PHYSICAL REVIEW D VOLUME 16, NUMBER 1 1 JULY 1977

Meson-baryon scattering lengths in a chiral Lagrangian model

A. M. Harun ar Rashid and K. A. Muttalib
Department of Theoretical Physics, University of Dacca, Dacca, Bangladesh
(Received 19 January 1977)

The chiral $SU(3)\times SU(3)$ Lagrangian including $(3,\bar{3})+(\bar{3},3)$-type symmetry breaking is applied to the investigation of meson-baryon scattering lengths. The effect of the $N^*(1236)$ is taken into account. It is shown that a consistent picture of pion-nucleon and K-meson–nucleon scattering lengths can be obtained for a range of values of the arbitrary parameter Z occurring in the πNN^* coupling. The value $Z=0$ is required by the asymptotic behavior of a certain combination of the pion-nucleon amplitudes, whereas $Z=-1/2$ seems to be preferred by the P-wave pion-nucleon scattering lengths. The value of the pion-nucleon coupling constant f^2 is also calculated as a function of Z.

I. INTRODUCTION

Since the introduction of the phenomenological chiral Lagrangians by Weinberg,[1] a great many applications of this method have been made in describing low-energy processes.[2] While the meson-meson scattering processes can be adequately described in this formalism, its application to meson-baryon processes has not met with unqualified success.

The basic $SU(3)\times SU(3)$-invariant chiral Lagrangian suitable for meson-baryon scattering was written down by Weinberg, Wess and Zumino, Gürsey, and others, but it was Peccei[3] who first made a systematic study of the low-energy pion-nucleon scattering parameters in the chiral Lagrangian formulation. He, in fact, extended the model by including a $\pi NN^*(1236)$ interaction which allowed him to calculate the N^* contribution to the S- and P-wave scattering lengths. However, his results for the P-wave scattering length in the isospin-odd and $J=\frac{1}{2}$ state and for the S-wave scattering length in the isospin-even state were rather unsatisfactory. He pointed out that a possible explanation of the S-wave result might be a failure to include a contact term for the isospin-even amplitude. This contact term was believed to be present in the phenomenological Lagrangian so as to cancel the unacceptable asymptotic behavior of the N^* amplitude.

More recently, new low-energy πN differential elastic cross-section data[4] have become available, making it possible to obtain more accurate values of the low-energy pion-nucleon scattering parameters.[5] Olsson and Osypowski[6] have constructed a simple current-algebra model for πN elastic scattering including the usual σ term, the nucleon Born term, a ρ exchange term, the N^* contribution, and another term called the diffractive term. This diffractive term in the B^+ amplitude is supposed to account for diffractive effects usually considered only at higher energies. It is interesting to observe that this diffractive term contributes 0.034 to a_S^+ and the σ term contributes 0.012, whereas the nucleon and the Δ total of -0.006. This is exactly analogous to the calculation of Peccei,[3] who found that the nucleon and Δ contributions to a_S^+ are -0.011 and -0.050, so that in order to obtain a scattering length $a_S^+=-0.001$, he needed a contact-term contribution of $+0.060$.

The question therefore naturally arises whether this contact term, in the language of Peccei,[3] or the diffractive term, in the language of Olsson and Osypowski,[6] is really necessary or whether there is an alternative way of explaining the a_S^+ scattering length in the current-algebra or chiral Lagrangian framework. The diffractive term, as emphasized by Olsson and Osypowski, is necessary to explain the detalied structure of the pion-nucleon scattering amplitude, but if we restrict ourselves to the consideration of *only* the scattering lengths, a chiral Lagrangian model should suffice without any *ad hoc* terms since after all the chiral Lagrangians were designed specifically for this purpose. It appears to us that a systematic way of studying this question is to write down the most general chiral Lagrangian including the symmetry-breaking terms. Indeed, we will show that the extra contribution to the a_S^+ scattering length needed to cancel the large-N^* contribution can indeed come from symmetry-breaking effects without disturbing any of the good results of Peccei and others on other amplitudes.

Since Gell-Mann[7] proposed the idea of an approximate chiral symmetry of the strong interactions, the most popular approach has been to assume that the symmetry-breaking interaction Hamiltonian belongs to the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$. Although several authors[8] have recently added a term belonging to the $(8,8)$ representation, we will confine our discussion here to the $(3,\bar{3})$

$+(\bar{3},3)$ which is motivated in part by analogy with the structure of mass terms in the quark model. However, the phenomenological chiral Lagrangian will be written down following the very general treatment given by McDonald and Rosen.[9] Indeed, they have written down the chiral Lagrangian for the meson-baryon scattering processes involving all possible types of symmetry breaking but unfortunately the number of parameters in this theory is much too large. Although one hopes to be able to determine these parameters from the masses of the baryons and the S-wave pion-nucleon and K-meson-nucleon scattering lengths, it is difficult to do so in view of the uncertainties of these scattering lengths.

There is an additional complication due to an unknown parameter in the πNN^* interaction Lagrangian which has recently attracted considerable attention[10] in the literature. This unknown parameter Z was taken to be equal to $-\frac{1}{4}$ by Peccei, while other authors have preferred it to be $+\frac{1}{2}$ or $-\frac{1}{2}$. It is one of our purposes to try to fix this parameter from three different angles. First, we will show that the asymptotic behavior of the pion-nucleon scattering amplitudes can constrain the value of Z, although unfortunately not uniquely. Secondly, we will reexamine the Adler-Weisberger theorem following Brown, Pardee, and Peccei[11] and calculate the pion-nucleon coupling constant as a function of Z. Again unfortunately, we will show that the dependence of Z is so weak that no firm conclusion can be drawn from this approach either. Thirdly, we will calculate the S-wave pion-nucleon and K-meson-nucleon scattering lengths using the effective Lagrangian and then predict the P-wave pion-nucleon scattering lengths. Again, we will find that the scattering lengths can not strongly constrain the value of Z.

However, taking all these approaches together, we will try to show that a value of Z equal to zero is not inconsistent with all the available experimental data. This value $Z=0$ has the great merit that it removes the necessity of the contact term advocated by Peccei, without destroying any of the beautiful features of this effective-Lagrangian model. This is achieved by simply attributing the correction to the scattering length to the symmetry-breaking terms which were not, to our knowledge, included in phenomenological-Lagrangian calculations before.

II. THE EFFECTIVE LAGRANGIAN

For meson-baryon scattering, the SU(3) × SU(3)-invariant chiral Lagrangian is, up to second-order terms,

$$-L_{\rm inv}=\bar{B}(\gamma\cdot\partial+m_0)B+\frac{1}{2F_\pi^2}\bar{B}_l\gamma_\mu if_{iml}B_m if_{ijk}M_j\partial^\mu M_k+\frac{1}{F_0}(d_{ijk}D+if_{ijk}F)\bar{B}_i\gamma_\mu\gamma_5\partial^\mu M_k\,, \tag{1}$$

where B and M are the usual 3×3 baryon and pseudoscalar-meson matrices, respectively, F_0 is the pion-decay constant, and D/F_0 and F/F_0 are the symmetric and antisymmetric coupling constants.

