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Analysis of hadronic decays of ψ/J particles in generalized Veneziano models

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Assuming that the ψ/J particle decays into ordinary hadrons through mixing with daughters of the ω and/or ϕ recurrences, we construct the amplitude for $\psi\to 3\pi$ from a five-point Veneziano amplitude for $K\bar{K}\to 3\pi$. This amplitude well describes the characteristic features of the experimental data on the 3π decay of ψ/J: (i) The $\rho\pi$ channel is dominant; (ii) signals of resonances at α_ρ = even are not seen in the 3π Dalitz plot.

I. INTRODUCTION

Recently there has been a great deal of interest in studies of the hadronic decays of ψ/J in connection with the fact that its hadronic decays into ordinary hadrons provide important clues for understanding the Okubo-Zweig-Iizuka rule.[1] Until now various attempts to explain the ψ hadronic decays have been made by many authors.[2] Especially, studies based on the dual unitarization scheme[3,4] give nice interpretations of some features of the ψ decays.

Experimentally the decay of ψ into multihadron channels has a large amount of branching ratio. Dalitz plots for those multihadronic decays such as $\psi\to 3\pi$ and $\psi\to K\bar{K}\pi$ have now become available.[5,6] Since the ψ is very heavy, kinematically allowed regions in Dalitz plots are much enlarged and enough energy is available to reach the higher resonances. Therefore detailed analyses of the distributions of final particles in the ψ hadronic decays may afford us a deeper insight into hadron dynamics.

The purpose of this paper is to present a speculation on the mechanism which governs the hadronic decays of ψ into ordinary hadrons and to study, as a first application, the $\psi\to 3\pi$ decay channel.

Analogously to the dual unitarization scheme, we assume that the ψ decays into ordinary hadrons through mixing with daughters of the ω and/or ϕ recurrences as illustrated in Fig. 1. Hence the amplitudes for $\psi\to$ ordinary hadrons may be written as

$$A(\psi\to \text{hadrons})=\sum_i\sum_\beta g_{\psi\omega_{i,\beta}}\frac{1}{m_\psi^2-\alpha_\omega^{-1}(i)}A(\omega_{i,\beta}\to\text{hadrons})+\sum_i\sum_\beta g_{\psi\phi_{i,\beta}}\frac{1}{m_\psi^2-\alpha_\phi^{-1}(i)}A(\phi_{i,\beta}\to\text{hadrons}) \qquad (1.1)$$

where i = positive integer and α_ω (α_ϕ) is the Regge trajectory of ω (ϕ). The $\omega_{i,\beta}$ ($\phi_{i,\beta}$) are daughters of the ω (ϕ) recurrences, satisfying the equation

$$\alpha_\omega(m_{\omega_{i,\beta}}{}^2)=i\ \ (\alpha_\phi(m_{\phi_{i,\beta}}{}^2)=i)\ ,$$

and having spin one and the same quantum numbers as ω (ϕ). In general, daughters are degenerate[7] and the subscript β distinguishes one state from another in the degenerate level. The constants $g_{\psi\omega_{i,\beta}}$ ($g_{\psi\phi_{i,\beta}}$) express the strength of the ψ mixing with $\omega_{i,\beta}$ ($\phi_{i,\beta}$).

It depends on the process which we consider whether the $\omega_{i,\beta}$ and/or $\phi_{i,\beta}$ contribute or not. For example, only daughters of the ω recurrences contribute to the channels $\psi\to$ odd pions, and both daughters of the ω and ϕ recurrences contribute to the channel $\psi\to K\bar{K}\pi$.

In the case when only the daughters of the ω

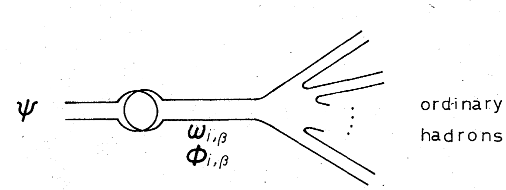

FIG. 1. Quark diagram for $\Psi\to 3\pi$. $\Psi-\omega_i$ mixing effect is assumed to be irrelevant to the final-state distributions in 3π.

recurrences contribute, we will find (in the following section) that the pole term of the daughters $\omega_{i=9,\beta}$ with mass of $\alpha_\omega^{-1}(i=9)$ dominates the decay amplitudes among others. Then the amplitudes for such channels will be proportional to the sum of products of the coupling constants $g_{\psi\omega_{i=9,\beta}}$ and the strong vertices for $\omega_{i=9,\beta}\to$ ordinary hadrons. We have now no means of estimation for $g_{\psi\omega_{i=9,\beta}}$. But when it is assumed *a priori* that the only one state among the degenerate states $\omega_{i=9,\beta}$ dominantly couples to ψ (and also to the $K\bar{K}$ channel), it may be possible to construct the decay amplitudes of ψ into certain channels from the generalized Veneziano amplitudes.

