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Scalar contribution to radiative decays of the η meson

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We study the role which scalar mesons play in the radiative decays of the η meson. A scalar-meson model involving the $\delta(970)$ is proposed to describe the three radiative η decays: $\eta\to\pi^0\gamma\gamma$, $\eta\to\pi^+\pi^-\pi^0\gamma$, and $\eta\to\pi^+\pi^-\gamma\gamma$. Using the experimental width for $\eta\to\pi^0\gamma\gamma$ we determine the δ-γ-γ coupling and calculate the decay rates for the four-body η decays. The results are compared with the experimental data.

I. INTRODUCTION

The vector-dominance model (VDM) has achieved a number of important successes in the area of pseudoscalar- and vector-meson decays. The original simplified VDM scheme proposed by Gell-Mann, Sharp, and Wagner[1] met with some qualitative success but failed to make predictions in agreement with the experimental data for such ratios as $\Gamma(\omega\to\pi\gamma)/\Gamma(\omega\to 3\pi)$ and $\Gamma(\eta\to\pi\pi\gamma)/\Gamma(\eta\to\gamma\gamma)$. An improved version of VDM was developed later by Brown, Munczek, and Singer[2] (BMS). In the BMS approach the effects of SU(3)-symmetry breaking in the vertex involving two vector mesons and one pseudoscalar meson were taken into account along with current mixing for the vector mesons. BMS found that the observed rates for the radiative η decays required sizable symmetry-breaking effects in their model.

Recently, there has been a renewed interest in the radiative decays of mesons kindled by recent measurements[3] of radiative decay widths. Motivated by these developments, Brown and Singer[4] refined their earlier model to include the effects of η-η' mixing. This updated version of the BMS model was successful in describing a large number of strong and radiative meson decays.[5] Absent from the work of Brown and Singer, however, was the η-meson decay $\eta\to\pi^0\gamma\gamma$. The failure of VDM to explain the observed decay width for $\eta\to\pi^0\gamma\gamma$ was first noted by Singer[6] and subsequently by Oppo and Oneda.[7] The VDM calculation yielded a value for the $\eta\to\pi^0\gamma\gamma$ width which was about two orders of magnitude below the experimental value.[3] In their paper Oppo and Oneda suggested an alternative mechanism for this decay which involved the exchange of a scalar-meson resonance which was coupled to the $\eta\pi$ system. Since such a scalar meson was, at that time, only hypothetical, no definitive test could be made of this model for describing $\eta\to\pi^0\gamma\gamma$ decay.

Recently, Gokhale, Patil, and Rindani[8] have made a new estimate for the decay rate for $\eta\to\pi^0\gamma\gamma$. Assuming the two photons are in an s-wave state, they relate the amplitude for this process to that for $\gamma\gamma\to K^+K^-$ in the $I=1$ state by crossing and SU(3) symmetry, which is then evaluated by saturating it with the δ scalar meson. They predict a width $\Gamma(\eta\to\pi^0\gamma\gamma)=48$ eV in comparison with the experimental value[3] 26 ± 9 eV. Thus, whereas VDM predicts a decay rate for $\eta\to\pi^0\gamma\gamma$ two orders of magnitude below the experimental result, a model characterized by scalar-meson dominance furnishes a prediction which is in rough agreement with experiment.

The purpose of this paper is to further explore the role which scalar mesons play in the radiative decays of the η meson. We restrict our discussion to contributions made by the $\delta(970)$ resonance, the only well-established scalar meson, which presumably has the quantum numbers $J^P=0^+$, and which decays predominantly into $\eta\pi$.

In Sec. II we preface our studies with a discussion of the BMS model and its incompatible predictions for the three radiative decays $\eta\to\gamma\gamma$, $\eta\to\pi\pi\gamma$, and $\eta\to\pi\gamma\gamma$. In Sec. III we introduce the scalar-meson model and determine the decay rate for $\eta\to\pi^0\gamma\gamma$ in terms of the δ-γ-γ coupling. This coupling is then evaluated using the experimental width for this decay. In Secs. IV and V we extend the application of the δ model to two rare radiative decay modes $\eta\to\pi\pi\pi\gamma$ and $\eta\to\pi\pi\gamma\gamma$ where scalar-meson contributions are important. In Sec. VI we summarize our results and present our conclusions.

II. THE BMS VECTOR-DOMINANCE MODEL AND RADIATIVE η DECAYS

We begin by discussing the vector-meson-dominance model proposed by Brown, Munczek, and Singer[2] (BMS) that includes the effects of SU(3)-symmetry breaking in the PVV vertex (two vector mesons and one pseudoscalar meson) as well as current mixing for the vector mesons.[9,10]

In the BMS model the effective Lagrangian responsible for the PVV interactions which emerge in the vector-meson-dominance picture is given by the general octet-broken form[2]

$$\mathcal{L}_{PVV}=\tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}(hD^{abc}V^a_{\alpha\beta}V^b_{\mu\nu}P^c+\lambda D^{ab}V^a_{\alpha\beta}P^bV^0_{\mu\nu}), \tag{2.1}$$

where

$$V^a_{\mu\nu}=\partial_\mu V^a_\nu-\partial_\nu V^a_\mu-g_\rho f^{abc}V^b_\mu V^c_\nu, \tag{2.2}$$

$$D^{abc}=d^{abc}+\sqrt{3}\,\epsilon_1 d^{abd}d^{d8c}+\frac{\sqrt{3}}{2}\epsilon_2(d^{acd}d^{d8b}+d^{bcd}d^{d8a})+\frac{\epsilon_3}{\sqrt{3}}\delta^{ab}\delta^{c8}, \tag{2.3}$$

$$D^{ab}=\delta^{ab}+\sqrt{3}\,\epsilon_4 d^{ab8}. \tag{2.4}$$

P^a ($a=1,\ldots,8$) represent the fields for the octet of pseudoscalar mesons. V^a_μ, V^0_μ represent the nine vector fields described by a Yang-Mills–type Lagrangian which, when diagonalized in terms of the physical particles, gives the relationships of the V^a_μ to the physical fields

$$\begin{aligned}
V^{1,2,3}_\mu&=\frac{1}{\sqrt{K_\rho}}\rho^{1,2,3}_\mu,\\
V^{4,5,6,7}_\mu&=\frac{1}{\sqrt{K_{K^*}}}K^{*4,5,6,7}_\mu,\\
V^8_\mu&=\frac{\sin\theta}{\sqrt{K_\omega}}\omega_\mu-\frac{\cos\theta}{\sqrt{K_\phi}}\phi_\mu,\\
V^0_\mu&=\frac{\cos\theta}{\sqrt{K_\omega}}\omega_\mu+\frac{\sin\theta}{\sqrt{K_\phi}}\phi_\mu,
\end{aligned} \tag{2.5}$$

$$K_i=\frac{m^2}{m_i^2}\quad(i=\rho,K^*,\omega,\phi),\qquad m=847\text{ MeV},\quad\theta=30^\circ. \tag{2.6}$$