The symmetry-breaking Lagrangian is

$$\begin{aligned}-L_{\rm SB}=m_0\Bigg\{&\bar{B}B\left[\gamma\left(1-\frac{m_0X}{8F_\pi^2}\right)+\gamma'\,\frac{\Pi_8}{F_\pi^2}\right]+\bar{B}(\mu D_k+\nu F_k)B\,\frac{\Pi_k}{F_\pi^2}\\&+\bar{B}(\mu' D_k+\nu' F_k)B\left[\delta_{8k}\left(1+\frac{X}{60F_\pi^2}(11-6m_2)\right)+\frac{1}{4F_\pi^2}d_{8kc}\,\Pi_c+\frac{1}{10F_\pi^2}M_8M_k(-2m_2+\tfrac{1}{3})\right]\\&+\bar{B}(\mu'' D_k+\nu'' F_k)B\left[\delta_{8k}\left(1+\frac{11X}{180F_\pi^2}(-2m_2+3)\right)+\frac{M_8M_k}{90F_\pi^2}(-2m_2+3)+d_{8kc}\frac{\Pi_c}{F_\pi^2}(\tfrac{2}{15}m_2+\tfrac{1}{4})\right.\\&\left.+\frac{8m_3^2}{5m_2(2m_2+3)}\left(\frac{X\delta_{8k}}{6F_\pi^2}-\frac{4M_8M_k}{3F_\pi^2}-d_{8kc}\frac{\Pi_c}{F_\pi^2}\right)\right]\Bigg\}\,,\end{aligned} \tag{2}$$

where

$$X=M_iM_i,\quad Y=d_{ijk}M_iM_jM_k,\quad \Pi_i=d_{ikl}M_kM_l\,,$$

$(F_k)_{ij}=-if_{kij}$, $(D_k)_{ij}=d_{kij}$, and m_2, m_3 are Casimir eigenvalues,

$$m_2=\tfrac{1}{3}[\mu_1^2+\mu_2^2+(\mu_1+\mu_2)^2+6(\mu_1+\mu_2)],\quad m_3=\tfrac{1}{9}(\mu_2-\mu_1)\,[(\mu_1+2\mu_2)(\mu_2+2\mu_1)+9(\mu_1+\mu_2+1)]\,,$$

for the representation (μ_1,μ_2) of SU(3).

From (1), the relevant Yukawa couplings are

$$-L_{NN\pi}=f_{NN\pi}\bar{N}\gamma_\mu\gamma_5\vec{\tau}\cdot\partial^\mu\vec{\pi}N\,,\quad -L_{N\Lambda K}=f_{N\Lambda K}\bar{N}\gamma_\mu\gamma_5\partial^\mu K\Lambda+\text{H.c.}\,,\quad -L_{N\Sigma K}=f_{N\Sigma K}\bar{N}\gamma_\mu\gamma_5\vec{\tau}\cdot\vec{\Sigma}\partial^\mu K+\text{H.c.}\,, \tag{3}$$

where

$$f_{NN\pi}=\frac{F+D}{\sqrt{2}F_0},\quad f_{N\Lambda K}=-\frac{1+2\alpha}{\sqrt{3}}f_{NN\pi},\quad f_{N\Sigma K}=(1-2\alpha)f_{NN\pi},$$

and

$$\alpha=\frac{F}{F+D}.$$

Also, we have the contact terms from (1):

$$-L_{NN\pi\pi}=\frac{1}{4F_\pi^2}\bar{N}\gamma_\mu\vec{\tau}\cdot\vec{\pi}\,\overleftrightarrow{\partial}_\mu(\vec{\tau}\cdot\vec{\pi})N,\qquad -L_{NNKK}=\frac{1}{2F_\pi^2}(\bar{N}\gamma_\mu K\overleftrightarrow{\partial}_\mu K^+N-\bar{N}\gamma_\mu NK^+\overleftrightarrow{\partial}_\mu K).\tag{4}$$

The relevant symmetry-breaking Lagrangians are from (2),

$$-L_{SB}=\frac{m_0}{F_\pi^2}(\bar{N}N)(\underline{\pi}\cdot\underline{\pi})\left[-\frac{m_2}{8}\gamma+\frac{1}{\sqrt{3}}\gamma'+\frac{-\mu+3\nu}{6}+\frac{-\mu'+3\nu}{20\sqrt{3}}(1-m_2)+\frac{-\mu''+3\nu''}{20\sqrt{3}}\left(1-\tfrac{5}{3}m_2+\frac{8m_3^2}{m_2(2m_2+3)}\right)\right]\tag{5}$$

and

$$-L_{SB}(KN)=\frac{m_0}{F_\pi^2}(\bar{N}N)K^+K\left[-\frac{m_0}{4}\gamma-\frac{1}{\sqrt{3}}\gamma'+\frac{-\mu+3\nu}{6}+\frac{-\mu'+3\nu'}{10\sqrt{3}}(\tfrac{9}{4}-m_2)+\frac{-\mu''+3\nu''}{10\sqrt{3}}(\tfrac{9}{4}-m_2)\right]$$
$$+\frac{m_0}{F_\pi^2}(\bar{N}\vec{\tau}N)\cdot(K^+\vec{\tau}K)\left[\frac{\mu+\nu}{2}+\frac{\mu'+\nu'}{8\sqrt{3}}+\frac{\mu''+\nu''}{2\sqrt{3}}\left(\tfrac{2}{15}m_2+\tfrac{1}{4}-\frac{8m_3^2}{5m_2(2m_2+3)}\right)\right].\tag{6}$$

On the other hand, the mass term is

$$-L_{\text{mass}}=m_0[\bar{B}B(1+\gamma)+(\mu'+\mu'')\bar{B}D_8B+(\nu'+\nu'')\bar{B}F_8B].\tag{7}$$

