Polarized-electron–nucleon scattering in gauge theories of weak and electromagnetic interactions

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The asymmetry in the scattering of longitudinally polarized electrons by nucleons is considered in the context of gauge theories of the weak and electromagnetic interactions. The predictions of a variety of gauge groups and assignments of fermion representations of present interest are given for deep-inelastic scattering, elastic scattering, and Δ electroproduction. It is shown that a combination of the three experiments carried out to a few parts in 10^5 at $Q^2 \sim 1$ GeV$^2/c^2$ can distinguish between various $SU(2)\times U(1)$ models and between various classes of models based on larger gauge groups.

I. INTRODUCTION

A dependence on the electron's helicity of the elastic or inelastic electron-nucleon scattering cross section would be a *prima facie* parity-violating effect. While this is not expected to any order of the parity-conserving electromagnetic interactions, the observation[1] of processes involving weak neutral currents leads one to expect such effects. They would arise in lowest order of the weak interactions from the interference of weak and electromagnetic amplitudes. In gauge theories which unify the weak and electromagnetic interactions, the dependence of the scattering cross section on electron helicity should occur at a level of approximately $q^2/M_Z{}^2 \approx G_F q^2/(4\pi\alpha)$. In high-energy experiments, where q^2 is typically of order a few GeV$^2/c^2$, the corresponding parity-violating effects are of order a few times 10^{-5}. An experiment[2] designed to reach such a level of accuracy is being undertaken at SLAC.

The consequences of weak-electromagnetic interference in a gauge-theory context for deep-inelastic electron-nucleon scattering have been explored previously.[3-5] In Sec. II of this paper we derive in a different and much simpler way the formula for the polarized-electron asymmetry. Our result agrees with the final consensus of the earlier work.[3-5] In addition to our derivation being physically transparent and permitting one to understand easily the overall sign of the asymmetry, we treat the asymmetries predicted by a variety of gauge groups and assignments of fermion representations of current interest which go beyond the simplest Weinberg-Salam[6] model. Such extensions of the simplest model are suggested by both recent experimental[7-9] and theoretical developments.

In Sec. III the asymmetry expected for polarized-electron–proton elastic scattering is discussed. While the formula for the asymmetry has again been derived previously by others[10] in order to calculate the predictions of the Weinberg-Salam model, we calculate the predictions for the wider class of fermion representations and gauge groups of present interest.

The asymmetry in the process $ep \to e\Delta^+$, which has not been considered previously, is treated in Sec. IV. The required electromagnetic and weak transition matrix elements are calculated in the quark model. Predictions are presented for the same class of gauge theories considered in Secs. II and III.

These calculations, discussed in Sec. V, lead to two interesting conclusions. First, parity-violating effects may very well be much larger or much smaller than expected on the basis of the Weinberg-Salam model. Put differently, the experiments to be done soon may rule out some now popular models relatively easily. Second, the combination of the three polarized asymmetry experiments, deep-inelastic eN, elastic ep, and $ep \to e\Delta^+$, has a much greater ability to distinguish between models than any one of them individually. We expect then that along with neutrino scattering and atomic parity-violation searches, these experiments will play a prominent role in determining the character of the neutral weak current.

II. DEEP-INELASTIC POLARIZED-ELECTRON–NUCLEON SCATTERING ASYMMETRIES

Our treatment of deep-inelastic scattering is phrased in the language of the quark-parton model.[11] We work in the high-energy and large-momentum-transfer regime where the electron and quark-parton masses are neglected. It follows that the Dirac operators $(1+\gamma_5)/2$ and $(1-\gamma_5)/2$ project out the right- and left-handed pieces of the fermions, respectively, and that vector (γ_μ) and axial-vector $(\gamma_\mu\gamma_5)$ interactions preserve helicity.

The photon couples to a fermion (f) through the vector current (γ_μ) with a strength given by its charge, Q_f^γ. We may use the trivial identity $\gamma_\mu = \gamma_\mu(1+\gamma_5)/2+\gamma_\mu(1-\gamma_5)/2$ to define left-and right-handed charges

$$Q_{Lf}^\gamma = Q_{Rf}^\gamma \equiv Q_f^\gamma . \qquad (1)$$

Similarly, the weak neutral intermediate vector boson, Z^0, is defined to couple with strengths Q_{Lf}^Z and Q_{Rf}^Z to left- and right-handed fermions (f), respectively. Hence the Dirac structure of the Z^0-fermion vertex is

$$Q_{Rf}^Z\gamma_\mu(1+\gamma_5)/2+Q_{Lf}^Z\gamma_\mu(1-\gamma_5)/2 .$$

The right- and left-handed weak charges are generally different.

It is familiar from treatments[11] of deep-inelastic neutrino scattering that a left-handed neutrino scatters from a left-handed quark with a distribution isotropic in the center-of-mass scattering angle and from a right-handed antiquark with a distribution $(1+\cos\theta)^2/4$ in the center-of-mass or $(1-y)^2$ in the lab (where $y=\nu/E=1-E'/E=$ the fraction of the incident lepton's energy which is transferred to the nucleon).[12] In both instances the handedness of the quark (and lepton) is the same after the scattering as it was before.

The same considerations apply to the case of longitudinally polarized-electron scattering which is of interest here. A left- (right-) handed electron scatters from a left- (right-) handed quark with a distribution in y which is $\propto 1$, while a left- (right-) handed electron scatters from a right- (left-) handed quark with a distribution $\propto(1-y)^2$. The contributions from the amplitudes due to γ and Z^0 exchange to the scattering of electrons and quarks of given helicities are coherent, but those for different helicities are incoherent. Since the amplitude arising from a given exchange is proportional to the product of the charges at the two vertices and the exchanged boson's propagator, we have the following expressions for the differential cross sections[12]:

