Properties of neutrinos in a class of gauge theories

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We discuss the properties of neutrinos in a class of gauge theories characterized by manifest left-right symmetry and spontaneous genesis of parity nonconservation. Topics discussed in detail include: currents with anomalous Lorentz structure (herein called class III currents), electromagnetic properties of neutrinos, and rare processes involving neutrinos which are forbidden in the two-component theory. Calculations of the relevant coupling parameters and reaction rates are presented and compared to the best limits available from terrestrial experiments and astrophysical considerations. Some cosmological implications are also discussed, and it is suggested that all long-lived low-mass neutrinos have been discovered.

I. INTRODUCTION

During the last few years, a number of authors have investigated the possibility that weak interactions admit of invariance under the operation of left-right conjugation and that parity-nonconservation effects stem from the spontaneous breakdown of this left-right symmetry.[1-3] Within the framework of the conventional procedure, using kinematical Higgs fields, for constructing unified gauge models of weak and electromagnetic interactions,[4] the notion of spontaneous parity nonconservation can be implemented in a vast variety of ways. For a given gauge group, these various realizations are expected to merge into one at very short distances[5]; however, at low frequencies they are very different indeed.

Our purpose in this note is to discuss some properties of neutrinos in a particular class of left-right-symmetric gauge theories, those which embody the principle of *manifest* left-right (LR) symmetry.[2] In such theories the physical left-handed and right-handed currents are obtainable from each other via a γ_5 flip; more precisely,

$$J^{\text{physical}}_{L,\rho} = J^{\text{physical}}_{R,\rho}(\gamma_5 \to -\gamma_5) + \text{finite loop-level corrections (manifest realization).} \quad (1.1)$$

Here ρ is a Minkowski index and we have omitted internal indices. Equation (1.1) is to be distinguished from the corresponding relationship in nonmanifest realizations of LR symmetry:

$$J^{\text{physical}}_{L,\rho} = J^{\text{physical}}_{R,\rho}(\gamma_5 \to -\gamma_5) + \text{tree-level corrections (nonmanifest realization).} \quad (1.2)$$

Our discussion of neutrino properties begins with the observation that Eq. (1.1) is incompatible with the two-component neutrino theory; to achieve manifest left-right symmetry one is perforce obliged to have four-component neutrinos. With four-component neutrinos it is necessary to reconsider a variety of effects that were legislated away by the two-component theory and which have not been fully discussed in the context of renormalizable gauge theories.[6] We should also point out that although our motivation for considering this possibility is derived from the concept of manifest left-right symmetry, our analysis is applicable to any gauge theory which incorporates four-component neutrinos.

This paper is organized as follows: In Sec. II we consider the present limits on neutrino masses and in Sec. III we discuss the genesis of what we call class III currents (i.e., currents involving the S, P, T covariants). Sec. IV is devoted to a discussion of electromagnetic properties of neutrinos. In Sec. V we discuss some rare processes which are exactly forbidden in the two-component theory. We conclude (Table I—Sec. VI) with a review of neutrino parameters as gleaned from theory, terrestrial experiments, and astrophysical considerations, and a brief discussion of the cosmological implications of four-component neutrinos.

II. NEUTRINO MASSES

With four-component fields in the game, there is no reason to believe that the neutrino cannot acquire a mass either at the tree level via a Higgs tadpole or through loop effects. (We remind the reader that two-component neutrinos are automatically massless provided the theory conserves lepton number.[7]) Indeed if any of the neutrinos turn out to be exactly massless, this masslessness would be a rather miserable experimental fact, i.e., "unnatural" in the technical jargon of gauge theories.

The best experimental limits on the masses of the three neutrinos[8] that have so far been observed are as follows[9]:

$$m(\nu_e) < 35 \text{ eV}, \quad (2.1)$$

$$m(\nu_\mu) < 510 \text{ keV}, \quad (2.2)$$

$$m(\nu_\tau) < 450 \text{ MeV}. \quad (2.3)$$

In addition to these laboratory bounds, there exists a well-known cosmological argument[10] against the existence of stable neutrinos with masses greater than about 40 eV and less than about 2 GeV.

