PHYSICAL REVIEW D VOLUME 17, NUMBER 5 1 MARCH 1978

First-order radiative corrections to asymmetry coefficients in neutron decay

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(Received 2 August 1977)

Electromagnetic radiative corrections to the electron-neutrino, electron, and neutrino asymmetry coefficients in neutron β decay are calculated. Strong-interaction effects are retained as suggested by Sirlin.

I. INTRODUCTION

The β disintegration of the free neutron is independent of the binding effects on nuclear neutrons and is the natural process to determine the axial-vector–to–vector coupling-constants ratio $\rho = G_A/G_V$. Of course, one must have a good estimation of the radiative corrections to first order. In the past, calculations[1–6] in this respect have been performed. The main concern has usually been with the energy spectrum of the emitted electron, whose shape has been shown[4] to be free of ultraviolet-cutoff or strong-interaction complications, although the absolute normalizations, i.e., the values of G_V and G_A do depend on them.

Unfortunately, the electron energy spectrum has not yet been accurately measured in this decay. Instead, fairly good measurements of the angular-distribution asymmetry coefficients are available, namely, the electron-neutrino angular coefficient, $A_{e\nu}$, and the electron-momentum–neutron-spin and the neutrino-momentum–neutron-spin asymmetry coefficients,[7] A_e and A_ν, respectively. Following the work of Sirlin,[4] Shann[5] computed the radiative corrections to A_e. It should therefore be interesting to know the radiative corrections to $A_{e\nu}$ and A_ν also.

We want to obtain such corrections in the present paper. In Sec. II we explain under what assumptions we work, and compute the radiative corrections due to the exchange of virtual photons. In Sec. III, we calculate the contribution due to inner bremsstrahlung. In Sec. IV we give our final results and discuss their usefulness.

II. VIRTUAL RADIATIVE CORRECTIONS

In our calculation we certainly cannot ignore two facts: First, there will be a contribution of the strong interactions. That is, the radiative corrections will have a part that is affected by them. Second, there will be an ultraviolet divergence, rendering the calculation cutoff dependent.[3] The first problem will hopefully be solved as our understanding of strong interactions is completed. The second one may already be solved with the so called "gauge theories",[8] although such a solution also requires a better understanding of strong interactions. Some time ago, Sirlin[4] analyzed this situation showing that one can, in a general fashion, separate a finite calculable part from another one containing all the problems just mentioned. This separation is approximate and is valid only as long as one can neglect contributions of the order of $\alpha(E/M)\ln(M/E)$ and $\alpha q/M$, where M is the nucleon mass, E is the electron energy, q is the magnitude of the momentum transfer to the leptons, and α is the fine-structure constant. In this approach one can compute part of the radiative corrections, keeping track of the problematic contributions, in a clear way. This is the point of view we shall adopt in the present paper.

The separation of the model-independent part is discussed in detail in Ref. 4. The transition amplitude for the virtual part consists of five terms,

$$A_{\text{virtual}} = M_0 + M_1 + M_2 + M_3^c + S\,. \tag{1}$$

The zeroth-order matrix element M_0 is given, as usual, by

$$M_0 = \frac{G_V}{\sqrt{2}}\,\bar{u}_e(l)\gamma_\mu(1+\gamma_5)v_\nu(p_\nu)\,\bar{u}_p(p_2)\gamma_\mu(1+\rho\gamma_5)u_n(p_1) \tag{2}$$

in a standard notation.[4] The first-order matrix elements can be expressed in compact notation as follows:

$$M_1 = M_0 P_1 + \frac{G_V}{\sqrt{2}}\;\bar{u}_e \not{p}_2 \boldsymbol{\gamma}_\lambda(1+\gamma_5)v_\nu \bar{u}_p \gamma_\lambda(1+\rho\gamma_5)u_n P_2\,, \tag{3}$$

$$M_2 = M_0 P_3\,, \tag{4}$$

$$M_3^c = M_0 P_4\,, \tag{5}$$

and

$$S = \frac{G_V}{\sqrt{2}}\,\frac{\alpha}{2\pi}\,\bar{u}_e\gamma_\lambda(1+\gamma_5)v_\nu\bar{u}_p(c\gamma_\lambda + d\gamma_\lambda\gamma_5)u_n\,. \tag{6}$$

The expressions for the different P_i are given in the Appendix. The terms M_1, M_2, and M_3 are independent of the strong interactions and of any intermediate-boson hypothesis, and each one is finite if the Landau gauge is used. The sum of the three is gauge invariant, and it contains the infrared divergence. Inasmuch as the approximation of neglecting terms of order $\alpha(E/M)\ln(M/E)$ and $\alpha q/M$ is valid, it is in c and d of Eq. (6) where all the complications due to strong interactions and to the existence of an intermediate vector boson and of the ultraviolet divergence are to be found. In this approximation c and d are constants.

