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New resonances in the linear chiral SU(4)$\times$ SU(4) model

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In the context of the linear chiral SU(4)$\times$SU(4) model, we study a Lagrangian of 16 pseudoscalar and 16 scalar fields interacting by means of the most general nonderivative chiral-SU(4)$\times$SU(4)-invariant interaction and any particular symmetry-breaking term. The three-point and four-point coupling constants in the model can simply be related to masses in the system. As an application, partial decay rates of η'' (0^- meson analog of ψ) such as $\eta''\to K+\bar{K}+\pi$, $\eta''\to\eta+\pi+\pi$, etc., are computed with a choice of the symmetry-breaking term of $(4,4^*)+(4^*,4)$ type. Numerical analyses of the pseudoscalar-meson mass spectrum and the partial decay rates of η'' are carried out.

I. INTRODUCTION

The charm model[1] has been the simplest model to explain the narrow resonances[2,3]. In particular, after the experimental discovery of charmed mesons was announced,[4] the model stood on a much firmer ground. Taking this picture seriously, we must investigate a whole particle spectrum based on its underlying group SU(4). In fact there have been many investigations in this direction[5-9].

As an extension of the linear chiral SU(3)$\times$SU(3) model[10-12] investigated some time ago, in this paper we are going to investigate the chiral SU(4) $\times$SU(4) symmetry applied to the pseudoscalar and scalar meson system. Our linear chiral SU(3) $\times$SU(3) model is a Lagrangian formulation of the Glashow-Weinberg approach[13] and has some advantage over the Gell-Mann–Oakes–Renner model[14] in the sense that it allows the possibility of having a nondegenerate vacuum based on the spontaneous breakdown of symmetry and, in another, that the mixing problem has been handled properly. The model was particularly suited to select a possible symmetry-breaking term. The symmetry-breaking term of $(3,3^*)+(3^*,3)$ type[14] could yield a mass formula[11,12,15] which is well satisfied experimentally. In the tree approximation, results obtained from the Lagrangian are known to be consistent with the standard current-algebraic ones in the soft-pion limit[10,12,16]. (Here, of course, we do not have to take the unphysical and often ambiguous soft-pion limit.) Recently the model has been extended to SU(4)$\times$SU(4) by Schechter and Singer[17] and the mass formula has been derived. The present work is a further study of the SU(4)$\times$SU(4) version of the model. Although the formalism of Refs. 11, 12, and 17 is quite general from the group-theoretical point of view, it has been criticized as "nonrenormalizable," since the chiral-invariant part of the interaction contains polynomials higher than fourth order, explicitly. This fact has led people to construct a model of less general nature.[18] In order to improve the situation, we construct a formalism without referring to the group invariants at all. Although the discussion of the renormalization[19] is beyond the scope of the present paper, our effective Lagrangian, (2.6), restricted up to fourth-order polynomials, satisfies the power-counting rule for renormalizability and is expected to be renormalizable.

We shall investigate the linear chiral SU(4) $\times$SU(4) model with the most general nonderivative chiral-SU(4)$\times$SU(4)-invariant interaction and some particular symmetry-breaking interaction. As a result of the invariance alone, the chiral-invariant part of the interaction must satisfy two basic equations. Because of the existence of the additional symmetry-breaking interaction, the ground state of the system can be determined by the extremum condition. Together with the extremum condition, differentiated expressions of the basic equations are powerful enough to supply us with information not only on masses but also on coupling constants involved in the system.[12]

With a simple choice of $(4,4^*)+(4^*,4)$ type of symmetry breaking we derive mass formulas agreeing with those of Ref. 17. In order to investigate the model further, we next derive various relations among three-point and four-point coupling constants in the system. Then as an application of the formalism we shall here mainly concentrate on the decay processes[20] such as

$$\eta''\to K^+ + K^- + \pi^0,$$

$$\eta''\to \eta + \pi^+ + \pi^-,\ \text{etc.}$$

Since these are strong decay modes, they are expected to dominate over weak or electromagnetic decay modes.

The mystery of the ψ and ψ' has been why its hadronic width is so narrow. Under the circumstances it is a pertinent question to ask if a similar situation holds for η'', which is an expected 0^- meson counterpart of ψ. An order-of-magnitude estimate has been given by the charmonium model.[21] According to this model the hadronic width should be an order of 100 times larger than that of ψ, because the η'' decay is the two-gluon emission process, contrary to the corresponding three for the ψ decay. (Of course the coupling constant is supposed to be small in this energy range.) As was mentioned earlier, our Lagrangian contains detailed information not only on masses but also on interactions; we can quantitatively discuss the above decay widths without much ambiguity, in spite of the lack of experimental information. Because of the fact that η'' is predominantly in the charm-anticharm pair state, the above hadronic width of η'' is narrow, but not so much as ψ or ψ', agreeing with the qualitative feature of the charmonium picture.

Finally, we would like to mention that the present model yields a sum rule for the leptonic decay constants:

$$\frac{F_F}{F_\pi}-\frac{F_D}{F_\pi}=\frac{F_K}{F_\pi}-1$$

(see Sec. V for details). This sum rule will easily be testable by future experiments.

In Sec. II a brief discussion of our formalism is given. In Sec. III mass formulas are derived and the mixing angles for pseudoscalar mesons are determined. In Sec. IV formulas for three-point and four-point coupling constants are derived and then partial decay rates such as $\eta'' \to K+\bar{K}+\pi_0$ are computed. In Sec. V numerical results are discussed.

II. FORMALISM

The system in which we are interested consists of 16 pseudoscalar mesons ϕ_a^b $(a,b=1-4)$ and 16 scalar fields S_a^b $(a,b=1-4)$. The Lagrangian density for this system is then

$$\mathfrak{L}=-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu\phi\partial_\mu\phi)-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu S\partial_\mu S)-V_0-V_{\mathrm{SB}}\,. \tag{2.1}$$

Here V_0 is the most general (nonderivative) chiral-$\mathrm{SU}(4)\times\mathrm{SU}(4)$-invariant interaction and V_{SB} is the symmetry-breaking term to be specified later.

The ground state of the system will be determined from

$$\left\langle\frac{\partial V_0}{\partial\phi}\right\rangle_0+\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial\phi}\right\rangle_0=0\,, \tag{2.2}$$

$$\left\langle\frac{\partial V_0}{\partial S}\right\rangle_0+\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S}\right\rangle_0=0\,, \tag{2.3}$$

where the notation $\langle\;\rangle_0$ means that the enclosed expression is evaluated at the ground state. Next we introduce the physical fields (denoted by a tilde)

$$\tilde{\phi}=\phi-\langle\phi\rangle_0\,, \tag{2.4}$$

$$\tilde{S}=S-\langle S\rangle_0\,, \tag{2.5}$$

and expand the Lagrangian density as

$$\begin{aligned}\mathfrak{L}=&-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu\tilde{\phi}\partial_\mu\tilde{\phi})-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu\tilde{S}\partial_\mu\tilde{S})-\frac{1}{2}\sum_{a,b,c,d}\left(\left\langle\frac{\partial^2V}{\partial\phi_a^b\partial\phi_c^d}\right\rangle_0\tilde{\phi}_a^b\tilde{\phi}_c^d+\left\langle\frac{\partial^2V}{\partial S_a^b\partial S_c^d}\right\rangle_0\tilde{S}_a^b\tilde{S}_c^d\right)\\&-\frac{1}{2}\sum_{a,b,c,d,e,f}\left\langle\frac{\partial^3V}{\partial S_a^b\partial\phi_c^d\partial\phi_e^f}\right\rangle_0\tilde{S}_a^b\tilde{\phi}_c^d\tilde{\phi}_e^f-\frac{1}{4!}\sum_{a,\ldots,h}\left\langle\frac{\partial^4V}{\partial\phi_a^b\partial\phi_c^d\partial\phi_e^f\partial\phi_g^h}\right\rangle_0\tilde{\phi}_a^b\tilde{\phi}_c^d\tilde{\phi}_e^f\tilde{\phi}_g^h\\&-\frac{1}{3!}\sum_{a,\ldots,f}\left\langle\frac{\partial^3V}{\partial S_a^b\partial S_c^d\partial S_e^f}\right\rangle_0\tilde{S}_a^b\tilde{S}_c^d\tilde{S}_e^f-\frac{1}{4}\sum_{a,\ldots,h}\left\langle\frac{\partial^4V}{\partial\phi_a^b\partial\phi_c^d\partial S_e^f\partial S_g^h}\right\rangle_0\tilde{\phi}_a^b\tilde{\phi}_c^d\tilde{S}_e^f\tilde{S}_g^h\\&-\frac{1}{4!}\sum_{a,\ldots,h}\left\langle\frac{\partial^4V}{\partial S_a^b\partial S_c^d\partial S_e^f\partial S_g^h}\right\rangle_0\tilde{S}_a^b\tilde{S}_c^d\tilde{S}_e^f\tilde{S}_g^h+(\cdot\;\cdot\;\cdot)\,,\end{aligned} \tag{2.6}$$

with $V=V_0+V_{\mathrm{SB}}$. The sets of coefficients $\langle\partial^2V/\partial\phi_a^b\partial\phi_c^d\rangle_0$, for example, represent the matrix of pseudoscalar-meson squared masses. The set of coefficients $\langle\partial^3V/\partial S_a^b\partial\phi_c^d\partial\phi_e^f\rangle_0$ represents the coupling constants of $\tilde{S}$-$\tilde{\phi}$-$\tilde{\phi}$ vertices. Similarly, $\langle\partial^4V/\partial\phi\partial\phi\partial\phi\partial\phi\rangle_0$ represent the effective four-point coupling constants. In this paper we are not interested in the remaining terms. The last (dotted) term in (2.6) represents possible contributions coming from the interactions higher than fourth order. As mentioned in the Introduction, however, when we wish the model to be renormalizable, we should disregard these (dotted) contributions entirely. (The main advantage of this approach over the so-called renormalizable model[18] is that we do not allow any approximation in the stage of determining the ground state of the system.)

