Chiral anomalies and quark masses*

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The relation between chiral-symmetry breaking and quark masses is studied in the framework of the recently proposed extended partially conserved axial-vector current hypothesis and using chiral anomalies relevant to $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$. Equations are obtained which relate the quark masses to the chiral-symmetry-breaking parameters Δ_π and Δ_K which measure the deviations from Goldberger-Treiman relations in $SU(2) \times SU(2)$ and $SU(3) \times SU(3)$, respectively. Although, numerically, it is only possible to obtain bounds for the quark masses, these seem to be higher than what has been recently estimated in model-dependent approaches. Finally, the effect of quark confinement is briefly discussed.

I. INTRODUCTION

Many efforts have been devoted to the problem of relating quark masses to chiral-symmetry breaking in the framework of the quark model.[1-4] The starting point in this kind of approach consists in assuming that the divergence of the axial-vector current is given by

$$\partial^\mu A_\mu^\alpha = 2 i m_u \bar{q}\gamma_5\lambda^\alpha q, \quad \alpha = 1, 2, 3 \tag{1}$$

$$\partial^\mu A_\mu^\alpha = 2 i \frac{m_u + m_s}{2} \bar{q}\gamma_5\lambda^\alpha q, \quad \alpha = 4, 5, 6, 7, \tag{2}$$

where the quark masses, m_u and m_s, characterize the chiral-symmetry breaking. Specific values for m_u and m_s may then be obtained by taking matrix elements of Eqs. (1) and (2) between hadrons.

Another way of carrying out this program could be to start directly from the partially conserved axial-vector current (PCAC) hypothesis, i.e.,

$$\partial^\mu A_\mu^\alpha = m_a^2 f_a \phi_a^\alpha, \quad a = \pi, K, \eta, \tag{3}$$

where now chiral-symmetry breaking is related to the nonzero value of the pseudoscalar-meson mass. However, since the model-dependent relations (1) and (2) are now lost, the values of the quark masses would have to be obtained by a different method. A conceivable approach, which is the subject of this paper, consists in studying triangle anomalies relevant to $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$.

It is well known that the soft-meson theorems,[5] valid to all orders of renormalized perturbation theory, state that

$$\mathfrak{F}_a(0) \propto S, \tag{4}$$

where $\mathfrak{F}_a(0)$ is the off-mass-shell amplitude for $a \to \gamma\gamma$ and S is given in terms of the charges of the bare constituents circulating around the triangle loop. Chiral-symmetry breaking is then contained in the extrapolation factor

$$E_a \equiv \frac{\mathfrak{F}_a(m_a^2)}{\mathfrak{F}_a(0)}. \tag{5}$$

It has been recently shown,[6] within the framework of extended PCAC (EPCAC),[7] that E_a is determined by the corrections to Goldberger-Treiman relations (GTR)[8] for $\Delta S = 0$ or $|\Delta S| = 1$ β decays, i.e.,[9]

$$E_a = \frac{1}{1 - \Delta_a}, \tag{6}$$

where

$$\Delta_a = 1 - \frac{(m_H + m_{H'}) g_H^A}{\sqrt{2} f_a g_{aHH'}} \tag{7}$$

are the corrections to GTR for the $H \to H' + l\bar{l}$ decay. Moreover, it turns out that Δ_π and Δ_K are separately universal, i.e., independent of the particular baryons undergoing the β decay.[10] Therefore, since Δ_a is a measure of chiral-symmetry breaking, one has the desired relation between this violation and the quark masses which will show up from triangle-loop integrations.

Our results indicate that the values of the quark masses are very sensitive to the amount of chiral-symmetry breaking and are consistently higher than what has been recently estimated in the framework of the quark model.[2] On the other hand, though large differences between quark masses may appear, they are not compelling.

The paper is organized as follows: In Sec. II we discuss $\pi^0 \to \gamma\gamma$ in order to obtain information about m_u and m_d, and in Sec. III we consider $\eta \to \gamma\gamma$ in connection with m_u, m_d, and m_s. Finally, Sec. IV is devoted to concluding remarks.