From (7) we identify the baryon masses as

$$m_N=m_0\left[(1+\gamma)-\frac{\mu'+\mu''}{2\sqrt{3}}+\frac{\sqrt{3}}{2}(\nu'+\nu'')\right],$$

$$m_\Xi=m_0\left[(1+\gamma)-\frac{\mu'+\mu''}{2\sqrt{3}}-\frac{\sqrt{3}}{2}(\nu'+\nu'')\right],$$

$$m_\Sigma=m_0\left[(1+\gamma)+\frac{\mu'+\mu''}{\sqrt{3}}\right],$$

$$m_\Lambda=m_0\left[(1+\gamma)-\frac{\mu'+\mu''}{\sqrt{3}}\right].\tag{8}$$

These expressions satisfy the Gell-Mann–Okubo mass formula as expected,

$$\frac{m_N+m_\Xi}{2}=\frac{m_\Sigma+3m_\Lambda}{4}.$$

Clearly there are too many parameters for our comfort and it is at this stage we make the assumption that the symmetry breaking occurs in $(m,\bar{m})$ with a triangular representation of SU(3). Specifically we assume that the symmetry-breaking Lagrangian belongs to the $(3,\bar{3})+(\bar{3},3)$ representation so that we have $\mu_1=1$, $\mu_2=0$ and hence $m_2=\frac{8}{3}$, $m_3=-\frac{20}{9}$. In this case, because of the relation

$$\frac{8m_3^2}{m_2(2m_2+3)}=\tfrac{2}{3}m_2,\tag{9}$$

the parameters μ'' and ν'' become redundant, as we can easily see from (5) and (6) and hence we can replace $\mu'+\mu''$ and $\nu'+\nu''$ by two parameters $\bar{\mu}$ and $\bar{\nu}$, respectively.

Thus in the case of $(3,\bar{3})+(\bar{3},3)$ symmetry-breaking, we have

$$-L_{SB}(\pi N)=\frac{m_0}{F_\pi^2}(\bar{N}N)\vec{\pi}\cdot\vec{\pi}\times\left(\frac{-\gamma+\sqrt{3}\gamma'}{3}+\frac{-\mu+3\nu}{6}-\frac{-\bar{\mu}+3\bar{\nu}}{12\sqrt{3}}\right)$$

$$-L_{SB}(KN)=\frac{m_0}{F_\pi^2}\bar{N}NK^+K\times\left(\frac{-2\gamma+\sqrt{3}\gamma'}{3}+\frac{-\mu+3\nu}{6}-\frac{\bar{\mu}+3\bar{\nu}}{24\sqrt{3}}\right)+\frac{m_0}{F_\pi^2}(\bar{N}\vec{\tau}N)\cdot(K^+\vec{\tau}K)\left(\frac{\mu+\nu}{2}+\frac{\bar{\mu}+\bar{\nu}}{8\sqrt{3}}\right)\tag{10}$$

and the masses are given by

$$m_N = m_0 + m_0\gamma + \frac{-m_0\bar{\mu} + 3m_0\bar{\nu}}{2\sqrt{3}},$$
$$m_\Xi = m_0 + m_0\gamma - \frac{m_0\bar{\mu} + 3m_0\bar{\nu}}{2\sqrt{3}},$$
$$m_\Sigma = m_0 + m_0\gamma + \frac{m_0\bar{\mu}}{\sqrt{3}}, \qquad (11)$$
$$m_\Lambda = m_0 + m_0\gamma - \frac{m_0\bar{\mu}}{\sqrt{3}}.$$

The four mass relations involve four unknown parameters, but on account of the Gell-Mann–Okubo mass formula, only three of these parameters can be determined:

$$m_0(1+\gamma) = \frac{m_\Sigma + m_\Lambda}{2} = 1.1540,$$
$$m_0\bar{\mu} = 0.0666,$$
$$m_0\bar{\nu} = -0.2192.$$

III. THE SCATTERING AMPLITUDES

Using the symmetric effective Lagrangians given above, the contributions to the invariant amplitudes A and B for meson-baryon scattering are easily obtained from the relevant Feynman diagrams. We get

$$A^+(\pi N) = 4m_N f_{NN\pi}^2,$$
$$A^-(\pi N) = 0,$$
$$B^+(\pi N) = -4m_N^2 f_{NN\pi}^2\left(\frac{1}{s-m_N^2} - \frac{1}{u-m_N^2}\right), \qquad (12)$$
$$B^-(\pi N) = \frac{1}{2F_\pi^2} - 2f_{NN\pi}^2 - 4m_N^2 f_{NN\pi}^2\left(\frac{1}{s-m_N^2} + \frac{1}{u-m_N^2}\right),$$

and

$$A(K^+p) = \sum_{y=\Lambda,\Sigma^0} f_{yNK}^2\left[m_N + m_y + \frac{(m_y - m_N)(m_y + m_N)^2}{u - m_y^2}\right],$$
$$B(K^+p) = -\frac{1}{F_\pi^2} + \sum_y f_{yNK}^2\left[1 + \frac{(m_N + m_y)^2}{u - m_y^2}\right], \qquad (13)$$
$$A(K^+n) = f_{\Sigma NK}^2\left[m_N + m_\Sigma + \frac{(m_\Sigma - m_N)(m_\Sigma + m_N)^2}{u - m_\Sigma^2}\right],$$
$$B(K^+n) = -\frac{1}{2F_\pi^2} + f_{\Sigma NK}^2\left[1 + \frac{(m_\Sigma + m_N)^2}{u - m_\Sigma^2}\right],$$

where s, t, u are the usual Mandelstam variables.

The symmetry-breaking Lagrangian give

$$A^+(\pi N) = \frac{2m_0}{F_\pi^2}\left(\frac{-\gamma + \sqrt{3}\gamma'}{3} + \frac{-\mu + 3\nu}{6} - \frac{-\bar{\mu} + 3\bar{\nu}}{12\sqrt{3}}\right),$$
$$A^-(\pi N) = B^+(\pi N) = B^-(\pi N) = 0. \qquad (14)$$

and

$$A_1(KN) = \frac{m_0}{F_\pi^2}\left(-\frac{2\gamma + \sqrt{3}\gamma'}{3} + \frac{\mu + 3\nu}{3} + \frac{\bar{\mu}}{6\sqrt{3}}\right),$$
$$(15)$$
$$A_0(KN) = \frac{m_0}{F_\pi^2}\left(-\frac{2\gamma + \sqrt{3}\gamma'}{3} - \frac{5\mu + 3\nu}{3} - \frac{2\bar{\mu} + 3\bar{\nu}}{6\sqrt{3}}\right),$$

where the subscripts in the KN amplitudes denote isospin. Using these scattering amplitudes, we can obtain the pion-nucleon and K-meson-nucleon scattering lengths. But before we do this, let us calculate the contribution of the spin-$\frac{3}{2}$ resonance. We shall write the πNN^* interaction Lagrangian in the form

$$L_{N^*N\pi} = \frac{4h}{m_\pi}\bar{\psi}^\mu\theta_{\mu\nu}\psi\partial^\nu\phi, \qquad (16)$$

where

$$\theta_{\mu\nu} = g_{\mu\nu} + a\gamma_\mu\gamma_\nu \qquad (17)$$

with $a = -\frac{1}{2}(1+2z)$ and the spin-$\frac{3}{2}$ propagator in the form

$$\mathcal{P}_{\mu\nu}(P) = \frac{(\not{P} + M)}{P^2 - M^2}\left[-g_{\mu\nu} + \frac{2}{3M^2}P_\mu P_\nu + \tfrac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M}(\gamma_\mu P_\nu - \gamma_\nu P_\mu)\right]. \qquad (18)$$