We want to obtain the corrections to the decay of a polarized neutron, so let us assume that it is polarized in the direction $\hat{s}$. The transition probability with the virtual radiative corrections, allowing for the observation of the electron and neutrino directions, is

$$\begin{aligned} d\omega(n\to pe\nu)_{\text{virtual}} = \frac{G_V^2}{(2\pi)^5}(E_m-E)^2 d\Omega_\nu d^3l\Big(&1+\frac{\alpha}{\pi}(\phi_1+c)+3\rho^2\Big[1+\frac{\alpha}{\pi}(\phi_1+d)\Big] \\ &+\beta\hat{l}\cdot\hat{p}_\nu\Big\{1+\frac{\alpha}{\pi}(\phi_2+c)-\rho^2\Big[1+\frac{\alpha}{\pi}(\phi_2+d)\Big]\Big\} \\ &+\beta\hat{l}\cdot\hat{s}\Big\{-2\rho^2\Big[1+\frac{\alpha}{\pi}(\phi_2+d)\Big]+2\rho\Big[1+\frac{\alpha}{\pi}\Big(\phi_2+\frac{d+c}{2}\Big)\Big]\Big\} \\ &+\hat{P}_\nu\cdot\hat{s}\Big\{2\rho^2\Big[1+\frac{\alpha}{\pi}(\phi_1+d)\Big]+2\rho\Big[1+\frac{\alpha}{\pi}\Big(\phi_1+\frac{d+c}{2}\Big)\Big]\Big\}\Big), \end{aligned} \tag{7}$$

where $E_m=M_n-M_p$ and $\beta=|\vec{l}\,|/E$. The functions ϕ_1 and ϕ_2 are functions of β. They are given by

$$\phi_1=2\ln\frac{\lambda}{m}\left(\frac{1}{\beta}\tanh^{-1}\beta-1\right)-\frac{1}{\beta}(\tanh^{-1}\beta)^2+\beta\tanh^{-1}\beta+\frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right)+\tfrac{3}{2}\ln\frac{M_p}{m}-\frac{11}{8}, \tag{8}$$

and

$$\phi_2=2\ln\frac{\lambda}{m}\left(\frac{1}{\beta}\tanh^{-1}\beta-1\right)-\frac{1}{\beta}(\tanh^{-1}\beta)^2+\frac{1}{\beta}\tanh^{-1}\beta+\frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right)+\tfrac{3}{2}\ln\frac{M_p}{m}-\frac{11}{8}, \tag{9}$$

where λ is the infrared cutoff; M_p is the proton mass; m is the electron mass, and $L(y)$ is the Spence function.[9] The similarity between ϕ_1 and ϕ_2 is striking.

The infrared-divergent term will be cancelled by a similar contribution from the inner-bremsstrahlung correction, to which we now turn.

III. INNER-BREMSSTRAHLUNG CORRECTION

The transition amplitude for the emission of a real photon accompanying the β decay of the neutron is

$$\begin{aligned} A_{\text{brems}} = &\frac{eG_V}{\sqrt{2}}\bar{u}_p(p_2)\gamma_\mu(1+\rho\gamma_5)u_n(p_1) \\ &\times\left[\left(\frac{2l\cdot\epsilon_i}{2l\cdot k+\lambda^2+i\epsilon}-\frac{2p_2\cdot\epsilon_i}{2p_2\cdot k+\lambda^2+i\epsilon}\right)\bar{u}_e(l)\gamma_\mu(1+\gamma_5)v_\nu(p_\nu)+\bar{u}_e(l)\frac{\gamma\cdot\epsilon_i\gamma\cdot k}{2l\cdot k+\lambda^2+i\epsilon}\gamma_\mu(1+\gamma_5)v_\nu(p_\nu)\right]. \end{aligned} \tag{10}$$