II. $\pi^0 \to \gamma\gamma$

The $\pi^0 \to \gamma\gamma$ decay amplitude, $\mathfrak{F}_\pi(q^2)$, is defined by

$$\frac{\mu_\pi^2 - q^2}{f_\pi \mu_\pi^2} \langle \gamma(k_1, \epsilon_1)\gamma(k_2, \epsilon_2)|D_3|0\rangle = \epsilon_{\mu\nu\alpha\beta} k_1^\mu k_2^\nu \epsilon_1^\alpha \epsilon_2^\beta \mathfrak{F}_\pi(q^2), \quad (8)$$

where $q = k_1 + k_2$ and $D_3 = \partial_\mu A_3^\mu$. The low-energy theorem, Eq. (4), reads

$$\mathfrak{F}_\pi(0) = -\frac{2\alpha}{\pi} \frac{1}{f_\pi} S, \quad (9)$$

and the extrapolation factor, Eq. (5), is given by[6]

$$E_\pi = \frac{\mathfrak{F}_\pi(\mu_\pi^2)}{\mathfrak{F}_\pi(0)} = \frac{1}{1 - \Delta_\pi}. \quad (10)$$

The present experimental value of Δ_π is[11]

$$\Delta_\pi = 1 - \frac{(m_p + m_n) g_A}{\sqrt{2} f_\pi g_{np\pi^+}} = 0.06 \pm 0.02. \quad (11)$$

From now on we shall assume that $S = \frac{1}{2}$, according to the three-color triplet quark model,[12] and that EPCAC correctly describes the on-mass-shell extrapolation. However, in Ref. 6, E_π was derived under the assumption of an infinite quark mass. Relaxing this hypothesis and coupling the pion family to the circulating bare quarks in the triangle loop with γ_5 coupling one obtains

$$\mathfrak{F}_\pi(q^2) = C \frac{m_u}{q^2} \left[\sin^{-1} \frac{(q^2)^{1/2}}{2m_u}\right]^2 \frac{\mu_\pi^2 - q^2}{\mu_\pi} \times \sum_{n=0}^{N} \frac{f_{\pi_n} \mu_{\pi_n}^2}{\mu_{\pi_n}^2 - q^2}, \quad (12)$$

where m_u has been set equal to m_d and C is a constant (later on we shall study the case $m_u \neq m_d$).

In this case Eq. (10) is modified as follows:

$$E_\pi = \left(\frac{\sin^{-1} z}{z}\right)^2 \frac{1}{1 - \Delta_\pi}, \qquad z = \frac{\mu_\pi}{2m_u}. \quad (13)$$

Recalling the definition of the decay rate,

$$\Gamma(\pi^0 \to \gamma\gamma) = (\mu_\pi^3/64\pi)[\mathfrak{F}_\pi(\mu_\pi^2)]^2,$$

one finds

$$\left(\frac{\sin^{-1} z}{z}\right)^4 \frac{1}{(1 - \Delta_\pi)^2} = \frac{\Gamma(\pi^0 \to \gamma\gamma)}{\Gamma_0(\pi^0 \to \gamma\gamma)}, \quad (14)$$

where $\Gamma_0(\pi^0 \to \gamma\gamma)$ is the rate calculated from the off-mass-shell amplitude, Eq. (9). Inserting experimental values[13] into the right-hand side of Eq. (14) one has

$$\left(\frac{\sin^{-1} z}{z}\right)^4 \frac{1}{(1 - \Delta_\pi)^2} = 1.03 \pm 0.06, \quad (15)$$

which is an equation relating the quark mass, $m_u = m_d$, to the chiral-symmetry-breaking universal parameter Δ_π. In other words, once Δ_π is fixed one can derive the value of the corresponding m_u and vice versa. However, from a numerical point of view this example is not very interesting because $\Gamma_0(\pi^0 \to \gamma\gamma)$ is very close to $\Gamma(\pi^0 \to \gamma\gamma)$, i.e., the low-energy theorem itself almost predicts the right decay rate. In any case, it is important to point out that if Eq. (11) is used for Δ_π then from Eq. (15) one obtains a value for m_u consistent with infinity [for the mean value, though, Eq. (15) has no real solution].

If one neglects Δ_π altogether one still has m_u consistent with being infinite, although the mean value is now $m_u = 321$ MeV, which is shifted to $m_u = 562$ MeV if $\Delta_\pi = 0.01$ is used.

