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Leptonic decays of J/ψ and exact spectral-function sum rules

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Five exact spectral-function sum rules for vector currents are presented. A discrepancy between the Weinberg sum rules and the leptonic decays of the J/ψ vector meson is resolved. This provides a concrete example illustrating the usefulness of the asymptotically-free-field-theory approach to formulate spectral-function sum rules.

I. INTRODUCTION

Weinberg's spectral-function sum rules[1] have been successfully applied in analyzing the leptonic decays of ρ, ω, and ϕ mesons[2] assuming the validity of an asymptotic SU(3) symmetry. Thus it is not surprising that a large amount of work has been dedicated to the analysis of the leptonic decays of the new particle $J/\psi(3.1)$ by applying either the first spectral-function sum rule for U(4) symmetry[3] or the first spectral function sum rule for SU(4) symmetry togethere with the modified second sum rule.[4] What is surprising is the outcome of such an analysis: If one assumes the electric charge of the charm quark (c) to be $\frac{2}{3}$, and the J/ψ to be a $c\bar{c}$ bound state, then the leptonic decay width of J/ψ predicted by the sum rules is only ~1.5 keV, which is three to four times smaller than the experimental value 4.7 ± 1.1 keV. To avoid this discrepancy it was then proposed that the electric charge of the c quark be $-\frac{4}{3}$ instead of $\frac{2}{3}$. However, the recent discovery[5] of a neutral charmed meson and the negatively charged charmed antibaryon makes it impossible to retain the $-\frac{4}{3}$ charge assignment. And so one is forced to consider other possible causes of the discrepancy. While the validity of Weinberg's spectral-function sum rules was questioned by theorists[6] soon after the discovery of the sum rules, the leptonic decays of J/ψ provide new experimental evidence to support a reexamination.

Recently Bernard, Duncan, Lo Secco, and Weinberg[7] (BDLW) have studied the spectral-function sum rules within the context of asymptotically free field theories. They formulated a general procedure for extracting exact spectral-function sum rules. The main feature which distinguishes the BDLW procedure from the previous work is that instead of restricting attention *ab initio* to specific combinations of current products, such combinations usually resulting from symmetry-group-theoretical considerations, they employed the operator-product expansion analysis to determine those linear combinations for which the leading short-distance singularities are sufficiently soft and the corresponding sum rules valid. The coefficients of the linear combinations thus determined are functions of ratios of quark masses. This transition in the form of spectral-function sum rules is an instance where the quark-gluon model supersedes the symmetry-group-theoretical approach.

In Sec. II of this paper we shall apply the BDLW procedure to derive a new sum rule relating the leptonic decay widths of ρ, ω, ϕ, and J/ψ vector mesons. Using the well-known ratios of quark masses we find in Sec. III a remarkable agreement between the sum rule and the experimental value of the J/ψ leptonic decay width. In Sec. IV we present four more sum rules for vector currents. We also clarify in this section the relation between the sum rules presented here and those obtained in Ref. 7. We have restricted ourselves to the first spectral-function sum rules for vector currents.

II. DERIVATION OF THE SUM RULE RELATING $\Gamma(V \to l\bar{l})$

The Kallén-Lehman spectral representation for the product of vector or axial-vector currents is

$$\langle 0 \,|J^{\mu}_{A}(x)J^{\nu}_{B}(0)\,|0\rangle = \int d\mu^2 \left\{ g^{\mu\nu}\rho^{(1)}_{AB}(\mu^2) - \left[\rho^{(0)}_{AB}(\mu^2) + \frac{\rho^{(1)}_{AB}(\mu^2)}{\mu^2}\right]\partial^{\mu}\partial^{\nu}\right\}\Delta^{(+)}(x;\mu^2)\,, \tag{2.1}$$

where $\rho^{(j)}_{AB}(\mu^2)$ are the spin-j spectral functions, and $\Delta^{(+)}(x;\mu^2)$ is the positive-frequency Green's function. The short-distance expansion of $\Delta^{(+)}(x;\mu^2)$ is[7]