We have neglected the effects of η-η' mixing in this model although they can be easily incorporated.[11]

The BMS model is completed by the introduction of the effective electromagnetic interaction[9]

$$\mathcal{L}_{em}=\frac{em^2}{g_\rho}\left(\frac{1}{\sqrt{K_\rho}}\rho_\mu{}^3+\frac{\sin\theta}{(3K_\omega)^{1/2}}\omega_\mu-\frac{\cos\theta}{(3K_\phi)^{1/2}}\phi_\mu\right)A_\mu, \tag{2.7}$$

where the quantity g_ρ is related to the $\rho\pi\pi$ form factor $g_{\rho\pi\pi}(p^2)$ by

$$g_\rho=g_{\rho\pi\pi}(0)K_\rho{}^{1/2}, \tag{2.8}$$

and is taken to have the value $g_\rho^2/4\pi=3.40$, corresponding to a ρ-meson width of 146 MeV when the momentum dependence of the form factor is neglected.

Using Lagrangians (2.1) and (2.7), BMS calculated the decay widths for the six decay processes $\omega\to3\pi$, $\phi\to3\pi$, $\omega\to\pi^0\gamma$, $\phi\to\pi^0\gamma$, $\pi^0\to\gamma\gamma$, and $\rho^-\to\pi^-\gamma$, all of which depend only upon the known coupling constant g_ρ and the factors $h(1+\epsilon_1)$ and $\lambda(1+\epsilon_4)$. Using the experimental widths for $\omega\to3\pi$ and $\phi\to3\pi$ as input, they obtained excellent agreement with experiment for the values

$$\frac{m_\pi{}^2h^2}{4\pi}(1+\epsilon_1)^2=0.108, \tag{2.9}$$

$$\frac{m_\pi{}^2\lambda^2}{4\pi}(1+\epsilon_4)^2=0.328, \tag{2.10}$$

except for the troublesome case[12] of $\rho^-\to\pi^-\gamma$.

To determine the possible values of the symmetry-breaking parameters ϵ_1 and $\epsilon'=\epsilon_2+\epsilon_3$ one must consider the decays $\eta\to\gamma\gamma$ and $\eta\to\pi^+\pi^-\gamma$. A comparison of these decay widths predicted by the model with the experimental widths gives the relations

$$(1-\epsilon_1+2\epsilon')^2=2.2(1+\epsilon_1)^2 \tag{2.11}$$

and

$$(1-\epsilon_1+\epsilon')^2=0.53(1+\epsilon_1)^2, \tag{2.12}$$

which yields the following solutions:

$$\epsilon_1=1.04,\quad\epsilon'=1.53, \tag{2.13a}$$

$$\epsilon_1=0.96,\quad\epsilon'=-1.47, \tag{2.13b}$$

$$\epsilon_1=-2.03,\quad\epsilon'=-2.28, \tag{2.13c}$$

$$\epsilon_1=-0.49,\quad\epsilon'=-1.12. \tag{2.13d}$$

Elimination of some of these solutions, as well as a determination of the parameter ϵ_4, can be achieved by examining the decays $\phi\to\eta\gamma$ and $\omega\to\eta\gamma$. The experimental width for $\phi\to\eta\gamma$ and the corresponding experimental upper limit for $\omega\to\eta\gamma$ yield the equations

$$\Gamma(\phi\to\eta\gamma)=(98.0\text{ keV})\times\left(\frac{1-\epsilon_1-\epsilon'}{1+\epsilon_1}-\frac{\sqrt{3}\,\lambda\sin\theta}{2h\cos\theta}\frac{(1-\epsilon_4)}{(1+\epsilon_1)}\right)^2=82.0\text{ keV} \tag{2.14}$$

and

$$\Gamma(\omega\to\eta\gamma)=(58.0\text{ keV})\left(-0.214\frac{1-\epsilon_1-\epsilon'}{1+\epsilon_1}-\frac{0.79\lambda\sqrt{3}}{2h\cot\theta}\frac{(1-\epsilon_4)}{(1+\epsilon_1)}\right)^2<50.0\text{ keV}. \tag{2.15}$$

Upon examining the solutions in Eqs. (2.13) one finds that solutions (2.13c) and (2.13d) are incompatible with Eqs. (2.14) and (2.15). For solution (2.13a), Eqs. (2.14) and (2.15) yield two possible values for ϵ_4 (depending on the undetermined sign of h/λ):

$$\begin{aligned}
&\epsilon_1=1.04,\quad\epsilon'=1.53,\quad\epsilon_4=1.50\quad(h/\lambda<0),\\
&\epsilon_1=1.04,\quad\epsilon'=1.53,\quad\epsilon_4=0.67\quad(h/\lambda>0).
\end{aligned} \tag{2.16a}$$

Likewise for solution (2.13b) one obtains

$$\begin{aligned}
&\epsilon_1=0.96,\quad\epsilon'=-1.47,\quad\epsilon_4=0.66\quad(h/\lambda<0),\\
&\epsilon_1=0.96,\quad\epsilon'=-1.47,\quad\epsilon_4=1.50\quad(h/\lambda>0).
\end{aligned} \tag{2.16b}$$

We now wish to apply the BMS model to the η decay

$$\eta(p) \to \pi^0(q) + \gamma(k) + \gamma(k') . \tag{2.17}$$

The gauge- and Lorentz-invariant amplitude for this process can be written as

$$\begin{aligned}\mathfrak{M} = \tilde{F}(k)\{&(p^2 - p\cdot k)[(k\cdot\epsilon')(k'\cdot\epsilon) - (k\cdot k')(\epsilon\cdot\epsilon')] + (p\cdot k')(p\cdot k)(\epsilon\cdot\epsilon') + (p\cdot\epsilon)(p\cdot\epsilon')(k\cdot k') \\ &- (p\cdot k')(p\cdot\epsilon)(k\cdot\epsilon') - (p\cdot k)(k'\cdot\epsilon)(p\cdot\epsilon')\} + (k \to k') .\end{aligned} \tag{2.18}$$

Calculating $\tilde{F}(k)$ using the BMS model gives

$$\tilde{F} = \sum_i \left(\frac{f_i}{m_\pi^2}\right) D_i \quad (i = \rho, \omega, \phi) , \tag{2.19}$$

where

$$D_i = [(p-k)^2 - m_i^2]^{-1} , \tag{2.20}$$

and

$$f_\rho = -\frac{4m_\pi^2 h^2 e^2}{3K_\rho g_\rho^2}(1+\epsilon_1)(1-\epsilon_1+\epsilon') , \tag{2.21}$$

$$f_\omega = \frac{e^2 m_\pi^2}{3\sqrt{3}\, g_\rho^2 K_\omega}[2h\sin\theta(1+\epsilon_1) - \sqrt{3}\,\lambda(1+\epsilon_4)\cos\theta][h\sin\theta(1+\cos^2\theta)(1-\epsilon_1-\epsilon') + \cos^3\theta\sqrt{3}\,\lambda(1-\epsilon_4)] , \tag{2.22}$$

$$f_\phi = \frac{e^2 m_\pi^2}{3\sqrt{3}\, g_\rho^2 K_\phi}[2h\cos\theta(1+\epsilon_1) + \sqrt{3}\,\lambda(1+\epsilon_4)\sin\theta][h\cos\theta(1+\sin^2\theta)(1-\epsilon_1-\epsilon') - \sin^3\theta\sqrt{3}\,\lambda(1-\epsilon_4)] . \tag{2.23}$$