Right-handed electron on right-handed quark of type i,

$$d\sigma \propto \left|\frac{Q_{Re}^\gamma Q_{Ri}^\gamma}{q^2}+\frac{Q_{Re}^Z Q_{Ri}^Z}{q^2+M_Z^2}\right|^2 , \qquad (2a)$$

right-handed electron on left-handed quark of type i,

$$d\sigma \propto \left|\frac{Q_{Re}^\gamma Q_{Li}^\gamma}{q^2}+\frac{Q_{Re}^Z Q_{Li}^Z}{q^2+M_Z^2}\right|^2 (1-y)^2 , \qquad (2b)$$

left-handed electron on left-handed quark of type i,

$$d\sigma \propto \left|\frac{Q_{Le}^\gamma Q_{Li}^\gamma}{q^2}+\frac{Q_{Le}^Z Q_{Li}^Z}{q^2+M_Z^2}\right|^2 , \qquad (2c)$$

left-handed electron on right-handed quark of type i,

$$d\sigma \propto \left|\frac{Q_{Le}^\gamma Q_{Ri}^\gamma}{q^2}+\frac{Q_{Le}^Z Q_{Ri}^Z}{q^2+M_Z^2}\right|^2 (1-y)^2 . \qquad (2d)$$

The asymmetry for longitudinally polarized electrons,

$$A=\frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} , \qquad (3)$$

may now be computed simply by multiplying Eqs. (2a) – (2d) by the probability $f_i(x)$ of finding a quark of type i with fractional longitudinal momentum $x=q^2/2M_N\nu$ in the nucleon and noting that the differential cross sections $d\sigma_R = d\sigma_R(x,y)$ and $d\sigma_L = d\sigma_L(x,y)$ involve an incoherent sum over left- and right-handed quark partons in the unpolarized nucleon. Keeping terms up to order q^2/M_Z^2 we have

$$A(x,y)=\frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}=\frac{q^2}{M_Z^2}\Big\{2\sum_i f_i(x)[(Q_{Re}^\gamma Q_{Ri}^\gamma Q_{Re}^Z Q_{Ri}^Z - Q_{Le}^\gamma Q_{Li}^\gamma Q_{Le}^Z Q_{Li}^Z) + (Q_{Re}^\gamma Q_{Li}^\gamma Q_{Re}^Z Q_{Li}^Z - Q_{Le}^\gamma Q_{Ri}^\gamma Q_{Le}^Z Q_{Ri}^Z)(1-y)^2]\Big\}$$
$$\times\Big(\sum_i f_i(x)\{[(Q_{Re}^\gamma)^2(Q_{Ri}^\gamma)^2+(Q_{Le}^\gamma)^2(Q_{Li}^\gamma)^2]+[(Q_{Re}^\gamma)^2(Q_{Li}^\gamma)^2+(Q_{Le}^\gamma)^2(Q_{Ri}^\gamma)^2](1-y)^2\}\Big)^{-1} . \qquad (4)$$

Recalling that $Q_{Le}^\gamma = Q_{Re}^\gamma = Q_e^\gamma = -e$ and $Q_{Li}^\gamma = Q_{Ri}^\gamma$, we simplify this to

$$A(x,y)=-\frac{q^2}{M_Z^2}\frac{\sum_i f_i(x)(Q_i^\gamma/e)[(Q_{Re}^Z Q_{Ri}^Z - Q_{Le}^Z Q_{Li}^Z)+(Q_{Re}^Z Q_{Li}^Z - Q_{Le}^Z Q_{Ri}^Z)(1-y)^2]}{\sum_i f_i(x)(Q_i^\gamma)^2[1+(1-y)^2]} \qquad (5)$$

This may also be expressed in terms of vector and axial-vector coupling constants of the Z^0,

$$g_V = (Q_R^Z + Q_L^Z)/2\,, \tag{6a}$$

$$g_A = (Q_R^Z - Q_L^Z)/2\,, \tag{6b}$$

as

$$A(x,y) = -\frac{2q^2}{M_Z^2}\frac{\sum_i f_i(x)(Q_i^\gamma/e)(g_{A,e}g_{V,i} + \{[1-(1-y)^2]/[1+(1-y)^2]\}g_{V,e}g_{A,i})}{\sum_i f_i(x)(Q_i^\gamma)^2}\,. \tag{7}$$

This agrees with the consensus of earlier results for the asymmetry in deep-inelastic scattering in the scaling limit.[3,4,5]

The sum over parton types, i, in Eqs. (4), (5), and (7) is understood to include both quarks and antiquarks, as appropriate to a given x value. An antiquark of particular type has weak and electromagnetic charges of given handedness which are the negatives of the charges of opposite handedness of the quark of the same type. Stated another way, the electromagnetic charge and g_V change sign on going from quark to antiquark, but g_A does not. Therefore quarks and their corresponding antiquarks contribute with the same sign to the first (y-independent) term in the numerator of Eq. (7) and with opposite sign to the second (y-dependent) term. Antiquarks are known to be unimportant, except at small values of x. In the following we calculate asymmetries in various models for $x \gtrsim 0.2$ where antiquarks can be safely neglected, but it is well to remember that their main effect, if present, would simply be to lessen the y dependence of the predicted asymmetries in deep-inelastic scattering.

Equation (7) shows that the asymmetry[13] at $y=0$ arises entirely from the term involving the axial vector coupling of the electron (and vector coupling of the quark parton). This is simply understood from our derivation: A glance at Eqs. (2) shows that at $y=0$ there will be a nonzero asymmetry only if $Q_{Le}^Z \neq Q_{Re}^Z$, i.e., if the electron has a weak axial-vector charge.

The simplest gauge theories are based on the gauge group SU(2) × U(1). In such theories, if the spontaneous symmetry breaking is of the usual form, there is the relation

$$\frac{G_F}{2\sqrt{2}\pi\alpha} = \frac{1}{4\sin^2\theta_W\cos^2\theta_W M_Z^2}\,, \tag{8}$$

where θ_W is the Weinberg angle. Data from several experiments suggest that[14] $\sin^2\theta_W \approx \frac{1}{3}$.

The weak charges of the fermions are given by[14]

$$Q_L^Z = \frac{e}{\sin\theta_W\cos\theta_W}(T_{3L} - Q^\gamma\sin^2\theta_W) \tag{9a}$$

and

$$Q_R^Z = \frac{e}{\sin\theta_W\cos\theta_W}(T_{3R} - Q^\gamma\sin^2\theta_W)\,. \tag{9b}$$

Here T_{3L} and T_{3R} are the third component of the weak isospin for left- and right-handed fermions, respectively. In the original Weinberg-Salam model fermions are in left-handed doublets: the "*up*" quark has $T_{3L}=+\frac{1}{2}$, while the "down" quark and electron have $T_{3L}=-\frac{1}{2}$. All fermions are right-handed singlets, i.e., $T_{3R}=0$. More generally, right-handed fermions need not be assigned to singlets. Indeed, it is precisely the right-handed assignments about which one hopes to gain information as a result of parity-violation experiments

If the u quark occurred in a right-handed doublet along with a quark of charge $-e/3$, T_{3R}^u would be $+\frac{1}{2}$. In general, nonzero values for T_{3R}^u, T_{3R}^d, or T_{3R}^e would signal the existence of new fundamental fermions.