$$\Delta^{(+)}(x,\mu^2) \underset{x\to 0}{\sim} -\frac{1}{4\pi^2}\frac{1}{x^2 - i\eta\epsilon(x^0)} + \frac{\mu^2}{8\pi^2}\left[\ln\tfrac{1}{2}\gamma(\mu^2\,|x^2|)^{1/2} - \tfrac{1}{2} + i\pi\epsilon(x^0)\theta(-x^2)\right] + O(x^2)\,, \tag{2.2}$$

where $\eta \to 0^+$ and γ is the Euler constant. Knowledge of the leading singularities in the operator-product expansion of $J_A^\mu(x)J_B^\nu(0)$ on the left-hand side of Eq. (2.1) places constraints on various spectral integrals on the right-hand side of Eq. (2.1). For the combinations

$$\sum_{A,B} C_{AB}\langle 0|J_A^\mu(x)J_B^\nu(0)|0\rangle \tag{2.3}$$

with a leading singularity in the operator-product expansion stronger than x^{-2} but weaker than x^{-4}, the first spectral-function sum rule holds:

$$\sum_{A,B} C_{AB}\int\left[\rho_{AB}^{(0)}(\mu^2)+\frac{\rho_{AB}^{(1)}(\mu^2)}{\mu^2}\right]d\mu^2=0\,. \tag{2.4}$$

If the leading singularity of the combination (2.3) at short distance is weaker than x^{-2} we obtain two additional spectral-function sum rules:

$$\sum_{A,B}\int\rho_{AB}^{(1)}(\mu^2)d\mu^2=0\,, \tag{2.5}$$

$$\sum_{A,B}\int\mu^2\rho_{AB}^{(0)}(\mu^2)d\mu^2=0\,. \tag{2.6}$$

Now in the quark-gluon model the short-distance singularity structure of current products is summarized in the following two equations[7]:

$$\begin{aligned}\Sigma_{AB}^{(+)\mu\nu}(x) &\equiv \langle 0\,|\bar\psi(x)\gamma^\mu(1+\gamma_5)A\psi(x)\bar\psi(0)\gamma^\nu(1+\gamma_5)B\psi(0)\,|0\rangle\\ &\xrightarrow[x\to 0]{}\sum_{l=0}^{\infty}f_l^+(x)\mathrm{Tr}(\{A,B\}m^{2l})+\sum_{l=0}^{\infty}g_l^+(x)[\mathrm{Tr}(Am^{2l})\mathrm{Tr}B+\mathrm{Tr}(Bm^{2l})\mathrm{Tr}A]\\ &\qquad +h_1^+(x)\mathrm{Tr}(Am^2)\mathrm{Tr}(Bm^2)+h_2^+(x)\mathrm{Tr}(Am^2Bm^2)\\ &\qquad +h_3^+(x)[\mathrm{Tr}(ZmA)\mathrm{Tr}B+\mathrm{Tr}(ZmB)\mathrm{Tr}A]+h_4^+(x)\mathrm{Tr}(Zm\{A,B\})\,,\end{aligned} \tag{2.7}$$

$$\begin{aligned}\Sigma_{AB}^{(-)\mu\nu}(x) &\equiv \langle 0\,|\bar\psi(x)\gamma^\mu(1+\gamma_5)A\psi(x)\bar\psi(0)\gamma^\nu(1-\gamma_5)B\psi(0)\,|0\rangle\\ &\xrightarrow[x\to 0]{}\sum_{l=0}^{\infty}f_l^-(x)\mathrm{Tr}(AmBm^{2l+1}+BmAm^{2l+1})+\sum_{l=0}^{\infty}g_l^-(x)[\mathrm{Tr}(Am^{2l})\mathrm{Tr}B+\mathrm{Tr}(Bm^{2l})\mathrm{Tr}A]\\ &\qquad +h_1^-(x)\mathrm{Tr}(Am^2)\mathrm{Tr}(Bm^2)+h_2^-(x)[\mathrm{Tr}(ZmA)\mathrm{Tr}B+\mathrm{Tr}(ZmB)\mathrm{Tr}A]\\ &\qquad +h_3^-(x)\mathrm{Tr}(ZAmB+ZBmA)\,,\end{aligned} \tag{2.8}$$