Squaring Eq. (2.18), summing over photon polarizations, and integrating over final momenta, give for the decay rate

$$\Gamma(\eta \to \pi^0\gamma\gamma) = \frac{m_\eta}{64(2\pi)^3}\left(\frac{m_\eta}{m_\pi}\right)^4 \tilde{G} , \tag{2.24}$$

where

$$\tilde{G} = \sum_{i,j} f_i f_j I_{ij} \quad (i, j = \rho, \omega, \phi) , \tag{2.25}$$

$$\begin{aligned}&I_{\rho\rho} = 1.83\times10^{-3} , \quad I_{\rho\omega} = I_{\omega\rho} = 1.67\times10^{-3} , \\ &I_{\omega\omega} = 1.52\times10^{-3} , \quad I_{\rho\phi} = I_{\phi\rho} = 0.871\times10^{-3} , \\ &I_{\phi\phi} = 0.414\times10^{-3} , \quad I_{\omega\phi} = I_{\phi\omega} = 0.793\times10^{-3} .\end{aligned} \tag{2.26}$$

Using the values (2.8), (2.9), (2.10), and the solutions (2.16a) and (2.16b) for the symmetry-breaking parameters gives for the $\eta \to \pi^0\gamma\gamma$ decay width

$$\Gamma(\eta \to \pi^0\gamma\gamma) = 0.063 \text{ eV} , \quad 0.033 \text{ eV} , \tag{2.27}$$

where the first value corresponds to $h/\lambda < 0$ and the second value corresponds to $h/\lambda > 0$. The prediction in Eq. (2.27) is to be compared with the experimental value[3] of 26 ± 9 eV. Thus the prediction for the decay width of $\eta \to \pi^0\gamma\gamma$ based on the BMS model is more than two orders of magnitude below the experimental value.

The present calculation using the BMS model demonstrates that including SU(3)-breaking effects at the strong vertices does not alter the fact that vector-meson-dominance cannot adequately describe $\eta \to \pi^0\gamma\gamma$ decay. Although the effects of η-η' mixing have not been included, it is not expected that this mixing will change the order of magnitude of the results in (2.27). We thus conclude that, although the BMS vector-meson-dominance model can easily accommodate the η decay modes $\eta \to \pi^+\pi^-\gamma$ and $\eta \to \gamma\gamma$, the observed decay width for $\eta \to \pi^0\gamma\gamma$ is clearly out of reach in any generalized vector-dominance scheme.

III. THE δ-MESON CONTRIBUTION TO $\eta \to \pi^0\gamma\gamma$

The results of the previous section indicate that there must be contributions to the decay $\eta \to \pi^0\gamma\gamma$ other than those of vector mesons. Motivated by the work of Gokhale, Patil, and Rindani,[8] we examine the δ-meson contribution using a Feynman-diagram approach. The advantage of our calculation over that done in Ref. 8 is that it exhibits directly the δ couplings to $\eta\pi$ and $\gamma\gamma$. Once these couplings are evaluated from the experimental data, it is then possible for us to give a prediction for the decay rate for $\delta \to \gamma\gamma$ and to calculate the four-body decays of the η meson in Secs. IV and V.

As indicated in Ref. 8, the scalar-meson contribution to the decay $\eta \to \pi^0\gamma\gamma$ is large compared to the vector-dominance contribution because the considerable angular momentum barrier due to the 1^- intermediate states is no longer present. Furthermore, the δ meson does not contribute to the decays $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$ and therefore does not disturb the fits made to these decays along with the other radiative decays of pseudoscalar and vector

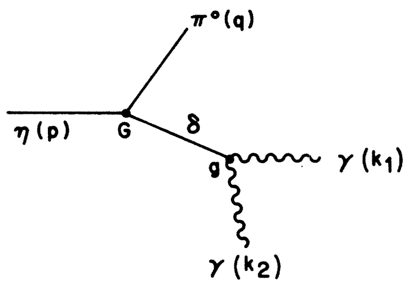


FIG. 1. Diagram for the decay $\eta \to \pi^0\gamma\gamma$ with the δ-meson intermediate state.

mesons using vector dominance.

The δ contribution to the decay $\eta \to \pi^0\gamma\gamma$ is shown in Fig. 1. The coupling at the δ-η-π vertex is given by the constant G, and the δ-γ-γ vertex is

$$g[(k_1 \cdot k_2) - (k_1 \cdot \epsilon_2)(k_2 \cdot \epsilon_1)]\,. \tag{3.1}$$

The polarization-averaged matrix element is then given by

$$\langle \mathfrak{M}^2 \rangle = \frac{2g^2G^2}{[(k_1+k_2)^2 - m_\delta^2]^2}(k_1 \cdot k_2)^2\,, \tag{3.2}$$

and from this we obtain the following expression for the $\eta \to \pi^0\gamma\gamma$ decay rate:

$$\Gamma(\eta \to \pi^0\gamma\gamma) = \frac{g^2G^2}{256\pi^3 m_\eta}\int_{\omega_{\min}}^{\omega_{\max}} \frac{(\omega^2-\mu^2)^{1/2}(m_\eta^2+\mu^2-2m_\eta\omega)^2}{(2m_\eta\omega + m_\delta^2 - m_\eta^2 - \mu^2)^2}\,d\omega\,, \tag{3.3}$$

where μ is the pion mass, $m_\delta = 976$ MeV, and

$$\omega_{\min} = \mu\,, \quad \omega_{\max} = \frac{m_\eta^2 + \mu^2}{2m_\eta}\,. \tag{3.4}$$

The integral in (3.3) has been evaluated numerically with the result being $0.0125\mu^2$.

The value of G can be obtained by calculating the decay rate $\Gamma(\delta \to \eta\pi)$ and fitting it to the measured value[3,13] 50 ± 20 MeV. The calculated expression is

$$\Gamma(\delta \to \eta\pi) = \frac{G^2}{16\pi m_\delta^3}(m_\delta^4 + m_\eta^4 + \mu^4 - 2m_\delta^2 m_\eta^2 - 2m_\delta^2\mu^2 - 2m_\eta^2\mu^2)^{1/2}\,, \tag{3.5}$$

and we find $G = 1.95 \pm 0.39$ GeV.

