# Dynamical constraints on the couplings of ground-state mesons—An estimate of the $\phi \rightarrow \pi\gamma$ decay

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We discuss an extension of the hypothesis of asymptotic level realization of SU(3) to a certain chiral SU(3)$\otimes$SU(3) charge–charge-density algebra, which is applied to the ground-state mesons. It produces interesting dynamical constraints for various couplings of the ground-state mesons which, among other results, permit us to compute the rate of the $\phi \rightarrow \pi\gamma$ decay. This decay is forbidden in this formulation if the masses of $\rho$ and $\omega$ are degenerate.

## I. INTRODUCTION

We present an extension of the hypothesis[1] of asymptotic level realization of SU(3) to a certain chiral SU(3)$\otimes$SU(3) charge–charge-density algebra. When applied to the ground-state mesons, it produces interesting dynamical constraints on both the strong and electromagnetic couplings of the ground-state mesons, i.e., the $1^{--}$ and $0^{-+}$ nonets. In particular, we give an estimate of the $\phi \rightarrow \pi\gamma$ decay which is consistent with experiment.

The $\phi \rightarrow 3\pi$ decay is strongly suppressed compared with the $\omega \rightarrow 3\pi$ decay. Similarly the $\phi \rightarrow \pi\gamma$ decay rate is much smaller than that of the $\omega \rightarrow \pi\gamma$ decay. Usually these processes are *made* forbidden by invoking either the quark-line selection rule[2] or Okubo's *ideal* nonet ansatz.[3] However, the origin, the precise form, and the extent of the workability of these so-called Okubo-Zweig-Iizuka (OZI) rules are not clearly known. A critical discussion has recently been given by Lipkin[4] about these rules. Experimentally it is known that they are certainly violated to some extent. Nevertheless the rules do not prescribe for us a simple recipe for estimating their degree of violation. Some theoretical justification of Okubo's ansatz was provided[5] in SU(6) for the $1^{--}$ mesons. However, besides the inevitable problem associated with the notion of exact SU(6), the violation of the ideal nonet structure is not prescribed even for the $1^{--}$ mesons in SU(6).

An entirely different and more theoretical approach to these problems was initiated by Matsuda and Oneda[6] in 1968. In this mainly algebraic approach, the striking suppression of the $\phi \rightarrow \rho\pi$ and $f' \rightarrow \pi\pi$ decays, etc. can be understood and actually be *calculated* as a consequence of an intimate dynamical *interplay* among the nonet masses, SU(3) mixing angles, and coupling constants (more precisely, the asymptotic matrix elements of axial-vector charges) which is characteristic in this theoretical framework. This dynamical mechanism turned out to be also useful[7] for explaining the new resonances in SU(4). In contrast no reference is explicitly made about the masses of mesons in the quark-line rules. In this paper we wish to treat the $1^{--} \rightarrow 0^{-+} + \gamma$ and $1^{--} \rightarrow 1^{--} + \gamma$ interactions in this theoretical framework. Our main concern is the suppression of the $\phi \rightarrow \pi\gamma$ decay.

## II. THE $\phi \rightarrow \pi\gamma$ DECAY RATE FROM THE SUM RULES INVOLVING THE ALGEBRA $[\dot{V}_{K^0}, A_{\pi^-}] = 0$

The theoretical ingredients (introduced so far) of this algebraic approach are[7,8] as follows:

(i) Asymptotic SU(3), which states that in broken SU(3) the SU(3)-multiplet classification should best be carried out in the infinite-momentum limit of the SU(3) multiplet, taking proper care of the possible SU(3) particle mixing,

(ii) The chiral SU(3)$\otimes$SU(3) charge algebras and their related algebras [valid in broken SU(3)], which play an essential role in making an extrapolation from the asymptotic world to the physical world.

(iii) A mechanism of SU(3) breaking characterized by the presence of the exotic commutation relations, $[\dot{V}_\alpha, V_\beta] = 0$ and $[\dot{V}_\alpha, A_\beta] = 0$, involving the time derivative of the SU(3) charge $V_\alpha$. Here $\alpha$ and $\beta$ stand for the *exotic* combinations of the physical SU(3) indices, i.e., $(\alpha, \beta) = (K^0, K^0)$, $(K^0, \pi^-)$, etc. $[\dot{V}_\alpha, V_\beta] = 0$ is nothing but the algebraic expression for the usual octet SU(3) breaking. The exotic commutator involving the axial charge $A_\beta$, $[\dot{V}_\alpha, A_\beta]=0$ [which plays a major role in producing dynamical information when combined with our hypothesis (i)], is a stronger assumption, although this algebra is certainly valid for the class of symmetry breaking usually considered.