For an isosinglet target such as the deuteron, where $f_u(x)=f_d(x)$, the x dependence of $A(x,y)$ drops out if we ignore antiquark contributions. In an SU(2) × U(1) gauge theory with conventional left-handed assignments, Eqs. (8) and (9) substituted in Eq. (7) yield

$$A_{ed}(x,y) = -\frac{G_F q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\Big[(1+2T_{3R}^e)(1-\tfrac{20}{9}\sin^2\theta_W + \tfrac{4}{3}T_{3R}^u - \tfrac{2}{3}T_{3R}^d) + (1-4\sin^2\theta_W - 2T_{3R}^e)(1-\tfrac{4}{3}T_{3R}^u + \tfrac{2}{3}T_{3R}^d)\Big(\frac{1-(1-y)^2}{1+(1-y)^2}\Big)\Big]\,. \tag{10}$$

The physical origin of the sign of the asymmetry in the simplest Weinberg-Salam model is easy to understand. Consider the limit where $\theta_W \to 0$ and the Z^0 couples only to left-handed fermions. Then the electron and up quark have opposite signs for both their electromagnetic and weak (left-handed) charges. Similarly the electron and down quark have the same signs for both types of charge. Therefore the γ and Z^0 exchanges interfere constructively and produce a larger cross section for scattering left-handed electrons (there is no Z^0 contribution for scattering of right-handed electrons in this limit). The resulting asymmetry, defined as $(d\sigma_R - d\sigma_L)/(d\sigma_R + d\sigma_L)$, is thus negative, in agreement with Eq. (10) in the corresponding limit.

With the left-handed fermions assigned to doublets in SU(2) × U(1), the most obvious extension of the simplest Weinberg-Salam model, where right-handed fermions are all in singlet representations, is to assign some or all fermions to right-handed doublets. We label such potential right-handed doublets for the electron, up quark and down quark as $(E^0, e)_R$, $(u, b)_R$, and $(t, d)_R$, respectively. The predicted asymmetries in deep-inelastic electron-deuteron scattering are shown in Fig. 1 for the eight possible combinations of right-handed e, u, and d assignments of interest.

A number of general features are apparent. First, if the electron is assigned to a right-handed as well as a left-handed doublet, it has a purely vector coupling to the Z^0 and the asymmetry vanishes at $y = 0$. The remaining asymmetry is due to the term in Eq. (7) involving $g_{V,e}$ and is proportional to $\sum_i Q_i^\gamma g_{A,i}$. Both the u and d quarks contribute to this term with the same sign when

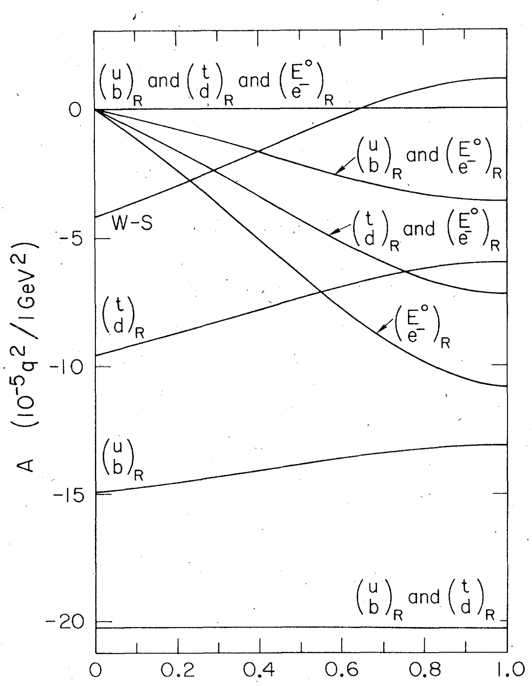

FIG. 1. The asymmetry, $A = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$, for deep-inelastic polarized-electron–deuteron scattering as a function of $y = (E - E')/E$ in various SU(2) × U(1) models. The asymmetry is given in units of $10^{-5} q^2/\mathrm{GeV}^2$, $\sin^2\theta_W = \frac{1}{3}$. Different models are labeled in terms of the assignment of the right-handed fermions. The Weinberg-Salam model (W-S) has only right-handed singlets. The other models have one or more of the electron, u quark, and d quark in right-handed doublets, as indicated.

$T_{3R}^u = T_{3R}^d = 0$. The addition of right-handed coupling for the u and/or d quarks makes their contribution smaller inasmuch as the u and/or d couplings then become purely vector in character. Thus theories with no right-handed couplings show the greatest dependence on y.

Second, if the electron is a right-handed singlet, as in the simplest theory, the asymmetry is controlled primarily by the term in Eq. (7) which survives at $y = 0$ and is proportional to $g_{A,e} \times (\sum_i Q_i^\gamma g_{V,i})$. The value of $g_{V,i}$ is dominated by the T_3 contributions. The magnitude of this term thus increases as right-handed couplings are introduced [see Eq. (10)]. It follows that the theory with quarks which are only right-handed singlets gives the smallest (in absolute magnitude) prediction, and the theory with both $(u, b)_R$ and $(t, d)_R$ gives the largest. Furthermore, the former right-handed doublet gives a larger effect than the latter because of the presence of the quark charge ($+2e/3$ for the u quark, $-e/3$ for the d quark).

It is significant that some theories give predictions for A of about 10^{-4} at $Q^2 = 1\ \mathrm{GeV}^2$. Such theories may be excluded by the data long before it is possible to check the Weinberg-Salam theory in detail. On the other hand, experiments[7] with atomic bismuth suggest that the electron has a purely vector coupling to the neutral weak current. If this is so, it will be difficult to distinguish between possibilities unless measurements are made at reasonably large values of y.