This gives

$$A^\pm = \frac{16h^2}{3m_\pi^2}\begin{pmatrix}2\\-1\end{pmatrix}\left[\frac{(M+m_N)q_R^2x + \frac{1}{3}(M-m_N)(E_R+m_N)^2}{s-M^2} + (a_0 + a_1 s) \pm (s \to u)\right],$$
$$(19)$$
$$B^\pm = \frac{16h^2}{3m_\pi^2}\begin{pmatrix}2\\-1\end{pmatrix}\left[\frac{q_R^2x - \frac{1}{3}(E_R+m_N)^2}{s-M^2} + (b_0 + b_1 s) \mp (s \to u)\right],$$

where $x = \cos\theta$,

$$a_0 = \frac{1}{6M^2}[2M^3 + m_N^3(-1 + 4Z + 4Z^2) + 3m_N M^2 + 2m_\pi^2(M + m_N) + 4m_N^2 M(Z + 2Z^2)],$$
$$a_1 = -\frac{2}{3M^2}[m_N(Z^2 + Z) + M(2Z^2 + Z)],$$
$$b_0 = \frac{1}{6M^2}[(M+m_N)^2 + 2m_N M(4Z + 8Z^2) + m_N^2(8Z + 4Z^2) - 4m_\pi^2 Z],$$
$$b_1 = \frac{2}{3M^2}Z^2,$$

and q_R and E_R are the pion-nucleon center-of-mass momentum and nucleon energy at the resonance energy.

Using these results, it is straightforward to calculate the S-wave and P-wave pion-nucleon scattering lengths, and we obtain

$$m_\pi a_S^- = \frac{1}{4\pi(1+m_\pi/m_N)}\left(\frac{2f^2}{4m_N^2/m_\pi^2-1}+\frac{m_\pi^2}{2F_\pi^2}\right)+\frac{16h^2m_\pi^2}{36\pi M^2(1+m_\pi/m_N)}(1-2Z)^2,$$

$$m_\pi a_S^+ = -\frac{1}{4\pi(1+m_\pi/m_N)}\frac{4m_N m_\pi f^2}{4m_N^2-m_\pi^2}-\frac{1}{4\pi(1+m_\pi/m_N)}\frac{128h^2m_\pi}{9M^2}[M(1-Z-2Z^2)+m_N(\tfrac{1}{4}+Z^2-Z)]$$
$$-\frac{1}{4\pi(1+m_\pi/m_N)}\frac{2m_\pi}{F_\pi^2}m_0\left(\frac{\gamma-\sqrt{3}\gamma'}{3}+\frac{\mu-3\nu}{6}+\frac{-\bar{\mu}+3\bar{\nu}}{12\sqrt{3}}\right), \qquad (20)$$

$$m_\pi a_{P\,3/2}^- = -\frac{2}{3}\frac{1}{4\pi(1+m_\pi/m_N)}\frac{4m_N^2m_\pi^2f^2}{[(m_N-m_\pi)^2-m_N^2]}$$
$$+\frac{2}{3}\frac{1}{4\pi(1+m_\pi/m_N)}\frac{16h^2m_\pi}{3}\left[\frac{1}{2}\left(\frac{1}{m_N+m_\pi-M}-\frac{1}{m_N-m_\pi-M}\right)+\frac{f^-(Z)}{m_N-m_\pi-M}\right],$$

$$m_\pi a_{P\,3/2}^+ = \frac{2}{3}\frac{1}{4\pi(1+m_\pi/m_N)}\frac{4m_N^2m_\pi^2f^2}{[(m_N-m_\pi)^2-m_N^2]}$$
$$-\frac{2}{3}\frac{1}{4\pi(1+m_\pi/m_N)}\frac{32h^2m_\pi}{3}\left[\frac{1}{2}\left(\frac{1}{m_N+m_\pi-M}+\frac{1}{m_N-m_\pi-M}\right)-\frac{f^+(Z)}{m_N-m_\pi-M}\right], \qquad (21)$$

$$m_\pi a_{P\,1/2}^- = m_\pi a_{P\,3/2}^- - \frac{m_\pi^3}{4m_N^2}a_s^- + \frac{m_\pi}{8\pi m_N}B_0^-,$$

$$m_\pi a_{P\,1/2}^+ = m_\pi a_{P\,3/2}^+ - \frac{m_\pi^3}{4m_N^2}a_s^+ + \frac{m_\pi}{8\pi m_N}B_0^+,$$

where

$$f^-(Z) = \frac{1}{12M^2}[4M^2(1+4Z^2+2Z)-2m_N^2(1+4Z^2+4Z)+2m_\pi^2(-1+4Z^2)$$
$$+m_N m_\pi(-4+8Z)+2m_N M(1-4Z^2)+2m_\pi M(1+12Z^2+4Z)],$$

$$f^+(Z) = f^-(Z),$$

$$B_0^+(Z) = -\frac{16m_N^3f^2}{m_\pi(4m_N^2-m_\pi^2)}$$
$$-\frac{32h^2}{9}\frac{m_N m_\pi}{4M^2q_R^2}\left[2M^2(-1+4Z^2)+4m_N^2(3-4Z^2)+4m_\pi^2(1-4Z^2)+\frac{4m_N}{M}(m_N^2-m_\pi^2)\right.$$
$$\left.+4m_N M+\frac{2(m_N^2-m_\pi^2)^2}{M^2}(-1+4Z^2)\right],$$

$$B_0^-(Z) = \frac{2f^2}{4m_N^2/m_\pi^2-1}+\frac{m_\pi^2}{2F_\pi^2}$$
$$+\frac{16h^2}{9M^4q_R^2}\{(Z^2+Z)(M^2-m_N^2-m_\pi^2)^2(2m_N^2+2M_N M+m_\pi^2)$$
$$+4m_N Z^2[M(M^2-m_N^2-m_\pi^2)^2-(2m_N^2m_\pi^2+m_\pi^4-m_N M^3)]$$
$$+2m_\pi^2Z[2M^2(m_N^2+m_\pi^2)-(M^4+5m_N^4)]+Mm_N[(M^2-m_\pi^2)^2-m_N^2(M^2+m_\pi^2)]$$
$$+\tfrac{1}{2}m_N^2(M^2-m_N^2)(3M^2-m_N^2)+\tfrac{1}{4}m_\pi^2[(M^2-m_\pi^2)^2-m_N^2(3m_N^2+10M^2)]\}.$$

All these results agree precisely with those of Peccei for $Z=-\frac{1}{4}$, as we can easily show by a straightforward, albeit tedious, calculation.