(iv) Hypothesis of asymptotic level realization of SU(3) in the algebra, for example, $[A_\alpha, A_\beta] = if_{\alpha\beta\gamma}V_\gamma$.

We note, however, that *no* reference is made in our theoretical framework about the chiral SU(3) ⊗ SU(3) and chiral SU(2) ⊗ SU(2) *symmetry*, although the chiral SU(3) ⊗ SU(3) *algebra* plays an important role. Also, no perturbation-theoretic treatment of broken SU(3) is involved. In the theoretical framework of (i), (ii), and (iii), the realization of the algebra $[\dot{V}_\alpha, A_\beta]=0$ in the asymptotic limit produces for mesons the following *general* interplay of the nonet-meson masses, SU(3) mixing angles, and asymptotic axial-vector matrix elements. Denoting a nonet by $(\pi_t, K_t, \eta_t, \eta_t')$ where $t$ denotes the $J^{PC}$ and other quantum numbers we obtain[7,8] (writing $\pi$ for the mass $m_\pi$, etc.)

$$\frac{\langle\eta_t|A_{\pi^-}|\pi_r^+(\vec{p})\rangle}{\langle\eta_t'|A_{\pi^-}|\pi_r^+(\vec{p})\rangle}=\tan\theta_{tt'}\frac{(\eta_t'^2-\pi_t^2)}{(\eta_t^2-\pi_t^2)},\quad \vec{p}\to\infty. \tag{1}$$

Here $t$ and $r$ are arbitrary except for $C_rC_t=1$. $\theta_{tt'}$ is the $\eta_t$-$\eta_t'$ mixing angle. From the realization of the exotic commutator $[\dot{V}_{K^0}, V_{\bar{K}^0}]=0$ in the asymptotic limit, $\theta_{tt'}$ is required to satisfy $\sin^2\theta_{tt'}=\frac{1}{3}(3\eta_t^2-4K_t^2+\pi_t^2)(\eta_t^2-\eta_t'^2)^{-1}$. Note the remarkable fact that the right-hand side of Eq. (1) is determined *solely* by the structure of the $t$ nonet. Equation (1) immediately implies that, in the limit of equal mass $\eta_t'^2=\pi_t^2$, there *appears* a general dynamical selection rule [so far, no approximation except for the neglect of possible intermultiplet SU(3) mixing is made] for the particular asymptotic axial-vector matrix elements involving the $\eta_t'$, i.e., $\langle\eta_t'|A_{\pi^-}|\pi_r^+(\vec{p})\rangle=0$ $(\vec{p}\to\infty)$ where $r$ is arbitrarily provided $C_rC_t=1$. Therefore, the possibility of having a *hidden* general (OZI-type) dynamical selection rule is already demonstrated in our theoretical framework.

However, the implication of Eq. (1) is much more significant. Equation (1), in fact, additionally provides us with a realistic estimate of the degree of violation of the dynamical selection rule which is exact in the *ideal* nonet mass limit $\eta_t'^2=\pi_t^2$. Restricting ourselves to the case $t=r=1^{--}$, Eq. (1) then predicts[6]

$$\frac{\langle\phi|A_{\pi^-}|\rho^+(\vec{p})\rangle}{\langle\omega|A_{\pi^-}|\rho^+(\vec{p})\rangle}=\tan\theta_{\phi\omega}\frac{(\rho^2-\omega^2)}{(\rho^2-\phi^2)}\equiv R,\quad \vec{p}\to\infty. \tag{2}$$

Using the partially conserved axial-vector current hypothesis (PCAC) for $A_{\pi^-}$ in Eq. (2) we obtain[6]

$$\frac{g_{\phi\rho\pi}}{g_{\omega\rho\pi}}=R\simeq-0.06, \tag{3}$$

where $g_{\phi\rho\pi}$ and $g_{\omega\rho\pi}$ are the coupling constants of the $\phi\to\rho\pi$ and $\omega\to\rho\pi$ couplings. (Strictly speaking for these couplings the pion is off the mass shell, i.e., $\pi^2=0$.) We have chosen $\theta_{\phi\omega}\simeq-40°$ (instead of $\simeq 40°$). The ideal value in our convention is $\theta_{\phi\omega}\simeq-35°$.