The coupling g cannot be obtained in a similar way since the decay $\delta \to \gamma\gamma$ has not been observed experimentally. In the calculation of Gokhale, Patil, and Rindani,[8] this coupling appears intrinsically in their SU(3) plus crossing relations, but is not given explicitly. Following their results, we assume that the scalar intermediate states produce a good fit for the $\eta \to \pi^0\gamma\gamma$ decay and proceed to calculate a value for g. This value is then used to predict the $\delta \to \gamma\gamma$ decay rate.

Using our values for G and the integral expression, Eq. (3.3) becomes

$$\Gamma(\eta \to \pi^0\gamma\gamma) = (1.98 \pm 0.79) \times 10^{-7} g^2 \text{ GeV}\,, \tag{3.6}$$

where the units of g are GeV^{-1}. From the experimental decay rate for $\eta \to \pi^0\gamma\gamma$ of 26 ± 9 eV, we find

$$g = 0.36 \pm 0.14 \text{ GeV}^{-1}\,. \tag{3.7}$$

Using the vertex (3.1), the $\delta \to \gamma\gamma$ decay rate is given by

$$\Gamma(\delta \to \gamma\gamma) = \frac{g^2}{64\pi} m_\delta^3\,, \tag{3.8}$$

and with the value (3.7) we find

$$\Gamma(\delta \to \gamma\gamma) = 610 \pm 460 \text{ keV}\,. \tag{3.9}$$

The branching ratio $\Gamma(\delta \to \gamma\gamma)/\Gamma(\delta \to \text{all})$ is approximately 1%, which compares well with the corresponding branching ratio of 2% observed[3] in the decay $\eta' \to \gamma\gamma$, the η' having mass and decay modes similar to those of the δ.

As indicated by Oppo and Oneda,[7] the scalar intermediate state can be used as a model for the decay $\eta \to 3\pi$. We review the model here in light of present day data.

The diagrams for $\eta \to 3\pi$ with a δ intermediate state are shown in Fig. 2. The coupling $g_{\eta\pi}$ is the strength of the $\eta \to \pi$ transition. The squared ma-

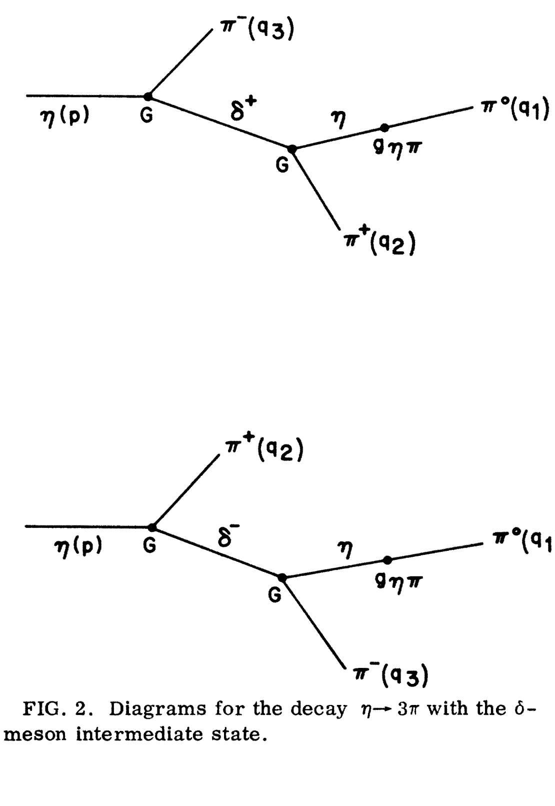


FIG. 2. Diagrams for the decay $\eta \to 3\pi$ with the δ-meson intermediate state.

trix element is

$$|\mathfrak{M}|^2 = \frac{G^4 g_{\eta\pi}^{\ 2}}{(m_\eta^{\ 2} - \mu^2)^2} \left[\frac{1}{(q_1+q_2)^2 - m_\delta^{\ 2}} + \frac{1}{(q_1+q_3)^2 - m_\delta^{\ 2}} \right]^2 , \tag{3.10}$$

giving the following expression for the decay rate:

$$\Gamma(\eta \to 3\pi) = \frac{1}{16\pi^3 m_\eta} \frac{G^4 g_{\eta\pi}^{\ 2}}{(m_\eta^{\ 2} - \mu^2)^2} \times \int_\mu^{\omega_{\max}} \frac{(\omega^2 - \mu^2)^{1/2} d\omega}{(2m_\eta \omega + m_\delta^{\ 2} - m_\eta^{\ 2} - \mu^2)^2} , \tag{3.11}$$

where

$$\omega_{\max} = \frac{m_\eta^{\ 2} - 3\mu^2}{2m_\eta} .$$

Numerically, the integral in Eq. (3.11) has the value $1.46 \times 10^{-4}\ \mu^{-2}$ and, using the value of G obtained previously, we find

$$\Gamma(\eta \to 3\pi) = (5.3 \pm 4.2) \times 10^{-3} g_{\eta\pi}^{\ 2}\ \text{GeV} , \tag{3.12}$$

where $g_{\eta\pi}$ is in units of GeV^2. The large uncertainty in (3.12) is due to the fact that G, obtained from the total width of the δ which has an uncertainty of 40%, appears twice in the amplitude for $\eta \to 3\pi$.

Various methods have been used for evaluating the coupling $g_{\eta\pi}$. Okubo and Sakita[14] consider the electromagnetic mass differences of the K and π mesons and obtain $g_{\eta\pi} = 0.0029\ \text{GeV}^2$. Equation (3.12) then gives $\Gamma(\eta \to 3\pi) = 45 \pm 36$ eV compared with the experimental value[3] of 201 ± 5 eV. Another value of $g_{\eta\pi}$ has been obtained by Riazuddin and Fayyazuddin,[15] who consider the difference between the coupling constants for charged and neutral pions coupled to nucleons. They obtain a value of $g_{\eta\pi} = 0.0072\ \text{GeV}^2$ which gives $\Gamma(\eta \to 3\pi) = 280 \pm 220$ eV, in better agreement with the experimental value. In spite of the large uncertainties, these predicted widths indicate that the δ model gives a reasonable description of the 3π decay of the η meson.