where ψ is a quark multiplet, A and B are square matrices acting on the flavors, m is the quark mass matrix, and the matrix $Z_{nm}\equiv\langle 0|\psi_n(0)\bar\psi_m(0)|0\rangle$. In perturbation theory, modulo logarithms, $f_0^+(x)$, $g_0^+(x)$, and $g_0^-(x)$ behave like x^{-6} for small x, $f_1^+(x)$, $g_1^+(x)$, $f_0^-(x)$, and $g_1^-(x)$ behave like x^{-4}, and all other coefficients behave like x^{-2}.

We shall restrict ourselves to the four-quark model:

$\psi=(u,d,s,c)$ with electric charges $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, and $\frac{2}{3}$, respectively,

$$m=\begin{pmatrix} m_0 & & & 0\\ & m_0 & & \\ & & m_s & \\ 0 & & & m_c\end{pmatrix},\quad Z=\begin{pmatrix} Z_0 & & & 0\\ & Z_0 & & \\ & & Z_s & \\ 0 & & & Z_c\end{pmatrix}.$$

The basic vector currents are

$$\begin{aligned}&\bar u\gamma_\mu u\,,\ \bar d\gamma_\mu d\,,\ \bar s\gamma_\mu s\,,\ \bar c\gamma_\mu c\,,\\ &\bar u\gamma_\mu d\,,\ \bar d\gamma_\mu u\,,\ \bar u\gamma_\mu s\,,\ \bar s\gamma_\mu u\,,\ \bar d\gamma_\mu s\,,\ \bar s\gamma_\mu d\,,\\ &\bar u\gamma_\mu c\,,\ \bar c\gamma_\mu u\,,\ \bar d\gamma_\mu c\,,\ \bar c\gamma_\mu d\,,\ \bar s\gamma_\mu c\,,\ \bar c\gamma_\mu s\,.\end{aligned} \tag{2.9}$$

The above choice of quark mass matrix leads to six conserved currents $\bar u\gamma_\mu u$, $\bar d\gamma_\mu d$, $\bar s\gamma_\mu s$, $\bar c\gamma_\mu c$, $\bar u\gamma_\mu d$, and $\bar d\gamma_\mu u$ which generate exact $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$ symmetry plus quark conservation. The rest are partially conserved currents. We shall not consider axial-vector currents. The currents listed in (2.9) can be classified into flavor-conserving currents and flavor-changing currents. Only the flavor-conserving vector currents are relevant to the leptonic decays of vector mesons. Also, the sum rule we shall present is a first spectral-function sum rule; henceforth, we shall ignore those functions of x in Eqs. (2.7) and (2.8) which behave like x^{-2}.

By a straightforward calculation we can derive from Eqs. (2.7) and (2.8)

$$\Sigma_{ii}^{(v)\mu\nu}(x) \equiv \langle 0|\bar{q}_i(x)\gamma^\mu q_i(x)\bar{q}_i(0)\gamma^\nu q_i(0)|0\rangle$$
$$\xrightarrow[x\to 0]{} [f_0^+(x)+g_0^+(x)+g_0^-(x)]+m_i^2[f_1^+(x)+g_1^+(x)+g_1^-(x)+f_0^-(x)]+O(x^{-2})\,, \tag{2.10}$$

where $q_i = u, d, s, c$ and $m_i = m_0, m_0, m_s, m_c$ for $i = 1, 2, 3, 4$. Thus the first spectral-function sum rule holds for the combination

$$\sum_{i=1}^{4} C_{ii}\Sigma_{ii}^{(v)\mu\nu}(x) \tag{2.11}$$

provided the C_{ii} satisfy the following constraints:

$$\sum_{i=1}^{4} C_{ii}=0\,, \tag{2.12}$$

$$\sum_{i=1}^{4} C_{ii}m_i^2=0\,. \tag{2.13}$$

It can be verified at once that the following combination satisfies Eqs. (2.12) and (2.13):

$$(m_c^2-m_s^2)[\Sigma_{11}^{(v)\mu\nu}(x)+\Sigma_{22}^{(v)\mu\nu}(x)]-2(m_c^2-m_0^2)\Sigma_{33}^{(v)\mu\nu}(x)+2(m_s^2-m_0^2)\Sigma_{44}^{(v)\mu\nu}(x)\,. \tag{2.14}$$