We are now interested in the evaluation of the rate of the $\phi\to\pi\gamma$ decay. By exploiting the idea of *vector-meson dominance*[9] ($\rho$ dominance for the isovector current) we obtain from Eq. (3)

$$\frac{g_{\phi\pi\gamma}}{g_{\omega\pi\gamma}}\simeq\frac{g_{\phi\rho\pi}}{g_{\omega\rho\pi}}=R\simeq-0.06. \tag{4}$$

The value of $R$ is sensitive (within a factor of 2) to the choice of the centere value of the $\rho$ mass. We obtain, for example, $|R|\simeq 0.06$ for the choice of the masses of $1^{--}$ mesons, $\rho=760$, $\omega=783$, $\phi=1020$, and $K^*=895$ MeV. Overall consistency of our result seems to prefer the value of $|R|$ in the range of 0.05–0.06. This favors the $\rho$ mass slightly smaller[10] than the value listed by the Particle Data Group. Experimentally, $\Gamma(\phi\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)\simeq(6\pm3)\times10^{-3}$. Equation (4) predicts this ratio to be around $8\times10^{-3}$. The relative sign of the $\phi\to\pi\gamma$ and $\omega\to\pi\gamma$ predicted by Eq. (4) is also consistent with the recent (preliminary) experiment by the Rochester group.[11]

## III. DYNAMICAL CONSTRAINTS FOR THE GROUND-STATE MESONS FROM THE HYPOTHESIS OF ASYMPTOTIC LEVEL REALIZATION OF ALGEBRA

We now study an extension of our hypothesis (iv) to the processes involving helicity change. Instead of charge algebra $[A_\alpha, A_\beta]=if_{\alpha\beta\gamma}V_\gamma$ previously studied, we now turn to a realization of the following charge–charge-density algebra:

$$[[j_3^0(0), A_{\pi^+}], A_{\pi^-}]=2j_3^0(0). \tag{5}$$

$j_3^0$ is the isovector charge density. We consider all the possible matrix elements of Eq. (5) inserted between the pseudoscalar meson $P(\vec{p})$ and the $1^{--}$ meson $V(\vec{k},\lambda=1)$ with helicity $\lambda=1$ and also between $V(\vec{p},\lambda=0)$ and $V(\vec{k},\lambda=1)$. We, of course, take the asymptotic limits $\vec{p}\to\infty$ and $\vec{k}\to\infty$,

$$\langle\pi^0|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|\omega(\lambda=1)\rangle=2\langle\pi^0|j_3^0|\omega(\lambda=1)\rangle, \tag{6}$$

$$\langle\pi^0|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|\phi(\lambda=1)\rangle=2\langle\pi^0|j_3^0|\phi(\lambda=1)\rangle, \tag{7}$$

$$\langle K^+|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|K^{*+}(\lambda=1)\rangle=2\langle K^+|j_3^0|K^{*+}(\lambda=1)\rangle, \tag{8}$$

$$\langle K^0|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|K^{*0}(\lambda=1)\rangle=2\langle K^0|j_3^0|K^{*0}(\lambda=1)\rangle, \tag{9}$$

$$\langle\eta|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|\rho^0(\lambda=1)\rangle=2\langle\eta|j_3^0|\rho^0(\lambda=1)\rangle, \tag{10}$$

$$\langle\eta'|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|\rho^0(\lambda=1)\rangle=2\langle\eta'|j_3^0|\rho^0(\lambda=1)\rangle\,, \quad (11)$$

and

$$\langle\rho^+(\lambda=0)|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|\rho^+(\lambda=1)\rangle =2\langle\rho^+(\lambda=0)|j_3^0|\rho^+(\lambda=1)\rangle\,, \quad (12)$$

$$\langle K^{*+}(\lambda=0)|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|K^{*+}(\lambda=1)\rangle =2\langle K^{*+}(\lambda=0)|j_3^0|K^{*+}(\lambda=1)\rangle\,, \quad (13)$$

$$\langle K^{*0}(\lambda=0)|[[j_3^0,A_{\pi^+}],A_{\pi^-}]|K^{*0}(\lambda=1)\rangle =2\langle K^{*0}(\lambda=0)|j_3^0|K^{*0}(\lambda=1)\rangle\,. \quad (14)$$