The expression for the asymmetry, Eq. (7), is general and applies to theories with more than one neutral weak vector boson. We need only note that the asymmetry, A, receives separate additive contributions from each such boson.

We now turn to examples of theories based on gauge groups other than SU(2) × U(1), which generally have more than one neutral weak vector boson. Theories based on $SU(2)_L \times SU(2)_R \times U(1)$ have been actively pursued in the last few years.[15-18] Some of the models[15-17] based on this gauge group have no parity violation in the interaction between electrons and nucleons since one of the two neutral weak vector bosons has only vector couplings and the other has only axial-vector couplings. The model of De Rújula, Georgi, and Glashow,[18] on the other hand, has one vector boson with purely vector couplings, and one which is just like the Z^0 in the Weinberg-Salam model. However, the contribution of this latter vector boson is reduced by $\cos^2\beta$, where β is a mixing angle. This would lead to predictions for the asymmetry similar to those of the Weinberg-Salam model, but reduced by a factor $\cos^2\beta$ (which is estimated to be $\sim\frac{1}{2}$).

Models based on SU(3) × U(1) have also been pursued recently.[19-21] In a version due to Lee and Weinberg[21] the fermions are assigned to both right- and left-handed triplets, except for the most positively charged member of each triplet which is a singlet when it has opposite handedness. There are two diagonal weak vector bosons, Y^0 and Z^0, and the relevant charges are (we denote by $\sin^2\theta_3$ the quantity called w in Ref. 21)

$$\begin{aligned} &g^Z_{V,e}=e(\tfrac{2}{3}-\cos^2\theta_3)/\sin\theta_3\cos\theta_3\,,\quad g^Z_{A,e}=0\,,\\ &g^Z_{V,u}=e(\tfrac{2}{3}\cot 2\theta_3)\,,\quad g^Z_{A,u}=-\frac{e}{3\sin\theta_3\cos\theta_3}\,,\\ &g^Z_{V,d}=-e\tfrac{1}{3}\cot\theta_3\,,\quad g^Z_{A,d}=0\,,\\ &g^Y_{V,e}=0\,,\quad g^Y_{A,e}=e/(\sqrt{3}\,\sin\theta_3)\,,\\ &g^Y_{V,u}=0\,,\quad g^Y_{A,u}=0\\ &g^Y_{V,d}=0,\quad g^Y_{A,d}=e/(\sqrt{3}\,\sin\theta_3)\,. \end{aligned} \tag{11}$$

Since th Y^0 has only axial-vector couplings, it cannot contribute to the asymmetry. The mass of the Z^0 is related to the mixing angle θ_3, to G_F, and to another parameter of the theory l, by

$$\frac{9G_F(1+l)}{8\sqrt{2}\pi\alpha}=\frac{1}{\sin^2\theta_3\cos^2\theta_3 M_Z{}^2}\,. \tag{12}$$

From Eqs. (7), (11), and (12) we find

$$A_{ed}(x,y)=\left(\frac{G_F q^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{9}{10}\right)2(1+l)(\tfrac{2}{3}-\cos^2\theta_3)\,\frac{1-(1-y)^2}{1+(1-y)^2}\,. \tag{13}$$

Lee and Weinberg find that data from weak-interaction experiments suggest $l\approx 0.2$, $\sin^2\theta_3\approx 0.2$. This would yield a prediction for A quite similar to that of the Weinberg-Salam model plus an extra E^0 so that $T^e_{3R}=-\frac{1}{2}$. Note, however, that the size of the effect is very sensitive to the value of $\cos^2\theta_3$.

A model based on $SU(3)\times SU(3)$ has been developed by Bjorken and Lane.[22] Restricted to the fermions e, u, and d, of interest for the asymmetry we are discussing here, it is equivalent to the $SU(2)\times U(1)$ model which has u and d as right-handed singlets but the electron in a doublet, $(E^0,e^-)_R$.

Some gauge theories with gauge groups larger than $SU(2)\times U(1)$ have physical vector bosons which can be expressed quite generally as orthogonal mixtures of simpler fields with pure V and pure A couplings.[15,16,17] Denoting the appropriate mixing angle between the two neutral bosons by ϕ, we have no parity violation when $\phi=0$. Such is the case, for example, in the model of Mohapatra and Sidhu.[17] More generally, the parity violations in atomic physics and polarized electron-nucleon scattering are proportional to $\sin 2\phi$. Thus limits on such parity violation provide bounds on the mixing in such models.

A related phenomenon occurs for the Lee-Weinberg $SU(3)\times U(1)$ model.[21] There, if the Y and Z bosons mix, additional parity violation results from the product of an axial coupling to the electron and a vector coupling to the quarks of the mixed boson fields. The magnitude of such mixing is limited by the parity-violation experiments with heavy atoms and would be tested by a measurement of the asymmetry in elastic electron-proton scattering and deep-inelastic electron-nucleon scattering at small y.

Up to this point we have considered the detailed predictions of various models for an isosinglet target such as the deuteron. Since $f_u(x)=f_d(x)$ in this case, the x dependence of the asymmetry drops out in the valence-quark approximation. For the sake of completeness we now indicate the results for a proton target. Rather than give qualitative results for all x and y, we have chosen $x=\frac{1}{3}$ to give quantitative predictions. Here the "valence" quarks dominate and furthermore, for the proton $f_u(x=\frac{1}{3})\simeq 2f_d(x=\frac{1}{3})$, as in the most naive quark-parton model.

Taking our general formula Eq. (5) and inserting the $SU(2)\times U(1)$ charges in Eq. (9), we then find

$$\begin{aligned} A_{ep}(x=\tfrac{1}{3},y)=\frac{d\sigma_R-d\sigma_L}{d\sigma_R+d\sigma_L}=-\left(\frac{G_F q^2}{2\sqrt{2}\pi\alpha}\right)\Big[&(1+2T^e_{3R})(\tfrac{5}{6}+\tfrac{4}{3}T^u_{3R}-\tfrac{1}{3}T^d_{3R}-2\sin^2\theta_W)\\ &+\left(\frac{1-(1-y)^2}{1+(1-y)^2}\right)(1-2T^e_{3R}-4\sin^2\theta_W)(\tfrac{5}{6}-\tfrac{4}{3}T^u_{3R}+\tfrac{1}{3}T^d_{3R})\Big]\,. \end{aligned} \tag{14}$$

The asymmetry predicted in the same $SU(2)\times U(1)$ models considered in connection with electron-deuteron deep-inelastic scattering is shown in Fig. 2 for electron-proton deep-inelastic scattering. It is typically negative, but not as large (in magnitude) as for the electron-deuteron case.