III. CLASS III CURRENTS

By class III currents we mean currents of the form $\bar\nu\nu$, $\bar\nu\gamma_5\nu$, and $\bar\nu\sigma_{\lambda\mu}\nu$; without a two-component constraint their participation in weak processes is no longer precluded by any general principle. Couplings involving class III currents may be expected to give rise to anomalous features in neutral-current processes.[6] However, if the underlying theory is a renormalizable gauge theory, such couplings are finite and calculable and—as we shall see—turn out to be rather small.

We consider the process $\nu + f \to \nu + f$ where f, for the sake of definiteness, is taken to be a spin-$\frac{1}{2}$ *hadron*. Also, we assume that all the Higgs fields in the theory are characterized by zero baryon number. We may then parametrize the ν-f interaction arising from class III currents, at low energies and momentum transfers, in terms of a phenomenological Lagrangian:

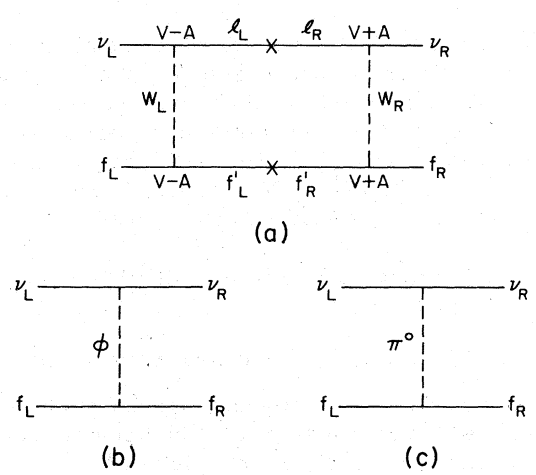

FIG. 1. Graphs contributing to class III couplings. The crosses in (a) indicate mass insertions.

$$\mathcal{L}^{\nu f} = G_S^{\nu f}(\bar\nu\nu)(\bar f f) + G_P^{\nu f}(\bar\nu\gamma_5\nu)(\bar f\gamma_5 f) + G_T^{\nu f}(\bar\nu\sigma_{\lambda\mu}\nu)(\bar f\sigma^{\lambda\mu} f). \quad (3.1)$$

In writing Eq. (3.1) we have only assumed CP invariance; however, for couplings of class III currents CP invariance actually implies invariance under *both* C and P.

To second order in the semiweak coupling, g, the parameter G_T vanishes; the first contribution arises in order g^4 through graphs of the type depicted in Fig. 1(a). However, G_S and G_P need not vanish in order g^2 [Figs. 1(b) and 1(c)]. If the same Higgs field, ϕ, couples to both ν and f, G_S will receive its major contribution from the exchange of the physical Higgs particle[11] described by the field $:\phi:$. Furthermore, G_P can also arise from the normal (class I) selfinteraction of the neutral axial current; indeed, if f admits of a Yukawa coupling to π^0 (as we assume hereafter), we expect G_P to be determined largely by the one-pion exhange graph, in much the same way as is the pseudoscalar term in muon capture.[12]

Our estimates of class III couplings, based on the above considerations, are as follows:

$$G_S^{\nu f} = \frac{1}{|\langle\phi\rangle|^2}(m_\nu m_f/m_\phi{}^2) + O(G_T^{\nu f}), \quad (3.2)$$

$$G_P^{\nu f} = G_F(g_A^f\sqrt{2})(m_\nu m_f/m_\pi{}^2) + O(G_T^{\nu f}), \quad (3.3)$$

$$G_T^{\nu f} \lesssim (0.1-1)G_F{}^2 m_l m_{f'} + O(G_F{}^2 m_\nu m_f), \quad (3.4)$$

where l and f' belong to the same weak isotopic doublets as ν and f, respectively. In deriving Eq. (3.3) we have used the neutral current of Ref. 3 in the limit $\beta = 0$ (no trimuon production) and $\gamma = 0$ (no parity violation in atomic physics). Also, in Eq. (3.4), we have used the bound[2] $m_{W_R}{}^2 \gtrsim 10 m_{W_L}{}^2$.

