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# On the decay $\pi^0 \to e^+e^-$ †

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Phenomenological aspects of the decay $\pi^0 \to e^+e^-$ are discussed in the presence of a general neutral-current interaction.

The high-flux pion beams in meson factories and other new experimental facilities open the possibility of observing the hitherto undetected rare decay $\pi^0 \to e^+e^-$.[1] This process is of interest as a probe of the $\pi^0\gamma\gamma$ vertex,[2] and also as a potential source of information on possible nonelectromagnetic interactions between electrons and hadrons.[3-6] Examples of the latter are the neutral-current interactions predicted by many unified theories of the weak and the electromagnetic interactions,[7] or the effective electron-hadron couplings arising from the exchange of leptoquark bosons (bosons causing quark $\leftrightarrow$ lepton transitions), which appear in theories attempting to unify the strong and the nonstrong interactions.[8] In the following we shall refer to any nonelectromagnetic electron-hadron coupling as "neutral-current interaction."

The purpose of this note is to give a phenomenological discussion of the process $\pi^0 \to e^+e^-$ in the presence of a general neutral-current interaction which is allowed to violate $CP$ invariance. The implications of the present experimental upper limit on the $\pi^0 \to e^+e^-$ decay rate on the coupling constants of the most general nonderivative effective Lagrangian are discussed, taking into account constraints provided by other available data. We point out that a large $CP$-violating amplitude, of the order of magnitude of the $CP$-conserving one, is not excluded in this decay by present data. Aspects of $\pi^0 \to e^+e^-$ decay in the presence of neutral-current interactions have previously been discussed in Refs. 3–6.

The decay $\pi^0 \to e^+e^-$ is expected to occur via the conventional electromagnetic interactions in fourth order, as shown in diagram (a) of Fig. 1. In the presence of a neutral-current interaction, it can also proceed according to diagram 1(b) and/or diagram 1(c).[9]

The most general matrix element for a $\pi^0$ decaying into an electron and a positron of four-momenta $p_-$ and $p_+$, respectively, can be written as[10]

$$M(\pi^0 \to e^+e^-) = a\bar{u}(p_-)\gamma_5 v(p_+) + ib\bar{u}(p_-)v(p_+)\,, \qquad (1)$$

where $a$ and $b$ are complex numbers. The amplitude $a$ represents the $P$- and $CP$-conserving part, while the $b$ term is $P$- and $CP$-violating. Hence, the decay is parity-conserving if $CP$ invariance holds.[11]

Since a common phase factor is irrelevant, the process $\pi^0 \to e^+e^-$ is in general characterized by three real numbers: the magnitudes of the amplitudes $a$ and $b$ and their relative phase. These can (in principle) be determined from the decay rate and from the various polarization effects. The differential decay probability in the $\pi^0$-rest frame is[12]

$$dW = \frac{m_\pi r}{128\pi^2}(|a|^2 + r^2|b|^2) \times [1 + \tilde{\alpha}\vec{n}\cdot\vec{\zeta}_+ \times \vec{\zeta}_- + \tilde{\beta}(\vec{n}\cdot\vec{\zeta}_+ + \vec{n}\cdot\vec{\zeta}_-) + \tilde{\gamma}\vec{\zeta}_+\cdot\vec{\zeta}_- + (1-\tilde{\gamma})\vec{n}\cdot\vec{\zeta}_+\,\vec{n}\cdot\vec{\zeta}_-]\,d\Omega_n\,, \qquad (2)$$

where $r = (1 - 4m^2/m_\pi^{\,2})^{1/2}$, $m$ and $m_\pi$ denote the electron and the pion mass, $\vec{\zeta}_-$ and $\vec{\zeta}_+$ are the electron and the positron polarization vectors in their respective rest frames, $\vec{n} \equiv \vec{p}_-/|\vec{p}_-| = -\vec{p}_+/|\vec{p}_+|$. The parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ are given by

$$\tilde{\alpha} = \frac{2r\,\mathrm{Re}\,ba^*}{|a|^2 + r^2|b|^2}\,, \qquad (3a)$$

$$\tilde{\beta} = \frac{2r\,\mathrm{Im}\,ba^*}{|a|^2 + r^2|b|^2}\,, \qquad (3b)$$

$$\tilde{\gamma} = \frac{|a|^2 - r^2|b|^2}{|a|^2 + r^2|b|^2}\,. \qquad (3c)$$

They are not independent but satisfy the relation $\tilde{\alpha}^2 + \tilde{\beta}^2 + \tilde{\gamma}^2 = 1$. The decay rate is

$$\Gamma = \frac{m_\pi r}{8\pi}(|a|^2 + r^2|b|^2)\,. \qquad (4)$$

Given $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$, one can compute the ratio of the amplitudes $a$ and $b$: $a/b = r(\tilde{\alpha} - i\tilde{\beta})/(1-\tilde{\gamma})$. The common multiplicative factor is then determined by $\Gamma$.

In the following discussion we shall treat the neutral pion as a pure $P = -1$, $C = +1$ isovector state. The effects of small admixtures of other states will be considered at the end of the paper.

$CPT$ invariance, which we shall assume to be valid,[13] requires that $b$ and $a$ be relatively real, apart from "unitarity phases" arising from the

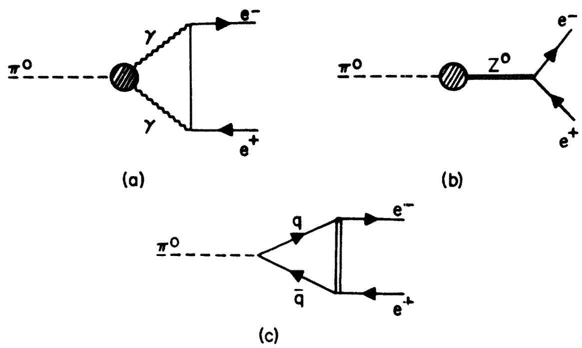


FIG. 1. Diagrams contributing to the decay $\pi^0 \to e^+e^-$. (a) The lowest-order electromagnetic contribution; (b) contribution due to the exchange of a neutral intermediate boson; (c) leptoquark contribution.

existence of real intermediate states.[14] The common phase of the amplitudes will be chosen so that both $b$ and $a$ are real in the absence of such phases.

We shall write

$$a = a^{(e)} + a^{(n)}, \quad b = b^{(e)} + b^{(n)}, \tag{5}$$

where $a^{(e)}, b^{(e)}$ and $a^{(n)}, b^{(n)}$ represent the contribution of the electromagnetic and the neutral-current interactions, respectively. We consider the process $\pi^0 \to e^+e^-$ in lowest order in the electromagnetic and the neutral-current interactions [diagrams (a), (b), and (c) of Fig. 1]. Since diagrams 1(b) and 1(c) have no absorptive parts, we have $\mathrm{Im}a^{(n)} = \mathrm{Im}b^{(n)} = 0$. Furthermore, there appears to be no term in the electromagnetic interactions which is $P$- and $CP$-violating,[15] so that $b^{(e)} = 0$. Hence,

$$\mathrm{Re}a = \mathrm{Re}a^{(e)} + a^{(n)}, \tag{6a}$$

$$\mathrm{Im}a = \mathrm{Im}a^{(e)}, \tag{6b}$$

$$\mathrm{Re}b = b^{(n)}, \tag{6c}$$

$$\mathrm{Im}b = 0, \tag{6d}$$

and the parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ simplify to

$$\tilde{\alpha} = \frac{2rb^{(n)}\mathrm{Re}a}{|a|^2 + r^2 b^{(n)2}}, \tag{7a}$$

$$\tilde{\beta} = -\frac{2rb^{(n)}\mathrm{Im}a^{(e)}}{|a|^2 + r^2 b^{(n)2}}, \tag{7b}$$

$$\tilde{\gamma} = \frac{|a|^2 - r^2 b^{(n)2}}{|a|^2 + r^2 b^{(n)2}}. \tag{7c}$$

The magnitude of $\mathrm{Im}a^{(e)}$ can be calculated in a model-independent way, using the unitarity relation. To order $\alpha^2$ only the two-photon intermediate state contributes and one obtains[16]

$$|\mathrm{Im}a^{(e)}| = \frac{1}{4r}\,\alpha\,\frac{m}{m_\pi}\left(\ln\frac{1+r}{1-r}\right)|F| \approx 2.6\times 10^{-7}. \tag{8}$$

In Eq. (8), $F$ is the $\pi^0 \to 2\gamma$ decay constant: $|F| = [64\pi\Gamma(\pi^0 \to 2\gamma)/m_\pi]^{1/2}$. The sign of $\mathrm{Im}a^{(e)}$ depends on the unknown sign of $F$.