Since our treatment is similar to the SU(3) σ model,[12] the discussion will be brief. Assuming parity invariance of the model, one finds

$$\langle\phi\rangle_0=\left\langle\frac{\partial V_0}{\partial\phi}\right\rangle_0=\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial\phi}\right\rangle_0\,. \tag{2.7}$$

Thus $\tilde{\phi}=\phi$ and (2.4) is identically satisfied. For the scalar mesons we may choose

$$\langle S_a^b\rangle_0=\delta_a^b\alpha_a \text{ (no sum)}, \qquad (2.8)$$

where δ_a^b is the Kronecker δ, and α_a are the four real constants characterizing the ground state of the model.

We now consider infinitesimal chiral SU(4) × SU(4) transformations. Under a vector transformation, the change in the fields is given by

$$\delta\phi=[E_V,\phi], \qquad (2.9a)$$

$$\delta S=[E_V,S], \qquad (2.9b)$$

where E_V is an arbitrary 4×4 infinitesimal matrix satisfying[22]

$$E_V^\dagger=-E_V. \qquad (2.10)$$

Then, since V_0 is invariant under the vector transformation,

$$\delta V_0=\mathrm{Tr}\left(\frac{\partial V_0}{\partial\phi}\delta\phi+\frac{\partial V_0}{\partial S}\delta S\right)=0.$$

This immediately leads to

$$[\phi,\partial V_0/\partial\phi]+[S,\partial V_0/\partial S]=0. \qquad (2.11)$$

Similarly, the change in the fields under an axial-vector transformation is given by

$$\delta\phi=-i[E_A,S]_+, \qquad (2.12a)$$

$$\delta S=i[E_A,\phi]_+, \qquad (2.12b)$$

where E_A again satisfies (2.10). Since we are interested in SU(4) × SU(4) transformations, we must in addition have a unimodular property so that

$$\mathrm{Tr}(E)=0. \qquad (2.13)$$

(2.13) does not have any effect on a vector transformation. In order to take into account (2.13), we use the Lagrange-multiplier method for an axial-vector transformation.

The invariance of V_0 under the axial-vector transformation yields

$$0=\delta V_0=\mathrm{Tr}\{E_A([\partial V_0/\partial S,\phi]_+-[\partial V_0/\partial\phi,S]_+-\lambda)\}.$$

Thus we must have

$$[\partial V_0/\partial S,\phi]_+-[\partial V_0/\partial\phi,S]_+=\lambda 1, \qquad (2.14a)$$

where 1 is a 4×4 unit matrix and the Lagrange multiplier λ is determined to be

$$\lambda=\tfrac{1}{4}\mathrm{Tr}([\partial V_0/\partial S,\phi]_+-[\partial V_0/\partial\phi,S]_+)$$
$$=\tfrac{1}{2}\mathrm{Tr}\left(\frac{\partial V_0}{\partial S}\phi-\frac{\partial V_0}{\partial\phi}S\right). \qquad (2.14b)$$

Equations (2.11), (2.14a), and (2.14b) are our basic equations. Differentiating with respect to fields and evaluating the resultant expression at the ground state, we can derive various relations between particle masses and the three- and four-point interaction vertices. In this paper the symmetry-breaking term is chosen to be

$$V_{\mathrm{SB}}=-2(A_1S_1^1+A_2S_2^2+A_3S_3^3+A_4S_4^4), \qquad (2.15)$$

which transforms according to the $(4,4^*)+(4^*,4)$ representation of SU(4) × SU(4).

III. MASS FORMULAS

The scalar-meson mass (squared) matrix which is to be compared with experiment is

$$\left\langle\frac{\partial^2 V}{\partial S_b^a\partial S_f^e}\right\rangle_0=\left\langle\frac{\partial^2 V_0}{\partial S_b^a\partial S_f^e}\right\rangle_0+\left\langle\frac{\partial^2 V_{\mathrm{SB}}}{\partial S_b^a\partial S_f^e}\right\rangle_0. \qquad (3.1)$$

Because of the basic equation (2.11) and the extremum condition (2.3), $\langle\partial^2V_0/\partial S_b^a\partial S_f^e\rangle_0$ must satisfy

$$(\alpha_a-\alpha_b)\left\langle\frac{\partial^2 V_0}{\partial S_b^a\partial S_f^e}\right\rangle_0=\delta_a^f\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_b^e}\right\rangle_0-\delta_e^b\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_f^a}\right\rangle_0. \qquad (3.2)$$

Therefore, scalar-meson (squared) masses are given by

$$\left\langle\frac{\partial^2 V}{\partial S_b^a\partial S_f^e}\right\rangle_0=\left\langle\frac{\partial^2 V_{\mathrm{SB}}}{\partial S_b^a\partial S_f^e}\right\rangle_0+\frac{1}{\alpha_a-\alpha_b}\left(\delta_a^f\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_b^e}\right\rangle_0-\delta_e^b\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_f^a}\right\rangle_0\right). \qquad (3.3)$$

Similarly, the pseudoscalar-meson mass (squared) matrix is

$$\left\langle\frac{\partial^2 V}{\partial\phi_b^a\partial\phi_f^e}\right\rangle_0=\left\langle\frac{\partial^2 V_0}{\partial\phi_b^a\partial\phi_f^e}\right\rangle_0+\left\langle\frac{\partial^2 V_{\mathrm{SB}}}{\partial\phi_b^a\partial\phi_f^e}\right\rangle_0, \qquad (3.4)$$

where $\langle\partial^2V_0/\partial\phi_b^a\partial\phi_f^e\rangle_0$ must satisfy the following relation:

$$(\alpha_a+\alpha_b)\left\langle\frac{\partial^2 V_0}{\partial\phi_b^a\partial\phi_f^e}\right\rangle_0-\tfrac{1}{2}\delta_a^b\sum_c\alpha_c\left\langle\frac{\partial^2 V_0}{\partial\phi_f^e\partial\phi_c^c}\right\rangle_0=-\delta_a^f\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_b^e}\right\rangle_0-\delta_e^b\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_f^a}\right\rangle_0+\tfrac{1}{2}\delta_a^b\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial S_f^e}\right\rangle_0. \qquad (3.5)$$

With a choice of Eq. (2.15), the scalar and pseudoscalar mass matrix is given by

$$(\alpha_a-\alpha_b)\left\langle\frac{\partial^2 V}{\partial S_b^a\partial S_f^e}\right\rangle_0=2\delta_a^f\delta_e^b(A_a-A_b) \qquad (3.6)$$

and

$$(\alpha_a+\alpha_b)\left\langle\frac{\partial^2 V}{\partial\phi_b^a\partial\phi_f^e}\right\rangle_0-\tfrac{1}{2}\delta_a^b\sum_c\alpha_c\left\langle\frac{\partial^2 V}{\partial\phi_f^e\partial\phi_c^c}\right\rangle_0=2\delta_a^f\delta_e^b(A_a+A_b)-\delta_a^b\delta_e^f A_e\,. \tag{3.7}$$

Nondiagonal components of Eq. (3.7) yield the following pseudoscalar-meson masses:

$$\begin{aligned}&(K^+)^2=\frac{2(A_3+A_1)}{\alpha_3+\alpha_1},\quad (K^0)^2=\frac{2(A_3+A_2)}{\alpha_3+\alpha_2},\\ &(D^+)^2=\frac{2(A_4+A_2)}{\alpha_4+\alpha_2},\quad (D^0)^2=\frac{2(A_4+A_1)}{\alpha_4+\alpha_1},\\ &(F^+)^2=\frac{2(A_4+A_3)}{\alpha_4+\alpha_3},\quad (\pi^+)^2=\frac{2(A_2+A_1)}{\alpha_2+\alpha_1}.\end{aligned} \tag{3.8}$$