IV. THE δ-MESON CONTRIBUTION TO $\eta \to \pi^+\pi^-\pi^0\gamma$

The contribution of the δ meson to the decay $\eta \to \pi^+\pi^-\pi^0\gamma$ is shown in Fig. 3. The δ coupling to the $\pi\pi\gamma$ is assumed to occur via vector mesons. The ω-γ and ϕ-γ couplings are given by $e^2 m_\omega^{\ 2}/2\gamma_\omega$ and $e^2 m_\phi^{\ 2}/2\gamma_\phi$, respectively, and we use the experimental values

$$\frac{\gamma_\omega^{\ 2}}{4\pi} = 4.6 , \quad \frac{\gamma_\phi^{\ 2}}{4\pi} = 2.8 . \tag{4.1}$$

The ρ-π-π vertex is $g_{\rho\pi\pi}\,[e_\rho \cdot (q_2 - q_1)]$ and from the experimental $\rho \to \pi\pi$ decay width[3]

$$\frac{g_{\rho\pi\pi}^{\ 2}}{4\pi} = 2.86 . \tag{4.2}$$

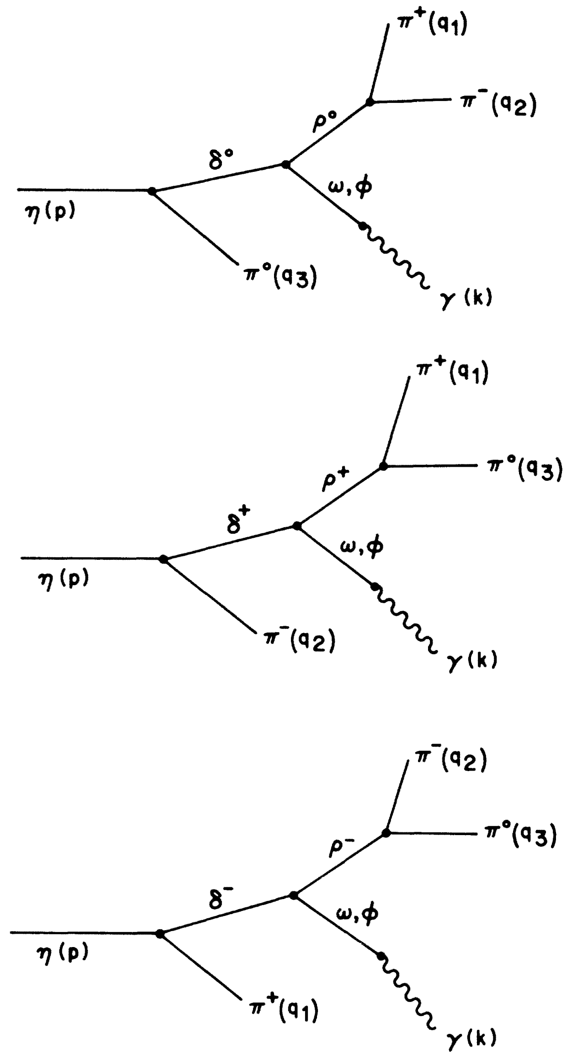


FIG. 3. The δ-meson contributions to $\eta \to \pi^+\pi^-\pi^0\gamma$.

The δ-ρ-ω vertex is given by

$$g_{\rho\omega\delta}[(e_\rho \cdot e_\omega)(p_\rho \cdot p_\omega) - (e_\rho \cdot p_\omega)(e_\omega \cdot p_\rho)] , \tag{4.3}$$

where e and p are the polarizations and momenta of the vector mesons. The coupling $g_{\rho\omega\delta}$ is obtained by assuming that the decay $\delta \to \gamma\gamma$, discussed in the previous section, is mediated by vector mesons. The δ-γ-γ coupling is then given by

$$g = \frac{\alpha}{2} g_{\rho\omega\delta} \left(\frac{\gamma_\rho^{\ 2}}{4\pi} \right)^{-1/2} \left[\left(\frac{\gamma_\omega^{\ 2}}{4\pi} \right)^{-1/2} + \beta \left(\frac{\gamma_\phi^{\ 2}}{4\pi} \right)^{-1/2} \right] , \tag{4.4}$$

where $\beta = g_{\rho\phi\delta}/g_{\rho\omega\delta}$ and experimentally

$$\frac{\gamma_\rho^{\ 2}}{4\pi} = 0.62 . \tag{4.5}$$

If the δ meson is part of an SU(3) octet then, on the basis of SU(3) symmetry, we expect the ratio β to be equal to the same ratio of vector-meson couplings to the pion. In vector-meson-dominance

$$\frac{\Gamma(\rho\to\pi\gamma)}{\Gamma(\phi\to\pi\gamma)}=\left(\frac{g_{\rho\omega\pi}}{g_{\rho\phi\pi}}\right)^2\left(\frac{\gamma_\rho}{\gamma_\omega}\right)^2\left(\frac{m_\phi}{m_\rho}\right)^3\left(\frac{m_\rho{}^2-\mu^2}{m_\phi{}^2-\mu^2}\right)^3, \tag{4.6}$$

and using the recent experimental data on these decays,[3] $g_{\rho\omega\pi}/g_{\rho\phi\pi}=0.095\simeq\beta$. Using this value of β and Eq. (4.5) in Eq. (4.4) we obtain

$$g_{\rho\omega\delta}\simeq 150\pm 60\ \text{GeV}^{-1}. \tag{4.7}$$

The decay rate obtained from the diagrams of Fig. 3 is

$$\Gamma(\eta\to 3\pi\gamma)=-\frac{9G^2 g_{\rho\omega\delta}{}^2 g_{\rho\pi\pi}{}^2 e^2}{128(2\pi)^8 m_\eta\gamma^2}\int\frac{d^3q_1}{\omega_1}\frac{d^3q_2}{\omega_2}\frac{d^3q_3}{\omega_3}\frac{d^3k}{k}\times\frac{[k\cdot(q_1+q_2)]^2(q_2-q_1)^2+[k\cdot(q_2-q_1)]^2(q_1+q_2)^2}{[(p-q_3)^2-m_\delta{}^2]^2[(q_1+q_2)^2-m_\rho{}^2]^2}\delta^4(p-q_1-q_2-q_3-k), \tag{4.8}$$

where we have defined

$$\frac{1}{\gamma}=\frac{1}{\gamma_\omega}+\frac{\beta}{\gamma_\phi}. \tag{4.9}$$

The integrations over the pion momenta are carried out in the center-of-mass system of the three pions using standard techniques.[16] The remaining integral has the form

$$\Gamma(\eta\to 3\pi\gamma)=\frac{3G^2 g_{\rho\omega\delta}{}^2 g_{\rho\pi\pi}{}^2}{64(2\pi)^4 m_\eta{}^2\gamma^2}\int_0^{k_{\max}}dk\int_\mu^{\omega_{\max}}d\omega\,\frac{E_k{}^2-2\omega E_k+\mu^2}{(E_k{}^2-2\omega E_k+\mu^2-m_\rho{}^2)^2}\left(\frac{E_k{}^2-2\omega E_k-3\mu^2}{E_k{}^2-2\omega E_k+\mu^2}\right)^{3/2}(T_1+T_2+T_3+T_4), \tag{4.10}$$

where

$$T_1=4qkm_\eta, \tag{4.11}$$

$$T_2=2k^2(E_k-q-\omega)^2 m_\eta{}^2\Delta_+, \tag{4.12}$$

$$T_3=-2k^2(E_k+q-\omega)^2 m_\eta{}^2\Delta_-, \tag{4.13}$$

$$T_4=-E_k(m_\eta{}^2+\mu^2-m_\delta{}^2-2km_\eta-2\omega E_k)\ln(\Delta_+/\Delta_-), \tag{4.14}$$

$$\Delta_\pm=[(m_\eta{}^2+\mu^2-m_\delta{}^2)E_k-2m_\eta(\omega m_\eta-k\omega\pm kq)]^{-1}, \tag{4.15}$$

$$E_k=[m_\eta(m_\eta-2k)]^{1/2},\quad q=(\omega^2-\mu^2)^{1/2}, \tag{4.16}$$

$$\omega_{\max}=\frac{E_k{}^2-3\mu^2}{2E_k},\quad k_{\max}=\frac{m_\eta{}^2-9\mu^2}{2m_\eta}. \tag{4.17}$$