The right-hand side of these equations can be parametrized, according to asymptotic SU(3), by the usual prescription of exact SU(3) plus SU(3) mixing where $\vec{p}\to\infty$ and $\vec{k}\to\infty$. The useful algebra to be used for this purpose is

$$[[j_3^0,V_{K^+}],V_{K^-}]+[[j_3^0,V_{K^0}],V_{\bar{K}^0}]=j_3^0.$$

On the left-hand side of Eqs. (6)–(14) we insert a complete set of single-particle intermediate states among the factors $j_3^0(0)$, $A_{\pi^+}$, and $A_{\pi^-}$. Among the above intermediate states, we distinguish the block contributions coming from various combination of the levels of mesons, i.e., the ones coming solely from ground state ($l=0$), from $l=0$ and $l=1$, $l=1$ only, .... In this paper we consider only the contribution coming from the ground state ($l=0$ level[12] in the simple $q\bar{q}$ quark model). For illustration, we show explicitly the ground-state contribution to Eq. (6),

$$\begin{aligned}&\langle\pi^0|j_3|\omega(\lambda=1)\rangle\langle\omega(\lambda=1)|A_{\pi^+}|\rho^-(\lambda=1)\rangle\langle\rho^-(\lambda=1)|A_{\pi^-}|\omega(\lambda=1)\rangle\\&\quad+\langle\pi^0|j_3|\phi(\lambda=1)\rangle\langle\phi(\lambda=1)|A_{\pi^+}|\rho^-(\lambda=1)\rangle\langle\rho^-(\lambda=1)|A_{\pi^-}|\omega(\lambda=1)\rangle\\&\quad-\langle\pi^0|A_{\pi^+}|\rho^-(\lambda=0)\rangle\langle\rho^-(\lambda=0)|j_3|\rho^-(\lambda=1)\rangle\langle\rho^-(\lambda=1)|A_{\pi^-}|\omega(\lambda=1)\rangle\\&\quad-\langle\pi^0|A_{\pi^-}|\rho^+(\lambda=0)\rangle\langle\rho^+(\lambda=0)|j_3|\rho^+(\lambda=1)\rangle\langle\rho^+(\lambda=1)|A_{\pi^+}|\omega(\lambda=1)\rangle\\&\quad+\langle\pi^0|A_{\pi^-}|\rho^+(\lambda=0)\rangle\langle\rho^+(\lambda=0)|A_{\pi^+}|\pi^0\rangle\langle\pi^0|j_3|\omega(\lambda=1)\rangle\\&\quad+\text{contributions from higher levels}=2\langle\pi^0|j_3|\omega(\lambda=1)\rangle.\end{aligned}$$

The parametrization of relevant matrix elements of $j_3$, $A_{\pi^+}$, and $A_{\pi^-}$ in the limit $\vec{p}\to\infty$ and $\vec{k}\to\infty$ is given by

$$\langle\pi^0|j_3|\omega(\lambda=1)\rangle\equiv(\tfrac{1}{2})^{1/2}s,\quad\langle\pi^0|j_3|\phi(\lambda=1)\rangle=(\tfrac{1}{2})^{1/2}d,$$

$$\langle K^+|j_3|K^{*+}(\lambda=1)\rangle=\tfrac{1}{2}(\tfrac{3}{2})^{1/2}(\cos\theta_{\phi\omega}d-\sin\theta_{\phi\omega}s),\quad\text{etc.},$$

$$\langle\eta^0|j_3|\rho^0\rangle=(\tfrac{1}{2})^{1/2}d',\quad\langle\eta'|j_3|\rho^0\rangle=(\tfrac{1}{2})^{1/2}s',$$

with a relation $\cos\theta_{\phi\omega}d-\sin\theta_{\phi\omega}s=\cos\theta_{\eta\eta'}d'-\sin\theta_{\eta\eta'}s'$,

$$\langle\rho^-(\lambda=0)|j_3|\rho^-(\lambda=1)\rangle=(\tfrac{1}{2})^{1/2}f,\quad\langle K^{*+}(\lambda=0)|j_3|K^{*+}(\lambda=1)\rangle=-\tfrac{1}{2}(1/2)^{1/2}f,\quad\text{etc.},$$

$$\langle\rho^-|A_{\pi^-}|\omega(\lambda=1)\rangle=S,\quad\langle\rho^-|A_{\pi^-}|\phi(\lambda=1)\rangle=D,$$

$$\langle K^{*+}|A_{\pi^+}|K^{*0}(\lambda=1)\rangle=(\tfrac{3}{2})^{1/2}(\cos\theta_{\phi\omega}D-\sin\theta_{\phi\omega}S),\quad\text{etc.},$$

$$\langle\pi^0|A_{\pi^-}|\rho^+(\lambda=0)\rangle=F,\quad\langle K^+|A_{\pi^+}|K^{*0}\rangle=-(\tfrac{1}{2})^{1/2}F,\quad\text{etc.}$$