Finally, the S-wave and P-wave K-N scattering lengths are given by

$$m_\pi a_S^1(KN) = -\frac{m_\pi}{4\pi(1+m_K/m_N)}\left[\frac{m_K}{F_\pi^2} + \sum_{y=\Lambda,\Sigma}\frac{m_K^2 f_{yNK}^2}{m_N+m_y-m_K} + \frac{G^2}{m_N-m_{y_1}-m_K} + \frac{m_0}{F_\pi^2}\left(\frac{2\gamma+\sqrt{3}\gamma'}{3} - \frac{\mu+3\nu}{3} - \frac{\bar{\mu}}{6\sqrt{3}}\right) - g_1(Z)\right],$$

$$m_\pi a_S^0(KN) = \frac{m_\pi}{4\pi(1+m_K/m_N}\left[-f_{NK\Lambda}^2\frac{m_K^2}{m_N+m_\Lambda-m_K} + 3f_{NK\Sigma}^2\frac{m_K^2}{m_N+m_\Sigma-m_K} + \frac{G^2}{m_N-m_{y_1}-m_K} - \frac{m_0}{F_\pi^2}\left(\frac{2\gamma+\sqrt{3}\gamma'}{3} + \frac{5\mu+3\nu}{3} + \frac{2\bar{\mu}+3\bar{\nu}}{6\sqrt{3}}\right) + 3g_1(Z)\right],$$

$$m_\pi a_{P\,3/2}^1 = \frac{2}{3}\frac{m_\pi^3}{4\pi(1+m_K/m_N)}\left[\sum_{y=\Lambda,\Sigma}\frac{f_y^2(m_N+m_y)^2}{(m_N+m_y-m_K)^2}\frac{1}{m_y-m_N+m_K} + \frac{G^2}{(m_N-m_{y_1}-m_K)^2}\frac{1}{-m_N-m_{y_1}+m_K} + g_2(Z)\right],\tag{22}$$

$$m_\pi a_{P\,1/2}^1 = m_\pi a_{P\,3/2}^1 - \frac{m_\pi^3}{4m_N^2}a_S^1 + \frac{m_\pi^3}{8\pi m_N}\left\{-\frac{1}{F_\pi^2} + \sum_{y=\Lambda,\Sigma} f_y^2\left[1+\frac{(m_N+m_y)^2}{(m_N-m_K)^2-m_y^2}\right] + \frac{G^2}{(m_N-m_K)^2-m_{y_1}^2} + g_3(Z)\right\},$$

where

$$g_1(Z) = \frac{H^2}{3m_N^2}\left(\frac{(M-m_N-m_K)(4m_N^2-q_R^2)+6Mq_R^2}{(m_N-m_K)^2-M^2} + \frac{1}{2M^2}\{2M^3+3m_NM^2-m_N^3+2m_K^2(M+m_N) + m_K(M+m_N)^2+4Z^2[2Mm_K(4m_N-m_K)+m_N^2(m_N+m_K)-(m_N-m_K)^3] + 4Z[4m_Nm_K(M+m_N)-m_K^2(M+m_N+m_K)]]\}\right),$$

$$g_2(Z) = \frac{H^2}{3m_N^2}\left\{\frac{(M-m_N-m_K)(4m_N^2-q_R^2)+6Mq_R^2}{[(m_N-m_K)^2-M^2]^2} + \frac{\frac{3}{2}(M+m_N+m_K)}{(m_N-m_K)^2-M^2} + \frac{2}{M^2}[(2M+m_N-m_K)Z^2+(M+m_N)Z]\right\},$$

$$g_3(Z) = \frac{H^2}{3m_N^2}\left(\frac{q_R^2-4m_N^2}{(m_N-m_K)^2-M^2} + \frac{1}{2M^2}\{(M+m_N)^2+4Z^2[m_N(m_N+4M)+(m_N-m_K)^2]+4Z[2m_N(m_N+M)-m_K^2]\}\right).$$

IV. ASYMPTOTIC BEHAVIOR OF THE N^* AMPLITUDE

Let us introduce the invariant amplitude A' by

$$A' = A + \frac{s-u}{t-4m_N^2}m_NB,\tag{23}$$

which at $t=0$ reduces to the amplitude $G(s,0)$ defined by Peccei. Now asymptotically

$$A'^{+}(s,0) \to -\frac{32h^2}{9m_\pi^2}\frac{s^2}{m_NM^2}[1+4a(1+a)].\tag{24}$$

For $a=-\frac{1}{4}$ which is the value used by Peccei, we get

$$A'^{+}(s,0) \to -\frac{4h^2s^2}{9m_\pi^2m_NM^2},$$

agreeing with the result of Peccei. If we now demand that $A'^{+}(s,0)$ should go to zero as $s\to\infty$, we must have

$$a=-\tfrac{1}{2},$$

i.e.,

$$Z=0.$$

It should be noted here that the choice $Z=0$ makes the amplitude $A'^{+}(s,0)$ calculated from the Lagrangian model vanish at high energy. This does not, of course, preclude the existence of the diffractive term[6] since the latter is related to nonresonant inelastic scattering.

On the other hand, the asymptotic behavior of $A'^{-}(s,0)$ comes out to be

$$A'^{-}(s,0) = -\frac{8h^2}{9m_\pi^2 m_N M^2}(s - m_N^2 - m_\pi^2) \times [(M+m_N)^2 + m_\pi^2(3+2a)(1+2a)], \quad (25)$$

which agrees with Peccei's result for $a=-\frac{1}{4}$. Thus, following Peccei,[11] we note that the Adler-Weisberger relation holds provided

$$f_0^2 = f^2 - \frac{h^2}{9M^2}[8(M+m_N)^2 + 8m_\pi^2(3+2a)(1+2a)], \quad (26)$$

and using $h^2=0.29$, and $f_0^2=0.722$ corresponding to $g_{\rho\pi\pi}^2/4\pi=3.53$ we find that

$$\left(\frac{f}{f^0}\right)^2 = 2.1006 + 0.004(3+2a)(1+2a).$$

The equation for a is

$$4a^2 + 8a + c = 0$$

where

$$c = 3 + \frac{2.1 - (f/f_0)^2}{0.004}.$$

For real a, we must have $4 \geq c$, which gives

$$(f/f_0)^2 \geq 2.0966.$$

Hence the minimum value of (f/f_0) is 1.4479 which occurs for $a=-1$, i.e., $Z=\frac{1}{2}$. Table I gives f/f_0 as a function of Z. Clearly no value of Z can produce the experimental value of $f/f_0=1.24$ from the asymptotic formula for $A'^{-}(s,0)$ including only the N^* contribution.

