Decays of a heavy lepton and an intermediate weak boson in quantum chromodynamics

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The renormalization-group method is applied to the calculation of total semileptonic decay of a heavy lepton and total hadronic decay of an intermediate weak boson. The prediction of the leptonic branching ratio of a heavy lepton is compared with the anomalous μe events observed at SPEAR.

I. INTRODUCTION

Application of renormalization-group considerations to the hadronic electromagnetic current has led to the prediction that the ratio[1]

$$R_{e^+e^-}(s)=\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} \tag{1}$$

should be independent of s at sufficiently high center-of-mass energy $\sqrt{s}=E$. Its value is calculable by perturbation in an asymptotically free gauge theory.[2] In the colored SU(3) quark-gluon model [now known as QCD (quantum chromodynamics)], the first two terms are[3]

$$R_{e^+e^-}(s)=3\left(\sum_i e_i^{\,2}\right)\left[1+\frac{\alpha(-|s|)}{\pi}\right], \tag{2}$$

where e_i is the quark charge of flavor i, and $\alpha(s)$ is the running coupling constant of the underlying gauge theory,[2]

$$\alpha(s)=\frac{12\pi}{(33-2N)\ln(-s/\mu^2)}\quad, \tag{3}$$

where N is the number of flavors which should be considered at energy s.

In a recent paper Shankar[4] has shown that, in addition to $R_{e^+e^-}(s)$, the renormalization-group method also allows one to calculate the integral

$$\Omega_{e^+e^-}(M^2)=\int_0^{M^2} ds\, R_{e^+e^-}(s) \tag{4}$$

for sufficiently large values of M^2.

Clearly these ideas are equally applicable to the hadronic weak current. In this paper, we consider only the charged current. In addition to the obvious application to the total hadronic decay rate of a charged weak intermediate vector boson (W^+ or W^-), we present in this paper an interesting prediction for yet another physical process: the total rate of the semileptonic decay of a heavy lepton. The latter requires a simple generalization of Shankar's result (4) to the higher moments. They are also calculable in QCD. These moment integrals for the analog of $R_{e^+e^-}$ in weak interaction appear directly in the rate of semileptonic decay of a heavy lepton. For definiteness, we will assume that the charged hadronic weak current in QCD is purely $V-A$ and it has the form

$$J_W^{\,\mu}=\sum_{(ij)} c_{ij}\bar{\psi}_i\gamma^\mu(1-\gamma_5)\psi_j\ , \tag{5}$$

where i,j refer to quark flavors. In the SU(4) quark model,[5] $J_W^{\,\mu}$ is

$$\begin{aligned}J_W^{\,\mu}&=\bar{u}\gamma^\mu(1-\gamma_5)(d\cos\theta+s\sin\theta)\\&\quad+\bar{c}\gamma^\mu(1-\gamma_5)(s\cos\theta-d\sin\theta)\ ,\end{aligned} \tag{6}$$

where u, d, s, and c are the so-called up, down, strange, and charm quarks, and θ is the Cabibbo angle. If the heavy lepton behaves like a heavy electron, the renormalization group predicts the following branching ratio:

$$\begin{aligned}R_L&=\frac{\sigma(L\to\nu_L+\text{hadrons})}{\sigma(L\to\nu_L+e+\bar{\nu})}\\&=3\sum_{(ij)}(c_{ij})^2\left[1+\frac{\alpha(-M_L^{\,2})}{\pi}\right],\end{aligned} \tag{7}$$

where L,ν_L denote the heavy lepton and its neutrino, respectively; $\alpha(-M_L^{\,2})$ is the value of the running coupling constant at $-s=M_L^{\,2}$, the square of the heavy-lepton mass. If M_L is below the charm threshold, only u, d, s quarks contribute and

$$\sum_{ij}(c_{ij})^2=\cos^2\theta+\sin^2\theta=1\ . \tag{8}$$

Both weak processes mentioned above have been discussed by other authors from the viewpoint of the parton model. Li and Paschos[6] have studied the decay of $W^\pm$, and Tsai[7] has considered in great detail the decays of a heavy lepton. One purpose of this paper is to confirm these parton-model results in QCD and also to give the first correction in such a theory. The decay of a heavy lepton provides a natural example of the idea[8] that the renormalization group can be useful in a process which involves low energy domain if one averages over a sufficiently large energy interval. Finally, we have compared the prediction of the semileptonic decay of a heavy lepton with the data of μe events observed in SPEAR, under the assumption that they are the decay products of a heavy lepton.