$\mathrm{Re}a^{(e)}$ is, on the other hand, model dependent. The available calculations[2] give values in the range

$$|\mathrm{Re}a^{(e)}| \approx 5\times 10^{-8} - 5\times 10^{-7}. \tag{9}$$

Since $|a|$ is unknown, to test for the presence of the $b$ amplitude (and thus for $CP$ violation), it is necessary to measure at least one of the parameters $\tilde{\alpha}$, $\tilde{\beta}$, or $\tilde{\gamma}$. As we can consider $\mathrm{Im}a^{(e)}$ to be known, the knowledge of $\tilde{\beta}$ and $\Gamma$ would already be sufficient to determine the magnitude of both $a$ and $b$. (See, however, the discussion at the end of the paper of possible contributions to $\mathrm{Im}b$ and possible additional contributions to $\mathrm{Im}a$.) The parameter $\tilde{\beta}$ can be determined through the measurement of the degree of longitudinal polarization of either $e^-$ or $e^+$, and thus it is relatively the most accessible. Summing the decay probability (2) over the spin states of , for example, the positron one obtains

$$\sum_{e^+\,\mathrm{spin}} dW = \frac{m_\pi r}{64\pi^2}\,(|a|^2 + r^2|b|^2)(1 + \tilde{\beta}\,\vec{n}\cdot\vec{\zeta}_-)d\Omega_n, \tag{10}$$

so that

$$\tilde{\beta} = \frac{N_R - N_L}{N_R + N_L}, \tag{11}$$

where $N_R$ ($N_L$) is the number of electrons emerging with positive (negative) helicity.

The only experimental information available at present on the decay $\pi^0 \to e^+e^-$ is an upper limit

$$B(\pi^0 \to e^+e^-) < 8\times 10^{-6} \tag{12}$$

on the branching ratio

$$B(\pi^0 \to e^+e^-) \equiv \Gamma(\pi^0 \to e^+e^-)/\Gamma(\pi^0 \to \mathrm{all}),$$

deduced from existing data which are sensitive to this process.[5] Hence [with $\Gamma(\pi^0 \to \mathrm{all}) \approx 7.8$ eV (see Ref. 17)]

$$\Gamma(\pi^0 \to e^+e^-) < 6.2\times 10^{-5}\ \mathrm{eV} \tag{13}$$

[for comparison, the unitarity lower bound for $\Gamma(\pi^0 \to e^+e^-)$ is $3.6\times 10^{-7}$ eV], so that

$$|a|^2 + r^2|b|^2 \approx |a|^2 + |b|^2 < 1.2\times 10^{-11}. \tag{14}$$

As a consequence, one obtains the following bounds on the magnitude of the amplitudes $a^{(n)}$ and $b^{(n)}$:

$$|a^{(n)}| < |\mathrm{Re}\,a^{(e)}| + [1.2\times10^{-11} - (\mathrm{Im}\,a^{(e)})^2 - b^{(n)2}]^{1/2} \quad (15a)$$

$$\leq |\mathrm{Re}\,a^{(e)}| + [1.2\times 10^{-11} - (\mathrm{Im}\,a^{(e)})^2]^{1/2}$$

$$\approx |\mathrm{Re}\,a^{(e)}| + 3.4\times10^{-6} \quad (15b)$$

and

$$|b^{(n)}| < [1.2\times10^{-11} - (\mathrm{Im}\,a^{(e)})^2 - (\mathrm{Re}\,a^{(e)} + a^{(n)})^2]^{1/2} \quad (16a)$$

$$\leq [1.2\times10^{-11} - (\mathrm{Im}\,a^{(e)})^2]^{1/2}$$

$$\approx 3.4\times10^{-6}. \quad (16b)$$

To get some feeling for the meaning of the above limits, we shall consider as a model of the electron-hadron neutral-current interaction the most general effective Lagrangian not containing derivatives of the lepton fields. For $\pi^0 \to e^+e^-$ we have to consider only an isovector term; also, as discussed before, there are no contributions from $P=+1$, $CP=-1$ and $P=-1$, $CP=+1$ parts. The $P=+1$, $CP=+1$ and the $P=-1$, $CP=-1$ components of the Lagrangian have the following form:

$$\mathcal{L}_n^{P=+1,CP=+1}(I=1) = \frac{G}{\sqrt{2}}\,(g_3^{VV}\bar{e}\gamma^\mu e J_{3\mu}^V + g_3^{AA}\bar{e}\gamma^\mu\gamma_5 e J_{3\mu}^A + g_3^{SS}\bar{e}eJ_3^S + g_3^{PP}\bar{e}i\gamma_5 e J_3^P + g_3^{TT}\bar{e}\sigma^{\mu\nu}eJ_{3\mu\nu}^T), \quad (17)$$

$$\mathcal{L}_n^{P=-1,CP=-1}(I=1) = \frac{G}{\sqrt{2}}\,(\tilde{g}_3^{VA}\bar{e}\gamma^\mu e\tilde{J}_{3\mu}^A + \tilde{g}_3^{AV}\bar{e}\gamma^\mu\gamma_5 e\tilde{J}_{3\mu}^V + g_3^{SP}\bar{e}eK_3^P + g_3^{PS}\bar{e}i\gamma_5 eK_3^S + g_3^{PTT}\bar{e}i\sigma^{\mu\nu}\gamma_5 eK_{3\mu\nu}^T). \quad (18)$$

In (17) and (18) $g_3^{VV}, g_3^{AA}, \tilde{g}_3^{VA}, \ldots$ are constants characterizing the strength of the corresponding terms relative to $2^{-1/2}G$ ($G \simeq 10^{-5}m_p^{-2}$). The quantities $J_{3\mu}^V$, $J_{3\mu}^A$, $J_3^S$, $J_3^P$, $J_{3\mu\nu}^T$ and similarly $\tilde{J}_{3\mu}^V$, $\tilde{J}_{3\mu}^A$, $K_3^S$, $K_3^P$, $K_{3\mu\nu}^T$ are hadronic densities transforming as a vector, axial-vector, scalar, pseudoscalar, and tensor, respectively. In isospin space they all transform as third components of $I$ (≡isospin) $=1$ operators. $J_{3\mu}^V$, $J_{3\mu}^A$, $J_3^S$, $J_3^P$, $J_{3\mu\nu}^T$, $K_3^P$, $K_3^S$, $K_{3\mu\nu}^T$ are first-class densities and $\tilde{J}_3^V, \tilde{J}_3^A$ are second-class densities according to Weinberg's classification.[18] The hadronic densities are chosen to be Hermitian, so that all the constants $g$ and $\tilde{g}$ are real. We normalize the currents $J_{3\mu}^A$ and $J_{3\mu}^V$ so that with $g_3^{AA}=1$,

$$g_3^{VV} = (1 - 4\sin^2\theta_W)(1 - 2\sin^2\theta_W)$$

(and $g_3^{SS} = g_3^{PP} = g_3^{TT} = 0$), (17) is identical to the parity-conserving part of the Lagrangian in the Weinberg model[19] in the case when $J_{3\mu}^V$ and $J_{3\mu}^A$ are third components of the isotriplet to which the usual charged weak currents belong.