The remaining integrals are carried out numerically with the result

$$\Gamma(\eta\to 3\pi\gamma)=0.056\pm 0.045\ \text{eV}, \tag{4.18}$$

which is an order of magnitude smaller than the experimental upper limit[3] of 0.5 eV. The large uncertainty in (4.18) is due to the uncertainties in G and $g_{\rho\omega\delta}$, both of which are caused by the 40% experimental uncertainty in the δ width.

By taking the branching ratio $\Gamma(\eta\to 3\pi\gamma)/\Gamma(\eta\to\pi\gamma\gamma)$ in the δ model, we eliminate the δ-γ-γ coupling and the δ-η-π coupling. Using the experimental value[3] for $\Gamma(\eta\to\pi\gamma\gamma)$ of 26 ± 9 eV, (4.18) yields

$$\frac{\Gamma(\eta\to 3\pi\gamma)}{\Gamma(\eta\to\pi\gamma\gamma)}=0.0022\pm 0.0008. \tag{4.19}$$

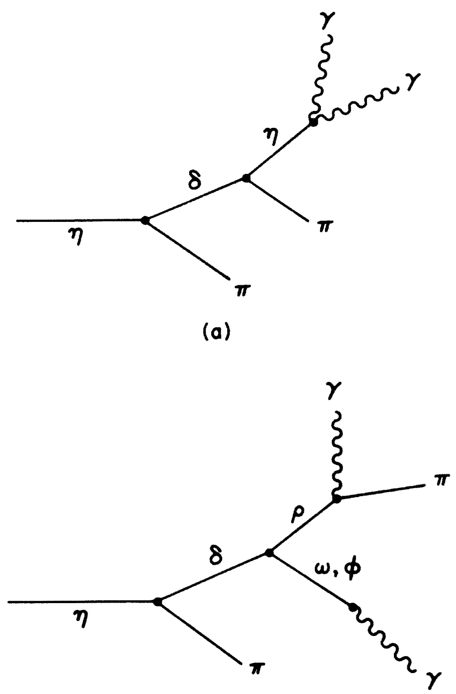
Previous calculations of this ratio using current algebra[17,18] are in agreement with this result. Also in agreement is a vector-dominance calculation of this ratio by Chatterjee.[19] From our discussion in Sec. II, the vector-dominance model is known to give a prediction for $\Gamma(\eta\to\pi\gamma\gamma)$ that is too small by at least two orders of magnitude, and therefore the vector-dominance contribution to $\eta\to 3\pi\gamma$ is at least two orders of magnitude smaller than the δ-meson contribution obtained above. If the δ intermediate state is the major contributor to the $\eta\to 3\pi\gamma$ decay, we predict that the actual decay rate will be an order of magnitude smaller than the present experimental upper limit.

V. THE δ-MESON CONTRIBUTION TO $\eta\to\pi^+\pi^-\gamma\gamma$

There are two types of diagrams which contribute to the decay $\eta\to\pi^+\pi^-\gamma\gamma$ in the δ model and these are shown in Figs. 4(a) and 4(b). In order to simplify the calculation, we approximate the δ propagator by a constant. Due to the large mass of the δ, this approximation is valid, particularly when we consider the large uncertainties in our results. We have tested this approximation by repeating the calculation of the previous section with a constant δ propagator and find that the results agree to within 5%.

In the η rest frame, the denominator of the δ propagator has the form

$$D=(p-q)^2-m_\delta{}^2=m_\eta{}^2+\mu^2-2\omega m_\eta-m_\delta{}^2, \tag{5.1}$$



(a)

(b)

FIG. 4. The δ-meson contributions to $\eta \to \pi^+\pi^-\gamma\gamma$.

where ω is the pion energy. In this frame, the minimum and maximum pion energy are μ and $m_\eta/2$, respectively, and we approximate ω as the mean of these two values, 207 MeV. The value of D is then $-0.90 m_\delta^2$ and we therefore replace the square of the δ propagator by $1.2\, m_\delta^{-4}$.

The matrix element for the contribution in Fig. 4(a) has the form

$$\mathfrak{M}_a = -\frac{1.1}{m_\delta^2} \frac{G^2 g_{\eta\gamma\gamma}^{\;2}}{[(k_1+k_2)^2 - m_\eta^2]} \epsilon_{\mu\nu\lambda\sigma} k_1^\mu \epsilon_1^\nu k_2^\lambda \epsilon_2^\sigma , \quad (5.2)$$

and for the contribution of Fig. 4(b)

$$\mathfrak{M}_b = -\frac{1.1}{m_\delta^2} \frac{G g_{\rho\pi\gamma} g_{\rho\omega\delta}\, e}{2\gamma} \frac{1}{[(q_1+k_1)^2 - m_\rho^2]} \times \epsilon_{\mu\nu\lambda\sigma} q_1^\mu k_1^\nu \epsilon_1^\lambda \times [k_2^\sigma \epsilon_2 \cdot (q_1+k_1) - \epsilon_2^\sigma k_2 \cdot (q_1+k_1)] . \quad (5.3)$$

In (5.2), the η-γ-γ vertex is given by

$$g_{\eta\gamma\gamma} \epsilon_{\mu\nu\lambda\sigma} k_1^\mu \epsilon_1^\nu k_2^\lambda \epsilon_2^\sigma . \quad (5.4)$$

The $\eta \to \gamma\gamma$ decay rate is

$$\Gamma(\eta \to \gamma\gamma) = \frac{g_{\eta\gamma\gamma}^{\;2}}{64\pi} m_\eta^{\;3} , \quad (5.5)$$

and from the data[3] one obtains $g_{\eta\gamma\gamma} = 0.020\ \mathrm{GeV}^{-1}$. Similarly, the ρ-π-γ vertex in (5.3) can be written as

$$g_{\rho\pi\gamma} \epsilon_{\mu\nu\lambda\sigma} q^\mu k^\nu e_\rho^\lambda \epsilon^\sigma , \quad (5.6)$$

and therefore

$$\Gamma(\rho \to \pi\gamma) = \frac{g_{\rho\pi\gamma}^{\;2}}{96\pi} \frac{(m_\rho^{\;2} - m_\pi^{\;2})^3}{m_\rho^{\;3}} . \quad (5.7)$$

Using the recent value[20] $\Gamma(\rho \to \pi\gamma) = 35$ keV, gives $g_{\rho\pi\gamma} = 0.16\ \mathrm{GeV}^{-1}$.