As a first illustration of our approach to the ψ hadronic decays, we shall study the $\psi\to 3\pi$ channel. An experimental plot for the $\psi\to 3\pi$ decays in Fig. 2 has the following features[5,6]: (i) The ψ decays into 3π mainly through the $\rho\pi$ channel (70%); (ii) the signals of ρ_f' [a daughter of $f(1270)$] in the 3π Dalitz plot are not seen; (iii) the signals of $g(1680)$ and ρ_g' (a daughter of g) seem to be present but are suppressed. In the following sections we shall show that our approach to this decay mode can well explain the above features of the experimental data.

Recently, Cohen-Tannoudji *et al.*[8] have proposed to use Virasoro amplitudes for certain decay channels such as $\psi\to 3\pi$ or $\psi\to K\bar{K}\pi$. The reason pointed out by them is that the duality properties of these processes are very different from the ones encountered in $\omega\to 3\pi$. Also, the Virasoro amplitude does not have the undesired poles at even integers of α_ρ. Our approach is quite different from theirs. We propose that the Dalitz plot for $\psi\to 3\pi$ is well described by the amplitude constructed from a five-point Veneziano amplitude for $K\bar{K}\to 3\pi$.[9] In fact our amplitude for $\psi\to 3\pi$ does not have the undesired even poles in the limit of zero pion mass.

In Sec. II we describe the way to construct the amplitude $A(\psi\to 3\pi)$. Evaluating this amplitude at the pole of $\alpha_\rho=1$, 2, and 3, we show in Sec. III that the gross features of the experimental data are well explained by our amplitude. Ratios among various coupling constants are also obtained. Section IV is devoted to some concluding remarks and discussions.

II. CONSTRUCTION OF THE AMPLITUDE FOR $\psi\to 3\pi$

As stated in Sec. I, only daughters of the ω recurrences contribute to the process $\psi\to 3\pi$. We determine the ω trajectory α_ω in the following way. We assume that trajectories are linear rising with universal slope α' and also that the exchange degeneracy of the ω and ρ trajectories is exact. We

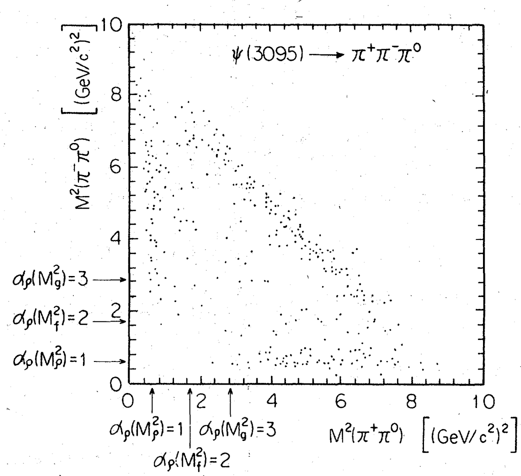

FIG. 2. Experimental 3π Dalitz plot.

fix the slope and the intercept $\alpha_\rho(0)$ by requiring[10,11] (1) $\alpha_\rho(m_\rho^2)=1$ and (2) $\alpha_\rho(m_\pi^2)=\frac{1}{2}$; the latter is implied by Adler's PCAC (partial conservation of axial-vector current) consistency condition.[12] Then we obtain

$$\alpha_\omega(s)=\alpha_\rho(s)=0.48+0.89s\ . \tag{2.1}$$

When we evaluate Eq. (2.1) at $s=m_\psi^2$, we find $\alpha_\omega(m_\psi^2)=9.0$. Therefore the contribution of the daughters $\omega_{i=9,\beta}$ with mass $\alpha_\omega^{-1}(i=9)$ dominates the amplitude for $\psi\to 3\pi$. If the level $\omega_{i=9,\beta}$ is nondegenerate (i.e., composed of only one state), we can construct the desired amplitude from a five-point Veneziano amplitude for the process $KK\to 3\pi$ by taking the pole residue at $\alpha_\omega=9$, projecting out the $J=1$ state, and finally using factorization. However, daughters may be degenerate[7] and the factorization does not hold in general. Here we make an *ad hoc* assumption: Only one state among the degenerate states $\omega_{i=9,\beta}$ dominantly couples to ψ and three pions, and also to the $K\bar{K}$ channel. Under this assumption, the factorization procedure is permitted and the amplitude for $\psi\to 3\pi$ is approximately proportional to that for $\omega_{i=9}\to 3\pi$ (hereafter we omit the subscript β),

$$A(\psi\to 3\pi)\propto A(\omega_{i=9}\to 3\pi)\ . \tag{2.2}$$

Incidentally, we have checked that the daughter $\omega_{i=9}$ (dominant one) is not a ghost state by using the amplitude for $K\bar{K}\to K\bar{K}$ trajectories given in Ref. 11.