The interactions (17) and (18) give the following contribution to the $\pi^0 \to e^+e^-$ amplitudes[20]:

$$a^{(n)} = 2\frac{Gm_\pi m}{\sqrt{2}}\,g_3^{AA}\kappa_A^{(\pi)} + \frac{Gm_\pi^2}{\sqrt{2}}\,g_3^{PP}\kappa_P^{(\pi)}$$

$$\approx (1.1\times10^{-9})g_3^{AA}\kappa_A^{(\pi)} + (1.4\times10^{-7})g_3^{PP}\kappa_P^{(\pi)}, \quad (19)$$

$$b^{(n)} = \frac{Gm_\pi^2}{\sqrt{2}}\,g_3^{SP}\lambda_P^{(\pi)}$$

$$\approx (1.4\times10^{-7})g_3^{SP}\lambda_P^{(\pi)}, \quad (20)$$

where the dimensionless constants $\kappa_A^{(\pi)}$, $\kappa_P^{(\pi)}$, and $\lambda_P^{(\pi)}$ are defined by

$$\langle 0|J_\mu^A|\pi^0(p)\rangle = m_\pi\kappa_A^{(\pi)}p_\mu, \quad (21)$$

$$\langle 0|J^P|\pi^0(p)\rangle = -im_\pi^2\kappa_P^{(\pi)}, \quad (22)$$

$$\langle 0|K^P|\pi^0(p)\rangle = im_\pi^2\lambda_P^{(\pi)}. \quad (23)$$

In terms of the coupling constants the inequalities (15b) and (16b) read (assuming that $|\mathrm{Re}\,a^{(e)}| < 6\times10^{-7}$, as suggested by Refs. 2)

$$|g_3^{AA}\kappa_A^{(\pi)} + 127g_3^{PP}\kappa_P^{\pi}| < 3640, \quad (24)$$

$$|g_3^{SP}\lambda_P^{(\pi)}| < 24. \quad (25)$$

These constraints summarize the information on the neutral-current interaction couplings which can be obtained from the existing upper limit on the $\pi^0 \to e^+e^-$ decay rate alone. We shall turn now to consider other available data which have a bearing on the possible size of the constants $g_3^{AA}$, $g_3^{PP}$, and $g_3^{SP}$.

Much more restrictive information than the bound (24) on the possible magnitude of $g_3^{AA}$ is provided by the hyperfine splitting $\nu$ of the ground state of the hydrogen atom,[21] which is known experimentally to an accuracy of one part in $10^{12}$ (see Ref. 22). Requiring that the contribution $\Delta\nu_{nc}/\Delta\nu_{em}$ of the $g_3^{AA}$ term in (17) be less than 2 ppm, which approximately is the theoretical uncertainty in the conventional calculation of the hyperfine splitting,[22] one obtains[23]

$$|g_3^{AA}\kappa_A^{(p)}| < 22, \quad (26)$$

where $\kappa_A^{(p)}$ is defined by

$$\langle p|J_{3\mu}^A|p\rangle = \bar{u}_p\kappa_A^{(p)}\gamma_\mu\gamma_5 u_p \quad (27)$$

($|p\rangle$ ≡ proton state). If $J_{3\mu}^A$ belongs to the same isotriplet as the usual charged weak axial-vector currents $A_{(1\pm i2)\mu}$, then $\kappa_A^{(p)} = \frac{1}{2}g_A \approx 0.62$ and $\kappa_A^{(\pi)} = f_\pi/m_\pi\sqrt{2} \approx 0.7$ ($f_\pi$ and $g_A$ are, respectively, the charged-pion decay constant and the neutron $\beta$-decay axial-vector coupling constant), so that from (26) one has

$$|g_3^{AA}| < 36 \quad (28)$$

and

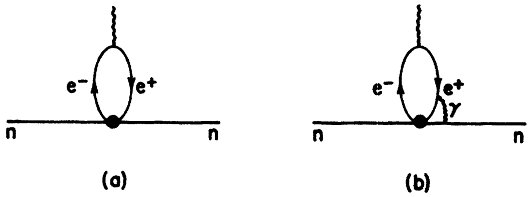


FIG. 2. Diagrams contributing to the electric dipole moment of the neutron. (a) Lowest-order contribution of the neutral-current interaction; (b) example of a diagram of order $Gg_3^{\Gamma\Gamma'}\alpha$.

$$|g_3^{AA}\kappa_A^{(\pi)}| < 25\,. \tag{29}$$

Further information on $g_3^{AA}$ is obtained by comparing data on deep-inelastic $e^-p$ and $e^+p$ scatterings,[24] where the presence of an axial-vector interaction leads to a difference in the corresponding cross sections.[25] Using the parton-model formulas given in Ref. 6, and taking the ratio of the $u$- and the $d$-quark momentum distribution functions to be $\approx \frac{1}{2}$, as suggested by the data,[26] the present experimental results[27] imply (for the case when $J_{3\mu}^A \equiv A_{3\mu}$) that

$$|g_3^{AA}| < 20 \tag{30}$$

and thus that

$$|g_3^{AA}\kappa_A^{(\pi)}| < 14\,. \tag{31}$$

This appears to be the best available limit on $g_3^{AA}$. However, in view of the model dependence, it is less reliable than (28) and (29). For comparison, recall that in the Weinberg model[19] $g_3^{AA}=1$ and $g_3^{AA}\kappa_A^{(\pi)} \approx 0.7$.

It may, of course, be that $J_{3\mu}^A$ is built up from quarks which are different from the valence quarks in the nucleon and the pion, in which case its effect on the $e^+p/e^-p$ ratio (and also on the scaling behavior of the structure functions[24]) is expected to be negligible in the kinematic region so far studied, and $\kappa_A^{(p)}$, $\kappa_A^{(\pi)} \ll 1$ presumably.[28] In any event, the magnitude of $g_3^{AA}$ (and also of $g_3^{PP}$ and $g_3^{SP}$) cannot be larger than about 100, without disturbing the approximate constancy of

$$R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

in the energy region 5 GeV $\lesssim E_{\text{c.m.}} \lesssim$ 7.8 GeV.[29]

The best limit on the possible size of the scalar-pseudoscalar coupling constant $g_3^{SP}$ is obtained by considering its contribution to the electric dipole moment of the neutron $D_n$.[30] Diagrams contributing to $D_n$ in orders $Gg_3^{SP}$ and $Gg_3^{SP}\alpha$ are shown in Fig. 2(a) and Fig. 2(b). It is easy to show that the $g_3^{SP}$ coupling does not contribute to diagram 2(a); consequently, the expected magnitude of $D_n$ is

$$D_n \approx \frac{1}{M}\frac{GM^2}{4\pi}\frac{\alpha}{\pi}g_3^{SP}\rho_P^{(n)}$$

$$\approx (4\times 10^{-23})g_3^{SP}\rho_P^{(n)}\ e\,\text{cm}\,, \tag{32}$$

where $M$ is the nucleon mass and $\rho_P^{(n)}$ is a number expected to be of the order of unity when $K_3^P$ contains valence quarks of the neutron and much smaller than unity otherwise.[28] Comparing (32) with present experimental upper limit[31]

$$D_n^{\text{exp}} < 3\times 10^{-24}\ e\,\text{cm} \tag{33}$$

we conclude that[32]

$$|g_3^{SP}\rho_P^{(n)}| < 0.1\,. \tag{34}$$

Thus, if $\rho_P^{(n)} \approx \lambda_P^{(\pi)}$, which is not implausible in the case when the densities $J_3^P$ and $K_3^P$ have similar structure, the contribution of the $g_3^{SP}$ coupling to the $\pi^0 \to e^+e^-$ amplitude is expected to obey

$$|g_3^{SP}\lambda_P^{(\pi)}| < 0.1\,. \tag{35}$$

Values of $g_3^{SP}\lambda_P^{(\pi)}$ somewhat larger than the upper bound in (35) are, of course, not excluded, since the magnitude of $D_n$ given by (32) represents only a very crude estimate.