II. SPECTRAL FUNCTIONS AND THEIR MOMENTS OF WEAK CURRENT

Define the spectral functions of the weak current

$$C^{\mu\nu}(q) = \int d^4x\, e^{-iqx}\langle 0|J_W{}^{\mu}(x)J_W{}^{\nu\dagger}(0)|0\rangle$$
$$= -(g_{\mu\nu}q^2 - q_\mu q_\nu)\rho_1(q^2) + q_\mu q_\nu \rho_2(q^2) \,. \tag{9}$$

These spectral functions ρ_1 and ρ_2 are absorptive parts of an analytic two-point function $\Pi_{\mu\nu}(q)$. In the deep Euclidean region $\Pi_{\mu\nu}(q)$ can be computed by the renormalization-group technique. To obtain predictions for ρ_1 and ρ_2, it is usually assumed that $\Pi_{\mu\nu}(q)$ so computed can be analytically continued to the physical (timelike) region of large $q^2 > 0$. In QCD this means that q^2 is large enough so that the running coupling constant α is small, and q^2 should not be near a quark-gluon threshold or a threshold of physical hadrons.

In QCD the calculation for ρ_1 and ρ_2 is almost identical to that for $R_{e^+e^-}$ of Eq. (2). The first two terms in the renormalization-group improved perturbation series are $(q^2 = s)$

$$\rho_W(s) \equiv \rho_1(s) = 3\sum_{ij}(c_{ij})^2 \frac{1}{3\pi}\left(1 + \frac{\alpha(-|s|)}{\pi}\right)$$
$$= \rho^{(0)}(s) + \alpha(-|s|)\rho^{(1)}(s) \,, \tag{10}$$

$$\rho_2(s) = O\left(\frac{m^2}{s}\right) \,, \tag{11}$$

where $\alpha(-|s|)$ is given by Eq. (3), and m is the quark mass.

To obtain moment sum rules of ρ_W, we follow closely the arguments of Shankar.[4] Consider the closed contour of Fig. 1; we have

$$\int_{C_1} s^n\Pi(s)ds = -\int_{C_2} s^n\Pi(s)ds \,, \tag{12}$$

where $\Pi(s)$ is the analytic function with the absorptive part $\rho_W(s)$. The first two terms for $\Pi(s)$ in QCD are

$$\Pi(s) = \Pi^{(0)}(s) + \alpha(s)\Pi^{(1)}(s) \,. \tag{13}$$

The left-hand side of Eq. (12) can be expressed in terms of the absorptive part of Π. For the right-hand side we can approximate the integrand by the theoretical formula (13). If s is large enough for $\alpha(s)$ to be small, this should be a good approxima-

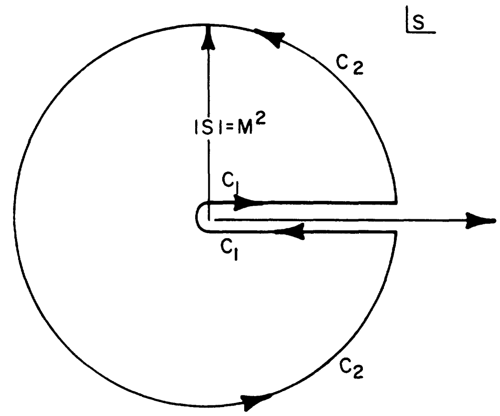

FIG. 1. The integral of Π along the closed contour $C_1 + C_2$ is zero.

tion over most of C_2 except for a small range near the real axis. As in the derivation of $R_{e^+e^-}(s)$, we will assume that $\Pi(s)$ does not have strong singularities in this region for the values of s under consideration. Let us choose $s = -M^2$, M^2 being the radius of C_2. We can ignore the variation of $\alpha(s)$ on C_2 and set it equal to $\alpha(-M^2)$. The small imaginary part it acquires away from the negative real axis is of the next order. Thus, the right-hand side of Eq. (12) can be related to the absorptive parts of $\Pi^{(0)}$ and $\Pi^{(1)}$. We now have the prediction

$$\Omega_W{}^{(n)}(M^2) = \int_0^{M^2} ds\, s^n \rho_W(s)$$
$$= \int_0^{M^2} ds\, s^n\rho^{(0)}(s) + \alpha(-M^2)\int_0^{M^2} ds\, s^n\rho^{(1)}(s) \,. \tag{14}$$

This is the generalization to Shankar's result where only the case $n = 0$ has been considered. As emphasized by Shankar, the validity of the result (14) depends on the upper limit M^2 being large enough so that $\alpha(-M^2)$ is small.