III. ELASTIC POLARIZED-ELECTRON-PROTON SCATTERING

The asymmetry in polarized-electron–proton scattering is determined by the electromagnetic and weak neutral-current form factors. These couplings are taken at the proton vertex to be the usual Dirac $[eF^\gamma_1(q^2)]$ and Pauli $[eF^\gamma_2(q^2)/2M_N]$ form factors for the electromagnetic current $[F^\gamma_{1p}(0)=1$ and $F^\gamma_{2p}(0)=1.79]$. For the neutral weak current $F^Z_1(q^2)$ replaces $eF^\gamma_1(q^2)$ and $F^Z_2(q^2)$ replaces $eF^\gamma_2(q^2)$, but there is an axial-vector (coefficient of

$\gamma_\mu\gamma_5$) form factor as well, $g^Z_{A,p}(q^2)$. A straightforward calculation shows that the polarized electron asymmetry is[10]

$$A_{ep\to ep}=\frac{d\sigma_R-d\sigma_L}{d\sigma_R+d\sigma_L}=\left(\frac{2q^2}{e^2M_Z{}^2}\right)\left\{-g_{A,e}\left[\frac{q^4}{4M_N{}^2}(F_1^\gamma+F_2^\gamma)(F_1^Z+F_2^Z)+\left(2EE'-\frac{q^2}{2}\right)\left(F_1^\gamma F_1^Z+\frac{q^2}{4M_N{}^2}F_2^\gamma F_2^Z\right)\right]\right.$$

$$\left.+g_{V,e}g_{A,p}(F_1^\gamma+F_2^\gamma)(E^2-E'^2)\right\}$$

$$\times\left\{\left(2EE'-\frac{q^2}{2}\right)\left[(F_1^\gamma)^2+\frac{q^2}{4M_N{}^2}(F_2^\gamma)^2\right]+\frac{q^4}{4M_N{}^2}(F_1^\gamma+F_2^\gamma)^2\right\}^{-1} \qquad (15)$$

For high energies and moderate values of q^2, $q^2/2M_NE\ll 1$. Since for elastic scattering $\nu=E-E'=q^2/2M_N$, we have $(E-E')/E=q^2/2M_NE\ll 1$ as well. With these approximations, applicable in the regime where the SLAC experiments will be done, the asymmetry takes on the much simpler form

$$A_{ep\to ep}\simeq-\left(\frac{2q^2}{e^2M_Z{}^2}\right)g_{A,e}\times\frac{F_1^\gamma(q^2)F_1^Z(q^2)+(q^2/4M_N{}^2)F_2^\gamma(q^2)F_2^Z(q^2)}{[F_1^\gamma(q^2)^2]+(q^2/4M_N^2)[F_2^\gamma(q^2)]^2} \qquad (16)$$

In terms of the Sachs form factors

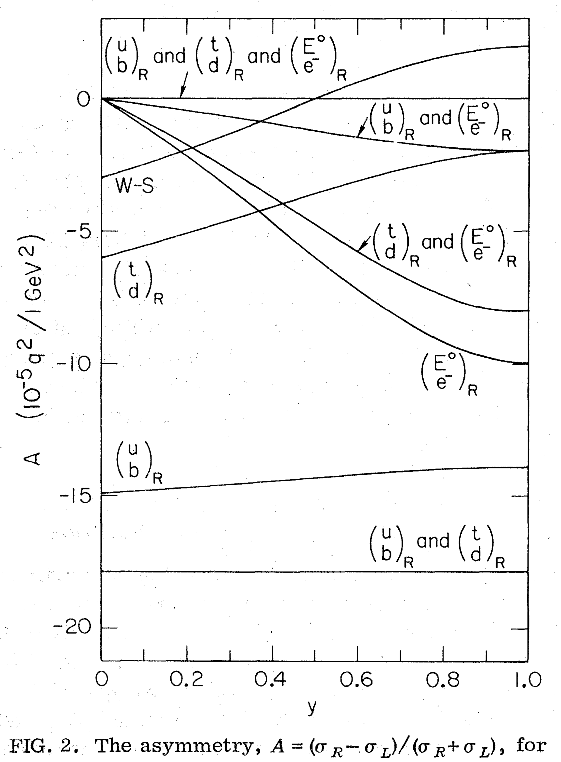

FIG. 2. The asymmetry, $A=(\sigma_R-\sigma_L)/(\sigma_R+\sigma_L)$, for deep-inelastic polarized electron-proton scattering at $x=\frac{1}{3}$ as a function of $y=(E-E')/E$ in various SU(2) $\times$ U(1) models. Notation and units as in Fig. 1.

$$G_E(q^2)=F_1(q^2)-\frac{q^2}{4M_N{}^2}F_2(q^2) \qquad (17a)$$

and

$$G_M(q^2)=F_1(q^2)+F_2(q^2)\,, \qquad (17b)$$

this may be rewritten as

$$A_{ep\to ep}\simeq-\left(\frac{2q^2}{e^2M_Z{}^2}\right)g_{A,e}\times\frac{G_E^\gamma(q^2)G_E^Z(q^2)+(q^2/4M_N{}^2)G_M^\gamma(q^2)G_M^Z(q^2)}{[G_E^\gamma(q^2)]^2+(q^2/4M_N{}^2)[G_M^\gamma(q^2)]^2}\,. \qquad (18)$$

Equation (18) shows that the asymmetry in elastic scattering is determined at high energies by the term proportional to the axial-vector Z^0 coupling at the electron vertex and the vector coupling at the proton vertex. Moreover, if we assume that all the Sachs form factors have the same (dipole) q^2 dependence, then we can write

$$A_{ep\to ep}\simeq-\frac{2q^2}{e^2M_Z{}^2}g_{A,e}\times\frac{G_E^\gamma(0)G_E^Z(0)+(q^2/4M_N{}^2)G_M^\gamma(0)G_M^Z(0)}{G_E^\gamma(0)^2+(q^2/4M_N{}^2)G_M^\gamma(0)^2}\,. \qquad (19)$$