The constraint (15a) with the bounds (29) and (35) leads to (assuming that $|\text{Re}a^{(e)}| < 6\times 10^{-7}$)

$$|g_3^{PP}\kappa_P^{(\pi)}| < 30\,, \tag{36}$$

which (provided that $\kappa_P^{(\pi)}$ is of order one) appears to be the best available limit on the pseudoscalar-pseudoscalar coupling.

The bound (35) allows for large $CP$-violating effects to be present. Thus, if $g_3^{SP}\lambda_P^{(\pi)} \approx 0.1$ and $a \approx \text{Im}a^{(e)}$, for example, the degree of longitudinal polarization of the electron would be about 10%. However, to have a chance to observe it even at such a level, the presently anticipated event rates[1] would have to be increased by at least five orders of magnitude.[33]

In our discussion so far, $\pi^0$ was assumed to be a pure $P=-1$, $C=+1$, $I=1$ state. We shall consider now briefly the possible effects of admixtures of states with other quantum numbers. Apart from isospin mixing, parity impurities should also be present in view of the evidence for parity violation in the effective hadronic interactions (at the level of first-order weak interactions or possibly stronger).[34] Additional impurities would be introduced by possible $CP$ violation in the effective hadronic interactions. In trying to estimate these effects, one is on extremely uncertain grounds, since with the exception of isospin mixing and apart from some data on $0^{++}$ states the nature and properties of the contributing states are unknown. Including the possible admixtures, the neutral pion state will be

$$\pi^0 = \pi_2 + \epsilon_1\pi_1 + \epsilon_1'\pi_1' + \epsilon_2'\pi_2' + \tilde{\epsilon}_2\tilde{\pi}_2 \;,$$

with $\pi_2$ representing the dominant $J^{PC} \equiv 0^{+-}$ state, $\pi_1$, $\pi_1'$, $\pi_2'$, and $\tilde{\pi}_2$ the $0^{--}$, $0^{+-}$, $0^{++}$, and other $0^{-+}$ states.

The most important from other $0^{-+}$ states is, presumably, the $\eta$ meson, for which $\mathrm{Re}\epsilon_\eta \approx 10^{-2}$, $\mathrm{Im}\epsilon_\eta \approx 10^{-7}$.[35] Lacking information, we shall represent the $\pi_1$, $\pi_1'$, and $\pi_2'$ states as resonances of masses $m_1$, $m_1'$, $m_2'$ and widths $\gamma_1$, $\gamma_1'$, and $\gamma_2'$. Thus

$$\epsilon_1 \approx \langle \pi_1 | H^{P=+1,CP=-1} | \pi_2\rangle / [(m_2 - m_1) - (i/2)(\gamma_2 - \gamma_1)] \,,$$

where $H^{P=+1,CP=-1}$ is the effective $P$-conserving, $CP$-violating hadronic interaction, etc. Regarding the admixture of $0^{--}$ states, the limit (33) on the electric dipole moment of the neutron suggests that the strength of $H^{P=+1,CP=-1}$ is less than $2\times10^{-4}$ relative to the strong interactions,[30] and therefore one expects

$$|\langle \pi_1 | H^{P=+1,CP=-1} | \pi_2\rangle| \leq 2\times10^{-4} m_x \,,$$

where $m_x$ is some mass. With $m_x = 1$ GeV, $\Delta m \equiv m_1 - m_2 \geq 1$ GeV, $\Delta\gamma/\Delta m \leq 0.5$ GeV, one would have $|\mathrm{Re}\epsilon_1| \leq 5\times10^{-5}$, $|\mathrm{Im}\epsilon_1| \leq 2\times10^{-4}$.[36]

A much more stringent limit is imposed on admixtures of $P=+1$, $C=+1$ states $\pi_2'$. In this case (33) leads to

$$|\langle \pi_2' | H^{P=-1,CP=-1} | \pi_2\rangle| \leq 10^{-10}\ \mathrm{GeV},$$

so that $|\mathrm{Re}\epsilon_2'| \leq 2.5\times10^{-11}$ and $|\mathrm{Im}\epsilon_2'| \leq 10^{-10}$.

For admixtures of $P=+1$, $C=-1$ states, present evidence on parity violations in nonleptonic nuclear processes[34] suggests

$$|\langle \pi_1' | H^{P=-1,CP=+1} | \pi_2\rangle| \approx 50\ G_{nl}^{\mathrm{p.v.}}\ m_x^3/4\pi \,.$$

Taking again $m_x = 1$ GeV, $\Delta m \geq 1$ GeV, and $\Delta\gamma/\Delta m \leq 0.5$ GeV one obtains $|\mathrm{Re}\epsilon_1'| \leq 5\times10^{-5}$, $|\mathrm{Im}\epsilon_1'| \leq 1.2\times10^{-5}$.

In the presence of the admixtures, the $\pi^0 \to e^+e^-$ amplitudes become

$$a = a_2 + a_{\mathrm{mixing}}$$
$$= a_2 + \epsilon_1 a_1 + \epsilon_1' a_1' + \epsilon_2' a_2' + \tilde{\epsilon}_2 \tilde{a}_2$$

and

$$b = b_2 + b_{\mathrm{mixing}}$$
$$= b_2 + \epsilon_1 b_1 + \epsilon_2' b_2' + \epsilon_1' b_1' + \tilde{\epsilon}_2 \tilde{b}_2 \;,$$

where $a_2$ and $b_2$ are given by Eqs. (19) and (20); $a_1, a_1', a_2', \tilde{a}_2$ and $b_1, b_1', b_2', \tilde{b}_2$ represent the $\pi_1, \pi_1', \pi_2', \tilde{\pi}_2 \to e^+e^-(^1S_0)$ and the $\pi_1, \pi_1', \pi_2', \tilde{\pi}_2 \to e^+e^-(^3P_0)$ amplitudes. Inspecting the possible types of couplings that could contribute in each case and ignoring contributions whose upper bound is relatively small, one obtains with the bounds for the mixing parameters given above,[37]

$$|\mathrm{Re}a_{\mathrm{mixing}}| \leq (2\times10^{-7}) m_\pi^{-2} |N_{PP}^{(\eta)}| + (3\times10^{-9}) m_\pi^{-2} |\tilde{N}_{PP}^{(\pi_1)}| + (7\times10^{-10}) m_\pi^{-2} |\tilde{N}_{PS}^{(\pi_1')}| \,, \qquad (37)$$

$$|\mathrm{Im}a_{\mathrm{mixing}}| \leq (7\times10^{-10}) m_\pi^{-2} |\tilde{N}_{PP}^{(\pi_1)}| + (2\times10^{-10}) m_\pi^{-2} |\tilde{N}_{PS}^{(\pi_1')}| \,, \qquad (38)$$

$$|\mathrm{Re}b_{\mathrm{mixing}}| \leq (4\times10^{-10}) m_\pi^{-2} |N_{SP}^{(\eta)}| + (3\times10^{-9}) m_\pi^{-2} |\tilde{N}_{SP}^{(\pi_1)}| + (7\times10^{-10}) m_\pi^{-2} |\tilde{N}_{SS}^{(\pi_1')}| \,, \qquad (39)$$

$$|\mathrm{Im}b_{\mathrm{mixing}}| \leq (7\times10^{-10}) m_\pi^{-2} |\tilde{N}_{SP}^{(\pi_1)}| + (2\times10^{-10}) m_\pi^{-2} |\tilde{N}_{SS}^{(\pi_1')}| \,, \qquad (40)$$

where, for example, $N_{PP}^{(\eta)} = \langle 0 | J^P | \eta(p^2 = m_\pi^2)\rangle$ with $J^P$ (≡first-class isoscalar hadronic density) coupled to a pseudoscalar leptonic density, $\tilde{N}_{SP}^{(\pi)} = \langle 0 | \tilde{J}^P | \pi_1(p^2 = m_\pi^2)\rangle$ with $\tilde{J}^P$ (≡second-class hadronic density of arbitrary isospin) coupled to a scalar leptonic density, etc.