The matrix elements of the form (5.2) give the following contribution to the decay rate:

$$\Gamma_a(\eta \to \pi\pi\gamma\gamma) = \frac{1.2}{m_\delta^4} \frac{G^4 g_{\eta\gamma\gamma}^{\;2}}{16(2\pi)^5 m_\eta} \int_0^{k_1\,\max} dk_1 \int_0^{k_2\,\max} dk_2 \left[\frac{E_k(E_k - 2k_2) - 4\mu^2}{E_k(E_k - 2k_2)}\right]^{1/2} \times \left[\frac{4k_1 k_2 (m_\eta E_k - 2k_1 k_2)}{(m_\eta E_k - 4k_1 k_2)} + m_\eta E_k \ln\left(1 - \frac{4k_1 k_2}{m_\eta E_k}\right)\right] , \quad (5.8)$$

where

$$E_k = [m_\eta(m_\eta - 2k_1)]^{1/2} ,$$

$$k_{1\,\max} = \frac{m_\eta^{\;2} - 4\mu^2}{2m_\eta} , \quad k_{2\,\max} = \frac{E_k^{\;2} - 4\mu^2}{2E_k} .$$

The integrations are performed numerically with the result

$$\Gamma_a(\eta \to \pi\pi\gamma\gamma) = (5.8 \pm 4.6) \times 10^{-5}\ \mathrm{eV} . \quad (5.9)$$

The integrals obtained for the contributions from matrix elements of the form (5.3) and the cross terms between (5.2) and (5.3) are much more complicated and are given in the Appendix. The results are

$$\Gamma_b(\eta \to \pi\pi\gamma\gamma) = (2.2 \pm 1.8) \times 10^{-5}\ \mathrm{eV} ,$$
$$\Gamma_{ab}(\eta \to \pi\pi\gamma\gamma) = (4.7 \pm 3.8) \times 10^{-5}\ \mathrm{eV} , \quad (5.10)$$

where Γ_b is calculated from (5.3) and Γ_{ab} is calculated from the cross terms. Thus, the total contribution to this decay in the δ model is

$$\Gamma(\eta \to \pi^+\pi^-\gamma\gamma) = (1.3 \pm 1.0) \times 10^{-4}\ \mathrm{eV} , \quad (5.11)$$

which is four orders of magnitude smaller than the present experimental upper limit[3] of 1.7 eV.

A previous calculation of this decay rate using current algebra has been made by Dreitlein and Mahanthappa.[21] They quote their result as

$$\frac{\Gamma(\eta\to\pi^+\pi^-\gamma\gamma)}{\Gamma(\eta\to\pi^+\pi^-\pi^0)}\simeq 0.0086\,,$$

which is equal to the present experimental upper limit of this ratio. The accuracy of this result is doubtful, however, since the authors used a constant matrix element and ignored the dynamics of the pions.

For the sake of comparison, we also estimate the contribution from the vector-dominance model to $\eta\to\pi^+\pi^-\gamma\gamma$ by assuming that for charged pions the dominant process is $\eta\to\rho^+\rho^-$, with each ρ then decaying to $\pi\gamma$. We eliminate the η-ρ-ρ coupling by comparing the rate for this decay to the rate for $\eta\to\pi^+\pi^-\gamma$ in vector dominance, where the main contribution comes from $\eta\to\rho\rho$ with one ρ decaying to $\pi^+\pi^-$ and the other ρ converting to a photon. For the purposes of this estimate, we ignore the details of the interaction and consider only four- and three-body phase space which have values $4.6\times10^{-7}\ \mu^3$ and $8.5\times10^{-5}\ \mu$, respectively. We therefore estimate the ratio of rates in vector dominance to be

$$\frac{\Gamma(\eta\to\pi^+\pi^-\gamma\gamma)}{\Gamma(\eta\to\pi^+\pi^-\gamma)}\sim 5.4\times10^{-3}\frac{g_{\rho\pi\gamma}{}^4\mu^4}{g_{\rho\pi\pi}{}^2(\alpha/4)(4\pi/\gamma_\rho{}^2)}$$

$$\sim 1.2\times10^{-8}\,. \tag{5.12}$$

The decay rate for $\eta\to\pi^+\pi^-\gamma$ is compatible with VDM and we therefore use the experimental value[3] of 42 eV in (5.12). The resulting vector-dominance estimate is $\Gamma(\eta\to\pi^+\pi^-\gamma\gamma)\simeq5\times10^{-7}$ eV, which is more than two orders of magnitude smaller than the δ-model results in (5.11).

VI. SUMMARY AND CONCLUSIONS

We have shown that a generalized vector-meson-dominance scheme, which includes the effects of SU(3)-symmetry breaking, cannot explain the observed decay rate for $\eta\to\pi^0\gamma\gamma$. Confronted by this failure, we have studied the role which scalar mesons play in the radiative decays of the η meson. In particular, we have focused our attention on the three decay modes $\eta\to\pi^0\gamma\gamma$, $\eta\to\pi^+\pi^-\pi^0\gamma$, and $\eta\to\pi^+\pi^-\gamma\gamma$ and on the contributions to these decays from the δ meson.

Using the experimental width for $\eta\to\pi^0\gamma\gamma$ and assuming δ-meson dominance for this decay, we first determined the δ-γ-γ coupling which is essential in applying the δ-meson model to other radiative decay modes. We then proceeded to calculate the decay widths for $\eta\to\pi^+\pi^-\pi^0\gamma$ and $\eta\to\pi^+\pi^-\gamma\gamma$ based on the δ-meson model. We found that in both cases the predicted rates are at least two orders of magnitude greater than the rates calculated in the vector-meson-dominance model. In addition, the δ-model prediction for the width of $\eta\to\pi^+\pi^-\pi^0\gamma$ is one order of magnitude below the current experimental upper limit, whereas in the case of $\eta\to\pi^+\pi^-\gamma\gamma$ the same model predicts a width which is about four orders of magnitude below the experimental upper limit.

We thus conclude that the δ scalar meson plays a major role in the dynamics of the radiative decays $\eta\to\pi^0\gamma\gamma$, $\eta\to\pi^+\pi^-\pi^0\gamma$, and $\eta\to\pi^+\pi^-\gamma\gamma$, whereas the generalized vector-dominance scheme cannot adequately describe these processes.

APPENDIX

We give the integrals associated with calculating the decay rate for $\eta\to\pi^+\pi^-\gamma\gamma$ from (5.3) and the cross terms between (5.2) and (5.3).