Diagonal components of Eq. (3.7) are more involved. All together we have 16 relations:

$$4\alpha_i M_{ij}-\sum_{c=1}^{4}\alpha_c M_{jc}=\begin{cases}6A_j & \text{for } i=j=1,\dots,4\\ -2A_j & \text{for } i\neq j;\ i=1,\dots,4;\ j=1,\dots,4,\end{cases}$$

where (3.9)

$$M_{ij}=\langle\partial^2 V/\partial\phi_i^i\partial\phi_j^j\rangle_0\,.$$

Or, equivalently, we can write the mass matrix in the form

$$M_{ab}=\begin{bmatrix}2\dfrac{A_1}{\alpha_1}+\dfrac{\alpha_2}{\alpha_1}M_{12} & M_{12} & \dfrac{\alpha_2}{\alpha_3}M_{12} & \dfrac{\alpha_2}{\alpha_4}M_{12}\\ M_{12} & 2\dfrac{A_2}{\alpha_2}+\dfrac{\alpha_1}{\alpha_2}M_{12} & \dfrac{\alpha_1}{\alpha_3}M_{12} & \dfrac{\alpha_1}{\alpha_4}M_{12}\\ \dfrac{\alpha_2}{\alpha_3}M_{12} & \dfrac{\alpha_1}{\alpha_3}M_{12} & 2\dfrac{A_3}{\alpha_3}+\dfrac{\alpha_1\alpha_2}{\alpha_3^2}M_{12} & \dfrac{\alpha_1}{\alpha_3}\dfrac{\alpha_2}{\alpha_4}M_{12}\\ \dfrac{\alpha_2}{\alpha_4}M_{12} & \dfrac{\alpha_1}{\alpha_4}M_{12} & \dfrac{\alpha_1}{\alpha_3}\dfrac{\alpha_2}{\alpha_4}M_{12} & 2\dfrac{A_4}{\alpha_4}+\dfrac{\alpha_1\alpha_2}{\alpha_4^2}M_{12}\end{bmatrix}. \tag{3.10}$$

Scalar-meson masses can be similarly discussed. For instance,

$$\begin{aligned}&(\kappa^+)^2=\frac{2(A_3-A_1)}{\alpha_3-\alpha_1},\quad (\kappa^0)^2=\frac{2(A_3-A_2)}{\alpha_3-\alpha_2},\quad (D_s^+)^2=\frac{2(A_4-A_2)}{\alpha_4-\alpha_2},\quad (D_s^0)^2=\frac{2(A_4-A_1)}{\alpha_4-\alpha_1},\\ &(F_s^+)^2=\frac{2(A_4-A_3)}{\alpha_4-\alpha_3},\quad \epsilon^2=\frac{2(A_2-A_1)}{\alpha_2-\alpha_1},\end{aligned} \tag{3.11}$$

where D_S, F_S, and ϵ are the scalar-meson analogs of D, F, and π, respectively.

Since we are interested in the limit of isospin invariance only, in this paper, we have $A_1=A_2$ and $\alpha_1=\alpha_2$. Then setting $\alpha_1=\alpha_2=\alpha$, $\alpha_3=\alpha W$, and $\alpha_4=\alpha W'$ we can express pseudoscalar-meson masses as

$$K^2=2(A_3+A_1)/\alpha(W+1)\,, \tag{3.12a}$$

$$\pi^2=2A_1/\alpha\,, \tag{3.12b}$$

$$F^2=2(A_4+A_3)/\alpha(W'+W)\,, \tag{3.12c}$$

$$D^2=2(A_4+A_1)/\alpha(W'+1)\,, \tag{3.12d}$$

and

$$M_{ab}=\begin{bmatrix} 2\frac{A_1}{\alpha}+M_{12} & M_{12} & \frac{M_{12}}{W} & \frac{M_{12}}{W'} \\ M_{12} & 2\frac{A_1}{\alpha}+M_{12} & \frac{M_{12}}{W} & \frac{M_{12}}{W'} \\ \frac{M_{12}}{W} & \frac{M_{12}}{W} & 2\frac{A_3}{\alpha W}+\frac{M_{12}}{W^2} & \frac{M_{12}}{WW'} \\ \frac{M_{12}}{W'} & \frac{M_{12}}{W'} & \frac{M_{12}}{WW'} & 2\frac{A_4}{\alpha W'}+\frac{M_{12}}{W'^2} \end{bmatrix}. \tag{3.13}$$

Physical masses are found as a solution of the secular equation $\det|M_{ab}-\lambda\delta_{ab}|=0$ ($a=1$–4). Since $\pi_0{}^2=2A_1/\alpha$ does not mix with the rest, we only have to solve a cubic equation to get η^2, η'^2, and η''^2:

$$\begin{vmatrix} 2M_{12}+\pi_0{}^2-\lambda & 2\frac{M_{12}}{W} & 2\frac{M_{12}}{W'} \\ \frac{M_{12}}{W} & 2\frac{A_3}{\alpha W}+\frac{M_{12}}{W^2}-\lambda & \frac{M_{12}}{WW'} \\ \frac{M_{12}}{W'} & \frac{M_{12}}{WW'} & 2\frac{A_4}{\alpha W'}+\frac{M_{12}}{W'^2}-\lambda \end{vmatrix}=0. \tag{3.14}$$

Or, explicitly,

$$\eta^2+\eta'^2+\eta''^2=\pi^2+2\frac{A_3}{\alpha W}+2\frac{A_4}{\alpha W'}+M_{12}\left(2+\frac{1}{W^2}+\frac{1}{W'^2}\right), \tag{3.15a}$$

$$\begin{aligned}\eta^2\eta'^2+\eta'^2\eta''^2+\eta^2\eta''^2=\pi^2\left(2\frac{A_3}{\alpha W}+2\frac{A_4}{\alpha W'}\right)+2\frac{A_3}{\alpha W}2\frac{A_4}{\alpha W'}\\ +M_{12}\left[4\frac{A_3}{\alpha W}+4\frac{A_4}{\alpha W'}+\frac{1}{W'^2}\frac{2A_3}{\alpha W}+\frac{1}{W^2}\frac{2A_4}{\alpha W'}+\pi^2\left(\frac{1}{W^2}+\frac{1}{W'^2}\right)\right],\end{aligned} \tag{3.15b}$$

$$\eta^2\eta'^2\eta''^2=\pi^2\frac{2A_3}{\alpha W}\frac{2A_4}{\alpha W'}+M_{12}\left[\pi^2\left(\frac{1}{W'^2}\frac{2A_3}{\alpha W}+\frac{1}{W^2}\frac{2A_4}{\alpha W'}\right)+2\left(\frac{2A_3}{\alpha W}\right)\left(\frac{2A_4}{\alpha W'}\right)\right]. \tag{3.15c}$$

We choose pseudoscalar-meson masses η^2, η'^2, K^2, and π^2 as inputs. Then, after eliminating A_4/a and M_{12} in (3.15), we can express η''^2 in terms of W and W' only. Of course, A_3/α is not a new parameter, but is related to W by

$$A_3/\alpha=\tfrac{1}{2}[(1+W)K^2-\pi^2]. \tag{3.16}$$

Explicitly, η''^2 satisfies the following quadratic equation:

$$\begin{aligned}(\eta''^2)^2\Delta\left[1-\left(2W^2+1+\frac{W^2}{W'^2}\right)\Delta\right]+\eta''^2\Delta\pi^2\left[\Delta\left(\frac{4A_3W}{\alpha\pi^2}+1\right)-\left(1+\frac{2A_3}{\alpha W\pi^2}\right)-\left(2+\frac{1}{W^2}+\frac{2}{W'^2}\right)\Omega\right]\\ +\Delta\pi^4\left[\frac{2A_3}{\alpha W}\frac{1}{\pi^2}+\left(\frac{4A_3}{\alpha W}\frac{1}{\pi^2}+\frac{1}{W^2}\right)\Omega\right]-\frac{\Omega^2\pi^4}{W^2W'^2}=0,\end{aligned} \tag{3.17}$$

where the dimensionless parameters, Δ and Ω, are defined by and are dependent upon W only as

$$\Delta(W)=\frac{1}{2(2A_3/\alpha-\pi^2W)^2}\left[\eta^2\eta'^2(2W^2+1)-(\eta^2+\eta'^2)\left(\frac{4A_3W}{\alpha}+\pi^2\right)+\pi^4+2\left(\frac{2A_3}{\alpha}\right)^2\right], \tag{3.18}$$

$$\Omega(W)=\frac{W^2}{2\pi^2(2A_3/\alpha-\pi^2W)^2}\left\{-\eta^2\eta'^2\left(\frac{4A_3}{\alpha}W+\pi^2\right)+(\eta^2+\eta'^2)\left[2\left(\frac{2A_3}{\alpha}\right)^2+\pi^4\right]-\left[2\left(\frac{2A_3}{\alpha}\right)^3\frac{1}{W}+\pi^6\right]\right\}. \tag{3.19}$$

Essentially the same mass formula was obtained earlier[17] in a slightly different way.

We first observe that in the limit $W'\to\infty$ (3.17) yields a trivial solution $\Delta(W)=0$, which is independent of η''^2. This corresponds to the case of chiral SU(3)×SU(3). In fact the mass formula

$$\Delta(W)=0 \tag{3.20}$$

was derived earlier in Refs. 11, 12, and 15.