We are interested in deriving constraints among $S$, $D$, and $F$ as well as among $s$, $d$, and $f$. With these parametrizations, Eqs. (6)–(14) become ($\theta$ now denotes the $\phi$-$\omega$ mixing angle)

$$1+\left(\frac{d}{s}\right)\left(\frac{D}{S}\right)+2\left(\frac{F}{S}\right)\left(\frac{f}{s}\right)+\left(\frac{F}{S}\right)^2+\cdots=\frac{2}{S^2}, \quad (6')$$

$$\left(\frac{D}{S}\right)+\left(\frac{d}{s}\right)\left(\frac{D}{S}\right)^2+2\left(\frac{F}{S}\right)\left(\frac{D}{S}\right)\left(\frac{f}{s}\right)+\left(\frac{F}{S}\right)^2\left(\frac{d}{s}\right)+\cdots=2\left(\frac{d}{s}\right)\frac{1}{S^2}, \quad (7')$$

$$3\left[\cos\theta\left(\frac{d}{s}\right)-\sin\theta\right]\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]^2+\left(\frac{F}{S}\right)\left(\frac{f}{s}\right)\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]+\cdots=4\left[\cos\theta\left(\frac{d}{s}\right)-\sin\theta\right]\frac{1}{S^2}, \quad (8')$$

$$\left(\frac{F}{S}\right)\left(\frac{f}{s}\right)\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]+\left(\frac{F}{S}\right)^2\left[\cos\theta\left(\frac{d}{s}\right)-\sin\theta\right]+\cdots=4\left[\cos\theta\left(\frac{d}{s}\right)-\sin\theta\right]\frac{1}{S^2}, \quad (9')$$

$$0+\cdots=2\langle\eta|j_3|\rho^0(\lambda=1)\rangle\,, \quad (10')$$

$$0+\cdots=2\langle\eta'|j_3|\rho^0(\lambda=1)\rangle\,, \quad (11')$$

and

$$\left(\frac{f}{s}\right)\left[1+\left(\frac{D}{S}\right)^2\right]+\left(\frac{F}{S}\right)+\left(\frac{F}{S}\right)\left(\frac{D}{S}\right)\left(\frac{d}{s}\right)+\cdots=2\left(\frac{f}{s}\right)\frac{1}{S^2}, \tag{12'}$$

$$3\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]\left(\frac{f}{s}\right)+3\left(\frac{F}{S}\right)\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]\left[\cos\left(\frac{d}{s}\right)-\sin\theta\right]+\cdots=4\left(\frac{f}{s}\right)\frac{1}{S^2}, \tag{13'}$$

$$3\left(\frac{F}{S}\right)\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]\left[\cos\theta\left(\frac{d}{s}\right)-\sin\theta\right]+\left(\frac{F}{S}\right)^2\left(\frac{f}{s}\right)+\cdots=4\left(\frac{f}{s}\right)\frac{1}{S^2}. \tag{14'}$$

We now extend the hypothesis of asymptotic level realization of SU(3) previously applied to the algebra $[A_\alpha,A_\beta]=if_{\alpha\beta\gamma}V_\gamma$ to the algebra, Eq. (5). We assume that for Eqs. (6)–(11) the *fractional* contributions from each level [i.e., ground states $(l=0)$, $l=0$ and $l=1$, states $l=1$ states, ...] to the left-hand side of Eqs. (6)–(11) are the same. For example, the ratio of the contribution from each level to Eqs. (6) and (7) should be equal to the ratio of 1 to $d/s$ unless the contribution is zero. If one equation has no contribution from a certain level, neither does any other equation. The same assumption can also be made for Eqs. (12)–(14). First of all, we immediately notice that from Eqs. (8′) and (9′) we obtain a constraint among $D$, $S$, and $F$, i.e.,

$$\left(\frac{F}{S}\right)^2=3\left[\cos\theta\left(\frac{D}{S}\right)-\sin\theta\right]^2. \tag{15}$$