For m_{ν_μ} as large as 150 keV (say), the largest class III coupling is the π^0-exchange contribution in Eq. (3.3):

$$G_P^{\nu N} \cong G_F \cdot (12 m_{\nu_\mu}/m_\pi) \sim 10^{-2} G_F \quad (m_{\nu_\mu} = 150 \text{ keV}). \quad (3.5)$$

IV. ELECTROMAGNETIC PROPERTIES

A. Preliminary considerations

The matrix element of the electromagnetic current between invariantly normalized neutrino states may be expressed in the form

$$\langle\nu(p_2,\lambda_2)|J_\mu^{\text{em}}(0)|\nu(p_1,\lambda_1)\rangle = \bar u_{\lambda_2}(p_2)\left\{[F_1(q^2)+\gamma_5 G_1(q^2)]\gamma^\rho\left(g_{\rho\mu} - \frac{q_\rho q_\mu}{q^2}\right) + [F_2(q^2)+\gamma_5 G_2(q^2)]i\sigma_{\mu\rho}q^\rho\right\}u_{\lambda_1}(p_1). \quad (4.1)$$

Here p denotes the momentum and λ denotes the helicity of the neutrino, $q \equiv p_2 - p_1$. The F_i and G_i are invariant form factors. Their values in the neighborhood of $q^2 = 0$ determine the "static" electromagnetic properties of the neutrino.

The charges carried by left-handed and right-handed neutrinos may be identified with $F_1(0) + G_1(0)$ and $F_1(0) - G_1(0)$, respectively. Hence the neutrality conditions:

$$F_1(0) = G_1(0) = 0. \tag{4.2}$$

Furthermore, if time-reversal invariance is assumed to be good, the neutrino cannot have an electric dipole moment, i.e.,

$$G_2(0) = 0. \tag{4.3}$$

Thus, the most important static parameter is the magnetic moment; following customary practice[13,14] we express it in units of Bohr magnetons:

$$F_2(0) = \kappa_\nu (e/2m_e)\,, \tag{4.4}$$

κ_ν being a dimensionless number.

The next important parameters are the radii

$$\begin{aligned} \langle r_V^2 \rangle &= +6F_1'(0), \\ \langle r_A^2 \rangle &= +6G_1'(0). \end{aligned} \tag{4.5}$$

Unfortunately the radii do not easily lend themselves to measurement. In terrestrial scattering experiments, for example, they are masked completely by weak-neutral-current effects; in astrophysical processes the typical values of q^2 are much too small to probe finite-radius effects. In spite of these shortcomings, one can infer from the observed neutral-current cross sections (for ν_e and ν_μ) that[13,14] $\langle r^2 \rangle \lesssim 10^{-30}$–$10^{-31}$ cm^2. This is to be compared with the usual range of values found in gauge theories[15]: $\langle r^2 \rangle \sim 10^{-33}$–$10^{-34}$ cm^2. (These values are independent of m_ν and valid for two-component neutrinos.)

B. Theoretical value of the magnetic moment

The magnetic moment may be readily calculated in any of the available gauge models[15]. We focus on a $U(1) \otimes SU(2)_L \otimes SU(2)_R$ model[16] with six leptons: (ν_e, e^-), (ν_μ, μ^-), and (ν_τ, τ^-). The leptons and Higgs fields are assigned to representations of the gauge group in such a way that the left-handed charge raising leptonic current is

$$J_\rho^L = \tfrac{1}{2}\left[\bar\nu_e \gamma_\rho (1-\gamma_5) e + \bar\nu_\mu \gamma_\rho (1-\gamma_5)\mu + \bar\nu_\tau \gamma_\rho (1-\gamma_5)\tau\right], \tag{4.6}$$

and the right-handed current satisfies Eq. (1.1).