We would like to comment on this result. First of all, it is not possible to invert Eq. (14) to recover the integrand for any $s < M^2$. Suppose the important mass scale for ρ is $m^2 \ll M^2$. The renormalization-group arguments assure that dependence on the ratio m^2/M^2 can be neglected. But if we try to invert the sum rules, we will generate ratios such as $m^2/M^2 \cdot M^2/s = m^2/s$ which is no longer small for $s < M^2$. Secondly, it has been argued[9] that direct application of quark-gluon perturbation theory in the physical region cannot be justified even at high energy where the running coupling constant is weak. In particular, in every order perturbation theory predicts production of quarks and gluons whereas only color-singlet bound states of gluons and quarks should be created. A prescription[9] has been suggested to "smear" the results, and only "prediction on the average" is to be confronted with experiment. While the necessity for smearing is obvious, a unique procedure has yet to be found. Our result, Eq. (14), which is equivalent to smearing over a large circle in the complex s plane, is a possible alternative. From this standpoint, the moment sum rules should be more reliable than the prediction for the integrand $\rho_W(s)$ at large s. Indeed, according to Shankar,[4] Eq. (4) agrees with the limited available data down to $s \simeq 0.8\ \mathrm{GeV}^2$ for the choice $\mu = 0.7$ GeV.

III. DECAYS OF HEAVY LEPTON AND W BOSON

We will now apply the results just obtained to the two physical processes mentioned in the Introduction. Consider the decay of the $W^\pm$ boson first. The W boson will be assumed to couple to the hadrons and the leptons via the conventional $V-A$ weak current. Equation (10) then predicts the following ratio for the hadronic decay rate of W to that of leptonic decay[10]

$$R_W = \frac{\Gamma(W^- \to \text{hadrons})}{\Gamma(W^- \to e\bar{\nu}_e)} = 3\sum_{ij}(c_{ij})^2\left[1+\frac{12}{(33-2N)\ln(M_W{}^2/\mu^2)}\right] \quad (15)$$

This is the analog of $R_{e^+e^-}(s)$ given by Eq. (2). In the SU(4) quark model, the weak current of Eq. (5) gives

$$\sum_{ij}(c_{ij})^2 = 2 \quad (16)$$

and

$$R_W = 6\left[1+\frac{12}{25\ln(M_W{}^2/\mu^2)}\right] . \quad (17)$$

For $M_W \simeq 40$ GeV and $\mu \sim 1$ GeV, the correction term is about 6%. If one neglects the correction term, we find the ratio

$$\frac{R_W}{R_{e^+e^-}} = \frac{\sum(c_{ij})^2}{\sum e_i{}^2} , \quad (18)$$

which is 1.8 in the SU(4) quark model.

We now turn to the semileptonic decay of a heavy lepton,

$$L \to \nu_L + \text{hadrons} . \quad (19)$$

The heavy lepton, L, will be assumed to have its own massless neutrino, ν_L, and its own lepton number. It has all the same characteristics as an electron and muon except for its heavier mass. Its interaction with the hadron is described by the effective Lagrangian

$$\mathcal{L}_I = \frac{G}{\sqrt{2}}\bar{\nu}_L\gamma_\mu(1-\gamma_5)LJ_W{}^{\mu\dagger} , \quad (20)$$

where G is the Fermi coupling constant. The total decay rate for (19) can be expressed in terms of the spectral functions ρ_1 and ρ_2 discussed in Sec. II (Ref. 7):

$$\Gamma(L\to\nu_L+h) = \frac{G^2}{32\pi^2M^3}\times\int_0^{M_L{}^2} ds(M_L{}^2-s)^2 \times[\rho_1(s)(M_L{}^2+2s)+\rho_2(s)M_L{}^2] , \quad (21)$$

where M_L is the mass of the heavy lepton. The corresponding leptonic decay rate is[7]

$$\Gamma(L\to\nu_L+l+\bar{\nu}_l) = \frac{G^2M_L{}^5}{(3)(2^6)\pi^3} , \quad (22)$$

where l is an electron or a muon, and m_l is set to zero.

