Comment on the analysis of Bethe-Salpeter scattering states by Hormozdiari and Huang

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The analysis of Bethe-Salpeter scattering states by Hormozdiari and Huang appears to contain invalid mathematical arguments. When these arguments are rectified, one arrives at substantially different conclusions. In particular, the prescription of Hormozdiari and Huang for constructing such states does not seem applicable to any process occurring in nature.

In a recent study of conditions under which the Bethe-Salpeter equation with scalar exchange will give rise to scattering states, Hormozdiari and Huang presented a prescription which they claimed is satisfactory provided the exchange particle is slightly unstable (i.e., has nonzero, but arbitrarily small, decay width).[1] Since all known scalar mesons have nonzero widths, the prescription of Ref. 1 (henceforth HH) was claimed to be viable for physical processes.

In the present work, we draw attention to certain arguments of HH which appear to be invalid. When these arguments are rectified, the prescription suggested by HH no longer seems viable.

For the sake of brevity, we assume the reader to be familiar with the work of HH, and we adopt the same notation here. The prescription of HH hinges on the requirement that certain functions $\Lambda_{B1}(x)$ and $\Lambda_{C1}(x)$ vanish faster than e^{-mr} as $r\to\infty$, where m denotes the common mass of the scattering particles, and r denotes the magnitude of the spatial part of the four-position x. According to Eq. (3.12) of HH, the r dependence of Λ_{B1} is given by

$$\Lambda_{B1}(x)\propto\Gamma(-\tfrac{1}{2},(\bar{\mu}+i\bar{\beta})r)\,, \tag{1}$$

where Γ denotes the incomplete gamma function, $\bar{\mu}$ denotes the imaginary part of the mass of the exchange particle, and $\bar{\beta}$ is real. HH expressed

$$\Gamma(-\tfrac{1}{2},(\bar{\mu}+i\bar{\beta})r)$$

in terms of a standard series which is asymptotic in r:

$$\Gamma(-\tfrac{1}{2},(\bar{\mu}+i\bar{\beta})r)=\frac{e^{-(\bar{\mu}+i\bar{\beta})r}G(r)}{[(\bar{\mu}+i\bar{\beta})r]^{3/2}\Gamma(\frac{3}{2})}\,, \tag{2}$$

where

$$G(r)\equiv\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(\frac{3}{2}+n)}{[(\bar{\mu}+i\bar{\beta})r]^n}\,. \tag{3}$$

After introducing the series (3), HH considered a sequence of related series and, by a comparison argument, claimed to show that $\Lambda_{B1}(x)$ decreases more rapidly than e^{-mr} as $r\to\infty$, for *any* $\bar{\mu}>0$. This claim was invalid, however, since every partial sum of the asymptotic series (3) tends to a nonzero constant as $r\to\infty$:

$$\operatorname*{limit}_{r\to\infty}\sum_{n=0}^{M}\frac{(-1)^n\Gamma(\frac{3}{2}+n)}{[(\bar{\mu}+i\bar{\beta})r]^n}=\Gamma(\tfrac{3}{2}) \tag{4}$$

for every non-negative integer M. By the definition of asymptotic series, this is the limiting value of the function represented by the series. Hence for large r,

$$\Gamma(-\tfrac{1}{2},(\bar{\mu}+i\bar{\beta})r)=\frac{e^{-(\bar{\mu}+i\bar{\beta})r}}{[(\bar{\mu}+i\bar{\beta})r]^{3/2}}\,[1+O(r^{-1})]\,, \tag{5}$$

and Eqs. (1) and (5) imply that

$$|\Lambda_{B1}(x)|\propto\frac{e^{-\bar{\mu}r}}{r^{3/2}} \tag{6}$$

in the limit of large r. Thus Λ_{B1} vanishes more rapidly than e^{-mr} only for $\bar{\mu}\geq m$, and *not* for arbitrarily small values of $\bar{\mu}$.[2]

A similar error occurs in the HH analysis of Λ_{C1}.[2,3] According to Eq. (3.30) of HH, the r dependence of Λ_{C1} is given by

$$\Lambda_{C1}(x)\propto\Gamma(-\tfrac{1}{2},(\bar{\mu}+z+i\gamma)r)\,, \tag{7}$$

where z is a real energy variable, and $\gamma=\mu_p\pm i\omega$, where μ_p is essentially the real part of the exchange particle mass, and ω is the center-of-mass energy. Again using the asymptotic series considered by HH, namely

$$\Gamma(-\tfrac{1}{2},(\bar{\mu}+z+i\gamma)r)=\frac{e^{-(\bar{\mu}+z+i\gamma)r}H(r)}{[(\bar{\mu}+z+i\gamma)r]^{3/2}\Gamma(\frac{3}{2})} \tag{8}$$

with

$$H(r)\equiv\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(\frac{3}{2}+n)}{[(\bar{\mu}+z+i\gamma)r]^n}\,, \tag{9}$$

we note that every partial sum of the series (9) approaches $\Gamma(\frac{3}{2})$ as $r\to\infty$, hence

$$|\Lambda_{C1}(x)|\propto\frac{e^{-(\bar{\mu}+z\mp\omega)r}}{r^{3/2}} \tag{10}$$

in the limit of large r. Thus Λ_{C1} vanishes more rapidly than e^{-mr} only if $(\bar{\mu}+z\mp\omega)\geq m$. Since z

may take on a minimum value of zero in the analysis of HH, this requires that $(\bar{\mu} \mp \omega) \geq m$, or $\bar{\mu} \geq (m+\omega)$. Such a requirement on $\bar{\mu}$ would seem to render the prescription inapplicable to situations of physical interest, and also raises doubts about the physical assumptions underlying the prescription. The ansatz Eq. (1.2) of HH seems especially deserving of scrutiny.

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[1]I. Hormozdiari and J. C. Huang, Phys. Rev. D 11, 348 (1975).

[2]The key error occurs in the sentence following Eq. (3.25) of HH. While b_n is greater than $|a_n^R|$ and $|a_n^I|$ in the limit as $r \to \infty$ for any fixed n, it is also true that $|a_n^R|$ and $|a_n^I|$ are greater than b_n in the limit as $n \to \infty$ for any fixed r. Hence the argument based on comparison of series is invalid for any fixed r, thus for every finite r. It should also be noted that a term-by-term comparison between an infinite series and an asymptotic series could never provide the basis for a valid argument: An infinite series must be summed to all orders, whereas an asymptotic series must *not* be summed to all orders (it would then diverge, i.e., be meaningless).

[3]In addition to an error such as that in the sentence following Eq. (3.25) of HH (see Ref. 2), the fact that γ has an imaginary part seems to have been overlooked in arriving at Eq. (4.2) of HH.