Assuming that

$$|N_{PP}^{(\eta)}|,\ |N_{SP}^{(\eta)}|,\ |\tilde{N}_{PP}^{(\pi_1)}|,\ |\tilde{N}_{SP}^{(\pi_1)}|,\ |\tilde{N}_{PS}^{(\pi_1')}|,\ |\tilde{N}_{SS}^{(\pi_1')}| \leq m_\pi^2 \,, \qquad (41)$$

one would have

$$|\mathrm{Re}a_{\mathrm{mixing}}| \leq 2\times10^{-7} \,, \qquad (42)$$

$$|\mathrm{Im}a_{\mathrm{mixing}}| \leq 9\times10^{-10} \,, \qquad (43)$$

$$|\mathrm{Re}b_{\mathrm{mixing}}| \leq 4\times10^{-9} \,, \qquad (44)$$

$$|\mathrm{Im}b_{\mathrm{mixing}}| \leq 9\times10^{-10} \,. \qquad (45)$$

Values larger than (41) for the matrix elements $N$, and consequently upper bounds for the mixing contributions exceeding (42)–(45), cannot be, of course, ruled out.

To summarize our discussion, with $CPT$ invariance assumed to be valid, the information one could obtain from a detailed experimental study of the decay $\pi^0 \to e^+e^-$ consists of the magnitude and the relative phase of the quantities

$$a = \mathrm{Re}a^{(e)} + (1.1\times10^{-9}) g_3^{AA}\kappa_A^{(\pi)} + (1.4\times10^{-7}) g_3^{PP}\kappa_P^{(\pi)} + \mathrm{Re}a_{\mathrm{mixing}} + i\,(\mathrm{Im}a^{(e)} + \mathrm{Im}a_{\mathrm{mixing}}) \,, \qquad (46)$$

$$b = (1.4\times10^{-7}) g_3^{SP}\lambda^{(\pi)} + \mathrm{Re}b_{\mathrm{mixing}} + i\,\mathrm{Im}b_{\mathrm{mixing}} \,, \qquad (47)$$

where $|\mathrm{Im}a^{(e)}| = 2.6\times10^{-7}$.

As far as one knows, the various terms contributing to $\mathrm{Re}a$ could be of comparable magnitude. The bounds (29) and (35) indicate, however, that the presence of $g_3^{AA}\kappa_A^{(\pi)}$ and/or $g_3^{SP}\kappa_P^{(\pi)}$ cannot increase the decay rate more than by a few percent (or perhaps about 30%, if we allow for the possibility that $g_3^{SP}\kappa_P^{(\pi)} \approx 1$) above the unitarity lower bound $\Gamma_u = (m_\pi r/8\pi)(\mathrm{Im}a^{(e)})^2 = 3.6\times10^{-7}$ eV. Hence, to the extent that (41) and thus the bounds

(42)–(45) hold, a $\pi^0 \to e^+ e^-$ decay rate considerably larger than $\Gamma_u$ would signal the presence of a large $\mathrm{Re}\,a^{(e)}$ or $g_3^{PP} \kappa_P^{(\pi)}$, or both. As follows from (42), the bound (36) on $g_3^{PP} \kappa_P^{(\pi)}$ is not affected appreciably by the possible presence of the mixing contributions. The limit (43) indicates that a possible violation of the unitarity bound should be limited to not more than about 1% in the rate, a deviation which would be difficult, if not impossible to isolate from possible effects of $P, C$-conserving higher-order corrections. A decay rate lower than $\Gamma_u$ by more than about 40% [corresponding to $\tilde{N}_{PS}^{(\pi)} \approx \tilde{N}_{PP}^{(\pi)} \approx 1\,\mathrm{GeV}^2$ in Eq. (38)] would suggest the possible presence of a $CPT$-violating neutral current interaction.[38]

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[1]Proposals to measure the decay rate for $\pi^0 \to e^+e^-$ including the following: J. S. Frank, C. M. Hoffman, R. E. Mischke and R. D. Werbeck, LAMPF Proposal No. 222, Los Alamos, 1975 (unpublished); J. D. Davies *et al.*, RHEL Proposal No. 170, summarized by B. Allardyce, T. Bressani, and N. Tanner, CERN Report No. CERN-PH-III-75/11 (unpublished).

[2]S. D. Drell, Nuovo Cimento 11, 693 (1959); S. M. Berman and D. A. Geffen, *ibid.* 18, 1192 (1960); C. Quigg and J. D. Jackson, Lawrence Radiation Laboratory Report No. UCRL-18487, 1968 (unpublished); I. K. Litskevich and V. A. Franke, Yad. Fiz. 10, 815 (1969) [Sov. J. Nucl. Phys. 10, 471 (1970)]; M. Pratap and J. Smith, Phys. Rev. D 5, 2020 (1972).

[3]F. C. Michel, Phys. Rev. 138, B408 (1965).

[4]A. Soni, Phys. Lett. 52B, 332 (1974); *ibid.* 53B, 280 (1974).

[5]J. D. Davies, J. G. Guy, and R. K. P. Zia, Nuovo Cimento 24A, 324 (1974).

[6]J. C. Pati and A. Salam, Phys. Rev. D 11, 1137 (1975).

[7]For a review, see for example M. A. Bég and A. Sirlin, Annu. Rev. Nucl. Sci. 24, 379 (1974).

[8]See, for example, J. C. Pati and A. Salam, Ref. 6; H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975).

[9]We shall not consider contributions from possible purely leptonic neutral-current interactions, since these would enter only in order $\alpha^2 G_{\rm nc}^{l}$. Similarly, the lowest order in which a $\Delta S = 0$, $P$- and $CP$-conserving and a $\Delta S = 0$, $P$- and $CP$-violating nonleptonic weak interaction could contribute is $\alpha^2 G_{\rm nl}^{P=+1, CP=+1}$ and $\alpha^2 G_{\rm nl}^{P=-1, CP=-1}$, respectively. One expects that $G_{\rm nl}^{P=-1, CP=-1} < 10^{-3}$ G, in view of the experimental upper limit on the electric dipole moment of the neutron (cf. Refs. 30 and 31). In diagram 1(b), $Z$ can be an axial-vector meson or a pseudoscalar meson, e.g., a pseudoscalar Higgs meson. The contribution of diagram 1(b) to the $\pi^0 \to e^+e^-$ amplitudes is given by Eqs. (19) and (20) even if $m_Z \le m_\pi$ [with $(G/\sqrt{2})g = f_l f_h/(m_\pi^2 - m_Z^2)$, where $f_l$ and $f_h$ are, respectively, the couplings of the intermediate boson to leptons and hadrons], but, of course, for other processes, if $q^2 \ge m_Z^2$, the effective interactions (17) and (18) would no longer represent the corresponding neutral-current interaction.