In deriving Eq. (15), we have assumed that $\cos\theta(d/s)-\sin\theta\neq 0$. Actually $\cos\theta(d/s)-\sin\theta=0$ contradicts experiment. [Experimentally $d/s\simeq 0$ and $|\sin\theta|\simeq(\frac{1}{3})^{1/2}$.] Impressively, we also find the same constraint Eq. (15) from Eqs. (13′) and (14′) provided $f/s\neq 0$. $f=0$ is physically unacceptable. According to our constraint Eq. (2) obtained from the commutator $[\dot{V}_{K^0},A_{\pi^-}]=0$,

$$\frac{D}{S}=\tan\theta\frac{\rho^2-\omega^2}{\rho^2-\phi^2}\equiv R. \tag{16}$$

Equations (15) and (16) reduce to a constraint

$$\left(\frac{F}{S}\right)^2=3\sin^2\theta\left(\frac{\rho^2-\omega^2}{\rho^2-\phi^2}-1\right)^2. \tag{17}$$

In the *ideal* limit[3] of the vector meson, i.e., $\rho^2=\omega^2$ and $\sin^2\theta=\frac{1}{3}$, Eq. (17) becomes

$$F^2=S^2 \quad (\text{in the } ideal \text{ limit of } 1^{--}). \tag{18}$$

Using PCAC in Eq. (18) we are led to a prediction:

$$\left(\frac{g_{\rho^+\pi^0\pi^-}}{\rho}\right)^2=g_{\omega\rho^+\pi^-}{}^2 \quad (\text{in the } ideal \text{ limit of } 1^{--}). \tag{19}$$

Without the approximation of *ideal* structure of $1^{--}$ we obtain instead a prediction from Eq. (17),

$$\left(\frac{g_{\rho^+\pi^0\pi^-}}{\rho}\right)^2\Big/g_{\omega\rho^+\pi^-}{}^2=3\sin^2\theta\left(\frac{\rho^2-\omega^2}{\rho^2-\phi^2}-1\right)^2 \simeq 0.986, \tag{20}$$

or using Eq. (16),

$$\left(\frac{g_{\rho^+\pi^0\pi^-}}{\rho}\right)^2\Big/g_{\phi\rho^+\pi^-}{}^2=3\sin^2\theta\left(\frac{1}{R^2}\right)\left(\frac{\rho^2-\omega^2}{\rho^2-\phi^2}-1\right)^2. \tag{21}$$

To simplify our notation, we define $\alpha\equiv\sqrt{3}\,[\cos\theta(D/S)-\sin\theta]$ ($\alpha=-1$ in the ideal limit of $1^{--}$) and $\beta\equiv\sqrt{3}\,[\cos\theta(d/s)-\sin\theta]$. Equation (15) then becomes

$$\left(\frac{F}{S}\right)^2=\alpha^2 \quad \text{or} \quad \left(\frac{F}{S}\right)=\epsilon\alpha, \quad \epsilon^2=1. \tag{22}$$

We also define $D/S\equiv\gamma$ ($\gamma=0$ in the *ideal* limit of $1^{--}$), $d/s\equiv x$ and $f/s\equiv y$. We now return to Eqs. (6′)–(11′). From Eqs. (10′) and (11′) we realize that the contributions of the ground-state mesons (i.e., $0^{-+}$ and $1^{--}$ nonet) to Eqs. (10) and (11) are, in fact, zero as long as we keep SU(2) symmetry. This implies, according to our hypothesis of asymptotic level realization of SU(3), that the net ground-state contributions to Eqs. (6)–(9) should also vanish. Hence, our independent realization conditions [in addition to the constraint Eq. (15) or Eq. (22) already obtained] are

$$(6')\to 1+\gamma x+2\epsilon\alpha y+\alpha^2=0, \tag{6''}$$

$$(7')\to \gamma+\gamma^2x+2\epsilon\alpha\gamma y+\alpha^2x=0, \tag{7''}$$

and

$$(8')\to \alpha^2(\beta+\epsilon y)=0 \quad (\alpha^2\neq 0). \tag{8''}$$

We also notice that the ground-state contribution to Eq. (14) is $\beta\epsilon\alpha^2+\alpha^2y=(\alpha^2/\epsilon)(\beta+\epsilon y)$ which *vanishes* according to Eq. (8″). Therefore, we see that the net contribution of the ground state to Eqs. (12)–(14) should also *vanish* according to the hypothesis of asymptotic level realization. Thus, Eq. (12′) gives rise to another constraint,