Since

$$\Sigma_{11}^{(v)\mu\nu}(x)+\Sigma_{22}^{(v)\mu\nu}(x)=\langle 0|\bar{u}(x)\gamma^\mu u(x)\bar{u}(0)\gamma^\nu u(0)|0\rangle+\langle 0|\bar{d}(x)\gamma^\mu d(x)\bar{d}(0)\gamma^\nu d(0)|0\rangle$$
$$=\left\langle 0\left|\frac{1}{\sqrt{2}}[\bar{u}(x)\gamma^\mu u(x)+\bar{d}(x)\gamma^\mu d(x)]\frac{1}{\sqrt{2}}[\bar{u}(0)\gamma^\nu u(0)+\bar{d}(0)\gamma^\nu d(0)]\right|0\right\rangle$$
$$+\left\langle 0\left|\frac{1}{\sqrt{2}}[\bar{u}(x)\gamma^\mu u(x)-\bar{d}(x)\gamma^\mu d(x)]\frac{1}{\sqrt{2}}[\bar{u}(0)\gamma^\nu u(0)-\bar{d}(0)\gamma^\nu d(0)]\right|0\right\rangle, \tag{2.15}$$

we see that the currents involved in (2.14) have the same quark structures as ω, ρ, ϕ, and J/ψ vector mesons. There is no spin-0 spectral function for a conserved current. Thus Eq. (2.14) implies the following spectral-function sum rule:

$$(m_c^2-m_s^2)\int\frac{\rho_\omega^{(1)}(\mu^2)}{\mu^2}d\mu^2+(m_c^2-m_s^2)\int\frac{\rho_\rho^{(1)}(\mu^2)}{\mu^2}d\mu^2$$
$$-2(m_c^2-m_0^2)\int\frac{\rho_\phi^{(1)}(\mu^2)}{\mu^2}d\mu^2+2(m_s^2-m_0^2)\int\frac{\rho_{J/\psi}^{(1)}(\mu^2)}{\mu^2}d\mu^2=0\,, \tag{2.16}$$

where $\rho^{(1)}_{\omega,\rho,\phi,J/\psi}$ are spin-1 spectral functions for the currents

$$\frac{1}{\sqrt{2}}[\bar{u}(x)\gamma^\mu u(x)+\bar{d}(x)\gamma^\mu d(x)]\,,\quad \frac{1}{\sqrt{2}}[\bar{u}(x)\gamma^\mu u(x)-\bar{d}(x)\gamma^\mu d(x)]\,,$$

$\bar{s}\gamma^\mu s$, and $\bar{c}(x)\gamma^\mu c(x)$,

respectively. Saturating[1,2,8] the spectral functions with ω, ρ, ϕ, and J/ψ vector mesons we transform Eq. (2.16) to an experimentally verifiable sum rule,

$$(m_c^2-m_s^2)\frac{m_\omega\Gamma(\omega\to l\bar{l})}{1}+(m_c^2-m_s^2)\frac{m_\rho\Gamma(\rho\to l\bar{l})}{9}$$
$$-2(m_c^2-m_0^2)\frac{m_\phi\Gamma(\phi\to l\bar{l})}{2}+2(m_s^2-m_0^2)\frac{m_{J/\psi}\Gamma(J/\psi\to l\bar{l})}{8}=0\,, \tag{2.17}$$

where $\Gamma(V\to l\bar{l})$ is the leptonic decay width of the vector meson V. Electron-muon universality requires $\Gamma(V\to e\bar{e})=\Gamma(V\to\mu\bar{\mu})$. Note that sum rules (2.16) and (2.17) remain true even if $m_u\neq m_d$. In that case m_0^2 would be the average of m_u^2 and m_d^2. Sum rules (2.16) and (2.17) can be easily generalized to include the lowest-lying neutral vector mesons made up of new quarks, if there are any.