Calculation of κ_ν then entails calculation of the four graphs in Fig. 2. (We ignore possible contributions involving *physical* Higgs fields.) The intermediate boson fields W_1 and W_2, in these graphs, are eigenstates of the mass matrix with predominantly left-handed and right-handed couplings, respectively.[2] In the notation of Ref. 2

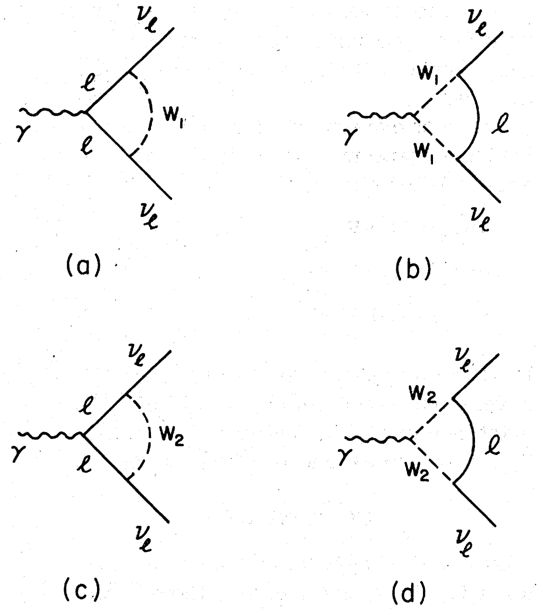

FIG. 2. Graphs contributing to magnetic moment of neutrinos.

$$W_1 = W_L \cos\zeta - W_R \sin\zeta\,, \tag{4.7}$$

$$W_2 = W_L \sin\zeta + W_R \cos\zeta\,, \tag{4.8}$$

where W_L and W_R are fields with pure $V-A$ and $V+A$ couplings, respectively, and ζ is a mixing angle. Evaluation of the graphs in Fig. 2 yields[15]

$$\kappa_{\nu_l} = \frac{G_F}{\pi^2\sqrt{2}} m_e \left[m_l \left(1 - \frac{m_{W_1}{}^2}{m_{W_2}{}^2}\right) \sin 2\zeta + \tfrac{3}{4} m_{\nu_l} \left(1 + \frac{m_{W_1}{}^2}{m_{W_2}{}^2}\right)\right]. \tag{4.9}$$

All gauge models will yield expressions of this form. The term proportional to m_l is due to left-right mixing in the physical charged current.[2] Of course, κ_{ν_l} vanishes for a two-component neutrino as is obvious from (4.1). Using the analysis of Ref. 2, to bound $(1 - m_{W_1}{}^2/m_{W_2}{}^2)\sin 2\zeta$ in Eq. (4.9), we obtain the upper limits (for m_{ν_l} small)

$$|\kappa_{\nu_l}| < \frac{0.12}{\pi^2\sqrt{2}} G_F m_e m_l = 2.6 \times 10^{-14}\left(\frac{m_l}{m_e}\right). \tag{4.10}$$

C. Experimental limits on magnetic moments (terrestrial)

Neutrino-electron scattering experiments have been used to set bounds on κ_ν. The Reines-Cowan experiment, on $\bar\nu_e + e^- \to \bar\nu_e + e^-$, yielded[13]

$$|\kappa_{\nu_e}| < 1.4 \times 10^{-9}, \tag{4.11}$$

whereas the Gargamelle measurement of $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ yielded[14]

$$|\kappa_{\nu_\mu}| < 8.1 \times 10^{-9}. \tag{4.12}$$

D. Astrophysical limits on magnetic moments

The best known example of a dynamical Higgs mechanism in operation is the acquisition of mass by photons propagating through a dense plasma. These massive "photons," commonly called plasmons,[17] may decay into $\nu\bar{\nu}$ pairs thereby providing a mechanism for energy loss from the plasma. Thus any known bound on the rate of energy loss from a hot, dense, plasma is convertible into a bound on κ_ν provided the plasma frequency (effective photon mass) exceeds $2m_\nu$.