TABLE I. f/f_0 as a function of Z.

a	Z	f/f_0
-1	$\frac{1}{2}$	1.448
$-\frac{1}{2}$	0	1.449
$-\frac{1}{4}$	$-\frac{1}{4}$	1.450
0	$-\frac{1}{2}$	1.454

V. CURRENT-ALGEBRA CONSTRAINTS AND THE VALUE OF f^2

Let us now define

$$\nu = \frac{s-u}{4m_N}, \quad \nu_B = \frac{t - m_\pi^2}{4m_N}. \quad (27)$$

Brown, Pardee, and Peccei[11] have derived, from current algebra, constraints on the amplitudes $A^+(\nu,\nu_B)$ and $G^-(\nu,\nu_B)$, where

$$G^-(\nu,\nu_B) = \frac{A^-(\nu,\nu_B)}{\nu} + B^-(\nu,\nu_B).$$

They have shown that current algebra requires

$$A^+(\nu,\nu_B) = \frac{\sigma_{\pi N}(t)}{F_\pi^2} + \frac{g^2}{m_N}\left[1 + \left(\frac{m_\pi}{m_N}\right)^4 a + \frac{2\nu_B}{m_N}b + \left(\frac{\nu}{m_N}\right)^2 c\right], \quad (28)$$

where $\sigma_{\pi N}$ is the pion-nucleon σ sigma term, a, b, c are dimensionless functions and the coupling constant g^2 is related to f^2 by $f^2 = g^2 m_\pi^2/16\pi m_N^2$. At $\nu=0$, $\nu_B=0$, this gives

$$A^+(\nu,\nu_B) = \frac{\sigma_{\pi N}(t=2m_\pi^2)}{F_\pi^2} + \frac{g^2}{m_N}\left[1 + \left(\frac{m_\pi}{m_N}\right)^4 a\right]. \quad (29)$$

The second current-algebra constraint is

$$G^-(\nu,\nu_B) = -\frac{g^2}{2m_N^2} + \frac{1}{2F_\pi^2}F_1^v(t) + \frac{1}{2F_\pi^2}\left[\left(\frac{m_\pi}{m_N}\right)^2 d + \left(\frac{2\nu_B}{m_N}\right)e + \left(\frac{\nu}{m_N}\right)^2 f\right], \quad (30)$$

where $F_1^v(t)$ is the isovector nucleon electromagnetic form factor and d, e, f are dimensionless functions. It is a simple matter to obtain the N^* contributions to $A^+(\nu,\nu_B)$ and $G^-(\nu,\nu_B)$ and we have

$$A_\Delta^+(0,0) = -\frac{32h^2}{9m_\pi^2}\frac{(2M+m_N)m_\pi^4}{M^2(M^2-m_N^2)} \quad (31)$$

and

$$G_\Delta^-(\nu,\nu_B) = \frac{16h^2}{9m_\pi^2 M^2}\Big\{\frac{1}{D}[24m_N^2M^2\nu_B^2 + 2m_N\nu_B(7M^4 + 10M^3m_N - 2Mm_N^3 + 4M^2m_\pi^2 + 2Mm_Nm_\pi + (m_N^2-m_\pi^2)^2) + m_\pi^4(M^2+m_N^2+4m_NM) + 2m_\pi^2(M+m_N)^2(M-m_N)(2M+m_N) + (M-m_N)^2(M+m_N)^4] - [(M+m_N)^2 - 4m_\pi^2 Z - 8m\nu_B Z^2]\Big\}, \quad (32)$$

where $D=(M^2-m_N^2+2m_N\nu_B)^2 - 4m_N^2\nu^2$. This expression agrees with that of Brown *et al.* for $Z=-\frac{1}{4}$. Since $A^+(0,0)$ is independent of Z, the value of the pion-nucleon coupling constant,

$$f^2 = \frac{g^2m_\pi^2}{16\pi m_N^2} = \frac{m_\pi^2}{16\pi m_N}\left[A^+(0,0) - \frac{\sigma_{\pi N}}{F_\pi^2}\right], \quad (33)$$

obtained by Brown *et al.* using Adler's determination of A^{+} and $\sigma_{\pi N} = 105$ MeV remains unchanged, i.e., $f^2 = 0.080$. On the other hand, the use of $G^{-}(\nu, \nu_B)$ for the evaluation of f^2 is dependent on Z. In fact, we find from the expression given above that

$$G^{-}(0,0) = 7.9468 + 1.3565Z\,,$$
$$G^{-}(0, -m_\pi^2/2m_N^2) = -14.8999 + 1.3565Z\,,$$
$$G^{-}(m_\pi, -m_\pi^2/2m_N^2) = 0.3375 + 1.3565Z - 1.3564Z^2\,.$$

The relevant formulas for f^2 are then

$$\text{(i) } f^2 = \frac{m_\pi^2}{16\pi F_\pi^2}\left[1 + 4.16\left(\frac{m_\pi}{m_N}\right)^2 - (6.1531 + 1.0503Z)\left(\frac{m_\pi}{m_N}\right)^2\right] - \frac{m_\pi^2}{8\pi}G^{-}(0,0)$$
$$= 0.0767 - 0.0011Z\,,$$

$$\text{(ii) } f^2 = \frac{m_\pi^2}{16\pi F_\pi^2}[1 + 0.2553 + 0.0232(Z^2 - Z)] - \frac{m_\pi^2}{8\pi}G^{-}(0, -m_\pi^2/2m_N)$$
$$= 0.0774 + 0.0011(Z^2 - Z)\,,$$

$$\text{(iii) } f^2 = \frac{m_\pi^2}{16\pi F_\pi^2}[1 + 0.0058 + 0.0232(Z^2 - Z) - \frac{m_\pi^2}{8\pi}G^{-}(m_\pi, -m_\pi^2/2m_N)$$
$$= 0.0744 + 0.0011(Z^2 - Z)\,.$$

Here we have used the nucleon's form factor as

$$F_1^v(t = 2m_\pi^2) = 1 + 4.16\left(\frac{m_\pi}{m_N}\right)^2, \quad F_1^v(0) = 1\,,$$

and the value of G^{-} used by Brown *et al.*,[12] i.e.,

$$-\frac{m_\pi^2}{8\pi}G(0, -m_\pi^2/2m_N) = 0.0206\,,$$
$$-\frac{m_\pi^2}{8\pi}[G(0,0) - G(0, -m_\pi^2/2m_N)] = 0.0126\,,$$
$$-\frac{m_\pi^2}{8\pi}[G(m_\pi, -m_\pi^2/2m_N)] = 0.0289\,.$$

TABLE II. Values of $10^3 f^2$.

