu^-\mu^+M_2^0\to\mu^-\mu^+\mu^-+X)B(b\to\mu^-\overline{M}_2^0+X\to\mu^-\mu^++X)\ . \tag{3.40}$$

The rates for all of these processes are estimated in Table IX.

5. *Antineutrino-induced events*

This model predicts very interesting multimuon events in high-energy antineutrino reactions. The reaction $\overline{\nu}_\mu u_R\to\mu^+b_R$ should lead to very large rates for $\mu^+\mu^-$ and $\mu^+\mu^-\mu^+$ events well above the b-quark threshold, as can be seen in Table VIII. Also, the B^0-$\overline{B}^0$ oscillation should produce $\mu^+\mu^+$ events at the relative rate,

$$\frac{R(\mu^+\mu^+)}{R(\mu^+\mu^-)}=\frac{B(b\to\overline{b}\to\mu^++X)}{B(b\to\mu^-+X)}=\frac{f}{2} \tag{3.41}$$

assuming b production as the dominant dimuon source.

Furthermore, there are multimuon events associated with $\overline{M}_1^0$ production. The initial reaction is $\overline{\nu}_\mu d_R\to\overline{M}_1^0 b_R$. The production cross section is identical to that for $\nu d_L\to M_1^0 b_L$, because the hadronic current is now $V+A$. Hence the ratio

$$\frac{\sigma(\overline{\nu}_\mu d_R\to\overline{M}_1^0 b_R)}{\sigma(\overline{\nu}_\mu N\to\mu^++X)}$$

will be higher than the corresponding ratio for neutrinos at any given energy. The $\overline{M}_1^0\, b$ final state can decay into multimuons by sequences similar to those in (3.36) through (3.40), except that the signs of the muons from the lepton vertex are reversed.

E. Phenomenology of the new leptons

There are six neutral heavy leptons in this model: $E_1^0, E_2^0, M_1^0, M_2^0, T_1^0, T_2^0$. The only strong constraint on their masses[33] comes from kaon decays. The M_2^0 and E_2^0 masses must be large enough to prevent the decays

$$K^-\to s_R\overline{u}_R\to\mu^-\overline{M}_2^0\to\mu^-\mu^+\pi^-\ ,$$
$$K^-\to s_R\overline{u}_R\to e^-\overline{E}_2^0\to e^-e^+\pi^- \tag{3.42}$$

which would otherwise proceed via the $W'^{\pm}$. Similarly, if the M_1^0, or E_1^0 were sufficiently light the transitions

$$K^-\to\mu^-\overline{M}_1^0\ ,$$
$$K^-\to e^-\overline{E}_1^0\ , \tag{3.43}$$

which proceed by either the W or W', could lead to the final states $e^-e^+\pi^-$, $\mu^-\mu^+\pi^-$, $e^-e^+e^-\overline{\nu}_e$, $e^-e^+\mu^-\overline{\nu}_\mu$, etc.

Similarly, if the E_1^0 or M_1^0 were much lighter than the charmed hadrons, the decay $c\to se^+E_1^0$, followed by $E_1^0\to e^-\pi^+$, $e^-e^+\nu_e$, etc., would be important. This, and a similar argument for the M_1^0, suggests that their masses should not be less than around 1 GeV. Of course, the trimuon phenomena require a large mass (5–8 GeV) for the M_1^0.

There are no corresponding limits for the T_1^0 or T_2^0, since decays into these leptons would have to be accompanied by a τ^- or T^-.

As has already been discussed, the M_1^0 can be produced in neutrino reactions, with the M_2^0 appearing as a decay product. They could also be produced in pairs weakly (via the Z bosons) in e^+e^- annihilation reactions. The E_1^0 and E_2^0 can also be produced in pairs (through the W, W', and Z bosons) in e^+e^- reactions. They may also appear as decay products of the M_1^0, M_2^0, and heavy quarks.

The decay modes of the M_1^0 and M_2^0 have been summarized in Table VI. Similarly, the decay modes of the E_1^0, assuming it is heavier than the E_2^0, with relative weights, are

$$\begin{aligned}
&E_{1R}^0\to e_R^-+X \quad (3G_F^2)\ ,\\
&E_{1R}^0\to e_R^-e^+\nu_e \quad (G_F^2)\ ,\\
&E_{1R}^0\to e_R^-\mu^+\nu_\mu \quad (G_F^2)\ ,\\
&E_{1L}^0\to e_L^-+X \quad (3G_F'^2)\ ,\\
&E_{1L}^0\to e_L^-e^+E_2^0 \quad (G_F'^2\Delta(m_{2E}/m_{1E}))\ ,\\
&E_{1L}^0\to e_L^-\mu^+M_2^0 \quad (G_F'^2\Delta(m_2/m_{1E}))\ ,
\end{aligned} \tag{3.44}$$

where X refers to hadrons and Δ is the appropriate phase-space factor, the subscript E having been used to distinguish the mass of $M_{1,2}^0$ and $E_{1,2}^0$. The E_2^0 will decay into e^-+X via the W'.

IV. MASSES

A. The Higgs potential

In our earlier description of the model in Sec. II, we introduced the spin-zero fields $\Omega, \Phi, \varphi, \chi$, and in (2.2) specified their vacuum expectation values. In this section, we would like to indicate how such a form arises from a potential. We will impose the constraint that the Lagrangian be invariant under CP as well as the discrete symmetry V, but not under C and P separately [CP invariance also requires that the Yukawa coupling in (2.11) involving Φ and Ω be real]. The most general SU(3)-invariant Higgs potential up to fourth order in the fields is then the sum of

$$\begin{aligned} P_1 &= a_1\Omega^+\Omega + a_2(\Omega^+\Omega)^2 ,\\ P_2 &= b_1\Phi^+\Phi + b_2(\Phi^+\Phi)^2 ,\\ P_3 &= c_1\,\mathrm{Tr}(\varphi^2+\chi^2) + c_2[\mathrm{Tr}(\varphi^2+\chi^2)]^2 + c_3\,\mathrm{Tr}\varphi^2\,\mathrm{Tr}\chi^2 \\ &\quad + c_4\,\mathrm{Tr}(\vec{\varphi}\cdot\vec{\chi})^2 + c_5\,\mathrm{Tr}(\varphi^2\chi^2) + c_6[\mathrm{Tr}(\vec{\varphi}\cdot\vec{\chi})]^2 ,\\ P_4 &= d_0\Omega^+\Phi\Phi^+\Omega + d_1\Omega^+\Omega\Phi^+\Phi \\ &\quad + d_2\Omega^+\Omega\,\mathrm{Tr}(\varphi^2+\chi^2) + d_3\Omega^+(\varphi^2+\chi^2)\Omega \\ &\quad + id_4\Omega^+(\varphi\chi-\chi\varphi)\Omega + d_5\Phi^+\Phi\,\mathrm{Tr}(\varphi^2+\chi^2) \\ &\quad + d_6\Phi^+(\varphi^2+\chi^2)\Phi + id_7\Phi^+(\varphi\chi-\chi\varphi)\Phi , \end{aligned} \tag{4.1}$$

where $\varphi = \frac{1}{2}(\vec{\lambda}\cdot\vec{\varphi})$ and $\chi = \frac{1}{2}(\vec{\lambda}\cdot\vec{\chi})$. We have not included a term $\mathrm{Tr}(\varphi^4+\chi^4)$ because it is a linear combination of the c_2 and c_3 terms. If we had not imposed CP invariance, P_3 would contain the additional terms $\mathrm{Tr}\varphi\chi\,\mathrm{Tr}(\varphi^2-\chi^2)$ and $\mathrm{Tr}(\varphi^3\chi-\chi^3\varphi)$.

The potential in (4.1) depends on 18 real parameters. Different segments of this 18 parameter space lead to drastically different physical consequences. For example, depending on the signs and relative magnitudes of the coefficients, electric charge and parity may or may not be conserved, the hadronic current in β decay may be left- or right-handed, etc. Our philosophy is simply to verify that there are finite regions of the parameter space that lead to the desired consequences.