Treating W as a free parameter, the sum rule (3.20) can be well satisfied with a choice of $W = 1.73$. According to the formula

$$W = 2F_K/F_\pi - 1\,, \tag{3.21}$$

$W = 1.73$ corresponds to $F_K/F_\pi = 1.37$, which is rather close to an experimental value $F_K/F_\pi = 1.28$.[23]

In this paper we are interested in a situation where η''^2 depends explicitly upon W and W'. Therefore, excluding the above singular case we assume $\Delta \neq 0$ hereafter. For a given W and W' we can determine η''^2 from Eq. (3.17). Then we can determine M_{12} and A_4/α from

$$M_{12} = \eta''^2 W^2 \Delta + \pi^2 \Omega\,, \tag{3.22}$$

$$\frac{A_4}{\alpha} = \frac{W'\pi^2}{2}\left\{\frac{\eta''^2}{\pi^2}\left[1 - \Delta\left(2W^2 + 1 + \frac{W^2}{W'^2}\right)\right] - \frac{\Omega}{W'^2} + \left(4W\frac{A_3}{\alpha\pi^2} + 1\right)\Delta\right\}. \tag{3.23}$$

To complete the discussion of masses, let us briefly mention our choice of mixing angles and their determination. The basis vectors for our mass matrix are ϕ_1^1, ϕ_2^2, ϕ_3^3, and ϕ_4^4. After identification of $\pi_0{}^2 = M_{11} - M_{12}$, we can express the characteristic equation in the form

$$\begin{bmatrix} \dfrac{M_{11} + M_{12} - \lambda_i}{2} & \dfrac{M_{12}}{W} & \dfrac{M_{12}}{W'} \\ \dfrac{M_{12}}{W} & M_{33} - \lambda_i & \dfrac{M_{12}}{WW'} \\ \dfrac{M_{12}}{W'} & \dfrac{M_{12}}{WW'} & M_{44} - \lambda_i \end{bmatrix} \begin{bmatrix} |\phi_1^1 + \phi_2^2\rangle \\ |\phi_3^3\rangle \\ |\phi_4^4\rangle \end{bmatrix} = 0\,, \tag{3.24}$$

where $\lambda_i = \eta^2$, η'^2, and η''^2 and

$$M_{11} = \pi^2 + M_{12}, \quad M_{33} \equiv 2\frac{A_3}{\alpha W} + \frac{M_{12}}{W^2}, \quad M_{44} \equiv 2\frac{A_4}{\alpha W'} + \frac{M_{12}}{W'^2}.$$

Then normalized basis vectors are given by

$$\begin{bmatrix} \left|\dfrac{\phi_1^1 + \phi_2^2}{\sqrt{2}}\right\rangle \\ |\phi_3^3\rangle \\ |\phi_4^4\rangle \end{bmatrix} = \begin{bmatrix} \dfrac{1}{N_i}\dfrac{1}{\sqrt{2}}[W^2 W'^2 (M_{33} - \lambda_i)(M_{44} - \lambda_i) - M_{12}{}^2] \\ \dfrac{1}{N_i} M_{12} W [M_{12} - W'^2 (M_{44} - \lambda_i)] \\ \dfrac{1}{N_i} M_{12} W' [M_{12} - W^2 (M_{33} - \lambda_i)] \end{bmatrix}, \tag{3.25}$$

where N_i are normalization constants and the explicit forms will be given shortly.

Physical fields π_0, η, η', and η'' and fields ϕ_i^i ($i = 1$–4) are related, via a 4×4 orthogonal matrix, as

$$\begin{bmatrix} \pi_0 \\ \eta \\ \eta' \\ \eta'' \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 & 0 \\ a & a & b & c \\ a' & a' & b' & c' \\ a'' & a'' & b'' & c'' \end{bmatrix} \begin{bmatrix} \phi_1^1 \\ \phi_2^2 \\ \phi_3^3 \\ \phi_4^4 \end{bmatrix}. \tag{3.26}$$

Inverting (3.26) and identifying it with (3.25) we determine the mixing parameter

$$a = \frac{1}{N_\eta}\frac{1}{2}\left\{\left[W^2\left(\frac{2A_3}{\alpha W} - \eta^2\right) + M_{12}\right] \times \left[W'^2\left(\frac{2A_4}{\alpha W'} - \eta^2\right) + M_{12}\right] - M_{12}{}^2\right\},$$

$$b = \frac{1}{N_\eta} M_{12} W W'^2 \left(\eta^2 - \frac{2A_4}{\alpha W'}\right),$$

$$c = \frac{1}{N_\eta} M_{12} W' W^2 \left(\eta^2 - \frac{2A_3}{\alpha W}\right),$$

where

$$N_\eta{}^2 = \frac{1}{2}\left\{\left[W^2\left(\frac{2A_3}{\alpha W} - \eta^2\right) + M_{12}\right] \times \left[W'^2\left(\frac{2A_4}{\alpha W'} - \eta^2\right) + M_{12}\right] - M_{12}{}^2\right\}^2 + (M_{12} W W'^2)^2 \left(\eta^2 - \frac{2A_4}{\alpha W'}\right)^2 + (M_{12} W' W^2)^2 \left(\eta^2 - \frac{2A_3}{\alpha W}\right)^2.$$

Similarly, a set of mixing parameters a', b', and c' (or a'', b'', and c'') can be obtained from a set of parameters a, b, and c by changing η into η'

(or η into η''). An exactly analogous situation holds for the scalar-meson system. Scalar mesons ϵ_0, σ, σ', and σ'' are mixed through

$$\begin{bmatrix}\epsilon_0\\ \sigma\\ \sigma'\\ \sigma''\end{bmatrix}=\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ a_\sigma & a_\sigma & b_\sigma & c_\sigma\\ a_{\sigma'} & a_{\sigma'} & b_{\sigma'} & c_{\sigma'}\\ a_{\sigma''} & a_{\sigma''} & b_{\sigma''} & c_{\sigma''}\end{bmatrix}\begin{bmatrix}\tilde{S}_1^1\\ \tilde{S}_2^2\\ \tilde{S}_3^3\\ \tilde{S}_4^4\end{bmatrix}.$$

Contrary to the pseudoscalar mesons, however, the mixing angles for scalar mesons remain undetermined.

IV. $S\phi\phi$ COUPLING AND ϕ^4 COUPLING

We are going to calculate decay processes such as $\eta''\to K+\bar{K}+\pi$. In order to do this we need information on $S\phi\phi$ coupling constants. Because of our choice of symmetry breaking (2.15), the coupling constants

$$\left\langle\frac{\partial^3 V}{\partial S_b^a\partial\phi_d^c\partial\phi_f^e}\right\rangle_0$$

introduced in (2.6) are exactly equal to

$$\left\langle\frac{\partial^3 V_0}{\partial S_b^a\partial\phi_d^c\partial\phi_f^e}\right\rangle_0 .$$

Differentiating (2.14a) once with respect to the pseudoscalar meson and once with respect to the scalar meson and evaluating at the ground state, we get the basic formula for $\langle\partial^3 V/\partial S\partial\phi\partial\phi\rangle_0$:

$$(\alpha_a+\alpha_b)\left\langle\frac{\partial^3 V_0}{\partial\phi_b^a\partial\phi_f^e\partial S_h^g}\right\rangle_0=\delta_e^b\left\langle\frac{\partial^2 V_0}{\partial S_h^g\partial S_f^a}\right\rangle_0+\delta_a^f\left\langle\frac{\partial^2 V_0}{\partial S_h^g\partial S_b^e}\right\rangle_0-\delta_g^b\left\langle\frac{\partial^2 V_0}{\partial\phi_f^e\partial\phi_h^a}\right\rangle_0-\delta_a^h\left\langle\frac{\partial^2 V_0}{\partial\phi_f^e\partial\phi_b^g}\right\rangle_0-\delta_a^b\left\langle\frac{\partial^2\lambda}{\partial S_h^g\partial\phi_f^e}\right\rangle_0, \tag{4.1}$$

where λ was already introduced in (2.14b). Or, more explicitly, $\langle\partial^2\lambda/\partial S_h^g\partial\phi_f^e\rangle_0$ could be written as

$$\left\langle\frac{\partial^2\lambda}{\partial S_h^g\partial\phi_f^e}\right\rangle_0=\frac{1}{2}\left(\left\langle\frac{\partial^2 V_0}{\partial S_h^g\partial S_f^e}\right\rangle_0-\left\langle\frac{\partial^2 V_0}{\partial\phi_h^g\partial\phi_f^e}\right\rangle_0\right)-\frac{1}{2}\sum_c\alpha_c\left\langle\frac{\partial^3 V_0}{\partial S_h^g\partial\phi_c^c\partial\phi_f^e}\right\rangle_0 .$$

From (2.11) we can similarly derive

$$(\alpha_a-\alpha_b)\left\langle\frac{\partial^3 V_0}{\partial\phi_h^g\partial\phi_f^e\partial S_b^a}\right\rangle_0=\delta_e^b\left\langle\frac{\partial^2 V_0}{\partial\phi_f^a\partial\phi_h^g}\right\rangle_0-\delta_a^h\left\langle\frac{\partial^2 V_C}{\partial\phi_f^e\partial\phi_b^g}\right\rangle_0-\delta_a^f\left\langle\frac{\partial^2 V_0}{\partial\phi_b^e\partial\phi_h^g}\right\rangle_0+\delta_g^b\left\langle\frac{\partial^2 V_0}{\partial\phi_f^e\partial\phi_h^a}\right\rangle_0 . \tag{4.2}$$