The plasma-neutrino process in stellar interiors has been widely discussed in the astrophysical literature.[18] The systems most convenient for our purpose are the objects known as degenerate dwarfs; in these objects the plasma frequency may be as high as 50 keV, thus permitting us to glean information about neutrinos as massive as ~10 keV. The energy-loss bounds envisaged by Sutherland *et al.*,[18] based upon the theoretical evolution of degenerate dwarfs, imply

$$|\kappa_{\nu_l}| < 8.5 \times 10^{-11} \quad (\text{for } m_{\nu_l} < 10 \text{ keV}) \tag{4.13}$$

where l could be e, μ, τ, or any other, as yet undiscovered, lepton.[19]

Another interesting result emerges in the (possibly hypothetical) situation in which e, μ, and τ numbers are not separate constants of the motion and only the total lepton number is a conserved entity.[20] For then, phase space permitting, the plasmon may dissociate into $\bar{\nu}_{l_1}\nu_{l_2}$ or $\bar{\nu}_{l_2}\nu_{l_1}$ pairs and one may set a bound on the transition magnetic moment operative in the process $\nu_{l_2} \to \nu_{l_1} + \gamma$:

$$|\kappa_{\nu_{l2}\to\nu_{l1}}| < 6.0 \times 10^{-11} \quad [\text{for } \tfrac{1}{2}(m_{\nu_{l1}} + m_{\nu_{l2}}) < 10 \text{ keV}]. \tag{4.14}$$

This bound limits the allowable decay rate for any light neutrino,[15] i.e.,

$$\Gamma(\nu_{l_2} \to \nu_{l_1} + \gamma) = \frac{\alpha}{8}\left(\frac{m_{\nu_{l2}}^2 - m_{\nu_{l1}}^2}{m_{\nu_{l2}}}\right)^3 \left(\frac{\kappa_{\nu_{l2}\to\nu_{l1}}^2}{m_e^2}\right) < 3.3 \times 10^{-24} \frac{1}{m_e^2}\left(\frac{m_{\nu_{l2}}^2 - m_{\nu_{l1}}^2}{m_{\nu_{l2}}}\right)^3. \tag{4.15}$$

E. A constraint on τ couplings

In the foregoing discussion we assumed that the couplings of the τ lepton are isomorphic to those of the electron and the muon. However, at present there is little experimental evidence to uphold this assumption, and it seems prudent to keep open the possibility that e-μ universality may not generalize into e-μ-τ universality. We find it worthwhile, therefore, to take note of a constraint on τ couplings that emerges from Eq. (4.13).

We do not commit ourselves to any specific gauge model but merely assume that the bulk of the weak interaction is mediated by a single pair of intermediate boson fields, $W^{(+)}$ and $W^{(-)}$, and that the effective $\tau\nu_\tau W$ coupling is of the form

$$\mathcal{L}_I = \frac{g}{\sqrt{2}}\bar{\nu}_\tau\gamma_\mu\left[a\left(\frac{1-\gamma_5}{2}\right) + b\left(\frac{1+\gamma_5}{2}\right)\right]\tau^- W^{(+)\mu} + \text{H.c.}. \tag{4.16}$$

Then the calculation[15] that led to Eq. (4.9) along with the astrophysical bound for κ_{ν_τ}, Eq. (4.13), implies

$$|ab| < 0.05 \quad (\text{for } m_\tau \cong 1.9 \text{ GeV},\ m_{\nu_\tau} < 10 \text{ keV}). \tag{4.17}$$

Thus if the mass of ν_τ is of the order of 10 keV or less, the ν_τ-τ current is either almost pure $V-A$ ($b=0$) or almost pure $V+A$ ($a=0$). A preliminary analysis of the so-called μ-e events indicates that the former solution is preferred to the latter.[21]

V. SOME RARE PROCESSES

A variety of decay processes, forbidden exactly in the two-component neutrino theory, become possible if the neutrino fields are endowed with four components. As examples we consider $\pi^0 \to \nu\bar{\nu}$ and $\nu_\mu \to \nu_e + \gamma$, the latter being of course still forbidden if muon number is indeed a constant of the motion.