In Eq. (10) we have kept only contributions of zeroth order in the recoil energy of the proton. This makes Eq. (10) free of strong-interaction uncertainties. In the above expression, k and ϵ_i are the momentum and polarization four-vectors of the emitted photon. In order to obtain the correct elimination of the infrared divergence in the full radiative correction to first order in α, the photon is handled as a massive vector boson of mass λ. This λ is the same one that appears in Eq. (8) and (9). Calculations can be performed consistently using the Coester representation for the massive photon.[10]

The bremsstrahlung transition probability, summed over the polarization and four-momentum of the photon, but again allowing for the observation of the electron and neutrino directions and polarization of the neutron, is

$$\begin{aligned} d\omega(n\to pe\nu)_{\text{brems}} = &\frac{G_V^2}{(2\pi)^5}\frac{\alpha}{\pi}(E_m-E)^2 d\Omega_\nu d^3l \\ &\times[(1+3\rho^2)\theta_1+\beta\hat{l}\cdot\hat{p}_\nu(1-\rho^2)\theta_2+\hat{s}\cdot\hat{p}_\nu(2\rho^2+2\rho)\theta_1+\beta\hat{s}\cdot\hat{l}(-2\rho^2+2\rho)\theta_2], \end{aligned} \tag{11}$$

in the same notation as Eq. (7). The functions θ_1 and θ_2 are given by

$$\theta_1 = \left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right)\left[\frac{2}{3}\,\frac{E_m - E}{E} + 2\ln\frac{2(E_m-E)}{\lambda} - 3\right] + \frac{(E_m-E)^2}{12E^2}\,\frac{1}{\beta}\tanh^{-1}\beta + 1$$
$$+\frac{1}{2\beta}\tanh^{-1}\beta\left[2+\ln\left(\frac{1-\beta^2}{4}\right)\right] + \frac{1}{\beta}[L(\beta) - L(-\beta)] + \frac{1}{2\beta}\left[L\left(\frac{1-\beta}{2}\right) - L\left(\frac{1+\beta}{2}\right)\right] \tag{12}$$

and

$$\theta_2 = \left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right)\left[\frac{2}{3}\,\frac{E_m - E}{E\beta^2} + \frac{(E_m-E)^2}{12E^2\beta^2} + 2\ln\frac{2(E_m-E)}{\lambda} - 3\right] + 1 + \frac{1}{2\beta}\tanh^{-1}\beta\left[2+\ln\left(\frac{1-\beta^2}{4}\right)\right]$$
$$+\frac{1}{\beta}[L(\beta) - L(-\beta)] + \frac{1}{2\beta}\left[L\left(\frac{1-\beta}{2}\right) - L\left(\frac{1+\beta}{2}\right)\right]. \tag{13}$$

IV. RESULTS AND DISCUSSION

The radiative correction to $n \to pe\nu$ to first order in α is given by the sum of Eqs. (7) and (11):

$$d\omega(n\to pe\nu) = \frac{G_V{}^2}{(2\pi)^5}(E_m - E)^2 d\Omega_\nu\, d^3l\Bigg(1 + \frac{\alpha}{\pi}(\phi_1 + \theta_1 + c) + 3\rho^2\left[1 + \frac{\alpha}{\pi}(\phi_1+\theta_1+d)\right]$$
$$+\beta\hat{l}\cdot\hat{p}_\nu\left\{1 + \frac{\alpha}{\pi}(\phi_2+\theta_2+c) - \rho^2\left[1+\frac{\alpha}{\pi}(\phi_2+\theta_2+d)\right]\right\}$$
$$+\beta\hat{l}\cdot\hat{s}\left\{-2\rho^2\left[1+\frac{\alpha}{\pi}(\phi_2+\theta_2+d)\right] + 2\rho\left[1+\frac{\alpha}{\pi}\left(\phi_2+\theta_2+\frac{d+c}{2}\right)\right]\right\}$$
$$+\hat{p}_\nu\cdot\hat{s}\left\{2\rho^2\left[1+\frac{\alpha}{\pi}(\phi_1+\theta_1+d)\right] + 2\rho\left[1+\frac{\alpha}{\pi}\left(\phi_1+\theta_1+\frac{d+c}{2}\right)\right]\right\}\Bigg), \tag{14}$$