$$(12')\to(1+\gamma^2)y+\epsilon\alpha+\epsilon\alpha\gamma x=0. \tag{12''}$$

From the constraint Eqs. (6″), (7″), (8″), (12″), and (22), we obtain simple relations,

$$x=\gamma, \quad \text{i.e.,} \quad \frac{d}{s}=\frac{D}{S} \quad (\text{or } \alpha=\beta) \tag{23}$$

and

$$y=-\epsilon\alpha, \quad \text{i.e.,} \quad \frac{f}{s}=-\epsilon\alpha, \tag{24}$$

whereas $F/S=\epsilon\alpha$ from Eq. (22). In the *ideal* limit of the $1^{--}$ nonet [i.e., $\rho^2=\omega^2$ which implies $D=0$ from Eq. (16)] where $\gamma\to 0$ and $\alpha\to -1$, we get $x\to 0$ (i.e., $d\to 0$) and $y\to\epsilon$. $x=0$ implies the presence of a general selection rule for the particular matrix elements of $j_3$, $\langle\pi^0(\vec{p})|j_3^0|\phi(\vec{k},\lambda=1)\rangle=0$ for $\vec{p}\to\infty$ and $\vec{k}\to\infty$. However, Eq. (23) actually *prescribes* the way in which this dynamical selection is violated, i.e.,

$$\frac{\langle\pi^0(\vec{p})|j_3^0|\phi(\vec{k},\lambda=1)\rangle}{\langle\pi^0(\vec{p})|j_3^0|\omega(\vec{k},\lambda=1)\rangle}=\frac{\langle\rho^-|A_{\pi^-}|\phi(\vec{k},\lambda=1)\rangle}{\langle\rho^-|A_{\pi^-}|\omega(\vec{k},\lambda=1)\rangle}=R$$

for $\vec{p}\to\infty$ and $\vec{k}\to\infty$. The last equality of the above equation is obtained by using Eq. (2). We define $g_{VP\gamma}(t)$ by

$$(4p_0k_0)^{1/2}\langle P(\vec{p})|j_3^0|V(\vec{k},\lambda=1)\rangle\equiv g_{VP\gamma}(t)\epsilon_{0\nu\rho\sigma}\epsilon_\nu(\vec{k})p_\rho k_\sigma,$$

where $t=(p-k)^2$ and $\epsilon_\nu(\vec{k})$ is the polarization vector of the vector meson with helicity $\lambda=1$ and $k_\sigma=(0,0,k,k_0)$. Then the above constraint reads

$$\frac{g_{\phi\pi\gamma}(t)}{g_{\omega\pi\gamma}(t)}=R. \tag{25}$$

For $t=0$, i.e., for the ratio of the coupling constants of the $\phi\to\pi\gamma$ and $\omega\to\pi\gamma$ decays, Eq. (25) takes the same form as Eq. (4) and, therefore, predicts the ratio of $\Gamma(\phi\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ consistent with experiment as discussed in Sec. II. However, there are important theoretical differences. In deriving Eq. (25) in this section, the assumption of vector-meson dominance is *not* made. Furthermore, PCAC is not used so that no soft-pion extrapolation is involved in deriving Eq. (25). Nevertheless, the result demonstrates that the vector-meson dominance[13] neatly connects our $[V_\alpha, A_\beta]=0$ constraint, Eq. (2) with Eq. (25) for $t=0$ obtained by exploiting the hypothesis of asymptotic level realization for the algebra, Eq. (5)—within the uncertainty involved in the soft-pion extrapolation.

Actually the implication of Eq. (25) is more general. Equation (25) predicts that for *any* momentum transfer $t$, the ratio of the amplitude for the $\phi\pi$ production to that of $\omega\pi$ production by a virtual photon is always suppressed by a factor $1/|R|$ if we neglect intermultiplet SU(3) mixing. $|R|$ is probably in the range of 0.05–0.06. This prediction can be tested experimentally. For another model to explain the rates of the $\phi\to\rho\pi$ and $\phi\to\pi\gamma$ decays see, for example, Ref. 14.

Finally our prediction on the strong-coupling constants of ground-state mesons, Eq. (20) or (21), is an important one. One can, for example, test Eq. (21) by using the rates of the $\rho\to\pi\pi$ and $\phi\to\rho\pi$ decays. For the value of $|R|$ in the range 0.05–0.06, Eq. (21) can be said to be well satisfied within the uncertainty associated with the soft-pion extrapolation.