III. LEPTONIC DECAYS OF J/ψ

We shall now apply Eq. (2.17) to analyze the leptonic decay of the J/ψ vector meson. First using quark mass ratios appropriate to current-algebra calculations, we have[9]

$$\frac{m_0+m_s}{2m_0}=\frac{m_K{}^2}{m_\pi{}^2}$$

or

$$\frac{m_s}{m_0}\approx 25\,. \tag{3.1}$$

Therefore, we can ignore $m_0{}^2$ in Eq. (2.17). After substituting in the experimental values of $m_{\omega,\rho,\phi,J/\psi}$ and $\Gamma(\rho,\omega,\phi\to e\bar{e})$ we get from Eq. (2.17) the following relation between $\Gamma(J/\psi\to l\bar{l})$ (in keV) and m_c/m_s:

$$\Gamma(J/\psi\to l\bar{l})=0.23(m_c/m_s)^2+1.49\,. \tag{3.2}$$

This is illustrated in Fig. 1. The experimental value 4.7 keV of $\Gamma(J/\psi\to l\bar{l})$ corresponds to the ratio

$$m_c/m_s=3.75\,. \tag{3.3}$$

This is in good agreement with the value $m_c/m_s=\frac{78}{20}=3.9$ obtained by Georgi and Politzer[10] using the renormalization-group technique. Note that a simple calculation based on partial conservation of axial-vector current (PCAC) and the mass of the charmed meson (~2 GeV) gives $m_c/m_s\approx m_D{}^2/m_K{}^2\approx 16$. Thus we expect PCAC to be badly violated in the charmed sector owing to the heaviness of charmed particles.

Alternatively, we can insert quark masses appropriate to the constituent model of hadrons. Here m_0 is not small enough to be ignored in contrast to the current-algebra limit. Several sets of constituent-quark masses are used in the literature. They are not identical nor very different. We try two alternatives[11] to show that our result does not depend sensitively on a particular choice.

(i) $m_0=0.34$ GeV, $m_s=0.48$ GeV. The relation between $\Gamma(J/\psi\to l\bar{l})$ (in keV) and m_c (in GeV) is

$$\Gamma(J/\psi\to l\bar{l})=2.01m_c{}^2+1.26 \tag{3.4}$$

(ii) $m_0=0.336$ GeV, $m_s=0.540$ GeV. The relation between $\Gamma(J/\psi\to l\bar{l})$ (in keV) and m_c (in GeV) is

$$\Gamma(J/\psi\to l\bar{l})=1.29m_c{}^2+1.35\,. \tag{3.5}$$

Equations (3.4) and (3.5) are illustrated in Fig. 2. The experimental value of $\Gamma(J/\psi\to l\bar{l})$ corresponds to (i) $m_c=1.3$ GeV, (ii) $m_c=1.6$ GeV, in remarkable agreement with the generally accepted value $m_c\approx 1.5$ GeV.

Thus we have shown that the sum rule (2.17) can successfully explain the leptonic decay width of the J/ψ vector meson. The value[3,4] of $\Gamma(J/\psi\to ll)$ predicted by the Weinberg sum rules can be reproduced by putting $m_c=m_0$ or m_s in (2.17), and so the factor of three to four discrepancy mentioned in the Introduction is revealed here as a consequence of the large mass difference between the charmed quark and noncharmed quarks.