where $\phi_1+\theta_1$ and $\phi_2+\theta_2$ can be simplified as follows:

$$\phi_1+\theta_1 = \tfrac{3}{2}\ln\frac{M_p}{m} - \frac{3}{8} + 2\left(\frac{\tanh^{-1}\beta}{\beta} - 1\right)\left[\frac{E_m-E}{3E} - \frac{3}{2} + \ln\frac{2(E_m-E)}{m}\right]$$
$$+\frac{2}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{2\beta}\tanh^{-1}\beta\left[2(1+\beta^2) + \frac{(E_m-E)^2}{6E^2} - 4\tanh^{-1}\beta\right] \tag{15}$$

and

$$\phi_2+\theta_2 = \tfrac{3}{2}\ln\frac{M_p}{m} - \frac{3}{8} + \left(\frac{\tanh^{-1}\beta}{\beta} - 1\right)\left[\frac{(E_m-E)^2}{12\beta^2E^2} + \frac{2(E_m-E)}{3E\beta^2} + 2\ln\frac{2(E_m-E)}{m} - 3\right]$$
$$+\frac{2}{\beta}L\left(\frac{2\beta}{1+\beta}\right) - \frac{2}{\beta}\tanh^{-1}\beta(\tanh^{-1}\beta - 1). \tag{16}$$

Sirlin[4] had already calculated the correction to the electron energy spectrum, that is, the first two terms in Eq. (14). Our result agrees exactly with his function $g(E, E_m, m)$. Shann[5] computed the corrections to the electron-momentum–neutron-spin angular distribution, that is, the fourth term in Eq. (14). Again, our result agrees exactly with his function $h(E, E_m, m)$. Actually it is the real parts of c and d that appear in Eq. (14), although we did not make it clear. The Coulomb-correction term is not yet incorporated into Eq. (14). On its inclusion a term equal[3] to π^2/β should be added to $\phi_1+\theta_1$ and $\phi_2+\theta_2$.

Equation (14) can be written more compactly by absorbing c and d in G_V and G_A, namely,

$$G_V' = G_V\left(1 + \frac{\alpha}{2\pi}c\right),$$

and

$$G_A' = G_A\left(1 + \frac{\alpha}{2\pi}d\right).$$

Neither the electron energy spectrum nor the three angular distributions have yet been accurately measured; it is the transition rate and the angular asymmetry coefficients that are available[7] with reasonable precision. The experimental values are usually quoted for these coefficients defined as

$$A^0_{e\nu} = \frac{1-\rho^2}{1+3\rho^2} = \frac{b}{a}\,\frac{2(N^+_{e\nu} - N^-_{e\nu})}{N^+_{e\nu} + N^-_{e\nu}}, \qquad (17)$$

$$A^0_{e} = \frac{-2\rho^2+2\rho}{1+2\rho^2} = \frac{b}{a}\,\frac{2(N^+_{e} - N^-_{e})}{N^+_{e} + N^-_{e}}, \qquad (18)$$

and

$$A^0_{\nu} = \frac{2\rho^2+2\rho}{1+3\rho^2} = \frac{2(N^+_{\nu} - N^-_{\nu})}{N^+_{\nu} + N^-_{\nu}}. \qquad (19)$$

The null index indicates that no radiative corrections are included. Here a and b are the integrals

$$a = \int_0^{l_m} \frac{l^3}{E}(E_m - E)^2 dl$$

and

$$b = \int_0^{l_m} l^2 (E_m - E)^2 dl,$$

with $l_m^2 = E_m^2 - m^2$.