Z	$\nu=0,\ \nu_B=0$	$\nu=0,\ \nu_B=-m_\pi^2/2m_N$	$\nu=m_\pi,\ \nu_B=-m_\pi^2/2m_N$
0	76.7	77.4	74.4
$-\frac{1}{4}$	77.0	77.7	74.7
$-\frac{1}{2}$	77.3	78.2	75.2
$\frac{1}{2}$	76.1	77.1	74.1

Thus we get the values of f^2 seen in Table II. We note that the threshold determinations give consistently lower values of f^2 while $Z = -\frac{1}{2}$ gives f^2 closest to its presently accepted value of 0.08.

VI. EVALUATION OF THE SCATTERING LENGTHS

For the evaluation of the scattering lengths, we have used as input[13]

$$m_\pi a_S^{+}(\pi N) = -0.005, \quad m_\pi a_S^{1}(KN) = -0.29,$$
$$m_\pi a_S^{0}(KN) = -0.01\,,$$

as well as the following values of the constants:

$$f_{NN\pi} = \frac{f}{m_\pi} = \frac{1.01}{0.1396} = 7.2362\ \text{GeV}^{-1}\,,$$
$$f_{N\Lambda K} = -\frac{1 + 2\alpha}{\sqrt{3}} f_{NN\pi} = -7.52\ \text{GeV}^{-1}\,,$$
$$f_{N\Sigma K} = (1 - 2\alpha) f_{NN\pi} = 1.4472\ \text{GeV}^{-1}\,,$$
$$\alpha = 0.4\,,$$
$$h^2 = 0.29, \quad G^2/4\pi = 0.32, \quad H^2/4\pi = 1.9\,,$$
$$F_\pi = 92.56\ \text{MeV}\,.$$

The three mass relations and the three S-wave scattering lengths determine six parameters,

$$m_0(1+\gamma) = 1.1540, \quad m_0\gamma' = 0.0156\,,$$
$$m_0\mu = 0.0725, \quad m_0\bar{\mu} = 0.0666\,,$$
$$m_0\nu = 0.0835\,, \quad m_0\bar{\nu} = -0.2192\,,$$

where m_0 can be estimated from the experimental value of the σ term,

$$\sigma_{\pi N} = \frac{2}{3}\left(1 + \frac{c}{\sqrt{2}}\right)\left(m_0\gamma + \frac{-m_0\bar{\mu} + 3m_0\bar{\nu}}{2\sqrt{6}c}\right), \tag{34}$$

where we have assumed the symmetry-breaking Hamiltonian to be of the form $H_{SB} = c_0 u_0 + c_8 u_8$ with $c = c_8/c_0$ and where u_0, u_8 are the even-parity SU(3) singlet operator and $T = Y = 0$ member of the octet of operators in $(3,\bar{3}) + (\bar{3},3)$. Comparing with (7)

$$c_0 u_0 = m_0\gamma \bar{B}B + \cdots\,,$$
$$c_8 u_8 = m_0\bar{\mu}\bar{B}D_8 B + m_0\bar{\nu}\bar{B}F_8 B\,, \tag{35}$$

so that the expression for $\sigma_{\pi N}$ in terms of u_0 and u_8,

$$\sigma_{\pi N} = \frac{2}{3}\left(c_0 + \frac{c_8}{\sqrt{2}}\right)\left(\langle N|u_0|N\rangle + \frac{1}{\sqrt{2}}\langle N|u_8|N\rangle\right),$$

immediately leads to the equation (34). The value of c is given by

$$c = 2\sqrt{2}\ \frac{m_\pi^2 - m_K^2}{m_\pi^2 + 2m_K^2} = -1.2513$$

Using the experimental determination of $\sigma_{\pi N}(t = 2m_\pi^2)$ and taking into account the correction given

TABLE III. πN scattering lengths as a function of Z.

	Nucleon contribution	Direct contribution	Δ contribution	Total	Experiment
$m_\pi a_S^-$	0.0008	0.1015	$Z=-\frac{1}{4}$: 0.0004	0.0800	0.087 ± 0.002
			$Z=-\frac{1}{2}$: 0.0001	0.0797	
			$Z=\frac{1}{2}$: 0.0001	0.0797	
			$Z=0$: 0.0005	0.0801	
$m_\pi a_S^+$	−0.0106	Symmetry-breaking term: 0.0441	$Z=-\frac{1}{4}$: −0.0502	−0.0167	−0.005 ± 0.002
			$Z=-\frac{1}{2}$: −0.0570	−0.0235	
			$Z=\frac{1}{2}$: 0	0.0335	
			$Z=0$: −0.0385	−0.0050	
$m_\pi a_{P\,3/2}^-$	−0.0550	⋯	$Z=-\frac{1}{4}$: −0.0273	−0.0823	−0.081 ± 0.002
			$Z=-\frac{1}{2}$: −0.0278	−0.0828	
			$Z=\frac{1}{2}$: −0.0373	−0.0923	
			$Z=0$: −0.0287	−0.0837	
$m_\pi a_{P\,3/2}^+$	0.0550	⋯	$Z=-\frac{1}{4}$: 0.0739	0.1289	0.133
			$Z=-\frac{1}{2}$: 0.0728	0.1278	
			$Z=\frac{1}{2}$: 0.0538	0.1088	
			$Z=0$: 0.0711	0.1261	
$m_\pi a_{P\,1/2}^-$	−0.0549	0.0081	$Z=-\frac{1}{4}$: 0.0295	−0.0191	−0.013 ± 0.002
			$Z=-\frac{1}{2}$: 0.0647	0.0161	
			$Z=\frac{1}{2}$: 0.0918	0.0432	
			$Z=0$: 0.0223	−0.0263	
$m_\pi a_{P\,1/2}^+$	−0.1082	⋯	$Z=-\frac{1}{4}$: 0.0506	−0.0579	−0.056
			$Z=-\frac{1}{2}$: 0.0487	−0.0598	
			$Z=\frac{1}{2}$: 0.0294	−0.0791	
			$Z=0$: 0.0479	−0.0606	

by Pagels,[14] the value of $\sigma_{\pi N}(t=0)$ is about 36 MeV, which requires

$$m_0 = 803 \text{ MeV}.$$

This shows that the bare nucleon mass is close to the physical nucleon mass and indicates that the symmetry-breaking contribution is small but not insignificant.

Finally, we calculate the P-wave pion-nucleon scattering lengths as a function of Z and compare them with experimental data. The results are given in Table III. The KN scattering lengths for $Z=0$ are given in Table IV.