Next, four-point coupling constants $\langle\partial^4 V/\partial\phi\partial\phi\partial\phi\partial\phi\rangle_0$ introduced in (2.6) are exactly equal to $\langle\partial^4 V_0/\partial\phi\partial\phi\partial\phi\partial\phi\rangle_0$ for our choice (2.15) and are expressible in terms of $S\phi\phi$ constants as

$$\begin{aligned}(\alpha_a+\alpha_b)\left\langle\frac{\partial^4 V_0}{\partial\phi_n^m\partial\phi_h^g\partial\phi_f^e\partial\phi_b^a}\right\rangle_0&=\delta_e^b\left\langle\frac{\partial^3 V_0}{\partial\phi_n^m\partial\phi_h^g\partial S_f^a}\right\rangle_0+\delta_g^b\left\langle\frac{\partial^3 V_0}{\partial\phi_n^m\partial\phi_f^e\partial S_h^a}\right\rangle_0+\delta_m^b\left\langle\frac{\partial^3 V_0}{\partial\phi_h^g\partial\phi_f^e\partial S_n^a}\right\rangle_0+\delta_a^f\left\langle\frac{\partial^3 V_0}{\partial\phi_n^m\partial\phi_h^g\partial S_b^e}\right\rangle_0\\&\quad+\delta_a^h\left\langle\frac{\partial^3 V_0}{\partial\phi_n^m\partial\phi_f^e\partial S_b^g}\right\rangle_0+\delta_a^n\left\langle\frac{\partial^3 V_0}{\partial\phi_h^g\partial\phi_f^e\partial S_b^m}\right\rangle_0-\delta_a^b\left\langle\frac{\partial^3\lambda}{\partial\phi_n^m\partial\phi_h^g\partial\phi_f^e}\right\rangle_0 .\end{aligned} \tag{4.3}$$

$\langle\partial^3\lambda/\partial\phi_n^m\partial\phi_h^g\partial\phi_f^e\rangle_0$ can be written explicitly in the form

$$\left\langle\frac{\partial^3\lambda}{\partial\phi_n^m\partial\phi_h^g\partial\phi_f^e}\right\rangle_0=\frac{1}{2}\left(\left\langle\frac{\partial^3 V_0}{\partial\phi_h^g\partial\phi_f^e\partial S_n^m}\right\rangle_0+\left\langle\frac{\partial^3 V_0}{\partial\phi_h^g\partial\phi_n^m\partial S_f^e}\right\rangle_0+\left\langle\frac{\partial^3 V_0}{\partial\phi_n^m\partial\phi_f^e\partial S_h^g}\right\rangle_0\right)-\frac{1}{2}\sum_c\alpha_c\left\langle\frac{\partial^4 V_0}{\partial\phi_n^m\partial\phi_h^g\partial\phi_f^e\partial\phi_c^c}\right\rangle_0 .$$

As an application we first calculate the strongly interacting decay process

$$\eta''(p)\to K^+(q_+)+K^-(q_-)+\pi^0(q_\pi) . \tag{I}$$

In the tree approximation relevant diagrams are shown in Fig. 1. Here the respective particle momentum is designated inside the brackets. We introduce appropriate isospin-invariant $S\phi\phi$ coupling constants and a four-point vertex by

$$\begin{aligned}-\mathcal{L}&=g_{\epsilon\eta''\pi}(\vec{\epsilon}\cdot\vec{\pi})\eta''+\frac{1}{\sqrt{2}}g_{\epsilon K\bar{K}}\bar{K}(\vec{\tau}\cdot\vec{\epsilon})K+\left(g_{\kappa\eta''K}(\bar{K}\kappa)\eta''+\frac{1}{\sqrt{2}}g_{\kappa K\pi}\bar{K}(\vec{\tau}\cdot\vec{\pi})\kappa+\text{H.c.}\right)\\&\quad+\left\langle\frac{\partial^4 V_0}{\partial\eta''\partial\pi^0\partial\phi_1^3\partial\phi_3^1}\right\rangle_0\eta''\pi^0K^+K^- ,\end{aligned} \tag{4.4}$$

then the scattering amplitude is

$$T=\left\langle\frac{\partial^4 V_0}{\partial\eta''\partial\pi^0\partial K^+\partial K^-}\right\rangle_0-\left(\frac{(1/\sqrt{2})g_{\epsilon K\bar{K}}\,g_{\epsilon\eta''\pi}}{\epsilon^2+(p-q_\pi)^2}+\frac{(1/\sqrt{2})\,g_{\kappa K\pi}\,g_{\kappa\eta''K}}{\kappa^2+(p-q_-)^2}+\frac{(1/\sqrt{2})g_{\kappa K\pi}g_{\kappa\eta''K}}{\kappa^2+(p-q_-)^2}\right) . \tag{4.5}$$

We must now express these coupling constants in terms of masses. We already know that[12]

$$\frac{1}{\sqrt{2}}g_{\kappa K\pi}=\left\langle\frac{\partial^3 V_0}{\partial\phi_1^3\partial\pi^0\partial S_3^1}\right\rangle_0=\frac{\kappa^2-\pi^2}{\sqrt{2}\,\alpha(1+W)}=\frac{K^2-\pi^2}{\sqrt{2}\,\alpha(W-1)}=\frac{\kappa^2-K^2}{\sqrt{2}\,2\alpha} \tag{4.6}$$

and

$$\frac{1}{\sqrt{2}}g_{\epsilon K\bar{K}}=\left\langle\frac{\partial^3 V_0}{\partial\phi_3^2\partial\phi_1^3\partial S_2^1}\right\rangle_0=\frac{\epsilon^2-K^2}{\sqrt{2}\,\alpha(1+W)}. \tag{4.7}$$

From (4.1) we can derive

$$g_{\kappa\eta''K}=\left\langle\frac{\partial^3 V_0}{\partial\phi_1^3\partial\eta''\partial S_3^1}\right\rangle_0=\frac{(a''+b'')(\kappa^2-\eta''^2)}{\alpha(1+W)} \tag{4.8}$$

and

$$g_{\epsilon\eta''\pi}=\left\langle\frac{\partial^3 V_0}{\partial\eta''\partial\pi_0\partial\epsilon_0}\right\rangle_0=\frac{a''}{\alpha}(\epsilon^2-\eta''^2). \tag{4.9}$$

The four-point vertex is

$$\begin{aligned}\left\langle\frac{\partial^4 V_0}{\partial\eta''\partial\pi^0\partial\phi_1^3\partial\phi_3^1}\right\rangle_0&=\sum_{g,m}\left\langle\frac{\partial^4 V_0}{\partial\phi_m^m\partial\phi_g^g\partial\phi_1^3\partial\phi_3^1}\right\rangle_0\frac{\partial\phi_g^g}{\partial\pi_0}\frac{\partial\phi_m^m}{\partial\eta''}\\&=\frac{1}{\alpha(1+W)}\left[\left\langle\frac{\partial^3 V_0}{\partial\eta''\partial\pi^0\partial S_1^1}\right\rangle_0+\left\langle\frac{\partial^3 V_0}{\partial\eta''\partial\pi^0\partial S_3^3}\right\rangle_0+\frac{1}{\sqrt{2}}\left\langle\frac{\partial^3 V_0}{\partial\eta''\partial\phi_1^3\partial S_3^1}\right\rangle_0+(a''+b'')\left\langle\frac{\partial^3 V_0}{\partial\pi^0\partial\phi_1^3\partial S_3^1}\right\rangle\right]\\&=\frac{1}{\alpha(1+W)}\left(\frac{g_{\epsilon\eta''\pi}}{\sqrt{2}}+\frac{g_{\kappa\eta''K}}{\sqrt{2}}+(a''+b'')g_{\kappa K\pi}\right)\\&=\frac{1}{\alpha(1+W)}\left(\frac{a''(\epsilon^2-\eta''^2)}{\sqrt{2}\,\alpha}+\frac{(a''+b'')}{\sqrt{2}\,\alpha(1+W)}(2\kappa^2-\pi^2-\eta''^2)\right).\end{aligned} \tag{4.10}$$

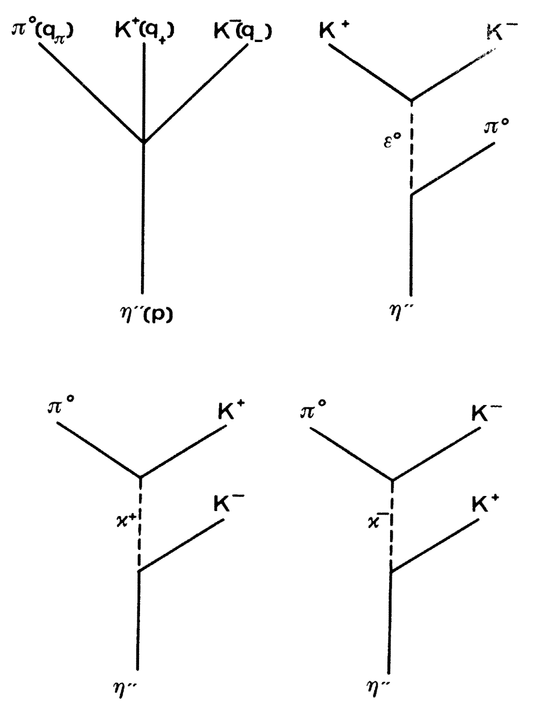

FIG. 1. Diagrams for $\eta''\to K^+ + K^- + \pi^0$ decay.