$N^{\pm}_{e\nu}$ is the number of events with the directions of e and ν at less than or more than $\pi/2$ from each other, respectively. $N^{\pm}_{e,\nu}$ are defined similarly but with respect to the neutron polarization direction. The expressions with radiative corrections to be compared with experiment are

$$A^{\exp}_{e\nu} = A^0_{e\nu}(\rho')\left[1 + \frac{\alpha}{\pi}\left(\frac{1}{a}\Phi_2 - \frac{1}{b}\Phi_1\right)\right], \qquad (20)$$

$$A^{\exp}_{e} = A^0_{e}(\rho')\left[1 + \frac{\alpha}{\pi}\left(\frac{1}{a}\Phi_2 - \frac{1}{b}\Phi_1\right)\right], \qquad (21)$$

$$A^{\exp}_{\nu} = A^0_{\nu}(\rho'), \qquad (22)$$

where

$$\Phi_1 = \int_0^{l_m} dl\, l^2 (E_m - E)^2 (\phi_1 + \theta_1) \qquad (23)$$

and

$$\Phi_2 = \int_0^{l_m} dl\, \frac{l^3}{E}(E_m - E)^2 (\phi_2 + \theta_2). \qquad (24)$$

The coefficients $A^0_{e\nu}(\rho')$, $A^0_e(\rho')$, and $A^0_\nu(\rho')$ refer to the first equality in Eqs. (17)–(19) only, but using $\rho' = G'_V/G'_A$ instead of ρ. As $\phi_1 + \theta_1$ and $\phi_2 + \theta_2$ are known, we can give the numerical values of the integrals Φ_1 and Φ_2,

$$\Phi_1 = 2.7771 \times 10^{-3}\ \mathrm{MeV}^5,$$

$$\Phi_2 = 1.974 \times 10^{-3}\ \mathrm{MeV}^5.$$

In these values we have included the Coulomb correction also. The second term in Eqs. (20) and (21) is then computed to be 0.0013.

For applications of the above formulas one must perform a reliable estimation of the constants c and d. The ultraviolet divergence contained in them could be eliminated either with a cutoff or by making use of the ideas of gauge theories along with some model for the strong interactions.[11] Conversely, one could use the experimental data, once enough precision is attained, along with some model to determine allowed ranges for c and d, for example, as done by Blin-Stoyle and Freeman[12] and very recently by Alburger and Wilkinson.[13] For this, one should fix the values of G_V and G_A within such model, because as mentioned before only G'_V and G'_A enter into the formulas and only these two can be experimentally determined.

As a final remark, let us say that in the present paper we have shown that the shape of the angular distributions given in Eq. (14) do not depend on the details of strong interactions or the intermediate boson. We see that the conclusion of Sirlin for the shape of the electron spectrum is extended just as well for the above angular distributions, except that $\phi_2 + \theta_2$, instead of $\phi_1 + \theta_1$, appears in the electron-neutrino and the electron-momentum–neutron-spin angular distributions. The model dependence has been absorbed into G'_V and G'_A, which makes the shapes model independent.

ACKNOWLEDGMENTS

One of the authors (A.G.) would like to thank S. Berman for a stimulating discussion. He is also grateful to A. Sirlin for a very encouraging conversation.

APPENDIX

After standard integration over the virtual-photon four-momentum, the factors P_i appearing in Eqs. (3)–(5) are

$$P_1 = \frac{\alpha}{2\pi}\left[\frac{2}{\beta}\ln\frac{\lambda}{m}\tanh^{-1}\beta - \frac{1}{\beta}(\tanh^{-1}\beta)^2 + \frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \ln\frac{M_p}{m} + \frac{1}{\beta}\tanh^{-1}\beta + \frac{1}{2} + \ln\frac{\Lambda}{M_p}\right],$$

$$P_2 = -\frac{\alpha}{2\pi}\,\frac{m}{M_p E_B}\tanh^{-1}\beta,$$

$$P_3 = \frac{\alpha}{2\pi}\left(\ln\frac{m}{\lambda} - \frac{9}{8} - \frac{1}{2}\ln\frac{\Lambda}{m}\right),$$

and

$$P_4 = \frac{\alpha}{2\pi}\left(\ln\frac{M_p}{\lambda} - \frac{3}{4} - \frac{1}{2}\ln\frac{\Lambda}{M}\right).$$

The ultraviolet cutoff Λ appears here because the

Feynman form for the photon propagator was used instead of the Landau form. Owing to the gauge invariance of A_{virtual}, the terms in $\ln\Lambda$ in the above expressions cancel each other. We would obtain cutoff-independent integrals if we use the Landau gauge. Contributions of the order of E/M to the P_i have been neglected in accordance with the approximation used in the text.

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