Next we construct the $A(\omega_{i=9}\to 3\pi)$ from a five-point function for the process $K\bar{K}\to 3\pi$ which is given by Bardakci-Ruegg as follows[9]:

$$A(K\bar{K}\to 3\pi)\propto \sum_{P(3,4,5)} K_2^{+}\tau_{i_3}\tau_{i_4}\tau_{i_5}K_1\,\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}B_5(\alpha_{12}^{\omega}-1,\alpha_{23}^{K^*}-1,\alpha_{34}^{\rho}-1,\alpha_{45}^{\rho}-1,\alpha_{51}^{K^*}-1)\ . \tag{2.3}$$

The indices $1,\ldots,5$ label the particles in Fig. 3. The sum is over all permutations of the three pions. The function B_5 is defined as follows:

$$B_5(\alpha_{12}-1,\alpha_{23}-1,\alpha_{34}-1,\alpha_{45}-1,\alpha_{51}-1)=\int_0^1 du_1du_4u_1^{-\alpha_{12}}(1-u_1)^{-\alpha_{23}}u_4^{-\alpha_{45}}(1-u_4)^{-\alpha_{34}}(1-u_1u_4)^{-\alpha_{51}+\alpha_{23}+\alpha_{34}-1}\ . \tag{2.4}$$

The Regge trajectory α_{ij} which is transmitted through the ij channel is written with the universal slope α' [$=0.89$ (GeV$/c)^{-2}$] and the intercept $\alpha_{ij}(0)$ as

$$\alpha_{ij}=\alpha_{ij}(0)+\alpha' s_{ij}\ , \tag{2.5}$$

where

$$s_{ij}=(P_i+P_j)^2\ . \tag{2.6}$$

The K^* trajectory $\alpha_{K^*}(s)$ is determined[11] by requiring the universal slope and $\alpha_{K^*}(m_K^{\,2})=\frac{1}{2}$, which is also implied by the Adler condition,

$$\alpha_{K^*}(s)=0.28+0.89s\ . \tag{2.7}$$

In the amplitude of Eq. (2.3), the $I=0$ part for three pions (and also for two kaons) is

$$A^{I=0}(K\bar{K}\to 3\pi)\propto\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\sum_{P(3,4,5)}B_5(\alpha_{12}^{\omega}-1,\alpha_{23}^{K^*}-1,\alpha_{34}^{\rho}-1,\alpha_{45}^{\rho}-1,\alpha_{51}^{K^*}-1)\ , \tag{2.8}$$

which contains the desired process $K\bar{K}\to\omega_{i=9}\to 3\pi$ corresponding to Fig. 4. Then evaluating the amplitude of Eq. (2.8) at the pole of $\alpha_{12}^{\omega}=9$, we obtain

$$A^{I=0}(K\bar{K}\to 3\pi)\big|_{\text{at }\alpha_{12}=9}\propto\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\frac{1}{9-\alpha_{12}}\sum_{P(3,4,5)}\frac{1}{8!}\left(\frac{d^8}{du_1^8}\tilde{B}_5\right)_{u_1=0}\ , \tag{2.9}$$

where $\tilde{B}_5$ is defined as

$$B_5\equiv\int_0^1 du_1u_1^{-\alpha_{12}}\tilde{B}_5\ . \tag{2.10}$$

The explicit form of Eq. (2.9) is shown in Appendix A.

In order to obtain the desired amplitude $A(\psi\to 3\pi)$, we next project the $J=1$ state out of the pole residue of Eq. (2.9) and factorize out the $K\bar{K}\omega_{i=9}$ vertex. In general the pole residue at $\alpha=l$ is composed of a superposition of spin states with $J=l$, $(l-1)$, $(l-2),\ldots,0$. In our case, because of a symmetry property of Eq. (2.9), only odd-spin states appear, i.e., $J=9$, 7, 5, 3, and 1. Projection of the $J=1$ state can be achieved by using the rotation matrix $\mathfrak{D}^1_{M0}(\theta,\phi)$.[13] From general arguments on the Lorentz structure, the

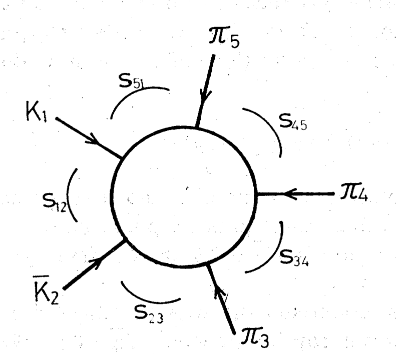

FIG. 3. Variables for the $K\bar{K}\pi\pi\pi$ reaction.