The d_0 term in P_4 determines the relative orientation of the vacuum expectation values of Φ and Ω. For $d_0>0$, their vacuum values must be orthogonal. By an overall SU(3) transformation $\langle\Omega\rangle_0$ can be chosen to lie in the one direction; this will be unchanged by an additional U-spin transformation that causes $\langle\Phi\rangle_0$ to be in the three direction, so that for $d_0>0$,

$$\langle\Omega\rangle_0 = v_\Omega\begin{bmatrix}1\\0\\0\end{bmatrix} , \quad \langle\Phi\rangle_0 = v_\Phi\begin{bmatrix}0\\0\\1\end{bmatrix} . \tag{4.2}$$

The orientations of $\langle\varphi\rangle_0$ and $\langle\chi\rangle_0$ are now fixed by the parameters and by (4.2). We have verified in a tedious calculation, that will not be reproduced, that for mild constraints on the parameters (such as $d_3>|d_4|$) the potential has a local (and probably global) minimum when the vacuum expectation values of $\varphi_1,\varphi_2,\varphi_4,\varphi_5$, and χ_1, χ_2, χ_4, and χ_5 are all zero. Hence, electric charge will be conserved in this region of the parameter space.

We now consider the orientations and magnitudes of $\langle\varphi\rangle_0$ and $\langle\chi\rangle_0$ assuming charge conservation. Define

$$\langle\varphi_i\rangle_0 = \sqrt{2}\,v_{1i} , \quad \langle\chi_i\rangle_0 = \sqrt{2}\,v_{2i} \tag{4.3}$$

so that $\mathrm{Tr}\langle\varphi\rangle_0^2 = \vec{v}_1\cdot\vec{v}_1$, $\mathrm{Tr}\langle\varphi\cdot\chi\rangle_0 = \vec{v}_1\cdot\vec{v}_2$, etc. The orientations of the vectors $\vec{v}_1$ and $\vec{v}_2$ are parametrized by

$$\vec{v}_i\cdot\vec{\lambda} = v_i\begin{bmatrix}2\gamma_i & 0 & 0\\ 0 & -(\beta_i+\gamma_i) & \alpha_i - i\delta_i\\ 0 & \alpha_i+i\delta_i & \beta_i-\gamma_i\end{bmatrix} , \tag{4.4}$$

where

$$\alpha_i^2+\beta_i^2+3\gamma_i^2+\delta_i^2 = 1 . \tag{4.5}$$

In terms of these parameters the Higgs potential is

$$\begin{aligned} V &= a_1|v_\Omega|^2 + a_2|v_\Omega|^4 + b_1|v_\Phi|^2 + b_2|v_\Phi|^4 + c_1(v_1^2+v_2^2) + c_2(v_1^2+v_2)^2 + c_3v_1^2v_2^2 \\ &\quad + \tfrac{1}{2}(c_4v_1^2v_2^2)[(\vec{v}_1\cdot\vec{v}_2)^2 + 2(\beta_1\beta_2-\gamma_1\gamma_2)\vec{v}_1\cdot\vec{v}_2 + 3(\beta_1\gamma_2-\beta_2\gamma_1)^2 + (\gamma_1^2+\gamma_2^2-\beta_1^2-\beta_2^2) - (\delta_1\alpha_2-\delta_2\alpha_1)^2] \\ &\quad + \tfrac{1}{2}(c_5v_1^2v_2^2)(1-2\gamma_1^2-2\gamma_2^2+4\gamma_1\gamma_2\vec{v}_1\cdot\vec{v}_2) + c_6(\vec{v}_1\cdot\vec{v}_2)^2 \\ &\quad + d_1|v_\Omega|^2|v_\Phi|^2 + (d_2|v_\Omega|^2 + d_5|v_\Phi|^2)(v_1^2+v_2^2) + 4d_3|v_\Omega|^2(\gamma_1^2v_1^2+\gamma_2^2v_2^2) \\ &\quad + \tfrac{1}{2}d_6|v_\Phi|^2[(\alpha_1^2+(\beta_1-\gamma_1)^2+\delta_1^2)v_1^2 + (\alpha_2^2+(\beta_2-\gamma_2)^2+\delta_2^2)v_2^2] + d_7|v_\Phi|^2v_1v_2(\alpha_1\delta_2-\delta_1\alpha_2) . \end{aligned} \tag{4.6}$$

We choose a_1, b_1, c_1 and $c_3<0$, and a_2, b_2, c_2, c_4, c_5, c_6, d_3, and $d_6>0$; d_1, d_2, and d_5 can be negative but are bounded.

Then, for $|d_7|$, d_3, and c_6 sufficiently large compared to d_6, c_4, and c_5, the potential is minimized for

$$\begin{aligned} &\beta_1=\beta_2=\gamma_1=\gamma_2=\vec{v}_1\cdot\vec{v}_2=0 ,\\ &v_1^2=v_2^2\neq 0 , \quad |v_\Phi|^2,\ |v_\Omega|^2\neq 0 . \end{aligned} \tag{4.7}$$

Finally, if we choose $d_7>0$, the condition $\vec{v}_1\cdot\vec{v}_2=0$ requires

$$\begin{aligned} &v_{16} = -v_{27} = v\cos\theta ,\\ &v_{17} = v_{26} = v\sin\theta , \end{aligned} \tag{4.8}$$

where the angle θ is not determined at the tree level. Then

$$\langle\varphi\rangle_0+i\langle\chi\rangle_0=\sqrt{2}\,ve^{i\theta}\begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}. \tag{4.9}$$

Except for the phase $e^{i\theta}$ this is the form postulated in (2.2).

We therefore see that there are ranges of the Higgs parameters for which the electric charge is conserved and parity is violated. The undetermined angle θ indicates a spontaneous breakdown of CP invariance. However, the W-boson mass matrix is independent of θ and the phase can be removed from the Fermion mass matrix by a redefinition of fields, so that the $\exp(i\theta)$ factor only affects higher-order loop diagrams involving heavy fermions (b, h, M_1^0, E_1^0, etc.)

It is clear that introducing a second field Ω' will cause no problem, but further octet fields σ and δ might. We have not worked out an expression for the general Higgs potential involving these fields, but it is plausible that the minimum of the potential would be for $\langle\sigma\rangle_0$ lying in a direction orthogonal to $\langle\varphi\rangle_0$; this would minimize terms such as $\mathrm{Tr}\langle(\varphi\cdot\sigma)^2\rangle_0$ if they appear with positive coefficient. We will simply assume that the vacuum expectation values of σ and δ approximately conserve CP, in accordance with experiment. Then $\langle\sigma\rangle_0$ will lie along the U_3 and U_0 directions, as discussed in Sec. II, and $\langle\delta\rangle_0$ will lie along U_2. Finally, we have verified that the parameters of the potential can be chosen so that either $v_\delta^2=0$ or $v_\delta^2=v_\sigma^2$, the two cases discussed in Sec. II.

B. Generation of small mass terms

Let us now return to the topic of Sec. II B namely, the effect of additional octet spin-zero fields σ,δ. Using the normalization of Eq. (2.5)

$$\begin{aligned}\langle\sigma\rangle_0&=\left\langle\frac{\vec{\lambda}\cdot\vec{\sigma}}{2}\right\rangle_0=\frac{v_\sigma}{\sqrt{2}}(\alpha U_1+\beta U_3+\gamma U_0),\\ \langle\delta\rangle_0&=\left\langle\frac{\vec{\lambda}\cdot\vec{\delta}}{2}\right\rangle_0=\frac{v_\delta}{\sqrt{2}}U_2,\end{aligned}\tag{4.10}$$

and, of course, a Yukawa coupling, restricted by the discrete symmetry to be of the form, e.g., q_u, $g_u\bar{q}_u[\sigma-i\gamma_5\delta]q_u$. We will take $\langle\sigma\rangle_0$ to be orthogonal to $\langle\varphi\rangle_0$, so that $\alpha=0$ in (4.6): There are several different cases we can discuss depending on whether $v_\delta^2=0$ or $v_\delta^2=v_\sigma^2$ and whether β or $\gamma=0$.