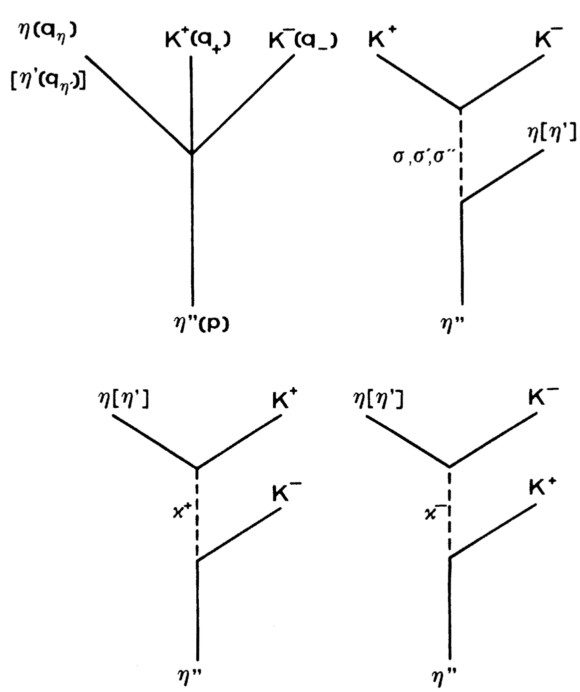

FIG. 2. Diagrams for $\eta''\to K^+ + K^- + \eta$ (or η') decay.

Substituting everything, we get

$$T=\frac{(a''+b'')}{\sqrt{2}F_K{}^2}\left[2\kappa^2-\pi^2-\eta''^2-(\kappa^2-\pi^2)(\kappa^2-\eta''^2)\left(\frac{1}{\kappa^2-\eta''^2-K^2-2(p\cdot q_-)}+\frac{1}{\kappa^2-\eta''^2-K^2-2(p\cdot q_+)}\right)\right]$$
$$+\frac{2a''(\epsilon^2-\eta''^2)}{\sqrt{2}F_K F_\pi}\left(1-\frac{\epsilon^2-K^2}{\epsilon^2-\eta''^2-\pi^2-2(p\cdot q_\pi)}\right), \tag{4.11}$$

where $2\alpha=F_\pi$ and $\alpha(1+W)=F_K$ were used.[24] These PCAC (partial conservation of axial-vector current) constants together with others will be discussed later.

Equation (4.11) is our final result. Unfortunately, experimental information on scalar mesons is rather scarce at present. In order to avoid ambiguities coming from scalar mesons, we first let all the scalar mesons be infinitely heavy. For all the known cases this procedure for the model yields the current-algebraic results.[12] Here we do not intend to claim the correctness of procedure, but merely regard it reasonable as a first try.

We next discuss another two strong-decay modes:

$$\eta''\to\begin{pmatrix}\eta\\ \eta'\end{pmatrix}+K^++K^-\quad\begin{pmatrix}\mathrm{II}_a\\ \mathrm{II}_b\end{pmatrix}$$

and

$$\eta''\to\begin{pmatrix}\eta\\ \eta'\end{pmatrix}+\pi^++\pi^-\quad\begin{pmatrix}\mathrm{III}_a\\ \mathrm{III}_b\end{pmatrix}.$$

Diagrams corresponding to processes (IIa) and (IIb) are shown in Fig. 2, while those of (IIIa) and (IIIb) are shown in Fig. 3.

Here we list the results for three-point and four-point vertices. Relevant $S\phi\phi$ coupling constants defined by

$$-\mathcal{L}=\sum_{j=\sigma,\sigma',\sigma''}\left[\tfrac{1}{2}g_{j\pi\pi}\,j(\vec{\pi}\cdot\vec{\pi})+g_{j\eta\eta''}\,j\eta\eta''+g_{jK\bar{K}}\,j(\bar{K}K)+g_{j\eta\eta'}\,j\eta\eta'\right]$$
$$+\sum_{l=\eta,\eta',\eta''}\left[(g_{\kappa lK}\bar{K}\kappa l+\text{H.c.})+g_{\epsilon l\pi}(\vec{\epsilon}\cdot\vec{\pi})l\right] \tag{4.12}$$

are related to masses as

$$g_{j\pi\pi}=\left\langle\frac{\partial^3V_0}{\partial j\,\partial\phi_1^2\,\partial\phi_2^1}\right\rangle_0=\frac{a_j}{\alpha}(j^2-\pi^2)\quad(j=\sigma,\sigma',\sigma''), \tag{4.13}$$

$$g_{jK\bar{K}}=\left\langle\frac{\partial^3V_0}{\partial j\,\partial\phi_1^3\,\partial\phi_3^1}\right\rangle_0=\frac{(a_j+b_j)}{\alpha(1+W)}(j^2-K^2)\quad(j=\sigma,\sigma',\sigma''), \tag{4.14}$$

$$\begin{bmatrix}g_{\epsilon\eta\pi}\\ g_{\epsilon\eta'\pi}\\ g_{\epsilon\eta''\pi}\end{bmatrix}=\begin{bmatrix}\frac{a}{\alpha}(\epsilon^2-\eta^2)\\ \frac{a'}{\alpha}(\epsilon^2-\eta'^2)\\ \frac{a''}{\alpha}(\epsilon^2-\eta''^2)\end{bmatrix}, \tag{4.15}$$

$$\begin{bmatrix}g_{\kappa\eta K}\\ g_{\kappa\eta' K}\\ g_{\kappa\eta'' K}\end{bmatrix}=\begin{bmatrix}\frac{(a+b)(\kappa^2-\eta^2)}{\alpha(1+W)}\\ \frac{(a'+b')(\kappa^2-\eta'^2)}{\alpha(1+W)}\\ \frac{(a''+b'')(\kappa^2-\eta''^2)}{\alpha(1+W)}\end{bmatrix}. \tag{4.16}$$

Coupling constants $g_{j\eta\eta'}$ ($j=\sigma$, σ', and σ'') are not related to masses. Four-point vertices are related to three-point vertices by

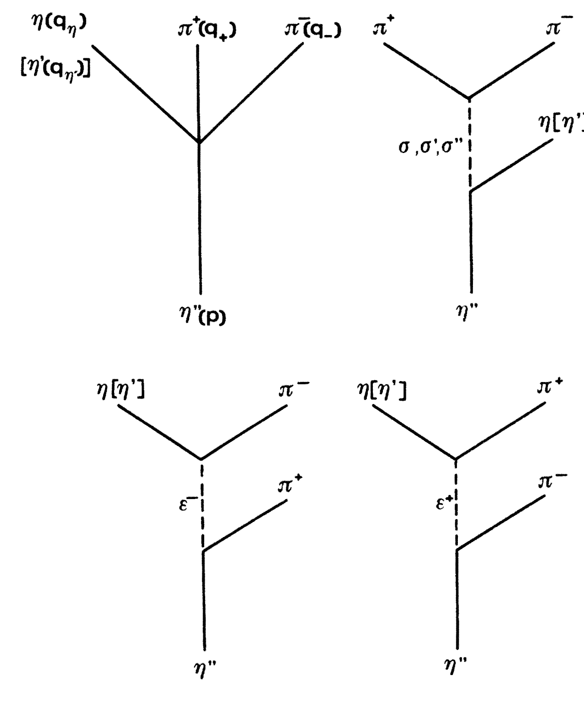

FIG. 3. Diagrams for $\eta''\to\pi^++\pi^-+\eta$ (or η') decay.

$$\left\langle\frac{\partial^4 V}{\partial\eta''\partial\eta\partial\phi_1^3\partial\phi_3^1}\right\rangle_0=\frac{1}{\alpha(1+W)}\left((a+b)g_{\kappa\eta''K}+(a''+b'')g_{\kappa\eta K}+\sum_{j=\sigma,\sigma',\sigma''}g_{j\eta''\eta}(a_j+b_j)\right), \tag{4.17}$$

$$\left\langle\frac{\partial^4 V}{\partial\eta''\partial\eta\partial\phi_1^2\partial\phi_2^1}\right\rangle_0=\frac{1}{\alpha}\left(ag_{\epsilon\eta''\pi}+a''g_{\epsilon\eta\pi}+\sum_{j=\sigma,\sigma',\sigma''}a_j\,g_{j\eta''\eta}\right). \tag{4.18}$$

$\langle\partial^4 V/\partial\eta''\partial\eta'\partial\phi_1^3\partial\phi_3^1\rangle$ can be obtained from (4.17) by changing η into η' everywhere including mixing angles $(a+b)$. Similarly, $\langle\partial^4 V/\partial\eta''\partial\eta'\partial\phi_1^2\partial\phi_2^1\rangle$ can be obtained from (4.18) by substituting η for η' everywhere. Finally, decay amplitudes are

$$T(\eta''(p)\to\eta(q_\eta)+K^+(q_+)+K^-(q_-))=\frac{(a+b)(a''+b'')}{F_K{}^2}\left[2\kappa^2-\eta^2-\eta''^2-(\kappa^2-\eta^2)(\kappa^2-\eta''^2)\left(\frac{1}{\kappa^2-\eta''^2-K^2-2(p\cdot q_-)}+\frac{1}{\kappa^2-\eta''^2-K^2-2(p\cdot q_+)}\right)\right]$$
$$+\sum_{j=\sigma,\sigma',\sigma''}\frac{(a_j+b_j)g_{j\eta''\eta}}{F_K}\left(1-\frac{j^2-K^2}{j^2-\eta''^2-\eta^2-2(p\cdot q_\eta)}\right) \tag{4.19}$$

and

$$T(\eta''(p)\to\eta(q_\eta)+\pi^+(q_+)+\pi^-(q_-))=\frac{4aa''}{F_\pi{}^2}\left[2\epsilon^2-\eta^2-\eta''^2\right.$$
$$\left.-(\epsilon^2-\eta^2)(\epsilon^2-\eta''^2)\left(\frac{1}{\epsilon^2-\eta''^2-\pi^2-2(p\cdot q_-)}+\frac{1}{\epsilon^2-\eta''^2-\pi^2-2(p\cdot q_+)}\right)\right]$$
$$+\frac{2}{F_\pi}\sum_{j=\sigma,\sigma',\sigma''}a_j g_{j\eta''\eta}\left(1-\frac{j^2-\pi^2}{j^2-\eta''^2-\eta^2-2(p\cdot q_\eta)}\right). \tag{4.20}$$