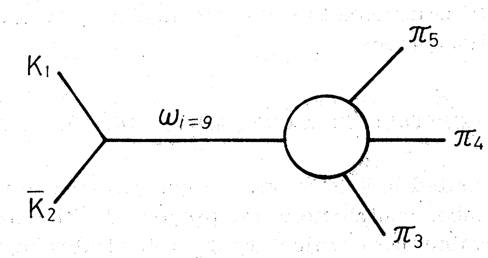

FIG. 4. The process $K\bar{K}\to\omega_{i=9}\to 3\pi$.

amplitude corresponding to Fig. 4 with the pole of the $J=1$ state $\omega_{i=9}$ has the form

$$A(K\bar{K}\to\omega_{i=9}\to 3\pi)=\text{const}\times\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\frac{1}{s_{12}-m_{\omega_i}{}^2}A(\omega_{i=9}\to 3\pi)\ , \tag{2.11}$$

where $A(\omega_{i=9}\to 3\pi)$ depends on the variables s_{34}, s_{45}, s_{35} only. The $K\bar{K}\omega_i$ coupling constant $f_{K\bar{K}\omega_i}$ is included in a normalization constant. Then we can pick up the amplitude $A(\omega_{i=9}\to 3\pi)$ by comparing the following two integrals:

$$\int d\Omega\mathfrak{D}_{M0}^{*1}(\theta,\phi)\times[\text{right-hand side of Eq. (2.9)}]\ , \tag{2.12a}$$

$$\int d\Omega\mathfrak{D}_{M0}^{*1}(\theta,\phi)\times[\text{right-hand side of Eq. (2.11)}]\ , \tag{2.12b}$$

where the integration is performed in the center-of-mass frame of the two kaons and θ and ϕ are the polar variables of one of the kaons. Integrations are very tedious but straightforward. Noticing that the $K\bar{K}\omega_{i=9}$ vertex is a mere constant and also that the amplitude for $\psi\to 3\pi$ is approximately proportional to that for $\omega_{i=9}\to 3\pi$, we find that the amplitude for $\psi\to 3\pi$ can be written in the form

$$A(\psi\to 3\pi)=C\epsilon_{\mu\nu\sigma\lambda}P_3^{\mu}P_4^{\nu}P_5^{\sigma}e^{\lambda}\left[\sum_{n=0}^{9}C_n(s,t,u)B(n-\alpha(s),\ 1-\alpha(t))+(t,u)\text{terms}+(u,s)\text{ terms}\right]\ , \tag{2.13}$$

where e is the polarization vector of ψ, C is a normalization constant, and we have redefined the variables as $s=s_{34}$, $t=s_{45}$, and $u=s_{53}$. The coefficient $C_n(s,t,u)$ are eighth degree polynomials in s, t, and u, but their expressions are very complicated. The pole structures come from the Euler beta functions. We can show that Eq. (2.13) is indeed symmetric in s, t, and u, although it does not seem to be so at first glance.

With this amplitude we can proceed further to introduce the imaginary part into the trajectory $\alpha_\rho(s)$ and to calculate the Dalitz-plot density for $\psi\to 3\pi$. This will be reported elsewhere. In the next section we evaluate $A(\psi\to 3\pi)$ at the poles of ρ, ρ'_f, and g and obtain the ratios among various coupling constants in order to see whether or not our amplitude can reproduce the gross features of the experimental data.

III. EVALUATION OF COUPLING CONSTANTS

To obtain the relations among coupling constants, we may straightforwardly evaluate Eq. (2.13) near the poles of $\alpha_\rho(s)=1$, 2, and 3. But we choose a different way here: We first evaluate the $I=0$ amplitude for $K\bar{K}\to 3\pi$, Eq. (2.8), at double poles of $\alpha_{12}^{\omega}=9$ and $\alpha_{34}^{\rho}=k$ (k is 1, 2, and 3); then we project out the $J=1$ state in the $K\bar{K}$ channel. This way it is much easier to see that the undesired even poles disappear in the limit of zero pion mass.