The contribution of σ,δ Yukawa couplings to the mass matrix, assuming (4.10) is an additional term

$$\begin{aligned}&v_\sigma\frac{\beta}{\sqrt{2}}[g_u(-\bar{d}_Lb_R+\bar{b}_Ld_R)+g_c(-\bar{s}_Lh_R+\bar{h}_Ls_R)\\ &\quad+g_e(-\bar{\nu}_{eL}E_{1R}^0+\bar{E}_{1L}^0E_{2R}^0)+g_\mu(-\bar{\nu}_{\mu L}M_{1R}^0+\bar{M}_{1L}^0M_{2R}^0)+g_\tau(-\bar{\nu}_{\tau L}T_{1R}^0+\bar{T}_{1L}^0T_{2R}^0)]\\ &\quad+v_\sigma\frac{\gamma}{\sqrt{2}}[g_u(2\bar{u}_Lu_R-\bar{d}_Lb_R-\bar{b}_Ld_R)+g_c(2\bar{c}_Lg_R-\bar{s}_Lh_R-\bar{h}_Ls_R)+g_e(2\bar{e}_Le_R-\bar{\nu}_{eL}E_{1R}^0-\bar{E}_{1L}^0E_{2R}^0)\\ &\quad+g_\mu(2\bar{\mu}_L\mu_R-\bar{\nu}_{\mu L}M_{1R}^0-\bar{M}_{1L}^0M_{2R}^0)+g_\tau(2\bar{\tau}_L T_R-\bar{\nu}_{\tau L}T_{1R}^0-\bar{T}_{1L}^0T_{2R}^0)]\\ &\quad+\frac{v_\delta}{\sqrt{2}}[g_u(\bar{d}_Ld_R-\bar{b}_Lb_R)+g_c(\bar{s}_Ls_R-\bar{h}_Lh_R)+g_e(\bar{\nu}_{eL}E_{2R}^0-\bar{E}_{1L}^0E_{1R}^0)+g_\mu(\bar{\nu}_{\mu L}M_{2R}^0-\bar{M}_{1L}^0M_{1R}^0)\\ &\quad+g_\tau(\bar{\nu}_{\tau L}T_{2R}^0-\bar{T}_{1L}^0T_{1R}^0)].\end{aligned}\tag{4.11}$$

We will always adjust our Higgs potential parameters so that $\langle\sigma\rangle_0/\langle\varphi\rangle_0\ll1$; this allows the v_δ contributions to the b,h,E_1^0,M_1^0,T_1^0 masses to be neglected and, as stated earlier, the gauge-boson mass shifts may also be neglected. The most straightforward case to treat is that in which $|v_\delta|=|v_\sigma|$; all the previously massless particles, except of course for the neutrinos, now acquire a mass and, e.g., $m_u\sim-2g_uv_\sigma\gamma/\sqrt{2}$, $m_d\sim-g_uv_\delta/\sqrt{2}$. These masses are all adjustable parameters, so we have little predictive power beyond such statements as ν_e-E_1^0 mixing is much smaller than ν_μ-M_1^0 since they are scaled by m_e/m_μ. We cannot in general even say whether m_d is larger or smaller than m_u since that depends on β and γ, restricted to the normalization condition $\beta^2+3\gamma^2=1$. In principle by making the coupling constant g_c large enough, one could drop the g particle altogether, setting $g_cv_\sigma\sim m_c$, with g_R replaced by c_R. Similarly, letting $g_\tau v_\sigma\sim m_\tau$, T^- could be dropped. We would probably need a very-large-mass h particle then to reduce the amount of s-h mixing, but m_h is a free parameter. Of course, without c and g singlet states one also has the problem of finding a way to mix u and c, i.e., how to generate a Cabibbo angle, without other undesirable mixings.

A more restricted scheme is for $\alpha=0$ and either β or $\gamma=0$. The case in which $\gamma=0$ and $v_\delta=0$ is of particular interest from a theoretical point of view so we shall proceed to discuss it at some length. We wish to make clear, however, that we have not found a potential in which this occurs, so what we say from now on is meant to be illustrative. It is conceivable that it could occur in a general potential involving couplings of σ, δ to φ, χ, etc.

Let us begin by concentrating on the quark sector, namely, considering the u, d, b, fields. From (2.3) and (4.7) we see what the mass matrix is

$$\sqrt{2}\,[\tfrac{1}{2}g_u v_\sigma(-\bar{d}_L b_R + \bar{b}_L d_R) + f_u v \bar{b}_L b_R]\,. \qquad (4.12)$$

If we call $g_u v_\sigma/\sqrt{2} = \Delta$, $f_u v\sqrt{2} = m$, we see that we have a two-by-two left-right matrix

$$(\bar{d}_L^0, \bar{b}_L^0)\begin{pmatrix} -\Delta & 0 \\ m & \Delta \end{pmatrix}\begin{pmatrix} b_R^0 \\ d_R^0 \end{pmatrix} = (\bar{d}_L^0, b_L^0)\begin{pmatrix} 0 & -\Delta \\ \Delta & m \end{pmatrix}\begin{pmatrix} d_R^0 \\ b_R^0 \end{pmatrix}, \qquad (4.13)$$

where we have introduced zero superscripts on the fields to indicate that they are not the eigenstates of the mass matrix. For consistency we should have had these superscripts all along. We now diagonalize (4.13) by performing separate rotations on q_L and q_R by angles ω_L and ω_R with transformation matrix $U(\omega)$, that is

$$U(\omega) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}. \qquad (4.14)$$

It turns out for the simple case of (4.3) that $\omega_L = -\omega_R$, and that $\tan 2\omega_R = 2\Delta/m$. The mass eigenstates are then

$$m_b = m\cos^2\omega_R + \Delta\sin 2\omega_R\,, \quad m_d = -m\sin^2\omega_R + \Delta\sin 2\omega_R \qquad (4.15)$$

so that for $\Delta/m \ll 1$, the case we are interested in, we find $\omega_R \sim \Delta/m \ll 1$ and

$$m_b \sim m\,, \quad m_d \sim \Delta^2/m\,, \quad \omega_R = -\omega_L \sim \Delta/m \sim (m_d/m_b)^{1/2} \qquad (4.16)$$

with the u quark remaining massless. As we shall see in the next section, however, the u does acquire a mass through the one-loop diagram present after d-b mixing. This mass will be of order $m_u \sim (\alpha/\pi)(\sin\omega)m_b$. Similar arguments may be made for q_c, l_e, l_μ, l_τ triplets. A clear problem in this formulation is the generation of a muon mass of 105 MeV, if we wish the M_1^0 and M_2^0 to have masses of only a few GeV, as seems indicated by our considerations of Sec. III. We could replace μ_R^- by M_R^- and have singlet fields μ_R^- and M_L^- as we have done for the τ^-, but this would be at variance with our interpretation of multimuon data in Sec. III.

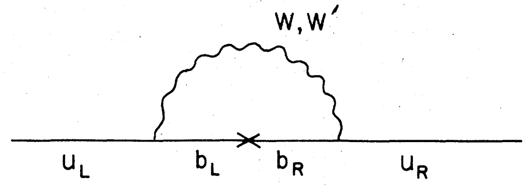

FIG. 11. One-loop diagram contributing to the u-quark mass.

C. One-loop diagrams

For the type of d-b mixing discussed in Sec. IV B, we obtain, through the one-loop diagram of Fig. 11, a contribution to the u mass. It is of course proportional to m_b, the fermion mass scale, and to the d-b mixing angle. Using $-\omega_L = \omega_R = \omega$, the result of Sec. IV B, the self-mass term is

$$\bar{u}_L(-\Sigma_{LR})u_R = \bar{u}_L\left[\frac{i g^2 m_b \cos\omega\,\sin\omega}{8\pi^2} \times \ln\left(\frac{M_+{}^2 + m_b{}^2}{M_+'^2 + m_b{}^2}\right)\right]u_R \qquad (4.17)$$

and defining $\alpha_g = g^2/4\pi$, we see that, since $m_b{}^2 \ll M_+{}^2, M_+'^2$

$$m_u^{(1\text{ loop})} \sim \frac{\alpha_g}{2\pi}\, m_b \cos\omega\,\sin\omega \ln\left(\frac{M_+'^2}{M_+{}^2}\right) \qquad (4.18)$$

and finally using (2.32) to relate α_g to α and our estimates from Sec. III of $M_+'^2/M_+{}^2$, we find

$$m_u^{(1\text{ loop})} \sim 0.35(\alpha/\pi)\cos\omega\,\sin\omega\, m_b\,. \qquad (4.19)$$

If we now take the version in which the d-b mixing angle is small and proportional to m_d/m_b we find

$$m_u^{(1\text{ loop})} \sim 0.35\,\frac{\alpha}{\pi}\left(\frac{m_d}{m_b}\right)^{1/2} m_b \qquad (4.20)$$

so that with $m_d \sim 10$ MeV, $m_b \sim 5$ GeV, we see that $m_u \sim \frac{1}{5}$ MeV. With the same sort of mechanism one can also get e and μ masses of the order of $\frac{1}{10}$ to 1 MeV.