VII. SUMMARY AND CONCLUSION

It is clear therefore that a consistent picture of low-energy meson-baryon scattering is possible within the framework of a chiral SU(3) × SU(3) effective Lagrangian including $(3,\bar{3})+(\bar{3},3)$ symmetry breaking. There are quite a few parameters in the theory coming from the symmetry-breaking term as well as one parameter from the πNN^* interaction Lagrangian. We have tried to constrain these parameters from the low-energy experimental data and it seems to us that the parameters occurring in the symmetry-breaking term can be

TABLE IV. KN scattering lengths ($Z=0$).

	Direct contribution	Nucleon contribution	$Y_1^*(1385)$ contribution	Symmetry-breaking contribution	Total	Experiment
$m_\pi a_S^1$	−0.4195	−0.0351	0.2506	−0.085 (input)	−0.29	−0.29
$m_\pi a_S^0$	⋯	−0.0880	0.7517	−0.6737 (input)	−0.01	−0.01
$m_\pi a_{P\,3/2}^1$	⋯	0.0143	−0.0023	⋯	0.012	0.024 ± 0.006
$m_\pi a_{P\,1/2}^1$	−0.0157	−0.0062	0.0021	−0.0005	0.020	0.026 ± 0.002

obtained from the baryon masses and three S-wave scattering lengths so as to agree with the pion-nucleon σ term. A great advantage of the phenomenological Lagrangian we have used is that we do not now need any more *ad hoc* assumptions to reproduce the value of a_S^+. It is to be remembered that Peccei tried to cancel the large-N^* contribution to this scattering length by an assumed contact term, whereas Olsson and Osypwski[6] introduced a diffractive term to serve essentially the same purpose. The point of view we have adopted is that the large-N^* contribution can be cancelled by the symmetry-breaking term, and, in fact, we can use the value of a_S^+ as input in order to determine the parameters of the symmetry-breaking terms, thus avoiding the necessity of introducing any *ad hoc* term to explain the very small, experimental value of this quantity. Similar is the case with the $Y_1^*(1385)$ contributions to the S-wave K-N scattering lengths.

We must emphasize here once again that we have concerned ourselves only with the scattering lengths and not with any detailed behavior of the scattering amplitude. It is probably difficult for a simple Lagrangian model like the one we have used to explain such detailed properties as the discrepancy between the threshold and crossing points in the forward amplitude. As Olsson and Osypowski have pointed out, the N^* resonance is alone responsible, in such simple models, for the amplitude $\Delta\bar{F} = \bar{F}^+(\nu=1,\ t=0) - \bar{F}^+(\nu=0,\ t=0)$, where $\bar{F} = A + \nu B$ with the nucleon pole removed. But the N^* can account for only $\frac{2}{3}$ of the total so that an extra contribution is necessary, which was called the diffractive term by Olsson and Osypowski. This problem cannot be solved in the Lagrangian model by any choice of symmetry-breaking parameters. Moreover, although by a suitable choice of symmetry breaking it is possible to obtain the meson-nucleon scattering lengths as we have shown, there are, of course, other requirements that must be met. For example, the t dependence of the pion-nucleon scattering amplitude $A^+(\nu=0, t)$ must agree with experiment but N^* alone cannot account for the proper t dependence. The symmetry-breaking term is important here but the one we have chosen is t-independent so that the observed[15] $A^+(0, t)$ cannot be fitted. A detailed amplitude analysis in the Lagrangian model will face these difficulties, but for scattering lengths only it seems to us that a consistent application of the effective Lagrangians is not only possible but also very useful for a unified treatment of all meson-baryon scattering.

The P-wave scattering lengths we have obtained (Table III) all agree with the experimental values for various values of Z, although $Z=\frac{1}{2}$ seems to be rather less satisfactory than $Z=0$, $-\frac{1}{2}$, and $-\frac{1}{4}$. But in view of the uncertainties of the experimental numbers, it is not possible to draw any firm conclusion on this point.

On the other hand, the asymptotic behavior of A'^+ amplitude requires $Z=0$ and it seems to us that such a value of Z is not inconsistent with the P-wave scattering lengths. Moreover, as we see from Table II and Table IV, this value of Z also gives a reasonable value of the coupling constant f^2 and of the P-wave KN scattering lengths, respectively. A firm statement on the value of Z is only possible when we have more accurate values of the scattering lengths, but in this work our primary concern has been to show that a reasonably consistent picture of low-energy meson-baryon scattering in the context of phenomenological chiral Lagrangians is possible without any *ad hoc* assumptions.

[1]S. Weinberg, Phys. Rev. Lett. 18, 188 (1967); J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967); P. Chang and F. Gürsey, *ibid*. 164, 1752 (1967); B. W. Lee and H. T. Nieh, *ibid*. 166, 1507 (1968).

[2]J. A. Cronin, Phys. Rev. 161, 1483 (1967); B. W. Lee, *ibid*. 170, 1359 (1968); A. J. Macfarlane and P. H. Weisz, Lett. Nuovo Cimento 2, 25 (1969); L. Cronin, Nucl. Phys. B11, 355 (1969).

[3]R. D. Peccei, Phys. Rev. 176, 1812 (1968).

[4]P. J. Bussey, J. R. Carter, D. R. Dance, D. V. Bugg, A. A. Carter, and A. M. Smith, Nucl. Phys. B58, 363 (1973).

[5]D. V. Bugg, A. A. Carter, and J. R. Carter, Phys. Lett. 44B, 278 (1973).

[6]M. G. Olsson and E. T. Osypowski, Nucl. Phys. B101, 136 (1975).

[7]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[8]A. Sirlin and M. Weinstein, New York Univ. Report No. 4/72 (unpublished); A. M. Harun ar Rashid and M. A. Rashid, Phys. Rev. D 3, 581 (1971).

[9]S. P. Rosen and A. McDonald, Phys. Rev. D 4, 1833 (1971); 6, 644 (1972).

[10]R. D. Peccei, Phys. Rev. 176, 1812 (1968). L. M. Nath, B. Etemadi, and J. D. Kimel, Phys. Rev. D 3, 2153 (1971); C. R. Hagen, *ibid*. D 4, 2204 (1971).

[11]L. S. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. D 4, 2801 (1971).

[12]Adler's value quoted by Brown *et al*. We have purposely taken the same numbers as those used by Brown *et al*. since our intention was not to calculate f^2, but rather to see the effect of Z.

[13]H. Pilkuhn, W. Schmidt, A. D. Martin, C. Michael, F. Steiner, B. R. Martin, M. M. Nagels, and J. J. de Swart, Nucl. Phys. B65, 460 (1973); Y.-M. P. Lam, and Y. Y. Lee, Phys. Rev. D 2, 2976 (1970).

[14]H. Pagels, Phys. Rep. 16C, 220 (1975).

[15]M. M. Nagels *et al*., Nuc. Phys. B109, 1 (1976).