Relaxing the restriction $m_u = m_d$ one finds, instead of Eq. (15),

$$\left[\frac{4(\sin^{-1} z_1)^2/z_1 - (\sin^{-1} z_2)^2/z_2}{4z_1 - z_2}\right]^2 \frac{1}{(1 - \Delta_\pi)^2} = 1.03 \pm 0.06, \quad (16)$$

where

$$z_1 = \frac{\mu_\pi}{2m_u}, \quad z_2 = \frac{\mu_\pi}{2m_d}. \quad (17)$$

Equation (16) has four types of solutions. The first one is obtained by starting from $z_2 = 0$ and $z_1 = z_1$ (maximum) fixed by the equation. Increasing z_2 from zero, z_1 decreases down to a minimum value compatible with the equation. The second solution is obtained by starting from $z_1 = z_2$ [which is equivalent to $z_2 = 0$ and $z_1 = z_1$ (maximum)] and having z_1 and z_2 decrease simultaneously. Both solutions join smoothly at the point $z_1 = z_1$ (minimum). The remaining two solutions correspond to interchanging z_1 and z_2 in the preceding analysis.

In Fig. 1 we show some solutions of Eq. (16) when $(1 - \Delta_\pi)^2 \Gamma(\pi^0 \to \gamma\gamma)/\Gamma_0(\pi^0 \to \gamma\gamma) = 1.09$, 1.03,

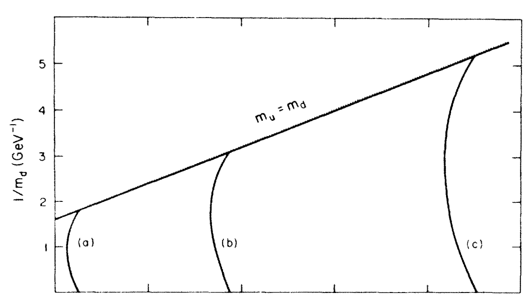

FIG. 1. Solutions to Eq. (16) for $(1-\Delta_\pi)^2\Gamma(\pi^0 \to \gamma\gamma)/\Gamma_0(\pi^0 \to \gamma\gamma) = 1.01$ (a), 1.03 (b), and 1.09 (c).

and 1.01. Since the situation $m_d = \infty$ (or alternately $m_u = \infty$) is clearly unrealistic one would expect physical values to lie in the upper portion of the curves. Although large mass differences might exist they are not strictly necessary.

It is clear that even if the decay rates and Δ_π were known with extreme accuracy so that one could single out a particular solution to Eq. (16), the values of m_u and m_d would lie within certain bounds. In other words, when the ratio m_u/m_d is fixed by other considerations one could not find a single value for both m_u and m_d. However, appealing to SU(2) symmetry one would not expect a large difference between m_u and m_d and therefore one would choose the solution corresponding to the upper portion of the curve in Fig. 1. In this case then, the bounds for the quark masses become narrower.

III. $\eta \to \gamma\gamma$

The $\eta \to \gamma\gamma$ decay amplitude, $\mathfrak{F}_\eta(q^2)$, is defined as

$$\mathfrak{M}(\eta \to \gamma\gamma) = \epsilon_{\mu\nu\alpha\beta} k_1^\mu k_2^\nu \epsilon_1^\alpha \epsilon_2^\beta \mathfrak{F}_\eta(q^2) , \tag{18}$$

where ϵ and k are the polarizations and momenta of the photons and $q = k_1 + k_2$. The low-energy theorem, Eq. (4), becomes

$$\mathfrak{F}_\eta(0) = -\frac{1}{\sqrt{3}} \frac{2\alpha}{\pi} \frac{1}{f_\eta} S . \tag{19}$$

In Ref. 6 it was shown that EPCAC predicts a decay rate in agreement with experiment without the need of invoking η-η' mixing. Therefore, we shall adopt this point of view in what follows.

Repeating the steps discussed in the previous section it turns out that the equivalent of Eq. (16) is

$$\left[\frac{5(\sin^{-1}x_1)^2/x_1 - 2(\sin^{-1}x_2)^2/x_2}{5x_1 - 2x_2}\right]^2 \frac{1}{(1-\Delta_K)^2} = \frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma_0(\eta \to \gamma\gamma)} = 1.8 \pm 0.3 , \tag{20}$$

where

$$x_1 = \frac{m_\eta}{2m_u} , \quad x_2 = \frac{m_\eta}{2m_s} , \tag{21}$$

and we have assumed $m_u = m_d$ and $\Delta_\eta = \Delta_K$.[10] Of the four types of solutions to Eq. (20) one is left with only two if the restriction $m_s \geq m_u$ is used. The experimental value of Δ_K is[11]

$$\Delta_K = 1 - \frac{(m_\Lambda + m_p) g_\Lambda^A}{\sqrt{2} f_K g_{\Lambda p K}} = 0.30 \pm 0.15 . \tag{22}$$

In Fig. 2 we show some solutions of Eq. (20) for $(1-\Delta_K)^2\Gamma(\eta\to\gamma\gamma)/\Gamma_0(\eta\to\gamma\gamma) = 2.1$, 1.8, 1.458, and 1.152.