For the case of u quark there is another pair of one-loop diagrams which contributes to its mass due to u-c mixing. These are the diagrams in which u_L goes to h_L via emission of $W^+, W^{+\prime}$, h_L goes to h_R, and h_R to u_R via reabsorption of the W. These presumably give a contribution to m_u of order $0.35(\alpha/\pi)(m_s m_h)^{1/2}\sin\theta_L\sin\theta_R$ where θ_L is the u_L-c_L mixing angle and θ_R the u_R-g_R mixing angle. There is no *a priori* reason why this term should not be as large or larger than the one-loop diagram involving the b meson.

A few comments are probably appropriate at this point. First m_b is an unrenormalized parameter so that although the one-loop integral is finite,

m_b in general need not be, so we emphasize again that our arguments for m_u, m_d are only qualitative. A second question is whether our one-loop diagram calculation is invalidated by the presence of terms proportional to U_0 induced in $\langle\sigma\rangle_0$: This type of argument has been stressed[34] by Georgi and Glashow in their discussion of attempts to calculate the electron mass. This would say that it is inconsistent to start by saying $\langle\lambda\cdot\sigma\rangle_0$ lies along the U_3 direction and then calculate the u mass by one-loop diagrams, as these very loop diagrams engender terms in our effective action which lead to terms in $\langle\sigma\rangle$ lying along the U_0 direction. Stated in another way it is wrong to think we can minimize the potential with $\langle\sigma\rangle_0=(v_\sigma/\sqrt{2})U_3$, for if $\beta\neq 0$, γ also must be nonzero in (4.6). The u mass is therefore not calculable; nevertheless, the possibility of a mass hierarchy $m_d\sim m_b\sin^2\omega$, $m_u\sim\alpha m_b\sin\omega$ is perhaps realizable and interesting enough to have warranted this digression.

V. ADDITIONAL FEATURES OF THE MODEL

In this section we will describe several secondary features of the model. These include small flavor-changing neutral currents, the K_L-K_S mass difference, the $K_L\to\mu^+\mu^-$ decay, nonleptonic decays, B^0-$\bar{B}^0$ oscillations, the muon magnetic moment, the decay $\mu\to e\gamma$, and CP violation.

A. Off-diagonal neutral currents

The quark triplets in this model are

$$\begin{bmatrix} u^0\\ d^0\\ b^0\end{bmatrix}_L, \begin{bmatrix} u^0\\ b^0\\ d^0\end{bmatrix}_R, \begin{bmatrix} c^0\\ s^0\\ h^0\end{bmatrix}_L, \begin{bmatrix} g^0\\ h^0\\ s^0\end{bmatrix}_R, c_R^0, g_L^0,$$

where the superscripts indicate that the fields are weak interaction eigenstates and not mass eigenstates. The neutral currents J_1 and J_2 of Eq. (2.28) are diagonal in this weak-interaction basis.

The charge $\frac{2}{3}$ quarks appear both in singlets and triplets, while the charge $-\frac{1}{3}$ quarks appear in two inequivalent locations within the triplets. Hence, any mixings between the triplets and singlets such as, e.g., u_R^0-c_R^0 or $u_L^0-g_L^0$ mixing or between the charge $-\frac{1}{3}$ components of one triplet will lead to off-diagonal terms in J_1 and J_2 when they are expressed in terms of the mass eigenstates.

The Higgs multiplets φ, χ, Ω, and Φ, which were introduced in Sec. II to give masses to the b, h, c, g, E_1^0, E_2^0, etc. and to produce the asymmetry between the right- and left-handed triplets, do not cause any mixing between the weak eigenstates (the discrete symmetry V was introduced, in part, to prevent such mixings). The triplet Ω' introduced in (2.13) generates the Cabibbo mixing between u_L^0 and c_L^0 (and between u_R^0 and g_R^0). However, the u_L^0 and c_L^0 are both members of triplets, so the Ω' does not lead to any off-diagonal neutral currents [the Glashow, Iliopoulos, and Maiani[35] (GIM) mechanism].

However, the additional octets σ and δ, which were introduced in Sec. IV to give masses to the light Fermions, do induce mixing between triplets and singlets and between the same charge members of one triplet. Hence, the σ and δ will produce small off-diagonal neutral currents between d and b, between s and h, and between c, g, and u. [The discrete symmetry V prevents any direct coupling between the pairs (d,b) and (s,h). Hence there are no $s\to d$ neutral-current components.]

In the lepton sector the σ and δ will lead to small off-diagonal neutral currents between the members of the sets (ν_e, E_1^0, E_2^0), (ν_μ, M_1^0, M_2^0), and (ν_τ, T_1^0, T_2^0).

The coefficients of all of these off-diagonal currents are very small. Hence, they do not cause any phenomenological difficulty. To see this in more detail, consider the Higgs couplings to d^0 and b^0 in Eqs. (4.11) and (2.3):

$$\mathcal{L}=(\bar{d}_L^0, \bar{b}_L^0)\begin{bmatrix} \frac{v_\delta g_u}{\sqrt{2}} & \frac{-v_\sigma g_u}{\sqrt{2}}(\beta+\gamma)\\ \frac{v_\sigma g_u}{\sqrt{2}}(\beta-\gamma) & \frac{-v_\delta g_u}{\sqrt{2}}+\sqrt{2}\,f_u v\end{bmatrix}\begin{bmatrix} d_R^0\\ b_R^0\end{bmatrix}$$

$$+\text{H.c.}$$

$$=-(d_L^0, b_L^0)M\begin{bmatrix} d_R^0\\ b_R^0\end{bmatrix}. \tag{5.2}$$

The weak eigenstates in (5.2) are related to the mass eigenstates d and b by unitary transformations U_L and U_R:

$$\begin{aligned}\begin{bmatrix} d_R^0\\ b_R^0\end{bmatrix}&=U_R\begin{bmatrix} d_R\\ b_R\end{bmatrix},\\ \begin{bmatrix} d_L^0\\ b_L^0\end{bmatrix}&=U_L\begin{bmatrix} d_L\\ b_L\end{bmatrix},\end{aligned} \tag{5.3}$$

where U_L and U_R are chosen so that

$$U_L^\dagger M U_R=\begin{bmatrix} m_d & 0\\ 0 & m_b\end{bmatrix}. \tag{5.4}$$

Of course, m_d and m_b are the bare masses of the d and b quarks. Assume for simplicity that the elements of M are real so that U_R and U_L become rotations by angles θ_R and θ_L. It is then straightforward to show (for $f_u v \gg g_u v_\sigma, g_u v_\delta$) that

$$\begin{aligned} -m_d &\approx \frac{v_\delta g_u}{\sqrt{2}}\,, \\ -m_b &\approx \sqrt{2}\ f_u v\,, \\ \theta_L &\approx \frac{-v_\sigma g_u(\beta+\gamma)}{\sqrt{2}\ m_b}\,, \\ \theta_R &\approx \frac{v_\sigma g_u(\beta-\gamma)}{\sqrt{2}\ m_b}\,. \end{aligned} \tag{5.5}$$

Recalling that the bare mass of the u quark is $m_u \approx -\sqrt{2}\ g_u v_\sigma \gamma$ and assuming that β and γ are of the same order of magnitude, we see that θ_L and θ_R are both of order $m_u/m_b \ll 1$. Consequently, when J_1 and J_2 are written in terms of d and b, the off-diagonal terms will be of order $\sin\theta_L \bar{d}_L\gamma_\mu b_L$ and $\sin\theta_R \bar{d}_R\gamma_\mu b_R$. Similarly, the off-diagonal $\bar{s}h$ terms will be of order $m_s/m_h \ll 1$. (This assumes $|v_\sigma| \approx |v_\delta|$.)