[10]The kinematics of pseudoscalar meson decays into massive lepton pairs has been discussed in A. Pais and S. B. Treiman, Phys. Rev. 176, 1974 (1968); see also L. A. Sehgal, *ibid.* 181, 2151 (1969).

[11]Angular momentum conservation forces the final $e^+e^-$ state to have $C = +1$. Consequently the parity-conserving amplitude is $CP$-conserving and the parity-violating amplitude is $CP$-violating.

[12]Cf. A. Pais and S. B. Treiman, Ref. 10.

[13]$CPT$ invariance could in principle be checked by looking for deviations of the relative phase of $a$ and $b$ from the "unitarity phase" (cf. L. M. Sehgal, Ref. 10). In practice this might be difficult, if not impossible, in view of additional contributions to the relative phase (due to admixtures of other states in the neutral-pion and higher-order diagrams) which cannot be reliably estimated (see the discussion at the end of the paper and Refs. 37 and 38).

[14]L. M. Sehgal, Ref. 10.

[15]Such an interaction with a strength $e\tilde{f}$ and isospin 0 or 1 would produce a neutron electric dipole moment $D_n$ of the order of $D_n \approx e\tilde{f}/M \approx 2 \times 10^{-14} \tilde{f}\, e\,\mathrm{cm}$, to be compared with the present experimental upper limit, $D_n^{\rm exp} < 3 \times 10^{-24}$ (cf. Refs. 30 and 31). Hence $\tilde{f} < 10^{-10}$. If the current is an isotensor, one would have $D_n \approx \alpha e \tilde{f}_2/\pi M \approx 2 \times 10^{-17} \tilde{f}_2\, e\,\mathrm{cm}$, so that $\tilde{f}_2 < 10^{-7}$.

[16]S. M. Berman and D. A. Geffen, Ref. 2; B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D 2, 179 (1970).

[17]Particle Data Group, Phys. Lett. 50B, 1 (1974).

[18]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[19]S. Weinberg, Phys. Rev. D 5, 1412 (1972).

[20]Note that the contribution of the $g_3^{AA}$ term is suppressed, since it is proportional to the electron mass (cf. Ref. 3). Parity conservation eliminates $J_3^S$, $K_3^S$, $J_{3\mu}^V$, $J_{3\mu}^V$, and the contribution of the tensor terms vanishes in view of $\bar{u}\sigma^{\mu\nu}v\langle 0|J_{3\mu\nu}^T|\pi^0\rangle \sim \bar{u}\sigma^{\mu\nu}p_\mu p_\nu v = 0$ (cf. Ref. 5) and $\bar{u}\sigma^{\mu\nu}\gamma_5 v\langle 0|K_{3\mu\nu}|\pi^0\rangle \sim \bar{u}\sigma^{\mu\nu}\gamma_5 p_\mu p_\nu = 0$. Finally, $G$-parity conservation excludes $J_{3\mu}^A$.

[21]J. Barclay Adams, Phys. Rev. 139, B1050 (1965); M. A. Bég and G. Feinberg, Phys. Rev. Lett. 33, 606 (1974); 35, (E)130 (1975). See also L. Wolfenstein, in *Particles and Fields—1974*, proceedings of the 1974 Williamsburg meeting of the Division of Particles and Fields of the American Physical Society, edited by C. E. Carlson (A.I.P., New York, 1975), p. 84; J. C. Pati and A. Salam, Ref. 6.

[22]D. E. Casperson *et al.*, Phys. Lett. 59B, 397 (1975); see also S. J. Brodsky, SLAC Report No. SLAC-PUB-1699, 1975 (unpublished).

[23]The contribution of the $g_3^{AA}$ term in (17) to the hydrogen ground-state hyperfine splitting is (cf. J. Barclay Adams, Ref. 21) $\Delta\nu_{\rm nc}/\Delta\nu_{\rm em} = 3Gg_3^{AA}\kappa_A^{(p)} mM/2\sqrt{2}\pi\alpha\mu_p$

($\mu_p$ =proton magnetic moment). It should be noted that in addition to the isovector axial-vector current interaction an isoscalar axial-vector coupling, as well as $I=1$ and an $I=0$ tensor-tensor coupling, could give similar contributions to the hyperfine splitting. The hyperfine splitting due to the $g_3^{PP}$ term is suppressed relative to the contribution of the $g_3^{AA}$ term by the factor $m/M \approx 10^{-3}$ [cf. L. B. Okun and V. I. Zakharov, Nucl. Phys. B57, 252 (1973)]. The bound (26) holds under the assumption that there is no significant cancellation between the $g_3^{AA}$ contribution and the other contributing terms.

[24]The presence of a neutral-current interaction leads also to deviations from scaling in deep-inelastic $e^{\pm}p$ scattering [A. Love, G. G. Ross, and D. V. Nanopoulos, Nucl. Phys. B49, 513 (1972); C. H. Llewellyn Smith and D. V. Nanopoulos, *ibid.* B78, 205 (1974); A. Soni, Ref. 4; I. I. Y. Bigi and J. D. Bjorken, Phys. Rev. D 10, 3697 (1974); M. A. Bég and G. Feinberg, Phys. Rev. Lett. 35, 130 (1975); J. C. Pati and A. Salam, Ref. 6]. Theoretical formulas for a general neutral-current coupling can be found in J. C. Pati and A. Salam, Ref. 6. With magnitudes of $g_3^{AA}$, $g_3^{PP}$, and $g_3^{SP}$ as large as 100, these effects are still only of the order of a few percent in the kinematic region so far studied and are thus not ruled out by the observed behavior of the structure functions. [For a review of the present data see R. E. Taylor, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California,* edited by W. T. Kirk (SLAC, Stanford, 1976), p. 679. It should be noted that these results have been analyzed in terms of the two structure functions $W_1$ and $W_2$ only, while if one allows for the presence of an axial-vector coupling, an additional structure function must be considered.]

[25]E. Derman, Phys. Rev. D 7, 2755 (1973); S. Berman and J. Primack, *ibid.* 9, 2171 (1974); J. C. Pati and A. Salam, Ref. 6. See also C. H. Llewellyn Smith and D. V. Nanopoulos, Ref. 24 and L. Wolfenstein, Ref. 21.

[26]W. B. Atwood, Ph.D. thesis, SLAC Report No. 185, 1975 (unpublished).

[27]L. S. Rochester *et al.*, Phys. Rev. Lett. 36, 1284 (1976).

[28]On the basis of the Zweig-Iizuka rule [G. Zweig, unpublished, 1964; J. Iizuka, Suppl. Prog. of Theor. Phys. 38, 21 (1966)], one expects in such a case $\kappa_A^{(p)}$, $\kappa_A^{(\pi)}$ (and also $\kappa_P^{(p)}$, $\kappa_P^{(\pi)}$ to be of the order of $10^{-1}$–$10^{-2}$.