The decay amplitude for (IIb) [or (IIIb)] can be obtained from (4.19) [or (4.20)] by merely substituting η for η' including mixing angles. Again we assume infinitely heavy scalar mesons. Coupling constants such as $g_{\sigma\eta''\eta}$ are not related to masses and are expected to be independent of mass. Then in the rest frame of η'' we have

$$T(\eta''\to\eta+K^++K^-)=[(a+b)(a''+b'')/F_K{}^2](\eta''^2+\eta^2-2K^2-2\eta''q_{\eta 0}), \tag{4.21}$$

$$T(\eta''\to\eta'+K^++K^-)=[(a'+b')(a''+b'')/F_K{}^2](\eta''^2+\eta'^2-2K^2-2\eta''q_{\eta'_0}), \tag{4.22}$$

$$T(\eta''\to\eta+\pi^++\pi^-)=(4aa''/F_\pi{}^2)(\eta''^2+\eta^2-2\pi^2-2\eta''q_{\eta 0}), \tag{4.23}$$

$$T(\eta''\to\eta'+\pi^++\pi^-)=(4a'a''/F_\pi{}^2)(\eta''^2+\eta'^2-2\pi^2-2\eta''q_{\eta' 0}). \tag{4.24}$$

Since the energy spectrum of the third particle [i.e., η for the process (IIa)] is directly proportional to $|T|^2$, our model will be easily testable by comparing it with the experimental energy spectrum determined from the Dalitz plot. At present we shall be satisfied with estimating the partial decay width of η''. Numerical results will be given in the next section after the discussion on masses.

V. NUMERICAL ANALYSIS

For the purpose of discussing masses, there are six parameters in the model:

$$\frac{M_{12}}{\alpha},\ \frac{A_1}{\alpha}\left(=\frac{A_2}{\alpha}\right),\ \frac{A_3}{\alpha},\ \frac{A_4}{\alpha},\ W,\ \text{and}\ W'.$$

With the choice of inputs of masses[25]

$\pi^2,\ K^2,\ \eta^2,$ and $\eta'^2,$

there remain two parameters free, which we have chosen to be W and W'. Of course W and W' will not be completely free, once PCAC constants are experimentally determined,

$$W=2\left(\frac{F_K}{F_\pi}\right)-1, \tag{5.1}$$

$$W'=2\left(\frac{F_D}{F_\pi}\right)-1. \tag{5.2}$$

Although experimentally F_K/F_π is known to be around 1.28, we shall not use this information here, since there is an ambiguity associated with it. At present F_D/F_π is not known. We shall have a further discussion on PCAC constants later.

Concerning the pseudoscalar-meson spectrum, contrary to the vector meson, the experimental situation is a little more ambiguous. On the possible assignment of η', we have chosen $\eta'=X\,(958)$

rather than $\eta'' = E(1420)$, since the latter assignment could not cope with the broad experimental width within the present scheme. (See our second paper in Ref. 8.) Experimentally η'' is yet to be observed, although there is a candidate at mass 2.75–2.8 GeV.[26] A D_0-like particle is observed at mass 1.865 GeV,[27] but its spin-parity property has yet to be confirmed.

In order to augment the scarce information on the pseudoscalar-meson spectrum we might rely on the quark model. According to the quark model, the quark-mass ratio,

$$R = \frac{m_c - m_u}{m_s - m_u}, \tag{5.3}$$

was determined from the analysis of the vector-meson mass spectrum. Numerically, we had[28]

$$R = 20.59 \,. \tag{5.4}$$

If the identical interaction is responsible for the pseudoscalar-meson system, our symmetry-breaking interaction (2.15) should satisfy the following relation:

$$\frac{A_4 - A_1}{A_3 - A_1} = R \,. \tag{5.5}$$

Since the present experimental situation is far from well established, we make three alternative assumptions in order to fix parameters W and W': (i) η'' mass = 2.8 GeV and validity of (5.5), (ii) D_0 mass = 1.865 GeV and validity of (5.5), (iii) D_0 mass = 1.865 GeV and η'' mass = 2.8 GeV. When the universality relation (5.5) is assumed, one more expression for A_4/α, besides (3.23), becomes available:

$$\frac{A_4}{\alpha} = R\left(\frac{A_3}{\alpha} - \frac{A_1}{\alpha}\right) + \frac{A_1}{\alpha}, \tag{5.6}$$

where $A_3/\alpha = \frac{1}{2}[(1+W)K^2 - \pi^2]$ and $A_1/\alpha = \frac{1}{2}\pi^2$ must be substituted. Equating (3.23) to (5.6) we can relate W to W'. It just happens that A_4/α in (3.23) is not sensitive to the choice of W' within the range of W of interest to us, and the identity should hold when W is close to $W = 1.727$ for any choice of W'.

In order to clarify this situation, listing of the approximate expressions might be helpful. According to the charm scheme, η'' as well as charmed mesons are expected to be much heavier than the rest of the pseudoscalar-meson 16-plet. After a little inspection of (3.17), (3.12d), and (3.23), it will be observed that large η'' and D masses are possible only for an extremely small Δ, (3.18). In other words, the reason why η'' or charmed mesons get heavy is due to an approximate validity of the SU(3) × SU(3) mass formula.

Some of the approximate expressions valid for small Δ are

$$\frac{\eta''^2}{\pi^2} = \frac{\Omega}{\sqrt{\Delta}\, W W'} + \frac{1}{2}\left[1 + \frac{2A_3}{\alpha W \pi^2} + \left(2 + \frac{1}{W^2} + \frac{2}{W'^2}\right)\Omega\right] + O(\Delta) \sim \frac{\Omega}{\sqrt{\Delta}\, W W'}, \tag{5.7a}$$

$$\frac{M_{12}}{\pi^2} \approx \left(\frac{W}{W'}\sqrt{\Delta} + 1\right)\Omega \sim \Omega \,, \tag{5.7b}$$

$$\frac{2A_4}{\alpha\pi^2} = \frac{W'\eta''^2}{\pi^2} - \frac{\Omega}{W'} + O(\Delta) \sim \frac{\Omega}{\sqrt{\Delta}\, W}. \tag{5.7c}$$

The leading term in A_4/α is seen not to depend upon W'. On the other hand, η'' is quite sensitive to the choice of W'. From the positivity of scalar-meson masses we should have a condition

$$W' \geq W \geq 1 \,. \tag{5.8}$$

(a) In order to get an idea about W' let us first assume $W' = W$, which corresponds to the assumption (i) as we shall see. Numerical results are

TABLE I. Numerical results for pseudoscalar- and scalar-meson masses with three different choices of W and W', corresponding to (5.9), (5.11), and (5.12) in the text.

$W = W' = 1.7273$	$W = 1.72696,\ W' = 2.79348$	$W = 1.7204,\ W' = 1$
$\eta'' = 2.793$ GeV	$\eta'' = 2.185$ GeV	$\eta'' = 2.714$ GeV
$D = 2.205$ GeV	$D = 1.865$ GeV	$D = 1.866$ GeV
$F = 2.006$ GeV	$F = 1.75$ GeV	$F = 1.672$ GeV
$D_S = 4.259$ GeV	$D_S = 2.709$ GeV	$D_S = \infty$
$F_S = \infty$	$F_S = 3.427$ GeV	$F_S =$ (arbitrary, see text)
$\frac{A_3}{\alpha\pi^2} = 18.045$	$\frac{A_3}{\alpha\pi^2} = 18.043$	$\frac{A_3}{\alpha\pi^2} = 17.998$
$\Delta = 2.77953 \times 10^{-4}$	$\Delta = 3.1469 \times 10^{-4}$	$\Delta = 1.03125 \times 10^{-3}$
$\Omega = 19.25$	$\Omega = 19.246$	$\Omega = 19.1751$
$M_{12} = 19.605$	$M_{12} = 19.492$	$M_{12} = 20.41$
$\frac{A_4}{\alpha\pi^2} = 362.7$	$\frac{A_4}{\alpha\pi^2} = 361.88$	$\frac{A_4}{\alpha\pi^2} = 190.77$

listed in Table I. With a choice

$$W' = W = 1.7273 \tag{5.9}$$

we get

$$\eta'' = 2.793 \text{ GeV}$$

and

$$D = 2.21 \text{ GeV}, \quad F = 2.01 \text{ GeV}.$$

These values agree with other theoretical predictions.[7,8] The above η'' mass is also close to the experimental candidate for η'' at 2.8 GeV.[26] If the experimentally observed resonance with mass[27] 1.865 GeV should be identified with a D^0 meson, then this choice of parameters should be discarded. At present the spin-parity property of this particle is unknown.