In the charge $\frac{2}{3}$ sector, the largest mixings are the Cabibbo mixings between u_L^0 and c_L^0 and between u_R^0 and g_R^0, which are characterized by the (arbitrary) left- and right-handed Cabibbo angles θ_{CL} and θ_{CR}. Write

$$\begin{aligned} \begin{bmatrix} u_L^0 \\ c_L^0 \end{bmatrix} &= U(\theta_{CL}) \begin{bmatrix} u_L' \\ c_L' \end{bmatrix}\,, \\ \begin{bmatrix} u_R^0 \\ g_R^0 \end{bmatrix} &= U(\theta_{CR}) \begin{bmatrix} u_R' \\ g_R' \end{bmatrix}\,, \end{aligned} \tag{5.6}$$

where u', c', and g' would be the mass eigenstates were it not for the σ octet. (Recall that the u' quark would be massless if σ were absent.) In terms of the primed fields, the mass terms can be written as

$$\begin{aligned} \mathcal{L} &= -m_c' \bar{c}_L' c_R' - m_g' \bar{g}_L' g_R' \\ &\quad + \sqrt{2}\ v_\sigma \gamma (g_u \bar{u}_L^0 u_R^0 + g_c \bar{c}_L^0 g_R^0) \\ &= -m_c \bar{c}_L' c_R' - m_g \bar{g}_L' g_R' \\ &\quad + \sqrt{2}\ v_\sigma \gamma [g_u \bar{u}_L'(\theta_{CL}) u_R'(\theta_{CR}) + g_c \bar{c}_L'(\theta_{CL}) g_R'(\theta_{CR})]\,. \end{aligned} \tag{5.7}$$

The last two terms in (5.7) mix the triplets and the singlets and lead to terms in $J_{1,2}$ of the form $\bar{u}_L\gamma_\alpha g_L \sin\varphi_L$, and $\bar{u}_R\gamma_\alpha c_R \sin\varphi_R$. The angles $\varphi_{L,R}$ may be determined from (5.7). To estimate them we use $-\sqrt{2}\ v_\sigma g_u \gamma \approx m_u$, and assume $v_\sigma^2 = v_\gamma^2$, $-\sqrt{2}\ v_\sigma g_c \gamma \approx m_s$ [see Eq. (4.11)]. We then have

$$\begin{aligned} \varphi_L &= O\left(\frac{m_u \sin\theta_{CR} \pm m_s \sin\theta_{CL}}{m_g}\right) \ll 1\,, \\ \varphi_R &= O\left(\frac{m_u \sin\theta_{CL} \pm m_s \sin\theta_{CR}}{m_c}\right) \ll 1\,. \end{aligned} \tag{5.8}$$

By similar means one can estimate the mixing angles for the leptons and the off-diagonal neutral currents. They are

$$\begin{aligned} \varphi(\nu_E - E_{1L}^0) &= O\left(\frac{m_e}{m_{E_1}}\right) \ll 1, \\ \varphi(E_{1L}^0 - E_{2L}^0) &= O\left(\frac{m_e m_{E_2}}{m_{E_1}^2}\right) \ll 1, \\ \varphi(E_{1R}^0 - E_{2R}^0) &= O\left(\frac{m_e}{m_{E_1}}\right) \ll 1 \end{aligned} \tag{5.9}$$

with similar forms for the μ and τ triplets.

B. Higher-order strangeness-changing effects

It is important to verify that there are no anomalously large contributions to the K_L-K_S mass difference or to the decay $K_L \to \mu^+\mu^-$. It will be seen that these and other effects are of normal (small) magnitude provided that the right-handed Cabibbo angle is fairly small. The key ingredient in the inhibition of these effects is the requirement (through the discrete V symmetry) that all Cabibbo mixings are caused by a Higgs triplet (the Ω') and not through Higgs octets.

We have seen that there is a small mixing between the triplet components (u_L^0, c_L^0) and the singlet g_L^0, as well as between (u_R^0, g_R^0) and c_R^0. The leading diagrams for hadronic $\Delta S=1$ transitions $\bar{s}d \to W\bar{W}, Z$ are shown in Fig. 12, where by leading we mean independent of the u, c, g masses. This means that both W vertices are either left-handed or right-handed. Despite mixing, the mass-independent part of the diagrams cancel, just as in SU(2)× U(1). To see this remember that the charged currents couple to the weak eigenstates $u_{L,R}^0$, c_L^0, g_R^0 and hence with

$$\begin{bmatrix} u_L^0 \\ c_L^0 \\ g_L^0 \end{bmatrix} = U_L \begin{bmatrix} u_L \\ c_L \\ g_L \end{bmatrix}, \quad \begin{bmatrix} u_R^0 \\ c_R^0 \\ g_R^0 \end{bmatrix} = U_R \begin{bmatrix} u_R \\ c_R \\ g_R \end{bmatrix} \tag{5.10}$$

the mass-independent part of, e.g.,

$$\bar{s}_L\gamma_\mu c_L^{0} \bar{u}^{0} \gamma^\mu d_L = \bar{s}_L\gamma_\mu (U_L)_{2i}\, q_i \bar{q}_j\, (U_L^\dagger)_{j1} d_L \tag{5.11}$$

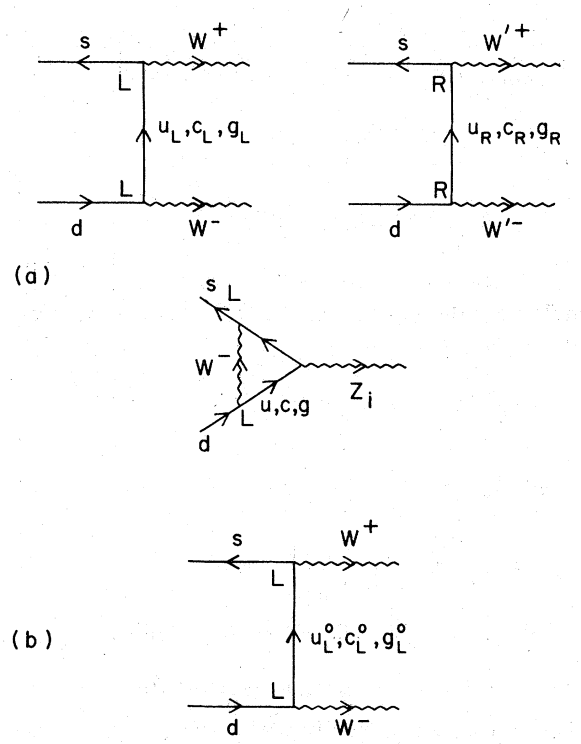

FIG. 12. (a) Several representative $|\Delta S|=1$ diagrams. The leading terms (independent of the u, c, and g masses) cancel. (b) The first diagram rewritten as a sum of weak eigenstates. The diagram, which has no mass insertions, vanishes because none of the weak eigenstates couples to both d and s.

is proportional to $(U_L)_{2j}(U_L^\dagger)_{j1}=0$. Similarly the RR diagrams are proportional to $(U_R)_{3j}\ (U_R^\dagger)_{j1}=0$. This can be restated using mass insertions, in simpler terms; the leading term can be written as a sum over weak eigenstates with no mass insertions. Since no weak eigenstate couples to both d and s, the diagram vanishes [Fig. 12(b)].

There is also a danger from diagrams involving both left- and right-handed currents. The one-mass-insertion diagrams of Fig. 13 would be large and proportional to $m_c \sin\theta_C$ or $m_g \sin\theta_C$ if there were any mass terms in the theory of the form $\bar{c}_L^0 u_R^0$ or $\bar{u}_L^0 g_R^0$. These would have to be due to octet or singlet Higgs fields (or direct-mass terms) that couple the q_u and q_c triplets. We have forbidden such terms by V symmetry so that one-mass-insertion diagrams vanish. Hence, only diagrams with two or more mass insertions contribute to effective $|\Delta S|=0$ neutral transitions.

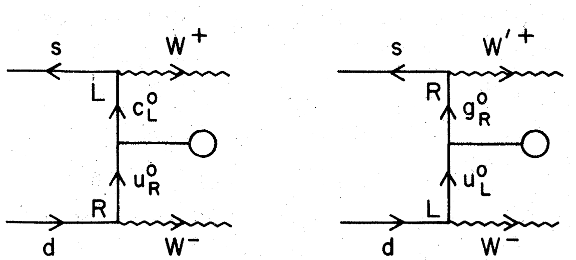

FIG. 13. Two $|\Delta S|=1$ diagrams with one mass insertion. Both diagrams vanish because of the discrete symmetry V which forbids direct couplings between the triplets q_u and q_c.