[29]For a summary of the experimental results see R. F. Schwitters, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California,* edited by W. T. Kirk (SLAC, Stanford, 1976), p. 5. Theoretical papers on $e^+e^- \to$ hadrons in the presence of neutral-current interactions include J. C. Pati and A. Salam, Phys. Rev. D 11, 1137 (1975); I. I. Y. Bigi and J. D. Bjorken, *ibid.* 10, 3697 (1974); A. Soni, Phys. Lett. 53B, 280 (1974); M. A. Bég and G. Feinberg, Phys. Rev. Lett. 33, 606 (1974); L. Palla and G. Pócsik, Lett. Nuovo Cimento 11, 541 (1974). If $s$ ($\equiv E_{\text{c.m.}}{}^2$) is in the asymptotic region, but $s \ll m_Z{}^2$, a general neutral-current interaction gives rise to terms in $R$ linear and quadratic in $s$ (cf. J. C. Pati and A. Salam, Ref. 6, Appendix B). In the presence of a neutral-current coupling of the form

$$\frac{G}{\sqrt{2}} g_3^{\Gamma'\Gamma} \bar{e}\Gamma' e \sum_{i=\text{color}} \tfrac{1}{2}(\bar{q}_i \Gamma q_i - \bar{q}'_i \Gamma q'_i)$$

$$(\Gamma, \Gamma' = \gamma_\mu\gamma_5, \gamma_5, \text{ or } 1)$$

with $g_3^{\Gamma\Gamma'} \approx 100$, $R$ is still constant ($\approx 5$) within 10% in the energy region 5 GeV $\lesssim E_{\text{c.m.}} \lesssim 7.8$ GeV. For $s$ in the asymptotic region, but $s \gtrsim m_Z{}^2$, the situation is more uncertain.

[30]For a recent discussion of the electric dipole moment of the neutron see L. Wolfenstein, Nucl. Phys. B77, 375 (1974).

[31]See N. F. Ramsey, Bull. Am. Phys. Soc. 21, 61 (1976).

[32]From the remaining terms in the Lagrangian (18) one expects on the basis of (33) an upper bound of the same order of magnitude as in (34) also for $g_3^{PS}$ and $\tilde{g}_3^{AV}$, since the contribution of these terms to Fig. 2(a) vanishes as well. However, more restrictive for $g_3^{PS}$ is the present experimental upper limit on the electric dipole moment of the cesium atom $D_{\text{Cs}}$ [M. Weisskopf *et al.*, Phys. Rev. Lett. 21, 1645 (1968)], from which one can deduce $|g_3^{PS}\lambda_S^{(p)}| \lesssim 10^{-3}$ [C. Bouchiat, Phys. Lett. 57B, 284 (1975)]. Note that the $g_3^{SP}$ couplings does not contribute to $D_{\text{Cs}}$ since it cannot induce parity mixing in the electronic states [the nonrelativistic electron-nucleon potential is of the form $\vec{\sigma}_N \cdot \vec{r}_N f(|\vec{r}_N - \vec{r}_e|)$]. The contribution of the $\tilde{g}^{AV}$ term to $D_{\text{Cs}}$ is suppressed relative to the contribution of $g_3^{PS}$ by the factor $m/M \approx 10^{-3}$. The interaction terms proportional to $\tilde{g}_3^{VA}$ and $g_3^{PTT}$ contribute to Fig. 2(a) and therefore one expects $|\tilde{g}_3^{VA}| \lesssim 2\times 10^{-4}$ and $|g_3^{PTT}| \lesssim 2\times 10^{-4}$. The contribution of the $g_3^{PTT}$ term to $D_n$ has been studied by S. P. Rosen (unpublished) and has been found to be logarithmically divergent and proportional to $m/M$. With a cutoff of $\approx 50M$ the less restrictive bound $|g_3^{PTT}| \lesssim 2\times 10^{-3}$ follows. The contribution of $g_3^{PTT}$ and $\tilde{g}_3^{VA}$ to $D_{\text{Cs}}$ is suppressed relative to the contribution of the $g_3^{PS}$ term by the factor $1/(Z-N) = \frac{1}{23}$ ($Z$ and $N$ are, respectively, the number of protons and neutrons in the atom), and thus $D_{\text{Cs}}^{\text{exp}}$ does not improve the corresponding limits. The interaction (18) gives rise also to an electric dipole moment of the electron. The bounds which follow are weaker than those from $D_n$: For the couplings which contribute in order $Gg_3^{\Gamma\Gamma'}$ one expects $D_e \approx Gg_3^{\Gamma\Gamma'} m/4\pi \approx 10^{-23} g_3^{\Gamma\Gamma'}\, e$ cm, to be compared with the present experimental upper limit $D_e^{\text{exp}} < 3\times 10^{-24}\, e$ cm [M. C. Weisskopf *et al.*, Phys. Rev. Lett. 21, 1645 (1968); M. A. Player and P. G. H. Sandars, J. Phys. B3, 1620 (1970)].

[33]To detect an electron longitudinal polarization with an accuracy of 30%, assuming that the degree of polarization is $\approx 0.1$, with an analyzing power of 0.05, and assuming that 1 electron in about 200 will undergo Møller scattering in the polarimeter [see for example D. M. Schwartz, Phys. Rev. 162, 1306 (1967)], the required number of events is $\approx 2\times 10^8$, to be compared with the presently anticipated event rates of 0.3–0.6 per hour, with $\Gamma(\pi^0 \to e^+e^-)$ approximately at the unitarity bound level (cf. Ref. 1).

[34]For a review, see for example, F. Boehm, in *High Energy Physics and Nuclear Physics and Nuclear Structure— 1975,* Proceedings of the Sixth International Conference, Santa Fe and Los Alamos, edited by D. E.

Nagle *et al.*, (AIP, New York, 1975), p. 488; P. Herczeg, in *ibid.*, p. 504. If the present experimental evidence persists, one may have to conclude that the strength $G^{\text{pv}}_{\text{nl},I=2}$ of the isotensor component of the effective parity-violating strangeness-conserving hadronic interaction is $G^{\text{pv}}_{\text{nl},I=2} \approx 50G$.

[35]P. C. McNamee, M. D. Scadron, and S. A. Coon, Nucl. Phys. A249, 483 (1975), and references quoted therein.

[36]We are assuming that the interactions are *CPT* invariant. Then $\langle\pi_1|H^{P=+1,CP=-1}|\pi_2\rangle$, $\langle\pi_2'|H^{P=-1,CP=-1}|\pi_2\rangle$ are pure imaginary and $\langle\pi_1'|H^{P=-1,CP=+1}|\pi_2\rangle, \langle\tilde{\pi}_2|H^{P=+1,CP=+1}|\pi_2\rangle$ are real.

[37]The contributions of possible neutral-current interactions to $a_{\text{mixing}}$ and $b_{\text{mixing}}$ are

$$a^{(n)}_{\text{mixing}} = \epsilon_1\left(\frac{2Gm}{\sqrt{2}}\tilde{g}^{AA}\tilde{N}^{(\pi_1)}_{AA} + \frac{G}{\sqrt{2}}\tilde{g}^{PP}\tilde{N}^{(\pi_1)}_{PP}\right) + \epsilon_1'\left(\frac{2Gm}{\sqrt{2}}g^{AV}N^{(\pi_1')}_{AV} + \frac{G}{\sqrt{2}}\tilde{g}^{PS}N^{(\pi_1')}_{PS}\right) + \epsilon_2'\left(\frac{2Gm}{\sqrt{2}}\tilde{g}^{AV}\tilde{N}^{(\pi_2')}_{AV} + \frac{G}{\sqrt{2}}g^{PS}N^{(\pi_2')}_{PS}\right) + \epsilon_\eta\left(\frac{2Gm}{\sqrt{2}}g_0^{AA}N^{(\eta)}_{AA} + \frac{G}{\sqrt{2}}g_0^{PP}N^{(\eta)}_{PP}\right),$$

$$b^{(n)}_{\text{mixing}} = \epsilon_1\left(\frac{G}{\sqrt{2}}\tilde{g}^{SP}\tilde{N}^{(\pi_1)}_{SP}\right) + \epsilon_1'\left(\frac{G}{\sqrt{2}}\tilde{g}^{SS}\tilde{N}^{(\pi_1')}_{SS}\right) + \epsilon_2'\left(\frac{G}{\sqrt{2}}g^{SS}N^{(\pi_2')}_{SS}\right) + \epsilon_\eta\left(\frac{G}{\sqrt{2}}g^{SP}N^{(\eta)}_{SP}\right).$$