IV. FOUR ADDITIONAL SUM RULES IN THE V SECTOR

We have shown that the new sum rule (2.17) can explain nicely the leptonic decays of the J/ψ vector meson. The derivation of the spectral-function sum rule (2.16), from which we get Eq. (2.17), provides a concrete example illustrating the usefulness of the BDLW procedure. Moreover, the sum rule (2.16) was not obtained in Ref. 7. In

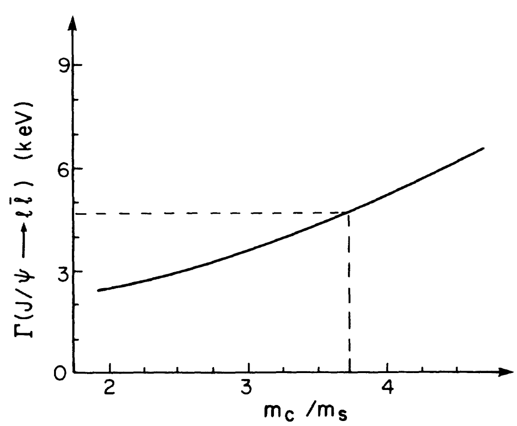

FIG. 1. The relation between the leptonic decay width $\Gamma(J/\psi\to l\bar{l})$ and the quark mass ratio m_c/m_s. $\Gamma(J/\psi\to l\bar{l})=4.7$ keV implies $m_c/m_s=3.75$.

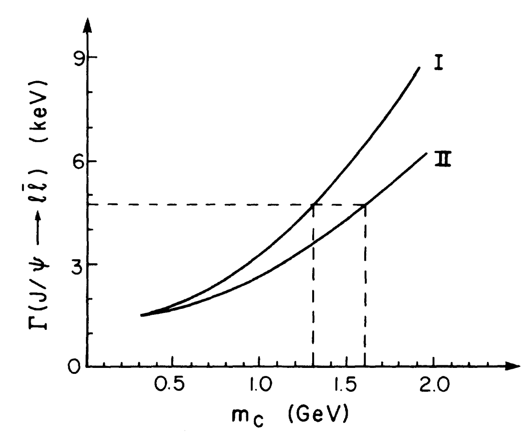

FIG. 2. The relation between the leptonic decay width $\Gamma(J/\psi\to l\bar{l})$ and the quark mass m_c. Curve I corresponds to $m_0=0.34$ GeV, $m_s=0.48$ GeV; $\Gamma(J/\psi\to l\bar{l})=4.7$ keV implies $m_c=1.3$ GeV. Curve II corresponds to $m_0=0.336$ GeV, $m_s=0.540$ GeV; $\Gamma(J/\psi\to l\bar{l})=4.7$ keV implies $m_c=1.6$ GeV.

Ref. 7 sum rules are formulated in $V+A$ and $V-A$ sectors separately. The expressions for the sum rules in the $V-A$ sector are much simpler than those in the $V+A$ sector. This disparity in the degree of simplicity in form together with the nonuniqueness of writing down a set of linearly independent spectral-function sum rules (a linear combination of spectral-function sum rules is again a spectral-function sum rule) makes it very difficult to tell from the $V+A$ and $V-A$ expressions how many sum rules can exist in the V sector. In this section we shall show that in addition to the sum rule (2.16), there exist four (and only four) additional linearly independent first spectral-function sum rules for vector currents.

Let us introduce the two kinds of current products

$$\Sigma_{ij}^{(v)\mu\nu}(x) \equiv \langle 0\,|\bar{q}_i(x)\gamma^\mu q_j(x)\bar{q}_j(0)\gamma^\nu q_i(0)|\,0\rangle\ , \quad (4.1)$$

$$\tilde{\Sigma}_{ij}^{(v)\mu\nu}(x) \equiv \langle 0\,|\bar{q}_i(x)\gamma^\mu q_i(x)\bar{q}_j(0)\gamma^\nu q_j(0)|\,0\rangle\ , \quad (4.2)$$

with $i \neq j$. They are symmetric with respect to the interchange of the subscripts i and j. Our specific choice of quark mass matrix and Z matrix allows the existence of the following identities concerning $\Sigma_{ii}^{(v)\mu\nu}(x)$, $\Sigma_{ij}^{(v)\mu\nu}(x)$, and $\tilde{\Sigma}_{ij}^{(v)\mu\nu}(x)$:

$$\Sigma_{11}^{(v)\mu\nu}(x)=\Sigma_{22}^{(v)\mu\nu}(x)=\Sigma_{12}^{(v)\mu\nu}(x)+\tilde{\Sigma}_{12}^{(v)\mu\nu}(x) \quad (4.3)$$

and

$$\left.\begin{aligned}\Sigma_{1i}^{(v)\mu\nu}(x)&=\Sigma_{2i}^{(v)\mu\nu}(x)\\ \tilde{\Sigma}_{1i}^{(v)\mu\nu}(x)&=\tilde{\Sigma}_{2i}^{(v)\mu\nu}(x)\end{aligned}\right\}, \quad \text{for } i>2\,. \quad (4.4)$$

With the aid of the above properties of Σ's a simple combinatorial counting shows that there are only 10 linearly independent Σ's; we choose the following ten:

$$\begin{aligned}&\Sigma_{11}^{(v)\mu\nu}(x)\ ,\ \Sigma_{33}^{(v)\mu\nu}(x)\ ,\ \Sigma_{44}^{(v)\mu\nu}(x)\\ &\Sigma_{12}^{(v)\mu\nu}(x)\ ,\ \Sigma_{13}^{(v)\mu\nu}(x)\ ,\ \Sigma_{24}^{(v)\mu\nu}(x)\ ,\ \Sigma_{34}^{(v)\mu\nu}(x) \qquad (4.5)\\ &\tilde{\Sigma}_{13}^{(v)\mu\nu}(x)\ ,\ \tilde{\Sigma}_{24}^{(v)\mu\nu}(x)\ ,\ \tilde{\Sigma}_{34}^{(v)\mu\nu}(x)\,.\end{aligned}$$

The short-distance behavior of $\Sigma_{ii}^{(v)\mu\nu}(x)$ is given by Eq. (2.10). Similarly we find that

$$\Sigma_{ij}^{(v)\mu\nu}(x)\underset{x\to 0}{\longrightarrow} f_0^+(x)+\tfrac{1}{2}(m_i^2+m_j^2)[f_1^+(x)+f_0^-(x)] -\tfrac{1}{2}(m_i-m_j)^2 f_0^-(x)+O(x^{-2}) \quad (4.6)$$

$$\tilde{\Sigma}_{ij}^{(v)\mu\nu}(x)\underset{x\to 0}{\longrightarrow}[g_0^+(x)+g_0^-(x)] +\tfrac{1}{2}(m_i^2+m_j^2)[g_1^+(x)+g_1^-(x)]+O(x^{-2}) \quad (4.7)$$

for $i\neq j$.

Using Eq. (2.10), Eq. (4.6), and Eq. (4.7), we get, in addition to Eq. (2.14), the following four linearly independent combinations of the Σ's for which the first spectral-function sum rule holds:

$$(m_c-m_s)(m_0+m_s+m_c)\Sigma_{12}^{(v)\mu\nu}(x)-(m_c^2+m_cm_0-m_0m_s-m_0^2)\Sigma_{13}^{(v)\mu\nu}(x) +(m_s^2+m_sm_0-m_0m_c-m_0^2)\Sigma_{24}^{(v)\mu\nu}(x)+m_0(m_c-m_s)\Sigma_{34}^{(v)\mu\nu}(x)\ , \quad (4.8)$$

$$(m_s^2-m_0^2)\tilde{\Sigma}_{13}^{(v)\mu\nu}(x)-(m_c^2-m_0^2)\tilde{\Sigma}_{24}^{(v)\mu\nu}(x)+(m_c^2-m_s^2)\tilde{\Sigma}_{34}^{(v)\mu\nu}(x)\ , \quad (4.9)$$

$$(m_c^2-m_s^2)\Sigma_{11}^{(v)\mu\nu}(x)-(m_c^2-m_0^2)\tilde{\Sigma}_{13}^{(v)\mu\nu}(x)+(m_s^2-m_0^2)\tilde{\Sigma}_{24}^{(v)\mu\nu}(x)-(m_c^2-m_s^2)\Sigma_{12}^{(v)\mu\nu}(x)\ , \quad (4.10)$$