Near the double poles at $\alpha_{12}^{\omega}=9$ and $\alpha_{34}^{\rho}=k$ ($k=1$, 2, and 3) the $I=0$ amplitude for $K\bar{K}\to 3\pi$ becomes

$$A^{I=0}(K\bar{K}\to 3\pi)\Big|_{\substack{\text{at }\alpha_{12}=9\\ \alpha_{34}=1}}\propto\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\frac{1}{9-\alpha_{12}}\frac{1}{1-\alpha_{34}}\frac{2}{8!}B_1\ , \tag{3.1a}$$

$$A^{I=0}(K\bar{K}\to 3\pi)\Big|_{\substack{\text{at }\alpha_{12}=9\\ \alpha_{34}=2}}\propto\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\frac{1}{9-\alpha_{12}}\frac{1}{2-\alpha_{34}}\frac{1}{8!}(\alpha_{35}+\alpha_{45}-8)B_1\ , \tag{3.1b}$$

$$\begin{aligned}A^{I=0}(K\bar{K}\to 3\pi)\Big|_{\substack{\text{at }\alpha_{12}=9\\ \alpha_{34}=3}}\propto\ &\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\frac{1}{9-\alpha_{12}}\frac{1}{3-\alpha_{34}}\\ &\times\left\{\left(\frac{1}{2}\times\frac{1}{8!}\right)[\alpha_{45}(\alpha_{45}+1)+\alpha_{35}(\alpha_{35}+1)]B_1+\frac{1}{7!}B_2+\left(\frac{1}{2}\times\frac{1}{6!}\right)B_3\right\}\ ,\end{aligned} \tag{3.1c}$$

where

$$B_1=\alpha_{15}(\alpha_{15}+1)\cdots(\alpha_{15}+7)+\alpha_{25}(\alpha_{25}+1)\cdots(\alpha_{25}+7)\ , \tag{3.2a}$$

$$\begin{aligned}B_2=&[\alpha_{45}(\alpha_{23}-\alpha_{15}-\alpha_{45}+1)+\alpha_{35}(\alpha_{24}-\alpha_{15}-\alpha_{35}+1)]\alpha_{15}(\alpha_{15}+1)\cdots(\alpha_{15}+6)\\ &+[\alpha_{45}(\alpha_{13}-\alpha_{25}-\alpha_{45}+1)+\alpha_{35}(\alpha_{14}-\alpha_{25}-\alpha_{35}+1)]\alpha_{25}(\alpha_{25}+1)\cdots(\alpha_{25}+6)\ ,\end{aligned} \tag{3.2b}$$

$$\begin{aligned}B_3=&[(\alpha_{23}-\alpha_{15}-\alpha_{45}+1)(\alpha_{23}-\alpha_{15}-\alpha_{45}+2)+(\alpha_{24}-\alpha_{15}-\alpha_{35}+1)(\alpha_{24}-\alpha_{15}-\alpha_{35}+2)]\alpha_{15}(\alpha_{15}+1)\cdots(\alpha_{15}+5)\\ &+[(\alpha_{13}-\alpha_{25}-\alpha_{45}+1)(\alpha_{13}-\alpha_{25}-\alpha_{45}+2)+(\alpha_{14}-\alpha_{25}-\alpha_{35}+1)(\alpha_{14}-\alpha_{25}-\alpha_{35}+2)]\alpha_{25}(\alpha_{25}+1)\cdots(\alpha_{25}+5)\ .\end{aligned} \tag{3.2c}$$

In general there is a relation among trajectories α_{ij}, i.e.,

$$\alpha_{45}+\alpha_{35}=\alpha_{12}-\alpha_{34}+1+\alpha' m_\pi^2 . \tag{3.3}$$

When $\alpha_{12}=9$ and $\alpha_{34}=2$,

$$\alpha_{45}+\alpha_{35}=8+\alpha' m_\pi^2 . \tag{3.4}$$

Hence in the limit of zero pion mass, we find from Eq. (3.1b) that the residue of $A^{I=0}(K\bar{K}\to 3\pi)$ at $\alpha_{12}=9$ and $\alpha_{34}=2$ turns out to be exactly zero. Also, in the same limit it can be shown that the residue of $A^{I=0}(K\bar{K}\to 3\pi)$ becomes zero at $\alpha_{12}=9$ (more generally any integer) and $\alpha_{34}=$ any even integer. This means that the ω recurrences and their daughters decouple with the even integer poles of the ρ trajectory in the limit $m_\pi^2=0$. This is just a generalization of the fact that the even poles disappear in the Veneziano amplitude for $\pi\pi\to\pi\omega$ under the condition[14]

$$\alpha_\rho(s)+\alpha_\rho(t)+\alpha_\rho(u)=2 . \tag{3.5}$$

Recall that we have assumed the exact exchange degeneracy of the ρ and ω trajectories and Adler's condition. Under these assumptions the above condition Eq. (3.5) is reduced to $m_\pi^2=0$.

When the amplitude $A(\psi\to 3\pi)$ is constructed from $A^{I=0}(K\bar{K}\to 3\pi)$, it takes over the above-mentioned property: All the even poles disappear from $A(\psi\to 3\pi)$ in the limit $m_\pi^2=0$.