The notation employed here is analogous to the one used in Eqs. (17) and (18). Thus $G\tilde{g}^{PS}/\sqrt{2}$, for example, represents the strength of a pseudoscalar-leptonic–second-class-scalar-hadronic coupling of arbitrary isospin, $Gg_0^{PP}/\sqrt{2}$ refers to the strength of a pseudoscalar–isoscalar-pseudoscalar coupling, etc.; $\tilde{N}^{(\pi_1)}_{SP} = \langle 0|\tilde{J}^P|\pi_1(p^2=m_\pi^2)\rangle$, where $\tilde{J}^P$ is a second-class pseudoscalar density coupled to a scalar leptonic density; $\bar{e}\gamma_\mu\gamma_5 e\langle 0|J^V_\mu|\pi_1'(p^2=m_\pi^2)\rangle = \bar{e}\gamma_\mu\gamma_5 e N^{(\pi_1')}_{AV} p_\mu$, etc. Note that $\text{Re}a_1^{(n)}$, $\text{Re}a_2^{(n)\prime}$, $\text{Im}a_1^{(n)\prime}$, and $\text{Im}a_\eta^{(n)}$ are the absorptive parts of $a_1^{(n)}$, $a_2^{(n)\prime}$, $a_1^{(n)\prime}$, and $a_\eta^{(n)}$ and therefore vanish in lowest order. With the mixing parameters given in the text, and with $|\tilde{g}^{AA}|, |\tilde{g}^{PP}|, |\tilde{g}^{SS}|, |g^{SS}|, |g_0^{PP}|, |\tilde{g}^{SP}|, |\tilde{g}^{PS}| \le 100$ (from data on $e^+e^- \to$ hadrons; see Ref. 29), $|g_0^{SP}| \le 0.1$ (from $D_\eta^{\text{exp}}$), $|g_0^{AA}| \le 40$ (suggested by the hydrogen ground-state hyperfine splitting), $|\tilde{g}^{AV}| \le 0.1$, $|g^{PS}| \le 10^{-3}$ (see Ref. 32), $g^{AV} \le 1$ (suggested by data on atomic parity violation [cf. P. E. G. Baird *et al.*, Nature 264, 528 (1976)]), one obtains the bounds (37)–(40), neglecting contributions whose upper bound is relatively small.

Inspection shows that the contributions from the two-photon intermediate state to $a_{\text{mixing}}$ and $b_{\text{mixing}}$ have negligible effect on the limits (42)–(45). The largest allowed contribution is to $\text{Re}a_{\text{mixing}}$, from the $\eta$ meson. Assuming $|\text{Re}a_\eta^{(e)}(p^2=m_\pi^2)| \approx |\text{Re}a_2^{(e)}| \le 10|\text{Im}a_2^{(e)}|$, one would have $|\text{Re}(\epsilon_\eta a_\eta^{(e)})| \approx |\text{Re}\epsilon_\eta \text{Re}a_\eta^{(e)}| \le 4\times 10^{-8}$. (The contribution to $\text{Im}a_{\text{mixing}}$ is much smaller: The term $\text{Re}\epsilon_\eta \text{Im}a_\eta^{(e)}$ in $\text{Im}(\epsilon_\eta a_\eta^{(e)})$ is already included in Eq. (8), since now $|F| = [(F_2 + F_\eta \text{Re}\epsilon_\eta)^2 + (F_\eta \text{Im}\epsilon_\eta)^2]^{1/2} \approx F_2 + F_\eta \text{Re}\epsilon_\eta$; consequently $|\text{Im}(\epsilon_\eta a_\eta^{(e)})| \le 3\times 10^{-13}$.)

[38]Present data do not rule out large violations of *CPT* invariance in the amplitudes $a$ and $b$. We shall denote the strength of the contributing *CPT*-violating neutral-current-interaction couplings by $(G/\sqrt{2})g_3^{AA\prime}$, $(G/\sqrt{2})g_3^{PP\prime}$ (contributes to $\text{Im}a$), and $(G/\sqrt{2})g_3^{SP\prime}$ (contributes to $\text{Im}b$) in analogy with the notation used in Eqs. (17) and (18). The most restrictive information on $g_3^{SP\prime}$ is obtained by considering its contribution to the quantity $\delta_K \approx [m(K^0) - m(\bar{K}^0)]/(m_L - m_S)$ [for a discussion of tests of *CPT* invariance see L. Wolfenstein, Nuovo Cimento 63A, 269 (1969)]. In lowest order $\delta_K \approx (\alpha/\pi)(g^{SP})(G^{\text{pv}}_{\text{nl}}/G)$, where $G^{\text{pv}}_{\text{nl}}$ represents the strength of the parity-violating strangeness-conserving hadronic interactions. Data on the $K^0$-$\bar{K}^0$ system imply $|\bar{\delta}| \equiv (1/\sqrt{2})|(\frac{1}{2}\delta_K + \beta_0)| \le 6\times 10^{-4}$ [cf. K. Kleinknecht, Annu. Rev. Nucl. Sci. 26, 1 (1976)], where $\beta_0 \approx [\gamma(K^0) - \gamma(\bar{K}^0)]/[\gamma(K_0) + \gamma(\bar{K}_0)]$, $\gamma(K^0)$ and $\gamma(\bar{K}^0)$ are the decay widths of $K^0$ and $\bar{K}^0$. Assuming that $[\gamma(K^0) - \gamma(\bar{K}^0)]/[\gamma(K^0) + \gamma(\bar{K}^0)] \approx [\gamma(K^+) - \gamma(K^-)]/[\gamma(K^+) + \gamma(K^-)] \equiv \Delta_K$ and using the experimental limit $\Delta_K \le 10^{-3}$ [F. Lobkowicz *et al.*, Phys. Rev. Lett. 17, 548 (1967)], one infers that $|\delta_K^{\text{exp}}| \le 4\times 10^{-3}$, and consequently that $|g_3^{SP\prime}| \le 4|G/G_{\text{nl}}|$. With $G_{\text{nl}} \approx 50G$ (cf. Ref. 34) one has $|g_3^{SP\prime}| \le \frac{1}{10}$. Note that if a *CP*-invariant parity-violating semileptonic neutral-current interaction of strength $Gg$ exists, one would have $|g_3^{SP\prime}| \le \min[(4\times 10^{-3})/g, 4G/G_{\text{nl}}]$.

For the *CP*-conserving couplings $g_3^{AA\prime}$ and $g_3^{PP\prime}$ no significant bound follows from $\delta_K$. The best upper limit appears to be $|g_3^{AA\prime}|, |g_3^{PP\prime}| \le 100$, indicated by data on $e^+e^- \to$ hadrons (cf. Ref. 29).

The strength of a possible *CPT*-violating electromagnetic interaction is limited in the *P*-conserving, *C*-conserving case by $D_\eta^{\text{exp}}$ to be less than $e/10$ (cf. Ref. 30), and in the *P*-violating, *C*-conserving case by $\delta_K^{\text{exp}}$ to be less than $10^{-7}e$.

Additional contributions would come from possible *CPT*-violating components in the interactions involved in $a_{\text{mixing}}$ and $b_{\text{mixing}}$. Inspection shows that these would not change significantly the bounds (42)–(45), with the exception of the bound (43) for $\text{Im}a_{\text{mixing}}$, which could be of the order of $10^{-7}$ in the presence of a *CPT*-violating $g_0^{PP}$-type neutral-current interaction.