In the framework of the model of Ref. 3 and on the basis of considerations similar to those that led to Eq. (3.4) for G_P, we find

$$\Gamma(\pi^0 \to \nu\bar{\nu}) = \frac{1}{\pi}G_F^2 m_\pi f_\pi^2 m_\nu^2\left(1 - \frac{4m_\nu^2}{m_\pi^2}\right)^{1/2}, \tag{5.1}$$

f_π being the usual pion decay constant. Hence the branching ratio[22]

$$\frac{\Gamma(\pi^0 \to \nu\bar{\nu})}{\Gamma(\pi^0 \to 2\gamma)} = \left(\frac{8\pi G_F f_\pi^2}{\alpha}\right)^2\left(\frac{m_\nu}{m_\pi}\right)^2\left(1 - \frac{4m_\nu^2}{m_\pi^2}\right)^{1/2} = 1.2 \times 10^{-7}\left(\frac{m_\nu}{m_\pi}\right)^2\left(1 - \frac{4m_\nu^2}{m_\pi^2}\right)^{1/2}. \tag{5.2}$$

Equation (5.2) provides a crude order-of-magnitude estimation of the ratio

$$R = \frac{\sigma(2\gamma \to \nu\bar{\nu})}{\sigma(2\gamma \to 2\gamma)}, \tag{5.3}$$

the estimate being, of course, exact at energies such that one is at the pion pole.[23]

The rate for the process $\nu_\mu \to \nu_e + \gamma$ can be expressed in terms of the relevant transition magnetic moment:

$$\Gamma(\nu_\mu \to \nu_e + \gamma) = \frac{\alpha}{8}\left(\frac{m_{\nu_\mu}{}^2 - m_{\nu_e}{}^2}{m_{\nu_\mu}}\right)^3 \frac{\kappa_{\nu_\mu \to \nu_e}{}^2}{m_e{}^2} . \quad (5.4)$$

Hence,

$$m_{\nu_\mu}\Gamma(\nu_\mu \to \nu_e + \gamma) < \left(\frac{m_{\nu_\mu}}{m_e}\right)^4 \alpha \frac{m_e{}^2}{8} \kappa_{\nu_\mu \to \nu_e}{}^2$$

$$< 1.3 \times 10^{-3} \left(\frac{m_{\nu_\mu}}{m_e}\right)^4 \text{ MeV/sec}, \quad (5.5)$$

the last inequality[24] following from the bound in Eq. (4.14).

The bound in Eq. (5.5) may be compared to the best bound available from terrestrial experiments[25]:

$$m_{\nu_\mu}\Gamma(\nu_\mu \to \nu_e + \gamma) < 3.3 \times 10^{-4} \text{ MeV/sec.} \quad (5.6)$$

We note that in its region of validity ($m_{\nu_\mu} < 10$ keV), the bound in (5.5) is at least 10^{-6} times smaller than in (5.6). This is because the indirect method used in (5.4) and (5.5) takes advantage of phase-space limitations while direct observation cannot.

VI. CONCLUDING REMARKS

The bounds on the various parameters are summarized in Table I. It is evident that if one endows neutrino fields with four components in order to satisfy the principle of manifest left-right symmetry in the context of a renormalizable gauge theory, the extra components do not disturb the equilibrium of the situation too much. Qualitatively new effects that show up in neutrino physics, at "moderate" energies are sharply limited in magnitude by the smallness of (a) the neutrino mass and (b) the mixing angle ζ introduced in Ref. 2 and Eqs. (4.7) and (4.8). This is in sharp contrast to the situation, considered by several authors,[6] in which one simply introduces four-component neutrino fields in an *ad hoc* manner without reference to any underlying gauge theory; quantities such as G_S, G_P, G_T, and κ_ν are then free parameters. However, the theory—at this time—does not provide any explanation for the smallness of m_ν and ζ. As noted elsewhere,[2,3] decisive tests of the theoretical ideas are expected to become feasible at superhigh energies which probe distances $\sim O((10^3 \text{ GeV})^{-1})$. At such energies, weak interactions become parity conserving and one can, in principle, set up a factory for manufacturing right-handed neutrinos.

It is amusing to note that, since the energy required to switch off parity nonconservation was naturally available in the early stages of the "big bang," we may well be immersed in a hitherto unperceived sea of right-handed neutrinos. Equally amusing is the possibility that weak interactions, in some sectors of the universe, froze[26] into a predominantly $(V+A)$ pattern; nuclear reactions going on in such sectors would continually feed "fresh" neutrinos of the wrong helicity into the background sea.

Finally, we note that considerations based on

TABLE I. Currently available upper bounds on neutrino parameters. The magnetic moments and the transition magnetic moments are expressed in units of Bohr magnetons.