It is also interesting to know that the residue of $A^{I=0}(K\bar{K}\to 3\pi)$ vanishes at the double pole of α_{12} = even integer and $\alpha_{34}=$ any integer in the limit $m_\pi^2=0$. The proof is illustrated in Appendix B for the case $\alpha_{12}=$ any even integer n and $\alpha_{34}=1$. Physically the pion mass is nearly equal to zero. Therefore the above argument approves of our picking up a single term with $i=9$ in Eq. (1.1) for the process $\psi\to 3\pi$. A little change in slope parameter makes the value of $\alpha_\omega(m_\psi^2)$ depart from 9.0, and it seems that the contributions of other terms with $i=7$, 8, 10, and 11 may be also important. However, both amplitudes $A(\omega_{i=8}\to 3\pi)$ and $A(\omega_{i=10}\to 3\pi)$ are proportional to m_π^2 (here we have assumed that only one state is dominant among the degenerate states $\omega_{i=8,\beta}$ and $\omega_{i=10,\beta}$), and their magnitudes are very small. The contributions of the terms with $i=7$ and 11 are expected to be small because of the propagators $1/(m_\psi^2-\alpha_\omega^{-1}(i))$ in Eq. (1.1). Hence we conclude that it is a very good approximation to take up a single contribution of $\omega_{i=9}$ for the amplitude $\psi\to 3\pi$.

Now going back to Eqs. (3.1a)–(3.1c), projecting out the $J=1$ state in the $K\bar{K}$ channel in just the same way as we did in Sec. II, and factorizing out the $K\bar{K}\omega_{i=9}$ vertex (this procedure amounts to changing only a normalization constant), we finally obtain the amplitude $A(\psi\to 3\pi)$ evaluated near poles of $\alpha_{34}=1$, 2, and 3, which correspond to ρ, ρ'_f, and $g(\rho'_g)$ poles. Calculations are done with the use of Eqs. (2.1) and (2.7) for the ρ and K^* trajectories (i.e., we use the value of physical pion mass). The results are

$$A(\psi\to 3\pi)\underset{\alpha(s)=1}{\sim} C_0\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_3^{\mu_1}P_4^{\mu_2}P_5^{\mu_3}e^{\mu_4}\times\frac{100}{1-\alpha(s)}, \tag{3.6a}$$

$$A(\psi\to 3\pi)\underset{\alpha(s)=2}{\sim} C_0\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_3^{\mu_1}P_4^{\mu_2}P_5^{\mu_3}e^{\mu_4}\times\frac{0.46}{2-\alpha(s)}, \tag{3.6b}$$

$$A(\psi\to 3\pi)\underset{\alpha(s)=3}{\sim} C_0\epsilon_{\mu_1\mu_2\mu_3\mu_4}P_3^{\mu_1}P_4^{\mu_2}P_5^{\mu_3}e^{\mu_4}\times\frac{3.0[\alpha(t)-\alpha(u)]^2-4.4}{3-\alpha(s)}, \tag{3.6c}$$

where C_0 is an appropriate normalization constant.

We have changed variables s_{34}, s_{45}, s_{35} to s, t, u. Because of cyclic symmetry for three pions, the amplitude has the same forms near the poles in the t and u channels as those in the s channel. Note that two resonances g($J=3$ state) and ρ'_g($J=1$ state) are overlapping at the pole $\alpha(s)=3$ in Eq. (3.6c).

The squares of the residues (including the kinematical factor) in Eqs. (3.6a)–(3.6c) roughly express the densities around

$$s=m_\rho^2,\ m_{\rho'_f}^2,\ \text{and } m_g^2\ (m_{\rho'_g}^2)$$

in the Dalitz plot of the $\psi\to 3\pi$ decay. The Eqs. (3.6a)–(3.6c) tell us that (i) the ρ'_f signal can hardly be seen; (ii) the g and ρ'_g signals are present but are suppressed in comparison with ρ. Thus our amplitude for the $\psi\to 3\pi$ decay can well explain the gross features of the experimental data, especially the fact that the ψ decays into three pions dominantly through $\rho\pi$ channel.