C. K_L-K_S mass difference

The leading contributions are the box diagrams with two mass insertions on each internal quark line, as shown in Fig. 14. The first diagram is essentially the same as in the $SU(2)_L\times U(1)$ model, while the second diagram has a similar structure. Hence we can take over the results of Gaillard and Lee[36] with appropriate modifications. The result (keeping only the dominant terms from the u-c-g mixing) is

$$\frac{m_{K_L}-m_{K_S}}{m_K}\approx\frac{f_K{}^2}{G\pi^2}\left(G_F{}^2 m_c{}^2\sin^2\theta_{CL}+G_F'^2 m_g{}^2\sin^2\theta_{CR}\right), \quad (5.12)$$

where f_K is the kaon decay constant ($f_K\approx f_\pi\approx 0.96m_\pi$). The experimental value for $(m_{K_L}-m_{K_S})/m_K$ is 7×10^{-15}. If we were to take Eq. (5.12) literally we would require

$$\frac{m_g{}^2}{m_c{}^2}\ \frac{\sin^2\theta_{CR}}{\sin^2\theta_{CL}}\approx 2.2\,, \quad (5.13)$$

where we have used $m_c\approx1.5$ GeV and $(G_F'/G_F)^2\approx0.23$. However, (5.12) should not be taken literally because only the short-distance contribution

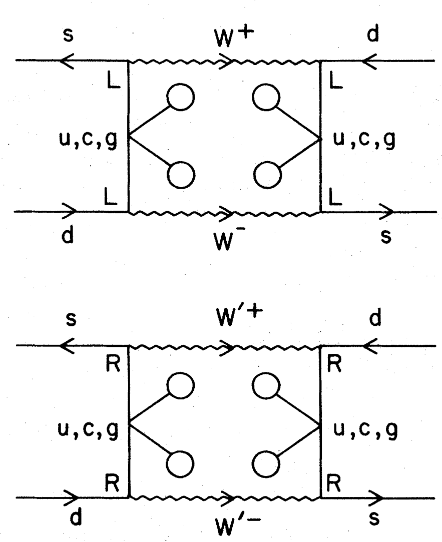

FIG. 14. The leading (two-mass-insertion) contributions to the K_L-K_S mass difference. Diagrams involving Z_i exchange are higher order in G_F.

to the mass difference is included in (5.12). Furthermore, the estimation by Gaillard and Lee[36] of the matrix element $\langle \bar{K}^0 | H_{\rm eff} | K^0 \rangle$, where $H_{\rm eff}$ is due to the diagrams in Fig. 14, is really only an order-of-magnitude estimate. Finally, we have not included the diagrams with three or more mass insertions on the quark lines.

Diagrams such as those of Fig. 14 but with one right-handed and one left-handed vertex, LR,[37] on an internal quark line vanish in the one-mass-insertion limit, as shown earlier. The next contribution comes from three mass insertions, but these appear to be no larger than the diagrams of Fig. 14. We conclude that for the quantity in (5.13) of order unity, the K_L-K_S mass difference is estimated to be of reasonable magnitude.

If we had introduced the Cabibbo angle through d^0-s^0 mixing (rather than u^0-c^0-g^0 mixing) there would be diagrams similar to Fig. 14, but involving the b and h quarks and the W^0 and $\bar{W}^0$. Then, we would have

$$\frac{m_{KL}-m_{KS}}{m_K}=O(f_K{}^2/3\pi^2)[G_F^{02}(m_h-m_b)^2\sin^2\theta_C]) \tag{5.14}$$

which could well be too large.

D. The decay $K_L \to \mu^+\mu^-$

Some possible contributions to the decay are shown in Fig. 15. There is a second pair of box diagrams (not shown) in which the W^+ and W^- are replaced by W'^+ and W'^-, the quark couplings are $V+A$, and the intermediate leptons are M^0_{1L} and M^0_{2R}.

The Z_i-exchange diagrams lead to an effective Hamiltonian of order

$$H^Z_{\rm eff}=O\left(G_F\,\alpha\,\frac{m_c{}^2}{M_Z{}^2}\sin\theta_{CL}\,\bar{s}\gamma_\alpha(1+\gamma_5)d\,\bar{\mu}\gamma^\alpha\mu\right) \tag{5.15}$$

plus a similar right-handed term. This does not contribute to $K\to 2\mu$, however, because the kaon matrix element is

$$\langle 0|\bar{s}\gamma_\alpha(1+\gamma_5)d|K^0\rangle = f_K P_{K\alpha}\,. \tag{5.16}$$

The contraction of the kaon momentum $P_{K\alpha}$ with the purely vector muon neutral current $\bar{\mu}\gamma_\alpha\mu$ vanishes.

Similarly, the leading contributions (with no mass insertions on the lepton lines) to the two box diagrams shown in Fig. 15 are identical, except for the helicities of the leptons. The first yields[35]

$$H_{\rm eff}\approx\left(\frac{G_F}{\sqrt{2}}\right)^2\frac{m_c^2}{\pi^2}\left(\ln\frac{M_+{}^2}{m_c{}^2}-1\right)\sin\theta_{CL}\times\bar{s}\gamma_\alpha d_L\,\bar{\mu}_L\gamma^\alpha\mu_L\,, \tag{5.17}$$

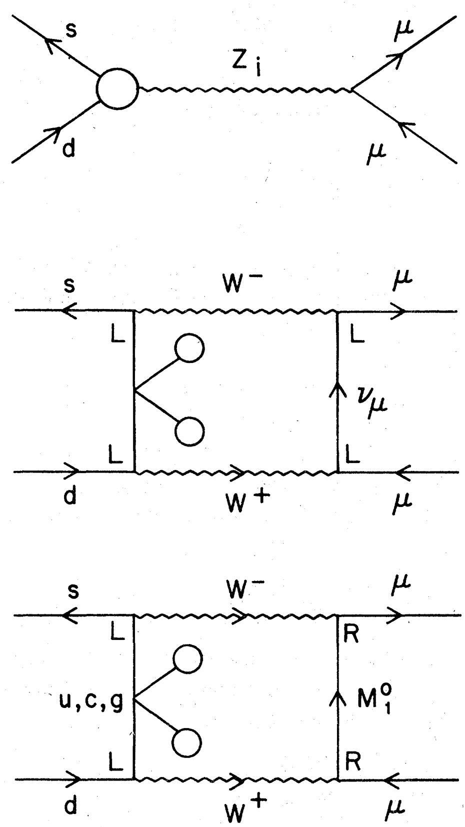

FIG. 15. Some contributions to $K_L\to\mu^+\mu^-$. The effective $\bar{s}\,\gamma_\mu d\,Z_i^\mu$ vertex is of order G_F, but the diagram does not contribute because the $\bar{\mu}\,\gamma_\mu\mu\,Z_i^\mu$ vertex is purely vector.

while the second is the same except $\mu_L\to\mu_R$. Hence, the sum of the two diagrams also has a purely vector effective muon current and does not contribute to K decay. The same of course holds for the two W'-exchange diagrams.

The diagrams with two mass insertions on the lepton line do contribute, however. We have roughly estimated their contribution to the branching ratio[38] for $K_L\to\mu^+\mu^-$ to be around 10^{-10}, so that the decay is probably dominated by $K_L\to 2\gamma\to\mu^+\mu^-$.

As we have seen, many cancellations have occurred here because the effective neutral muon current is vectorlike and hence vanishes when dotted into the K_L momentum. This cancellation does not take place for decays in which the effective hadronic current is vectorlike such as, e.g., $K^+\to\pi^+e^+e^-$, $K^+\to\pi^+\nu\bar{\nu}$; they are, nevertheless, no larger in our model than in the usual[36] SU(2)×U(1) model. The reason is the general arguments given in Sec. VB for $\Delta S=1$ transitions $s\bar{d}\to W,\bar{W}$ and $s\bar{d}\to Z$.