The couplings F_1^Z and F_2^Z or equivalently G_E^Z and G_M^Z depend on the choice of gauge model. To obtain them we note that in terms of quark fields the electromagnetic current is

$$\tfrac{2}{3}e\bar{u}\gamma_\mu u-\tfrac{1}{3}e\,\bar{d}\gamma_\mu d-\tfrac{1}{3}e\bar{s}\gamma_\mu s\,.$$

Neglecting the contribution of the strange quarks in the nucleon to its static electromagnetic properties we then have[23]

$$\langle p|+\tfrac{2}{3}e\,\bar{u}\gamma_\mu u-\tfrac{1}{3}e\bar{d}\gamma_\mu d|p\rangle=eG_p^\gamma \qquad (20a)$$

and with an isospin rotation,

$$\langle p|-\tfrac{1}{3}e\bar{u}\gamma_\mu u+\tfrac{2}{3}e\,\bar{d}\gamma_\mu d|p\rangle=eG_n^\gamma\,. \qquad (20b)$$

Inverting these equations gives

$$\langle p|\bar{u}\gamma_\mu u|p\rangle = 2G_p^\gamma + G_n^\gamma \quad (21a)$$

and

$$\langle p|\bar{d}\gamma_\mu d|p\rangle = G_p^\gamma + 2G_n^\gamma . \quad (21b)$$

Now the weak neutral vector current of relevance is

$$\tfrac{1}{2}(Q_{L,u}^Z + Q_{R,u}^Z)\bar{u}\gamma_\mu u + \tfrac{1}{2}(Q_{R,d}^Z + Q_{R,d}^Z)\bar{d}\gamma_\mu d .$$

From Eqs. (21) we immediately obtain

$$G_E^Z = \tfrac{1}{2}(Q_{L,u}^Z + Q_{R,u}^Z)(2G_{E,p}^\gamma + G_{E,n}^\gamma) + \tfrac{1}{2}(Q_{L,d}^Z + Q_{R,d}^Z)(G_{E,p}^\gamma + 2G_{E,n}^\gamma) , \quad (22a)$$

and similarly

$$G_M^Z = \tfrac{1}{2}(Q_{L,u}^Z + Q_{R,u}^Z)(2G_{M,p}^\gamma + G_{M,n}^\gamma) + \tfrac{1}{2}(Q_{L,d}^Z + Q_{R,d}^Z)(G_{M,p}^\gamma + 2G_{M,n}^\gamma) . \quad (22b)$$

All that remains is to substitute in the weak charges in a particular model. For example, with the gauge group SU(2) × U(1) we have

$$G_E^Z(0) = \frac{e}{\sin\theta_W \cos\theta_W}(\tfrac{1}{4} - \sin^2\theta_W + T_{3R}^u + \tfrac{1}{2}T_{3R}^d) , \quad (23a)$$

$$G_M^Z(0) = \frac{e}{\sin\theta_W \cos\theta_W} \times\left(\frac{\mu_p - \mu_n}{4} - \mu_p \sin^2\theta_W + \frac{2\mu_p + \mu_n}{2}T_{3R}^u + \frac{\mu_p + 2\mu_n}{2}T_{3R}^d\right) , \quad (23b)$$

and

$$g_{A,e} = \frac{e}{\sin\theta_W \cos\theta_W}\frac{1}{4} , \quad (23c)$$

where we have used the $q^2 = 0$ values $G_{E,p}^\gamma = 1$, $G_{E,n}^\gamma = 0$, $G_{M,p}^\gamma = \mu_p = 2.79$, and $G_{M,n}^\gamma = \mu_n = -1.91$.

The predictions for the asymmetry can be understood from Eqs. (23). For the original Weinberg-Salam model, the factor in parentheses in Eq. (23a) equals $-\frac{1}{12}$ if we take $\sin^2\theta_W = \frac{1}{3}$. Adding a $(u, b)_R$ doublet changes this to $+\frac{5}{12}$. Adding a $(t, d)_R$ doublet instead makes the parenthetical term $-\frac{1}{3}$, while adding both right-handed doublets gives $+\frac{1}{6}$.

In Fig. 3, the predicted asymmetry is shown as a function of q^2. For small q^2, the terms involving G_E dominate, and as just indicated G_E^Z has one sign for the original Weinberg-Salam and t-quark models, and the opposite sign for the b-quark and b-quark-plus-t-quark models. For large q^2, the G_M terms dominate the elastic cross section and the elastic asymmetry, and all models have the same negative sign but differing magnitudes for $A_{ep\to ep}$. As a result, the Weinberg-Salam and t-quark models yield predicted asymmetries which change sign and are still small ($\lesssim 10^{-5}$) in magnitude at $q^2 = 1$ GeV^2, while the models with b or b and t quarks give large negative asymmetries ($\approx -10^{-4}$) at $q^2 = 1$ GeV^2.

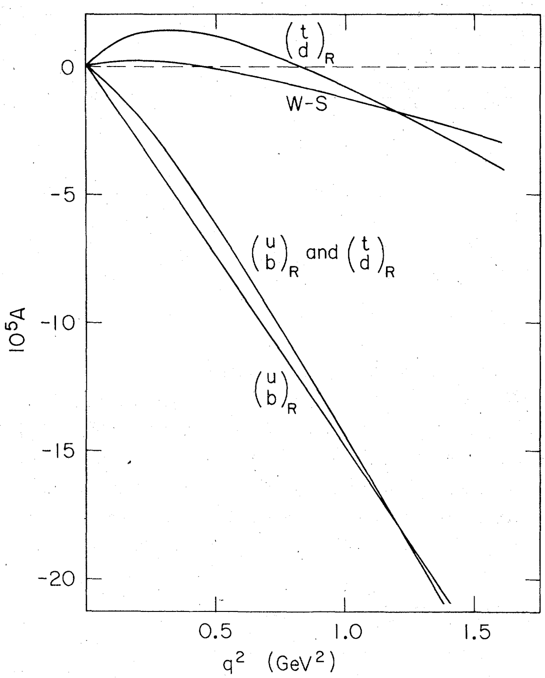

FIG. 3. The asymmetry, A, for polarized elastic-electron–proton scattering at high energies, in units of 10^{-5}, as a function of q^2. The SU(2) × U(1) models are labeled as in Figs. 1 and 2, and $\sin^2\theta_W = \frac{1}{3}$. The asymmetry is negligible in the high-energy regime if the electron is in a right-handed doublet.