From Eqs. (3.6a)–(3.6c) we can further obtain the relative strength of the coupling constants, whose normalizations are defined as follows[15]:

$$f_{\psi\rho\pi}\epsilon_{\mu\nu\lambda\sigma}(\partial^\mu\psi^\nu)(\partial^\lambda\rho_i^\sigma)\pi_i , \tag{3.7a}$$

and the same forms for the $\psi\rho'_f\pi$ and $\psi\rho'_g\pi$ couplings,

$$f_{\rho\pi\pi}\tfrac{1}{2}\epsilon_{ijk}(\pi_i\overleftrightarrow{\partial}_\mu\pi_j)\rho_k^\mu \tag{3.7b}$$

and the same forms for the $\rho'_f\pi\pi$ and $\rho'_g\pi\pi$ couplings, and

$$f_{\psi g\pi}\tfrac{1}{8}\epsilon_{\mu\nu\lambda\sigma}(\pi_i\overleftrightarrow{\partial}_\kappa\overleftrightarrow{\partial}_\tau\overleftrightarrow{\partial}_\sigma\psi^\nu)(\partial^\lambda g_i^{\mu\kappa\tau}), \tag{3.8a}$$

$$f_{g\pi\pi}\tfrac{1}{8}\epsilon_{ijk}(\pi_i\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_\lambda\pi_j)g_k^{\mu\nu\lambda}. \tag{3.8b}$$

With these coupling constants, we obtain the following relation:

$$f_{\psi\rho\pi}{}^2 f_{\rho\pi\pi}{}^2 : f_{\psi\rho'_f\pi}{}^2 f_{\rho'_f\pi\pi}{}^2 : f_{\psi\rho'_g\pi}{}^2 f_{\rho'_g\pi\pi}{}^2 : f_{\psi g\pi}{}^2 f_{g\pi\pi}{}^2$$
$$= 1 : 2.2\times10^{-5} : 2.6\times10^{-2} : 1.4\times10^{-1}\ \text{GeV}^{-8}. \tag{3.9}$$

The ratios $f_{g\pi\pi}{}^2/f_{\rho\pi\pi}{}^2$ and $f_{\rho'_g\pi\pi}{}^2/f_{\rho\pi\pi}{}^2$ can be obtained, for example, from the Veneziano amplitude for the $\pi\pi$ elastic amplitude with the $I=1$ state in the s channel[10]:

$$A^{I=1}(s,t,u) = -\beta\left[\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} - \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(u))}{\Gamma(1-\alpha(s)-\alpha(u))}\right]. \tag{3.10}$$

They are

$$\frac{f_{g\pi\pi}{}^2}{f_{\rho\pi\pi}{}^2} = 2\alpha'^2 \approx 1.58\ \text{GeV}^{-4} \tag{3.11}$$

and

$$\frac{f_{\rho'_g\pi\pi}{}^2}{f_{\rho\pi\pi}{}^2} = \tfrac{3}{40}[1+3\alpha(0)]^2 + \tfrac{3}{8}[1+\alpha(0)]^2 - \tfrac{3}{2}[1+\alpha(0)] + 1$$
$$\approx 0.048. \tag{3.12}$$

Using these values, we obtain the ratios

$$\frac{f_{\psi g\pi}{}^2}{f_{\psi\rho\pi}{}^2} \approx 9\times10^{-2}\ \text{GeV}^{-4} \tag{3.13}$$

and

$$\frac{f_{\psi\rho'_g\pi}{}^2}{f_{\psi\rho\pi}{}^2} \approx 5.4\times10^{-1}. \tag{3.14}$$

Finally, we can predict the ratios among the partial decay widths of ψ into $\rho\pi$, $g\pi$, and $\rho'_g\pi$. Using the width formula

$$\Gamma(\psi\to\rho\pi) = \frac{f_{\psi\rho\pi}{}^2}{12\pi}P_\rho{}^3,$$
$$\Gamma(\psi\to g\pi) = \frac{f_{\psi g\pi}{}^2}{45\pi}P_g{}^7\frac{m_\psi{}^4}{m_g{}^4}, \tag{3.15}$$

we find

$$\frac{\Gamma(\psi\to g\pi)}{\Gamma(\psi\to\rho\pi)} \approx 0.16 \tag{3.16}$$

and

$$\frac{\Gamma(\psi\to\rho'_g\pi)}{\Gamma(\psi\to\rho\pi)} \approx 0.23. \tag{3.17}$$

IV. CONCLUDING REMARKS AND DISCUSSIONS

In this paper we have made a speculation on the mechanism of the hadronic decays of ψ into ordinary hadrons and have analyzed, as a first application, the final-state distributions in the $\psi\to3\pi$ channel. In constructing the amplitude for $\psi\to3\pi$, we have started from the five-point Veneziano function for the $K\bar{K}\to3\pi$ process. Our numerical results give good explanations of the characteristic features of the Dalitz plot for the $\psi\to3\pi$ decay. In particular, the absence of signals at $\alpha_\rho(m_{\pi\pi}{}^2)$ = even is a consequence of the very small contribution of the odd daughter trajectories (in the limit $m_\pi{}^2=0$, there is no contribution). The signals of g and ρ'_g mesons appear, but they are suppressed in comparison with those of the ρ meson.