E. Nonleptonic decays

There are two major contributions to nonleptonic strangeness-changing decays. The first is

$$H^1_{\rm eff}=\frac{4G_F}{\sqrt{2}}\,\bar{d}_L\gamma_\alpha u_L\,\bar{u}_L\gamma^\alpha s_L\cos\theta_{CL}\sin\theta_{CL}+\text{H.c.} \tag{5.18}$$

which is the same as occurs in $SU(2)_L\times U(1)$. The second contribution involves W' exchange and is $V+A$:

$$H^2_{\rm eff}=\frac{4G'_F}{\sqrt{2}}\,\bar{d}_R\gamma_\alpha u_R\bar{u}_R\gamma^\alpha s_R\cos\theta_{CR}\sin\theta_{CR}+\text{H.c.} \tag{5.19}$$

The two terms transform oppositely under the chiral $SU(3)\times SU(3)$ of the strong interactions. That is

$$\frac{[{}^5F^i,H^1_{\rm eff}]}{[F^i,H^1_{\rm eff}]}=-\frac{[{}^5F^i,H^2_{\rm eff}]}{[F^i,H^2_{\rm eff}]}\,, \tag{5.20}$$

where F^i and ${}^5F^i$ are the $SU(3)\times SU(3)$ generators. In order to avoid conflicts[39] with current algebra results we require

$$\frac{G'_F\sin\theta_{CR}\cos\theta_{CR}}{G_F\sin\theta_{CL}\cos\theta_{CL}}\ll 1. \tag{5.21}$$

In practice, it is probably sufficient if the ratio is no more than 20%. (The ratio G'_F/G_F is around $\frac{1}{2}$.) This is the strongest restriction on $\sin\theta_{CR}$ in the model.

As in most other models, there is no natural enhancement of the $\Delta I=\frac{1}{2}$ amplitude. One must rely on short distance effects[40] or other dynamical enhancements.

F. Semileptonic decays

We note briefly that the mixings between b_R and d_R and between ν_e and E^0_{IL} induce a very small right-handed component in β decay $d_R\to u_R e^-\bar{\nu}_e$. (If a single octet of Higgs fields had components in both the U_1 and U_3 directions, the ensuing W-W' mixing would lead to right-handed contributions of the same order.) The b_R-d_R mixing is of order m_u/m_b. For $m_u\approx 5$ MeV and $m_b\approx 5$ GeV, this would merely induce a 0.1% change in the effective axial coupling g_A.

G. B^0-$\bar{B}^0$ mixing

There has been a good deal of discussion of D^0-$\bar{D}^0$ mixing.[37,41] In the standard four-quark model[1,35] the off-diagonal matrix elements in the mass matrix are small compared to the decay rates. If we call λ the average of the decay rates of D^0 and $\bar{D}^0$ and $\Delta\lambda$ the difference and Δm the mass difference, we find $\Delta\lambda/\lambda\ll 1$, $\Delta m/\lambda\ll 1$, the suppression being at least of order $\tan^2\theta_c$ (θ_c is the Cabibbo angle).[37] Models were considered,[42] however, in which $\Delta\lambda/\lambda$, $\Delta m/\lambda\sim 1$ and, by introducing large charm-changing neutral currents, models[37] in which $\Delta m/\lambda\gg 1$. Recent data, however,[43] rule out a large D^0-$\bar{D}^0$ mixing.

The D^0-$\bar{D}^0$ mixing in our model is due to the very small charm-changing neutral currents and to box diagrams with internal b and h quark lines. Although Δm and λ depend on unknown parameters, it appears that D^0-$\bar{D}^0$ mixing is unimportant.

On the other hand, there is a large effective Hamiltonian for changing the b quantum number by two units, because the W^0 couples to both $\bar{b}d$ and $\bar{d}b$. It is

$$H_{\rm eff}=\frac{g^2}{2M_0^2}\bar{b}_L\gamma_\alpha d_L\bar{b}_R\gamma^\alpha d_R+\text{H.c.} \tag{5.22}$$

Let us call B^0, $\bar{B}^0$ the analogs of the D^0, $\bar{D}^0$ or K^0, $\bar{K}^0$ states, namely, the pseudoscalar $b\bar{d}$ and $\bar{b}d$ states (presumably the lowest mesonic states with a b quark). The B^0, $\bar{B}^0$ mass matrix has a term Δm proportional to G^0_F because of (5.22), while the terms in λ are all proportional to some G^2_F; we therefore see that $\Delta m/\lambda|_{B^0,\bar{B}^0}\gg 1$ and complete mixing occurs, i.e., B^0 and $\bar{B}^0$ oscillate into each other many times before decaying and

$$r=\frac{\Gamma(B^0\to\mu^-\bar{\nu}_\mu+X)}{\Gamma(B^0\to\mu^+\nu_\mu+X)}\approx 1\,. \tag{5.23}$$

The analog would be that the superweak[44] interaction is $\lesssim G_F$ rather than $\lesssim 10^{-9}\,G_F$.

The consequences of these B^0-$\bar{B}^0$ oscillations in neutrino scattering have already been discussed in Sec. III and need not be repeated, except perhaps to reemphasize the possibility of a relatively large $\mu^+\mu^+$ production cross section in $\bar{\nu}_\mu N$ scattering, as estimated in Eq. (3.41).

B^0-$\bar{B}^0$ mixing could lead to same-sign leptons in e^+-e^- collisions:

$$\begin{aligned} e^+e^-\to B^0\bar{B}^0\to \mu^+\mu^+ &+X\,,\\ \mu^-\mu^- &+X\,,\\ e^+e^+ &+X\,,\\ e^-e^- &+X \end{aligned} \tag{5.24}$$

though the cross section is presumably small.

$$\frac{\sigma(e^+e^-\to\mu^+\mu^++X)}{\sigma(e^+e^-\to\mu^+\mu^-)}\sim\frac{(s-4m_b^2)^{1/2}}{\sqrt{s}}\left(1+\frac{2m_b^2}{s}\right)(Q_{b,\rm eff})^2\times\tfrac{1}{2}fB(\bar{b}\to\mu^++X)B(\bar{B}^0\to\mu^++X)\,, \tag{5.25}$$

where $Q_{b,\rm eff}$ is equal to $-\frac{1}{3}$, the b charge, if we imagine the sequence occurring by $e^+e^-\to b\bar{b}$, $\bar{b}$ decaying into $\mu^+ + X$, and b picking up a $\bar{d}$ to form a B^0 which then oscillates into a $\bar{B}^0$ and then decays. Our estimate for the ratio in (5.25) is $\approx 10^{-3} f$ for $s \gg m_b^2$.

H. The muon magnetic moment

A recent high-precision measurement of the muon magnetic moment[45] requires $-4.2\times10^{-8} < a_\mu^w < 1.6\times10^{-8}$, where a_μ^w is the weak contribution to $\frac{1}{2}(g_\mu - 2)$.

There are two important contributions to a_μ^w in this model. The Z-boson contribution, shown in Fig. 16(a), is around 1×10^{-8}. The leading contribution from the heavy leptons are shown in Fig. 16(b). They give[32]

$$a_\mu^w = \frac{G_F m_\mu}{\sqrt{2}\,4\pi^2}\left(\frac{-v_\sigma g_\mu(\beta+\gamma)}{\sqrt{2}}\right) + \frac{G_F' m_\mu}{\sqrt{2}\,4\pi^2}\left(\frac{v_\sigma g_\mu(\beta-\gamma)}{\sqrt{2}}\right) + Z\text{-boson term}. \quad (5.26)$$

If we assume that β and γ are of the same order of magnitude, then the mass insertions are of the magnitude of $m_\mu \approx -\sqrt{2}\,v_\sigma g_\mu \gamma$; then the heavy-lepton contribution to a_μ^w is

$$a_\mu^w = O\left(\frac{G_F m_\mu{}^2}{\sqrt{2}\,4\pi^2}\right) + Z\text{-boson term} \approx 3\times10^{-9} + Z\text{-boson term}. \quad (5.27)$$

Hence, there is no problem with $g_\mu - 2$.

I. The decay $\mu \to e\gamma$

The Higgs couplings and discrete symmetries we have introduced in Secs. II and IV do not contain any mixing between the electron and muon triplets.

However, the Higgs structure can be modified, if desired, to allow a small $\mu - e\gamma$ rate. Just as in the quark sector, we insist that any mixing between the l_e, l_μ, and l_τ triplets be due to Higgs triplets and not through Higgs octets or singlets.