Once again, a large mass difference between

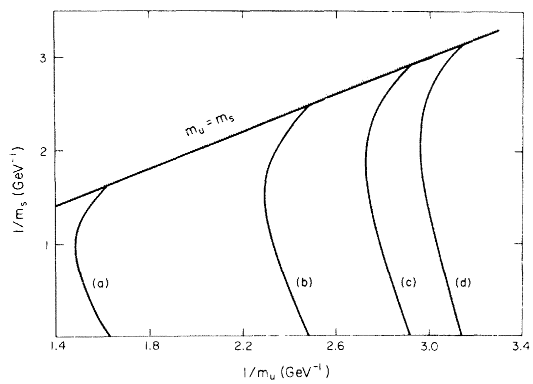

FIG. 2. Solutions to Eq. (20) for $(1-\Delta_K)^2\Gamma(\eta\to\gamma\gamma)/\Gamma_0(\eta_0\to\gamma\gamma) = 2.1$ (a), 1.458 (b), 1.8 (c), and 2.1 (d).

m_u and m_s is not compelling. Also, even in the case of extremely accurate experimental values for the rates and for Δ_K, it is only possible to obtain bounds on the quark masses.

IV. CONCLUDING REMARKS

The relation between chiral-symmetry breaking and the mass of the quarks obtained here from EPCAC and chiral anomalies seems to be a promising alternative to the usual model-dependent approaches in which chiral-symmetry violations are due to explicit quark-mass terms. These are absent in our formulation since we have started from PCAC, where the chiral-symmetry breaking parameter is the boson mass. However, it is clear that from a numerical point of view it would be necessary to have a much better experimental accuracy in the decay rates of $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ as well as in Δ_π and Δ_K before definite predictions for the quark masses could be established. Furthermore, even if a particular curve in Figs. 1 and 2 could be singled out, the prediction for the quark masses would be in the form of bounds rather than specific values. For the time being, one reads from Figs. 1 and 2 quark masses consistently higher than what has been recently estimated in the framework of the quark model and also that large mass differences, though not excluded, are not compelling.

As a final point we would like to point out that if a quark-confinement mechanism is introduced one would not expect major modifications in our results. The quark masses, though, would have to be interpreted as being renormalized by the confinement mechanism.

This assertion is based on a model-dependent estimation[13] of the corrections to the $\pi^0 \to \gamma\gamma$ amplitude introduced by quark confinement which is of the order $(\mu_\pi^2/2m_u^2)^4$.

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[9]Pagels has estimated that the derivations of E_π from unity, due to continuum contributions, is approximately 5×10^{-3} (H. Pagels, private communication). Equation (6), therefore, is valid up to terms of this order.

[10]Since Δ_η is hypothetical one may assume $\Delta_\eta \simeq \Delta_K$. Since $m_\eta^2 - m_K^2 \simeq 0.06\ \mathrm{GeV}^2$ and $\Delta_K = 0(m_K^2/1\ \mathrm{GeV}^2)$ one expects this assumption to be good within 6%. This has already been used in Ref. 6.

[11]We have used $g_A = -(1.25 \pm 0.09)$, $g_\Lambda^A/g_\Lambda^V = -(0.66 \pm 0.05)$, and $f_K/f_+(0)f_\pi = 1.26 \pm 0.02$ with $f_+(0) = 0.97$, as given in Particle Data Group, Rev. Mod. Phys. 48, S1 (1976), H. Pagels, Phys. Rep. 16C, 219 (1975); $g_{np\pi^+} = -\sqrt{2}\, g_{pNN}$, where $g_{\pi NN}/4\pi = 14.64^{+0.54}_{-0.72}$, and $g_{\Lambda pK}/4\pi = 13.5 \pm 2.5$, given by H. Pilkuhn *et al*., Nucl. Phys. B65, 460 (1973), T. W. Rogers *et al*., Phys. Rev. 178, 2478 (1969); and finally, $\sqrt{2} f_\pi = (0.93251 \pm 0.00144)\, \mu_{\pi^+}$, from R. J. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A150, 369 (1970).

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