Finally we comment on the work by Cohen-Tannoudji *et al.*[8] They proposed to use a Virasoro amplitude for $\psi\to3\pi$ rather than a Veneziano amplitude. This Virasoro amplitude does not have poles at even integers of α_ρ, which is consistent with the experimental data. However, this amplitude predicts an enhancement in the central region $[\alpha(s)\approx\alpha(t)\approx3]$ of the 3π Dalitz plot. Although the signals of the g meson are predicted to be small, the ρ'_g signals are predicted to be larger than that of ρ. This fact is inconsistent with the experimental data. They also proposed that the Dalitz plot for $\psi\to K\bar{K}\pi$ exhibits characteristic structures described by a Virasoro amplitude.

In our approach, the amplitude for $\psi\to K\bar{K}\pi$ may be constructed from the five-point Veneziano amplitude for the process $K\bar{K}\to K\bar{K}\pi$. The kaon mass is far from zero, so the appearance of K^{**} signals can be expected. It may be also possible to apply our idea to the study of final-state distributions with more than three particles.

A further complete description of the Dalitz plot for $\psi\to3\pi$ and the analysis of the final-state distribution for the $\psi\to K\bar{K}\pi$ decay will be reported elsewhere.

Note added. After the completion of this work we found that a similar analysis had been done by Z. Z. Aydin,[16] using the Virasoro amplitude. He did not discuss the decay width of $\psi\to\rho'_g\pi$. But we find that its predicted value by the Virasoro amplitude is much different from ours. Experimental data seem to prefer our result.

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APPENDIX A

The explicit form of Eq. (2.9) is

$$
\begin{aligned}
A(K\bar{K}\to\omega_{i=9}\to 3\pi) \propto\ & \epsilon_{\mu_1\mu_2\mu_3\mu_4}P_1^{\mu_1}P_2^{\mu_2}P_3^{\mu_3}P_4^{\mu_4}\,\frac{1}{9-\alpha_{12}}\,\frac{1}{8!}\\
&\times\Bigg[\binom{8}{0}\alpha_{23}(\alpha_{23}+1)(\alpha_{23}+2)\cdots(\alpha_{23}+7)B(1-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{1}\alpha_{23}(\alpha_{23}+1)\cdots(\alpha_{23}+6)\alpha B(2-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{2}\alpha_{23}(\alpha_{23}+1)\cdots(\alpha_{23}+5)\alpha(\alpha+1)B(3-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{3}\alpha_{23}(\alpha_{23}+1)\cdots(\alpha_{23}+4)\alpha\cdots(\alpha+2)B(4-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{4}\alpha_{23}(\alpha_{23}+1)\cdots(\alpha_{23}+3)\alpha\cdots(\alpha+3)B(5-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{5}\alpha_{23}\cdots(\alpha_{23}+2)\alpha\cdots(\alpha+4)B(6-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{6}\alpha_{23}(\alpha_{23}+1)\alpha(\alpha+1)\cdots(\alpha+5)B(7-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{7}\alpha_{23}\alpha(\alpha+1)\cdots(\alpha+6)B(8-\alpha_{45},1-\alpha_{34})\\
&+\binom{8}{8}\alpha(\alpha+1)\cdots(\alpha+7)B(9-\alpha_{45},1-\alpha_{34})\\
&+(\text{other five permutations of the three pions})\Bigg],
\end{aligned}
$$

where $B(x,y)$ is the Euler beta function and $\alpha\equiv\alpha_{51}-\alpha_{23}-\alpha_{34}+1$.

APPENDIX B

We show that the residue of $A^{I=0}(K\bar{K}\to 3\pi)$ at the double poles of $\alpha_{12}=$ even integer and $\alpha_{34}=1$ vanishes in the limit $m_\pi^2=0$.

The residue of $A^{I=0}(K\bar{K}\to 3\pi)$ at the double poles of $\alpha_{12}=n$ and $\alpha_{34}=1$ is proportional to

$$
\begin{aligned}
G=\ &\alpha_{15}(\alpha_{15}+1)\cdots(\alpha_{15}+n-2)\\
&+\alpha_{25}(\alpha_{25}+1)\cdots(\alpha_{25}+n-2).
\end{aligned}
$$

When $n=$even, G is proportional to

$$(\alpha_{15}+\alpha_{25}+n-2).$$

On the other hand, there is a general relation,

$$\alpha_{15}+\alpha_{25}=\alpha_{34}-\alpha_{12}+1+\alpha' m_\pi^2.$$

So we get in the limit $m_\pi^2=0$,

$$\alpha_{15}+\alpha_{25}=2-n.$$

Therefore G becomes zero.

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