Recall that the triplet Φ was introduced to give mass to the E_2^0, M_2^0, and T_2^0. From (2.3), the mass terms are

$$\mathcal{L}_\Phi = k_e v_\Phi \bar{E}_{2L}^0 E_{2R}^0 + k_\mu v_\Phi \bar{M}_{2L}^0 M_{2R}^0 + k_\tau v_\Phi \bar{T}_{2L}^0 T_{2R}^0. \quad (5.28)$$

We could also introduce additional triplets Φ', Φ'', etc., with different transformation properties under V, which could yield mixing terms such as $\bar{E}_{2L}^0 M_{2R}^0$, $\bar{M}_{2L}^0 E_{2R}^0$, $\bar{E}_{2L}^0 T_{2R}^0$, etc. into the mass matrix. (This is analogous to the way the Ω' triplet was used to generate the Cabibbo angle.)

The $\mu \to e\gamma$ decay can take place through these E_2^0-M_2^0 mixing terms. Typical two- and three-mass-insertion terms are shown in Fig. 17(a). These are of the same structure as the terms studied by Cheng and Li[46] in an SU(2)$\times$U(1) model, and yield a branching ratio for $\mu \to e\gamma$ of $\lesssim 10^{-9}$. The one-mass-insertion diagram of Fig. 17(b) would be dangerously large if the mass insertion were nonzero. However, an $\bar{E}_{1L}^0 M_{2R}^0$ (or $\bar{M}_{1L}^0 E_{2R}^0$) coupling would require a Higgs octet or singlet to couple q_{eL} to $q_{\mu R}$. This is forbidden by V symmetry.

To rephrase the argument W^+, $W^{+\prime}$ couple $e_{L,R}$ and $\mu_{L,R}$ only to members of $q_{eL,R}$ and $q_{\mu L,R}$, respectively, but there is no Higgs coupling which takes $q_{eL,R}$ to $q_{\mu R,L}$ so one-mass-insertion diagrams are forbidden. The leading terms are therefore two-mass-insertion LL and RR diagrams and three-mass-insertion LR diagrams, both of which

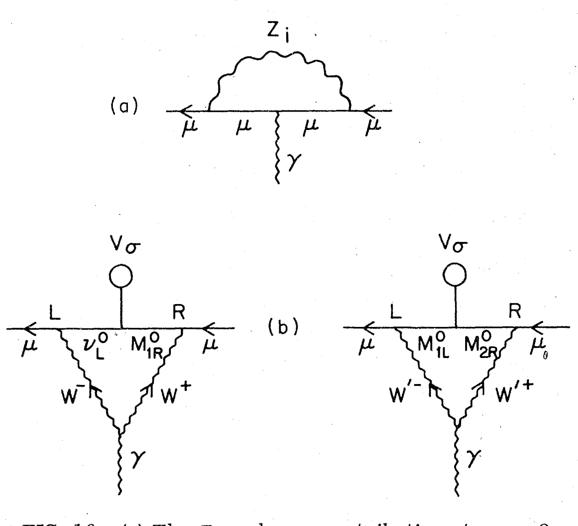

FIG. 16. (a) The Z-exchange contributions to $g_\mu - 2$. (b) The leading (one-mass-insertion) contributions of the heavy leptons to $g_\mu - 2$. There are two more diagrams with the left- and right-handed vertices interchanged.

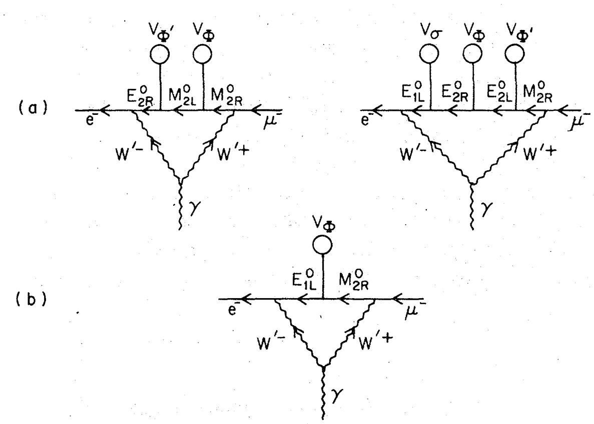

FIG. 17. (a) Two- and three-mass-insertion diagrams leading to $\mu \to e\gamma$. (b) A one-mass-insertion diagram that vanishes because of the discrete symmetry V.

give contributions to $\mu \to e\gamma \lesssim 10^{-9}$.

A second way of inducing some general mixing among the charged leptons is to let the Ω' triplet couple the singlets τ_R^-, T_L^- to the triplets q_e and q_μ. This situation would be quite similar to the u-c-g mixing we discussed in Sec. V A. The triplet-singlet mixing, combined with the effect of the σ, δ mesons would lead to off-diagonal neutral-current coupling; just as we had u_L-g_L and u_R-c_R terms in $J_{1,2}$, we would now have e_L^--T_L^-, μ_L^--T_L^-, e_R^--τ_R^-, μ_R^--τ_R^-, and τ^--T^- terms in $J_{1,2}$. Since these are caused by the presence of singlet-triplet couplings *and* σ, δ terms, one might expect these mixings to be very small, since in general the σ, δ couplings are small, i.e., using Eq. (4.11) we see $g_{e,\mu} v_\sigma \sim m_{e,\mu}$. There is no *a priori* reason, however, why $g_\tau v_\sigma$ could not be much larger than m_μ and hence lead to appreciable mixing. At a higher order of mixing, we find this mechanism leads to a μ-e mixing term in $J_{1,2}$ which in turn would lead to $\mu \to 3e$ decays via the neutral-current interaction. Since there does not appear to be a natural way to suppress this (postulating small mixing angles is unnatural), we regard it as preferable to allow μ-e mixing to occur via Φ' if at all and forbid by discrete symmetry l_e, l_μ couplings to τ_R^-, T_L^- via Ω or Ω'.

VI. CONCLUSIONS

The gauge model of weak and electromagnetic interactions we have considered here has been shown to have many virtues, among which are the following: (1) an essentially null effect for parity violation in bismuth (independently of Higgs mechanisms), (2) vectorlike theory, (3) agreement with neutral-current data, and (4) mechanism for trimuon production.

It is by no means unique; other SU(3)$\times$U(1) models with many of these features have been presented,[17] but it seemed to us to be the model of its type which most economically fits present data. In a separate publication we shall examine the merits of alternative classifications of the leptons and fermions.[47] There are significant differences, e.g., if the leptons are in the $\underline{3}$ or the $\underline{3}^*$ representation. Both models give reasonably good fits to the inclusive neutral-current data, but the Higgs structures are different and hence the couplings of the fermions to neutral currents are different: This shows up, for example, in different predictions for $\nu + N \to \nu + N + \pi$. In this case the $\underline{3}^*$ model considered in this paper agrees with the analysis, whereas the model with leptons in the $\underline{3}$ does not. Even with our specific assignment of leptons to the $\underline{3}^*$ and quarks to the $\underline{3}$ representation, there is still considerable freedom via mixing with singlets or possibly among triplets: for instance, one could introduce singlets u_R and t_L and replace u_R by t_R. In this case there is more parity violation in the neutral current, the lowest-mass state of E_2^0, M_2^0, and T_2^0 decays via u_R-t_R mixing, e.g., $E_2^0 \to e^- u_R \bar{d}_R$ and in general more leptons are produced in decays of the heavy leptons and of the b quark.

It is also important to look for tests of these SU(3)$\times$U(1) models, ways in which they differ from, e.g., SU(2)$_L\times$SU(2)$_R\times$U(1) models. Several such examples occur in the lepton sector; in the hadronic sector, $\bar{B}^0$-B^0 mixing is one such example.

Finally, we turn to the question of a true unification of weak, strong, and e.m. interactions. SU(6) is a promising candidate with the maximal subgroup SU(3)$\times$SU(3)$\times$U(1) including both weak and color SU(3) groups. Several papers[15] have already appeared on this topic with the fermions being in the $\underline{15} = (\bar{3},1)+(3,3)+(1,\bar{3})$ representation.

Two extremely encouraging results of embedding our SU(3)$\times$U(1) in SU(6) (Ref. 48) are that the parameters $\cos^2\theta$ (Weinberg-like angle) and $\eta'(|v_\Omega/v_\Phi|^2)$ are fixed. The first is determined by the group theory to be 0.20 and the second to be zero since an Ω field cannot be present. From Table VI, we see that these values are the same as those we obtained by a phenomenological fit to the neutral-current data. In addition the proton can be made absolutely stable.

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*Present address: Department of Physics, Arya-Mehr University of Technology, Tehran, Iran.

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