The asymmetry in Eq. (18) clearly vanishes if $g_{A,e} = 0$. This happens if $T_{3R}^e = -\frac{1}{2}$ in SU(2) × U(1) and is also the case for the Lee-Weinberg SU(3) × U(1) model.[21] The SU(2) × SU(2) × U(1) models[15-17] with a parity-conserving electron-nucleon interaction also give zero asymmetry, while that of De Rújula *et al.*[18] based on the same gauge group gives, as for deep-inelastic scattering, the predictions of the original Weinberg-Salam model reduced by $\cos^2\beta \simeq \frac{1}{2}$.

IV. ASYMMETRY IN ELECTROPRODUCTION OF THE Δ(1236) BY POLARIZED ELECTRONS

As in the case of elastic scattering, experiments measuring the asymmetry in electroproduction of the Δ(1236) by polarized electrons are likely to be done at high-beam energy and small or moderate values of q^2. Therefore $q^2/2M_N E$ and $(q^2 + M_\Delta^2 - M_N^2)/2M_N E = (E - E')/E$ will be small ($\ll 1$), and to a good approximation only the terms involving g_A^Z at the electron vertex and g_V^Z at the hadron vertex contribute to the polarized asymmetry.

Thus we expect an expression for the asymmetry in $ep\to e\Delta^+$ which is in direct analogy to Eq. (18). However, there is a further simplification in that experiment tells us that the electric quadrupole and longitudinal quadrupole electromagnetic couplings between a nucleon and $\Delta(1236)$ are negligible compared to the magnetic dipole coupling.[24] Calling $G_M^{*\gamma}(q^2)$ this magnetic dipole coupling [with a factor of e removed, so it is the analog of $G_M^\gamma(q^2)$ for the nucleon], the appropriate transcription of Eq. (18) to $ep\to e\Delta^+$ is

$$A_{ep\to e\Delta^+}\simeq-\left(\frac{2q^2}{e^2 M_Z{}^2}\right)g_{A,e}\frac{G_M^{*\gamma}(q^2)G_M^{*Z}(q^2)}{[G_M^{*\gamma}(q^2)]^2}$$
$$\simeq-\left(\frac{2q^2}{e^2 M_Z{}^2}\right)g_{A,e}\frac{G_M^{*Z}(q^2)}{G_M^{*\gamma}(q^2)}\,. \qquad (24)$$

Here $G_M^{*Z}(q^2)$ is the magnetic dipole Z^0 coupling of the nucleon to the $\Delta(1236)$.

To proceed further we need a method to compute $G_M^{*\gamma}$ and G_M^{*Z}. We of course choose the quark model, which is known to predict $G_M^{*\gamma}$ relative to G_M^γ for the nucleon approximately correctly, as well as predicting the vanishing of the electric quadrupole and longitudinal quadrupole amplitudes in photoproduction and electroproduction of the $\Delta(1236)$. In the quark model, magnetic moments and transition moments are proportional to matrix elements of $\sum_i Q_i^\gamma \vec{\sigma}^i$. For the sake of definiteness we take states with $J_z=\frac{1}{2}$, and evaluate the z component of the magnetic moment operator. Up to a common factor we find

$$eG_{M,p}^\gamma\propto e\,, \qquad (25a)$$

$$eG_{M,n}^\gamma\propto-\tfrac{2}{3}e\,, \qquad (25b)$$

and

$$eG_M^{*\gamma}\propto\tfrac{2}{3}\sqrt{2}\,e\,, \qquad (25c)$$

where we have used the constituent-quark-model wave functions for the nucleon and $\Delta(1236)$. The resulting predictions, $G_{M,n}/G_{M,p}=-\frac{2}{3}$ and $G_M^*/G_{M,p}=2\sqrt{2}/3$ are standard predictions arising from the quark model, and are in rather good agreement with experiment.[25]

For the calculation of G_M^{*Z}, we then assume it is proportional to matrix elements of $\sum_i g_{V,i}^Z\vec{\sigma}^i$. For the SU(2) × U(1) gauge theory,

$$g_{V,i}^Z=\frac{e}{\sin\theta_W\cos\theta_W}\left(\tfrac{1}{2}T_{3L}^i+\tfrac{1}{2}T_{3R}^i-Q_i\sin^2\theta_W\right). \qquad (26)$$

Taking matrix elements between constituent-quark wave functions, we find

$$G_{M,p}^Z\propto\frac{e}{\sin\theta_W\cos\theta_W}\left(\tfrac{5}{12}+\tfrac{2}{3}T_{3R}^u-\tfrac{1}{6}T_{3R}^d-\sin^2\theta_W\right) \qquad (27a)$$

and

$$G_M^{*Z}\propto\frac{e(\frac{2}{3}\sqrt{2})}{\sin\theta_W\cos\theta_W}\left(\tfrac{1}{2}+\tfrac{1}{2}T_{3R}^u-\tfrac{1}{2}T_{3R}^d-\sin^2\theta_W\right), \qquad (27b)$$

where the proportionality constant is the same in Eqs. (25). It may easily be checked that Eq. (27a) is equivalent to Eq. (23b) if we take into account the proportionality constant of Eqs. (25) and the quark-model relation $\mu_n/\mu_p=-\frac{2}{3}$. Combining Eq. (25c), Eq. (27b), and $g_{A,e}=(e/4\sin\theta_W\cos\theta_W)\times(1+2T_{3R}^e)$, and substituting in Eq. (24), we calculate the asymmetry in SU(2) × U(1) gauge theories to be

$$A_{ep\to e\Delta^+}\simeq-2\left(\frac{G_F q^2}{2\sqrt{2}\,\pi\alpha}\right)(1+2T_{3R}^e)\times\left[\left(\tfrac{1}{2}+\tfrac{1}{2}T_{3R}^u-\tfrac{1}{2}T_{3R}^d-\sin^2\theta_W\right)\right]. \qquad (28)$$