$$(m_c-m_s)(m_s^2+m_cm_s-m_0m_s-m_0^2)\Sigma_{11}^{(v)\mu\nu}(x)+m_0(m_c-m_0)(m_c-m_s)\Sigma_{33}^{(v)\mu\nu}(x)-(m_c-m_0)^2(m_0+m_s)\tilde{\Sigma}_{13}^{(v)\mu\nu}(x) +(m_s+m_0)(m_s-m_0)^2\tilde{\Sigma}_{24}^{(v)\mu\nu}(x)-(m_s+m_0)(m_c-m_0)^2\Sigma_{13}^{(v)\mu\nu}(x)+(m_s+m_0)(m_s-m_0)^2\Sigma_{24}^{(v)\mu\nu}(x)\,. \quad (4.11)$$

One can show that the combinations (2.14) and (4.8)–(4.11) are consistent with the corresponding combinations for $V\pm A$ sectors obtained in Ref. 7. The proof of the consistency involves tedious linear combinations owing to the fact that we have used a different set of Σ's [given by (4.5)].

For the time being there are not enough experimental data for evaluating most of the spectral functions involved in the spectral-function sum rules associated with the combinations (4.8)–(4.11), even in the pole-dominance approximation. The spectral functions for $\Sigma_{ij}^{(v)\mu\nu}(x)$ $(i\neq j)$ generally involve the spin-1 pieces as well as the spin-0 pieces. We do not have the relevant data for charmed vector mesons; our knowledge concerning scalar mesons is also poor in general. The spectral functions for $\tilde{\Sigma}_{ij}^{(v)\mu\nu}(x)$ $(i\neq j)$ are even more formidable because they are associated with the graphs in which the quark lines originated at the point $x=0$ are not connected to the quark lines ended at $x\neq 0$, i.e., the Iizuka-Okubo-Zweig-rule[12] forbidden graphs, and so are related to the mechanism responsible for the breaking of the rule. The pole-dominance approximation applied to the spectral functions of $\tilde{\Sigma}_{ij}^{(v)\mu\nu}(x)$ $(i\neq j)$ would require some new vector meson(s) purely made out of gluons; one possible candidate would be the O meson[13] proposed by Freund and Nambu.

V. CONCLUSIONS

In this paper we set out to find a way to remove the discrepancy between the Weinberg spectral-function sum rules and the leptonic decays of J/ψ

vector meson. To this end we specialized the BDLW procedure to the derivation of the first spectral-function sum rules for vector currents. We find five sum rules consequently. Among them, one [Eq. (2.16)] appears to relate the leptonic decays of ρ, ω, ϕ, and J/ψ vector mesons.

In contrast to the traditional Weinberg sum rules, the sum rules resulting from the BDLW procedure generally involve the ratios of quark masses explicitly. Thus the values of these ratios are needed when one applies the sum rules. We have shown in Sec. III that the discrepancy between the Weinberg sum rules and the leptonic decays of J/ψ is resolved by applying the sum rule (2.17), which results from Eq. (2.16) by local saturation of the spectral functions, and the usual estimates of quark mass ratios. The discrepancy is thus revealed as a consequence of the large mass difference between the charmed quark and noncharmed quarks.

We are not able to apply the other four first spectral-function sum rules because of insufficient experimental data. More detailed discussion is presented in Sec. IV.

We emphasize that it is by the involvement of the ratios of quark masses that the sum rule (2.17) is able to take into account the effect of the symmetry breaking appropriately. To our surprise, however, both the current-quark mass ratios and the constituent-quark masses (effectively only their ratios are involved) work equally well. The derivation of the spectral-function sum rules from a consideration of short-distance behavior of current products makes it clear that one is concerned with the effective quark mass ratios in the deep Euclidean region—i.e., with current-algebra (or bare) quark masses. Thus the fact that the constituent-quark masses also fit the sum rule (2.17) is most likely a numerical accident.

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