The decay rate from (5.3) is given by

$$\Gamma_b(\eta\to\pi\pi\gamma\gamma)=\frac{1.2}{m_\delta{}^4}\,\frac{G^2g_{\rho\omega\delta}{}^2g_{\rho\pi\gamma}{}^2e^2}{128(2\pi)^5m_\eta\gamma^2}\int_0^{k_{1\,\max}}k_1dk_1\int_0^{k_{2\,\max}}k_2dk_2\int_0^2F(y,k_1,k_2)dy\,, \tag{A1}$$

with

$$k_{1\,\max}=\frac{m_\eta{}^2-4\mu^2}{2m_\eta}\,,\quad k_{2\,\max}=\frac{E_k{}^2-4\mu^2}{2E_k}\,, \tag{A2}$$

$$E_k=[m_\eta(m_\eta-2k_1)]^{1/2}\,, \tag{A3}$$

$$F(y,k_1,k_2)=\sum_{i=1}^{9}C_iI_i\,, \tag{A4}$$

where

$$C_1=\frac{m_\eta{}^2k_1{}^2k_2{}^2}{E_k{}^4}[E_k{}^6-2E_k{}^4(E_kk_2+2\mu^2)y+E_k{}^2(4m_\eta{}^2k_1{}^2+k_2{}^2E_k{}^2+4\mu^2k_2E_k+8\mu^2m_\eta k_1+8\mu^4)y^2$$
$$-8m_\eta k_1k_2E_k(m_\eta k_1+\mu^2)y^3+4m_\eta{}^2k_1{}^2k_2{}^2y^4]\,, \tag{A5}$$

$$C_2=\frac{2m_\eta k_1k_2{}^2}{E_k{}^3}[E_k{}^5-E_k{}^3(k_2E_k+2\mu^2)y+4m_\eta k_1E_k(m_\eta k_1+\mu^2)y^2-4m_\eta{}^2k_1{}^2k_2y^3]\,, \tag{A6}$$

$$C_3=\frac{2m_\eta{}^2k_1{}^2k_2}{E_k{}^2}[E_k{}^3-2E_k(k_2E_k+\mu^2)y+k_2(k_2E_k+2\mu^2)y^2]\,, \tag{A7}$$

$$C_4=\frac{k_2{}^2}{E_k{}^2}(E_k{}^4+4m_\eta{}^2k_1{}^2y^2)\,, \tag{A8}$$

$$C_5=\frac{m_\eta{}^2k_1{}^2}{E_k{}^2}(E_k{}^2-2k_2E_ky+k_2{}^2y^2)\,, \tag{A9}$$

$$C_6=\frac{4m_\eta k_1k_2}{E_k}[E_k{}^2-(k_2E_k+\mu^2)y]\,, \tag{A10}$$

$$C_7=2k_2E_k\,, \tag{A11}$$

$$C_8=\frac{2m_\eta k_1}{E_k}(E_k-k_2y)\,, \tag{A12}$$

$$C_9=1\,, \tag{A13}$$

and

$$I_1=\frac{2b}{A_1}\,, \tag{A14}$$

$$I_2=\frac{1}{A_1}(B-2ab)\,, \tag{A15}$$

$$I_3=-\frac{A_3}{A_1}I_2\,, \tag{A16}$$

$$I_4=\frac{2}{A_1}(\varphi A_1-aB+a^2b)\,, \tag{A17}$$

$$I_5=\frac{b}{A_1{}^3}(A_1{}^2A_2{}^2-A_3{}^2)\varphi+\left(\frac{3A_3{}^2-A_1{}^2A_2{}^2}{A_1{}^4}\right)\frac{I_4}{2}\,, \tag{A18}$$

$$I_6=-\frac{A_3}{A_1{}^2}I_4\,, \tag{A19}$$

$$I_7=-\frac{aA_3}{A_1{}^3}(3aB-4A_1\varphi-2a^2b)\,, \tag{A20}$$

$$I_8=\frac{1}{2}\frac{A_2{}^2}{A_1{}^3}[(\varphi^2A_1{}^2-3a^2)B+6a\varphi A_1]+\frac{1}{2}\frac{A_3{}^2}{A_1{}^5}[(9a^2-\varphi^2A_1{}^2)B-14a\varphi A_1-4a^3b]\,, \tag{A21}$$

$$I_9=\frac{\varphi^2}{A_1{}^3}(A_1{}^2A_2{}^2-A_3{}^2)(\varphi A_1-aB+a^2b)-\frac{1}{A_1{}^5}(A_1{}^2A_2{}^2-3A_3{}^2)[\tfrac{1}{3}\varphi^2A_1{}^2b(4a^2-\varphi^2A_1{}^2)+4a^2\varphi A_1-2a^3B]\,, \tag{A22}$$

and

$$a=\mu^2-m_\rho{}^2+m_\eta k_1-\frac{m_\eta k_1k_2y}{E_k}\,, \tag{A23}$$

$$A_1=m_\eta k_1-\frac{m_\eta k_1k_2y}{E_k}\,, \tag{A24}$$

$$A_2=k_2E_k\,, \tag{A25}$$

$$A_3=-m_\eta k_1k_2E_k(1-y)-m_\eta k_1k_2{}^2y\,, \tag{A26}$$

$$\varphi=\left[\frac{E_k(E_k-2k_2)-4\mu^2}{E_k(E_k-2k_2)}\right]^{1/2}\,, \tag{A27}$$

$$B=\ln\left(\frac{a+A_1\varphi}{a-A_1\varphi}\right)\,, \tag{A28}$$

$$b=\frac{\varphi A_1}{a^2-\varphi^2A_1{}^2}\,. \tag{A29}$$

The integrals are performed numerically with the result given as Γ_b in (5.10).

The contribution from (5.2) and (5.3) cross terms is

$$\Gamma_{ab}(\eta\to\pi\pi\gamma\gamma)=\frac{1.2}{m_\delta^{\,4}}\,\frac{G^3 g_{\eta\gamma\gamma}g_{\rho\omega\delta}g_{\rho\pi\gamma}e}{8(2\pi)^5 m_\eta\gamma}\,(J_1+J_2)\,, \tag{A30}$$

where

$$J_1=\int_0^{k_1\,\max}dk_1\int_0^{k_2\,\max}dk_2\frac{m_\eta\varphi}{4E_k}\left[8k_1^{\,2}k_2^{\,2}+4E_k m_\eta k_1k_2+E_k^{\,2}m_\eta^{\,2}\ln\left(1-\frac{4k_1k_2}{m_\eta E_k}\right)\right], \tag{A31}$$

$$J_2=\int_0^{k_1\,\max}k_1dk_1\int_0^{k_2\,\max}k_2dk_2\int_0^2\frac{k_1^{\,2}k_2^{\,2}y^2}{(2k_1k_2y-m_\eta^{\,2})}\,\frac{m_\rho^{\,2}}{A_1}\ln\left(\frac{a+\varphi A_1}{a-\varphi A_1}\right). \tag{A32}$$

The results of these integrals are given as Γ_{ab} in (5.10).

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