(b) Alternatively, corresponding to assumption (ii) we can identify the meson with mass 1.865 GeV to be a D^0 meson. Then in addition to the two expressions for A_4/α, (3.23) and (5.6), we now have one more expression for A_4/α,

$$A_4/\alpha = \tfrac{1}{2}[(1+W')D^2 - \pi^2]. \tag{5.10}$$

Three expressions for A_4/α determine W and W' as

$$W = 1.7270 \text{ and } W' = 2.7935. \tag{5.11}$$

This choice yields a rather low value for η'',

$$\eta'' = 2.19 \text{ GeV}.$$

Here the F-meson mass is predicted to be

$$F = 1.755 \text{ GeV}.$$

(c) Corresponding to the assumption (iii), we next consider the problem whether the D^0 mass with 1.865 GeV is compatible with the η'' mass with 2.8 GeV. This turned out to be impossible as long as $W' \geq W$.[29,30] However, if we are allowed to consider $W' \leq W$, such a possibility can be realized. A choice, for instance,

$$W' = 1 \text{ and } W = 1.7204 \tag{5.12}$$

yields

$$\eta'' = 2.714 \text{ GeV}, \quad D = 1.866 \text{ GeV}, \text{ and } F = 1.672 \text{ GeV}.$$

Unfortunately, this choice of parameters makes one of the scalar-meson masses in the system, $\tilde{S}_3^4$, imaginary. In order to avoid such an awkard situation, we must modify our symmetry-breaking interaction (2.15).[31] The addition of the symmetry breaking of a quadratic form

$$\delta V^{\text{SB}} = \sum_{a,b} C_{ab} S_a^b S_b^a \quad (C_{ab} \text{ a constant}) \tag{5.13}$$

might solve the problem.

If we want to keep the pseudomeson sector undisturbed, however, the simplest modification might be to add a term to (2.15),

$$\delta V^{\text{SB}} = C S_3^4 S_4^3 \quad (C \text{ a constant}). \tag{5.14}$$

[According to (3.3) and (3.4), only the $\tilde{S}_3^4$ mass will be increased by an amount of C.] With this understanding we accept a choice (5.12). Partial decay rates for three different choices of parameters, (5.9), (5.11), and (5.12), are tabulated in Table II.

According to our results, η'' comes out predominantly in a pure charm-anticharm pair state. This situation holds true for all three choices of W', since mixing angles happen to be insensitive to W' within our range of interest. This is the reason why the η'' particle decaying into ordinary hadronic channels has narrow (partial) widths in general. As far as the actual estimate is concerned, though, two different choices of parameters (a) and (b),

TABLE II. Numerical results for mixing angles of the η, η', and η'' and partial decay widths of the η''. Here $\Gamma(a,b,c)$ represents the width for the decay $\eta'' \to a+b+c$.

	$W = W' = 1.7273$	$W = 1.72696$, $W' = 2.79348$	$W = 1.7204$, $W' = 1$
a	0.39971	0.39975	0.4004
b	−0.82485	−0.82482	−0.82406
c	-8.8887×10^{-3}	-9.1656×10^{-3}	-1.7042×10^{-2}
a'	−0.58253	−0.58237	−0.58001
b'	−0.56506	−0.56504	−0.56543
c'	4.49×10^{-2}	4.9282×10^{-2}	8.6418×10^{-2}
a''	2.974×10^{-2}	3.2405×10^{-2}	5.7169×10^{-2}
b''	1.8058×10^{-2}	2.0312×10^{-2}	3.4955×10^{-2}
c''	0.998952	0.99874	0.99611
$\Gamma(\eta' K^+K^-)$	0.232 MeV	0.0061 MeV	0.626 MeV
$\Gamma(\eta K^+K^-)$	0.143 MeV	0.019 MeV	0.426 MeV
$\Gamma(\eta' \pi^+\pi^-)$	2.9 MeV	0.346 MeV	8.33 MeV
$\Gamma(\eta \pi^+\pi^-)$	4.61 MeV	1.03 MeV	14.2 MeV
$\Gamma(\pi^0 K^+K^-)$	0.692 MeV	0.198 MeV	2.17 MeV

TABLE III. Numerical results for PCAC constants.

	$W = W' = 1.7273$	$W = 1.72696$ $W' = 2.79348$	$W = 1.7204$ $W' = 1.0$	Available experimental data
F_K/F_π	1.36	1.36	1.36	1.28
F_D/F_π	1.36	1.9	1.0	
F_F/F_π	1.72	2.26	1.36	

yield moderately different values for the decay width. The main reason for this is that the decay width is sensitive to the η'' mass.

Among various decay modes, the largest contribution comes from the decay $\eta'' \to \eta + \pi^+ + \pi^-$ independent of the choice of parameters. This is simply due to the largest available phase space. The decay mode $\eta'' \to \pi^0 + K^+ + K^-$ has also a large phase space available but, owing to the existence of the destructive interference, the matrix element turns out small and the net result is smaller than the naive expectation. Our calculation of decay rates became current-algebra-like after the scalar-meson masses were taken to be infinitely heavy. Comparison with future experiments will tell us if further refinement of the calculation or the model itself is needed.

So far we have not discussed PCAC constants. In order to distinguish different sets of parameters we can also rely on PCAC constants.

The leptonic decays of pseudoscalar mesons such as

$$\begin{pmatrix} \pi^+ \\ K^+ \end{pmatrix} \to l^+ + \nu \qquad (l = \mu \text{ or } e)$$

$$\begin{pmatrix} D^+ \\ F^+ \end{pmatrix} \to l^+ + \nu$$

provide us with information on PCAC constants defined by

$$\begin{aligned} \sqrt{2q_0}\langle 0 | A_{\mu}{}^2_1 | \pi^+(q)\rangle &= F_\pi i q_\mu , \\ \sqrt{2q_0}\langle 0 | A_{\mu}{}^3_1 | K^+(q)\rangle &= F_K i q_\mu , \\ \sqrt{2q_0}\langle 0 | A_{\mu}{}^2_4 | D^+(q)\rangle &= F_D i q_\mu , \end{aligned} \tag{5.15}$$

and

$$\sqrt{2q_0}\langle 0 | A_{\mu}{}^3_4 | F^+(q)\rangle = F_F i q_\mu .$$

They are related to our parameters W and W' as[12,17]

$$\begin{aligned} &F_\pi = 2\alpha, \quad F_K = \alpha(1+W), \\ &F_D = \alpha(1+W'), \quad F_F = \alpha(W+W'). \end{aligned} \tag{5.16}$$

Or, equivalently, writing these in the form

$$\frac{F_K}{F_\pi} = \frac{1+W}{2}, \tag{5.17a}$$

$$\frac{F_D}{F_\pi} = \frac{1+W'}{2}, \tag{5.17b}$$

$$\frac{F_F}{F_\pi} = \frac{W+W'}{2}, \tag{5.17c}$$

we shall see that the experimental deviation from 1 indicates immediately the existence of a non-SU(3)- or non-SU(4)-symmetric vacuum. [They are all equal to 1 in the SU(4) limit.]

Independent of any choice of W and W', we can derive the sum rule mentioned in the Introduction,

$$\frac{F_F}{F_\pi} - \frac{F_D}{F_\pi} = \frac{F_K}{F_\pi} - 1 . \tag{5.18}$$

Experimentally the right-hand side is 0.28. The left-hand side is unknown at present. Since this sum rule is an immediate consequence of our potential with nonderivative type (2.1), it is expected to hold rather generally. Its experimental confirmation is highly desirable. Numerical values of PCAC constants corresponding to our previous choices of parameters are given in Table III.

According to our prediction, leptonic decay modes of charmed mesons will not differ much from those of ordinary noncharmed mesons. This should be contrasted with the prediction of large enhancement of charmed-meson modes in the renormalizable model.[32] Since PCAC constants are rather sensitive to our choice of W and W', future experimental measurements of the leptonic decay modes such as $D^+ \to \mu^+ + \nu$ will soon settle the issue.

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[22]Equation (2.10) comes from the requirement that $1+E_V$ be unitary to first order in E_V.

[23]Although $W=1.73$ gives a best fit for the SU(3)×SU(3) mass formula, a slightly different choice such as $W=1.56$, corresponding to the experimental $F_K/F_\pi=1.28$, can still fit the mass formula within 2–3% accuracy.

[24]Our notation of PCAC constants is the same as in Ref. 12,

$$F_\pi \approx 135 \text{ MeV} \approx m_{\pi^0}.$$

The width Γ is related to T by

$$\Gamma=\frac{1}{64(3.1416)^3\eta''}\int dq_{+0}\,dq_{-0}\,|T|^2.$$

[25]We have chosen masses of neutral particles as our inputs: $\pi_0=134.96$ MeV, $K_0=477.7$ MeV, $\eta'=957.6$ MeV, $\eta=548.8$ MeV.

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[28]See V. Mathur, S. Okubo, and S. Borchardt, Phys. Rev., Ref. 7, and our first paper in Ref. 8. The number here is taken from Eqs. (2.18) and (2.20) in our second paper in Ref. 8.

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