If $M_L{}^2$ is sufficiently large, then the moment sum rules (14) predict the ratio[11]

$$R_L = 3\sum_{ij}(c_{ij})^2\left[1+\frac{\alpha(-M_L{}^2)}{\pi}\right] . \quad (23)$$

In QCD the correction to this result is of order $m_q{}^2/M_L{}^2$ or $\alpha(-M_L{}^2)^2$ (m_q is the quark mass).

The existence of a heavy lepton is suggested by the anomalous μe events observed in e^+e^- annihilation at SPEAR.[12] These μe particles are interpreted as products of leptonic decays of a heavy lepton of mass in the range 1.6–2.0 GeV. With the assumption of equal decay rates to the e and μ modes, $V-A$ coupling, and $M_L = 1.8$ GeV, the data[12] yield the leptonic branching ratios

$$\frac{\Gamma(L\to\nu_L+e+\bar{\nu}_e)}{\Gamma(L\to\text{all})} = \frac{\Gamma(L\to\nu_L+\mu+\bar{\nu}_\mu)}{\Gamma(L\to\text{all})} = 0.17^{+0.06}_{-0.03} . \quad (24)$$

Suppose we accept the heavy-lepton interpretation of these μe events. The theoretical value for the leptonic branching ratios depends crucially on whether the mass M_L is above or below the charm threshold. According to Eq. (5) for $J_W{}^\mu$, the Cabibbo-favored charm decay mode is

$$L \to \nu_L + F ,$$

where F is the lowest strange charmed meson. Naive-quark counting places the mass of F at

$$M_F \sim 2.05\text{–}2.10\ \text{GeV} , \quad (25)$$

which is outside the experimental mass range for the heavy lepton. Thus, we may assume that the charm quark does not contribute to the result (23) for R_L.[13] With $M_L = 1.8$ GeV, and $\mu \simeq 0.7$ GeV as adopted in Shankar's analysis,[4] we find

$$R_L = 3.70 \tag{26}$$

and

$$\frac{\Gamma(L \to \nu_L + e + \overline{\nu}_e)}{\Gamma(L \to \text{all})} = \frac{\Gamma(L \to \nu_L + \mu + \overline{\nu}_\mu)}{\Gamma(L \to \text{all})} = \frac{1}{2 + R_L} = 0.17\,. \tag{27}$$

This ratio would be 0.2 if one neglects the logarithmic correction of asymptotic freedom. The numerical value of (27) could be further reduced for theoretical reasons. Around $s = 9\ \text{GeV}^2$ the value of $R_{e^+e^-}$ predicted by (2) with $\mu = 0.7$ GeV is lower by about 10% as compared with the data. A similar error may occur in the calculation of R_L.

Equations (24) and (27) agree within the errors. Because of the large experimental uncertainty in (24) it is difficult to make a more quantitative comparison between theory and experiment. Nevertheless, we cannot help noticing that the experimental branching ratio is of the order expected of a heavy lepton and it is even likely to be less than 20% as predicted by QCD. The theoretical result (27) appears to support the interpretation of a heavy lepton being the main source of the observed μe events.[14]

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[12]M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975); M. L. Perl *et al.*, Phys. Lett. 63B, 466 (1976).

[13]If M_L is above but not far away from the charm threshold, the theoretical value for R_L would be difficult to compute reliably because of threshold mass effects.

[14]More recent data from both SPEAR and DESY appear to be in favor of a charged heavy lepton of mass about 2 GeV: G. J. Feldman *et al.*, Phys. Rev. Lett. 38, 117 (1977); H. Meyer, talk given at the 1977 Coral Gables conference (unpublished); and R. Felst, talk given at the 1977 Meeting of the American Physical Society, Chicago (unpublished). Also, more evidence is seen in a new SPEAR experiment using a lead-glass detector added to the magnetic detector. We thank Professor M. Perl for informing us of these new experimental developments. We should mention two earlier reports on inclusive anomalous lepton production in e^+e^- annihilation: M. Cavalli-Sforza *et al.*, Phys. Rev. Lett. 36, 558 (1976); W. Braunschweig *et al.*, Phys. Lett. 63B, 471 (1976).