The ideal limit ($g_{\phi\rho\pi}=0$, etc.) of this sum rule, Eq. (19), is in fact an $SU(6)_W$ result. However, Eq. (19) could be one of the good results of $SU(6)_W$ because this result, as shown in this paper, can also be derived by using a much weaker assumption than $SU(6)_W$. We may illustrate the somewhat delicate situation in an algebraic way. If we insert the algebra $[A_i, A_j]=if_{ijk}V_k$ between the ground-state mesons [corresponding to the $35\oplus 1$-plet of SU(6)], i.e., between $\langle V(\vec{k},\lambda)|$ and $|V'(\vec{k},\lambda)\rangle$ or between $\langle P(\vec{k})|$ and $|P'(\vec{k})\rangle$ with $\vec{k}\to\infty$, and demand that the algebra is saturated by the *same* ground-state mesons, we then also obtain Eq. (19) assuming $g_{\phi\rho\pi}=0$.
However, the saturation assumption used can hardly be justified. If we use, instead of saturation, our hypothesis of asymptotic level realization of SU(3) to the *same* algebra, we do not obtain Eq. (19) but we do obtain some other good constraint.[1] To obtain Eq. (19) we need to resort to the algebra Eq. (5). Equation (19) then implies that the fractional contribution of the ground state to the realization *under consideration* is independent of the helicity of the vector mesons. The fraction is in fact around 50% which hardly supports the saturation by the ground states.

## IV. FURTHER REMARKS

We have shown that the hypothesis of asymptotic level realization of SU(3) in the algebra, Eq. (5), produces remarkably consistent and simple dynamical constraints and enables us to make a realistic estimate of the rate of the $\phi\to\pi\gamma$ decay which turns out to be forbidden dynamically in the *ideal* limit where $\rho^2=\omega^2$. The predicted rate (based on the actual $\omega-\rho$ mass difference) is in reasonably good agreement with experiment. We stress that only the concept of *levels* of hadrons (which *groups* the $0^{-+}$ and $1^{--}$ nonets together as the ground-state mesons) is added to the purely algebraic theoretical framework. No higher symmetry such as $SU(6)_W$ is imposed from the outset upon our mesons. SU(6)-like constraints are, in fact, *discovered* in the process of realizing algebra by levels of mesons. Since SU(6) is certainly badly broken our method seems to provide a promising approach to the problem of finding SU(6)-like constraints among hadron interactions. The study of realization by higher-lying levels which is not undertaken in this paper should be interesting and will provide useful information on the pseudoscalar-meson and photon couplings of the mesons belonging to the levels $l=1, 2\ldots$. Its computation is not simple, but we do not need to make any apology for assuming exact symmetry such as SU(6). Our result seems to suggest that the notion of *levels of* mesons is more fundamental and information on the couplings among various levels may be extracted from the algebra as we have illustrated in Sec. III. The choice of our algebra, Eq. (5), which is essentially a type of chiral $SU(2)\otimes SU(2)$ algebra, was made because of the expectation that the effect of SU(3) breaking will be least important for the matrix elements of the axial charge $A_\pi$ as compared with the ones involving the charges $A_K$ and $A_8$. There is a growing feeling[4,15] that for the ground-state pseudoscalar mesons (especially for $\eta$ and $\eta'$) the effect of SU(3) mixing with the radially excited states could be important. Our result is free from this problem since in our Eqs. (6)–(14) no intermediate states involving $\eta$ and $\eta'$ appear. The information which comes from Eqs. (10) and (11) is based on $G$-parity conservation and is independent of the presence of possible mixing among $\eta$, $\eta'$ and their radially excited counterparts.

With a suitable consideration about (the possibly important) intermultiplet SU(3) mixing, the idea of equal fractional contribution may be extended to other commutators. In general, the fractional contribution will be labeled by the helicity, orbital and radial quantum numbers of the external and internal particle *levels* and *not* those of the individual particles themselves. Clearly, this is a great simplification since particles then enter into particular commutation relations in groups (i.e., levels) only. This paper shows—at least for the ground-state mesons—that our extension applied to the commutator $[[j_3^0, A_{\pi^+}], A_{\pi^-}] = 2j_3^0$ is powerful and produces results consistent with experiment. Further applications of this idea will be discussed in the future.

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[12]The radial quantum number should also be added to label levels.

[13]We note that both Eqs. (2) and (25) refer to the ratio of coupling constants. The ratio may be less sensitive to the existence of higher-lying vector mesons and to the off-mass-shell extrapolation involved.

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