	Upper bound		
Quantity	Gauge theory with manifest left-right symmetry	Experiment (terrestrial)	Astrophysics
m_{ν_e}	···	35 eV	40 eV
m_{ν_μ}	···	510 keV	40 eV
m_{ν_τ}	···	450 MeV	40 eV
$\lvert\kappa_{\nu_e}\rvert$	2.6×10^{-14}	1.4×10^{-9}	8.5×10^{-11}
$\lvert\kappa_{\nu_\mu}\rvert$	5.4×10^{-12}	8.1×10^{-9}	8.5×10^{-11}
$\lvert\kappa_{\nu_\tau}\rvert$	9.7×10^{-11}		8.5×10^{-11}
$\lvert\kappa_{\nu_\mu \to \nu_e}\rvert$		1.4×10^{-9}	6.0×10^{-11}
$\lvert\kappa_{\nu_\tau \to \nu_e}\rvert$			6.0×10^{-11}
$\lvert\kappa_{\nu_\tau \to \nu_\mu}\rvert$			6.0×10^{-11}

helium production,[27] in the standard big-bang cosmology, put an upper limit on the number of light ($m_\nu < 300$ keV) metastable ($t_\nu > 3$ min[28]) four-component neutrinos; the maximum number of such neutrinos is *three*.[29] Hence, if the masses and lifetimes of ν_e, ν_μ, and ν_τ satisfy the above bounds we may infer that *all stable and metastable light neutrinos have been discovered*; other neutrinos that might be lurking around would have to be either short-lived or—if stable—have masses in excess of 2 GeV.[10]

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[1]E. M. Lipmanov, Yad. Fiz. 6, 541 (1967) [Sov. J. Nucl. Phys. 6, 395 (1968)]; M. A. B. Bég and A. Zee, Phys. Rev. Lett. 30, 675 (1973); Phys. Rev. D 8, 1460 (1973); P. Fayet, Nucl. Phys. B78, 14 (1974); J. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61 (1976); R. N. Mohapatra and D. P. Sidhu, Phys. Rev. Lett. 38, 667 (1977); A. De Rújula, H. Georgi, and S. L. Glashow, Harvard report, 1977 (unpublished); J. Pati, S. Rajpoot, and A. Salam, Imperial College reports, 1977 (unpublished). The principle of *manifest* left-right symmetry was formulated in Ref. 2.

[2]M. A. B. Bég, R. V. Budny, R. Mohapatra, and A. Sirlin, Phys. Rev. Lett. 38, 1252 (1977); 39, 54(E) (1977).

[3]M. A. B. Bég, R. Mohapatra, A. Sirlin, and H.-S. Tsao, Phys. Rev. Lett. 39, 1054 (1977).

[4]See, for example, M. A. B. Bég and A. Sirlin, Annu. Rev. Nucl. Sci. 24, 379 (1974).

[5]See M. A. B. Bég and S.-S. Shei, Phys. Rev. D 12, 3092 (1975) and references cited therein.

[6]For discussions outside the framework of renormalizable theories see B. Kayser, G. T. Garvey, E. Fischbach, and S. P. Rosen, Phys. Lett. 52B, 385 (1974); R. L. Kingsley, F. Wilczek, and A. Zee, Phys. Rev. D 10, 2216 (1974); S. L. Adler, E. W. Colglazier, J. B. Healy, I. Karliner, J. Lieberman, Y. J. Ng. and H.-S. Tsao, *ibid.* 12, 3501 (1975); K. Winter, Phys. Lett. 67B, 236 (1977).

[7]See, for example, T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. 15, 381 (1965).

[8]We are assuming that ν_τ, the neutrino associated with the τ lepton observed by M. Perl *et al.* [Phys. Rev. Lett. 35, 1489 (1975) and Phys. Lett. 63B, 466 (1976)], is distinct from ν_e and ν_μ, the neutrinos associated with the electron and the muon, respectively.

[9]The bounds quoted are from E. F. Tretjakov *et al.*, I.T.E.F. Report No. 15, 1976 (unpublished); M. Daum *et al.*, [in SIN Physics Report No. 1, 1976 (unpublished), p. 13; and G. Kries, paper presented at the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977 (unpublished). We note that an experiment by H. Anderhub *et al.* [in, SIN Physics Report 1, 1976 (unpublished), p. 21] is designed to yield $m_{\nu_\mu} < 200$ keV.