The predicted asymmetries in various models are shown in Fig. 4. Again we graph only those cases where the electron is a right-handed singlet as in the original Weinberg-Salam model. When the electron is in a right-handed doublet, $g_{A,e}=0$ and the asymmetry (in the $q^2/2ME\ll1$ regime) vanishes. Because of our quark-model assumptions we have a magnetic dipole transition with the same form factor for the γ and Z^0, and the asymmetry is simply proportional to q^2. It is negative in

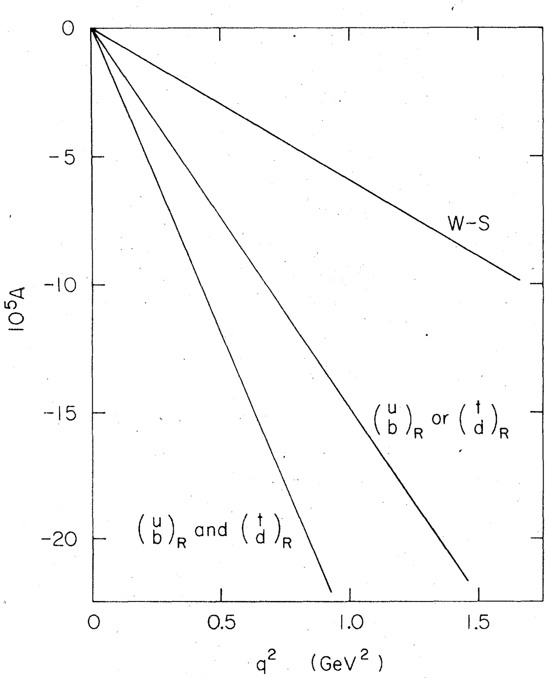

FIG. 4. The asymmetry, A, for $ep\to e\Delta^+$ at high energies, in units of 10^{-5}, as a function of q^2. The SU(2)×U(1) models are labeled as in Figs. 1 and 2, and $\sin^2\theta_W=\frac{1}{3}$. The asymmetry is negligible in the high-energy regime if the electron is in a right-handed doublet.

all cases. Adding either a $(u,b)_R$ or a $(t,d)_R$ doublet to the standard model results in the same prediction, and adding both doublets makes the asymmetry even larger in absolute magnitude. At $q^2 = 1$ GeV^2 the predictions range from about -0.6×10^{-4} to about -2.4×10^{-4}. Thus as long as $g_{A,e}$ has the value it has in the original model of Weinberg and Salam, the expected asymmetry for $ep \to e\Delta^+$ for any quark assignment should be relatively easily accessible in the coming round of experiments. We recall again, however, that it is currently popular to construct models so that $g_{A,e} = 0$ (as suggested by the atomic bismuth experiments) and to good approximation, the asymmetry in $ep \to e\Delta^+$ then vanishes.

V. DISCUSSION

Measurement of parity-violating effects in electron-nucleon scattering is not only of general interest because it arises from interference between weak and electromagnetic amplitudes, but particularly because it involves the neutral-current coupling of the electron in a clean and well-defined manner. The exact vector and axial-vector nature of this coupling occupies a central place at present in determining the direction that unified gauge theories of the weak and electromagnetic interactions will take in the future.

In theories where the gauge group is SU(2) × U(1), our results show that the combination of $ep \to ep$, $ep \to e\Delta$, and deep-inelastic asymmetry measurements can, in principle, determine whether each of the electron, the u quark, and d the quark appear in right-handed doublets as well as in the left-handed weak doublets to which they are assigned in the simplest theory. If the electron lies in a right-handed singlet (as in the original Weinberg-Salam model), it will be relatively easy to determine the assignments for the right-handed u and d quarks: Measuring the asymmetry in $ep \to ep$ and $ep \to e\Delta$ (Figs. 3 and 4) to roughly $\pm 4 \times 10^{-5}$ at $Q^2 = 1$ GeV^2 would suffice. If, as suggested by the experiments with atomic bismuth, the electron lies in a right-handed doublet and couples purely through the vector weak neutral current, then the situation is somewhat more difficult to resolve experimentally. In the high-energy regime where $q^2/2M_N E \ll 1$ the asymmetry in both $ep \to ep$ and $ep \to e\Delta$ is negligible. At $y = 0.5$ it ranges from 0 to about -7.5×10^{-5} in inelastic electron-deuteron scattering, depending on the assignment of the right-handed u and d quarks. The value of zero, which corresponds to a vector model with the electron, the u quark, and the d quark all in right-handed doublets, is in fact already excluded for SU(2) × U(1) by data showing that $\sigma(\nu N \to \nu X) \neq \sigma(\bar{\nu} N \to \bar{\nu} X)$. An asymmetry less than $\sim 3 \times 10^{-5}$ in magnitude for all three experiments then implies within SU(2) × U(1) models that only the unlikely and ungainly assignment of the electron and u quark, but not the d quark, to right-handed doublets is possible.

For theories with gauge groups larger then SU(2) × U(1), a wide range of possibilities presents itself. There are theories with no parity violation in electron-nucleon interactions such as some versions of SU(2) × SU(2) × U(1).[15,16,17] It is also possible to have a purely vector neutral current for the electron and hence a negligible asymmetry in $ep \to ep$ and $ep \to e\Delta$, as in recent models[21,22] involving SU(3) × U(1) and SU(3) × SU(3). There still would be a measurable asymmetry in deep-inelastic scattering in these models. There are finally models, such as that of De Rújula *et al.*[18] based on SU(2) × SU(2) × U(1), which have (reduced) asymmetries in all three types of electron scattering experiments.

The polarized e-N asymmetry experiments should still be able to single out one of these three classes as the only allowed type. The deep-inelastic asymmetry measurements in particular, done with sufficient accuracy over a range of y values, allow one to separately determine the vector and axial-vector couplings of the electron and of the quarks. As such, they are analogous in purpose and in power to the atomic physics experiments to be done on hydrogen.[26] We expect that polarized e-N experiments, along with atomic experiments, will likely play a decisive role in determining the gauge group of weak and electromagnetic interactions and the assignment of fundamental fermions to representations of that group.

ACKNOWLEDGMENTS

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[13] Technically, if we fix $\nu=E-E'$ and q^2 to be greater than certain minimum values to ensure being in the deep-inelastic region, then $y\to0$ only as $E\to\infty$. At SLAC energies, the minimum value of y obtainable for deep-inelastic scattering is ~0.1.

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