[10]For a recent application of this argument along with references to the original literature, see B. W. Lee and S. Weinberg, Phys. Rev. Lett. 39, 165 (1977).

[11]In theories with the attribute of manifest left-right symmetry, Higgs fields which develop vacuum expectation values cannot have pseudoscalar couplings to fermions. Consequently the q-number parts of such fields can contribute to G_S but not to G_P.

[12]M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).

[13]J. Bernstein, M. Ruderman, and G. Feinberg, Phys. Rev. 132, 1227 (1963).

[14]J. E. Kim, V. S. Mathur, and S. Okubo, Phys. Rev. D 9, 3050 (1974); D. Bardin and O. Mogilevsky, Lett. Nuovo Cimento 9, 549 (1974).

[15]J. E. Kim, Phys. Rev. D 14, 3000 (1976); W. J. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977). Although the charge radius is not in general a gauge-independent quantity for non-Abelian gauge theories, the leading contribution in this case which we quote $\sim eG_F \ln(M_W/m_l)$ is gauge independent and comes from the diagram in Fig. 2(a).

[16]This model is a trivial extension of the four-lepton model discussed in Ref. 2. See also Mohapatra and Sidhu, Ref. 1.

[17]P. W. Anderson, Phys. Rev. 112, 1900 (1958). Note that this paper appeared six years prior to the work of Higgs.

[18]P. Sutherland, J. N. Ng, E. Flowers, M. Ruderman, and C. Inman, Phys. Rev. D 13, 2700 (1976) and references cited therein.

[19]Note that Eq. (4.12) may be used to improve the bound in Eq. (4.11) if one accepts the relationship $\kappa_{\nu_e}/m_e = \kappa_{\nu_\mu}/m_\mu = \kappa_{\nu_\tau}/m_\tau$ implied by Eq. (4.9). The bound so obtained is $\kappa_{\nu_e} < 3.9 \times 10^{-11}$. Furthermore, if m_{ν_τ} is indeed <10 keV, then this line of reasoning when applied to Eq. (4.13) yields $\kappa_{\nu_e} < 2 \times 10^{-14}$.

[20]The possibility that muon number is not exactly conserved has recently been the focus of considerable theoretical speculation and experimental activity. For an extensive list of references see B. Humpert, SLAC report, 1977 (unpublished).

[21]S. Y. Pi and A. I. Sanda, Phys. Rev. Lett. 40, 286 (1978).

[22]We use the Adler formula [Phys. Rev. 177, 2426 (1969)] appropriate to a three-color quark model, for $\pi^0 \to 2\gamma$.

[23]See M. Gell-Mann, Phys. Rev. Lett. 6, 70 (1960); E. Fischbach, S. P. Rosen, H. Spivack, J. Gruenwald, A. Halprin, and B. Kayser, Phys. Rev. D 16, 2377 (1977).

[24]Other astrophysical bounds on neutrino decay rates have been quoted in a number of recent papers. For

example D. Dicus, E. Kolb, and V. Teplitz, Phys. Rev. Lett. 39, 168 (1977); T. Goldman and G. J. Stephenson, Jr., Phys. Rev. D 16, 2256 (1977); R. Cowsik, Phys. Rev. Lett. 39, 784 (1977).

[25]V. E. Barnes *et al.*, Phys. Rev. Lett. 38, 1049 (1977); E. Bellotti *et al.*, Lett. Nuovo Cimento 17, 553 (1976).

[26]Compare S. Weinberg, Phys. Rev. D 9, 3357 (1974).

[27]G. Steigman, D. Schramm, and J. Gunn, Phys. Lett. 66B, 202 (1977).

[28]S. Weinberg, *The First Three Minutes* (Basic, New York, 1977).

[29]The authors of Ref. 27 find a (generous) upper limit of seven on the number of light two-component neutrinos permitted by the agreement between observed and calculated helium production from the big bang. This limit would become three for four-component neutrinos.