PHYSICAL REVIEW D VOLUME 16, NUMBER 3 1 AUGUST 1977

Extraction of average multiplicities of high-energy off-mass-shell $\pi\pi$ and $K\pi$ scattering*

J. Erwin, R. D. Kass, J. H. Klems, Winston Ko, R. L. Lander, D. E. Pellett, and P. M. Yager
Department of Physics, University of California, Davis, California 95616

M. Alston-Garnjost
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 8 November 1976)

Inclusive $\Delta^{++}(1236)$ and Λ^0 production in π^+p and pp interactions is examined in terms of a one-meson-exchange model. We find inclusive Δ^{++} production to be consistent with one-pion exchange, and if one demands a gap of at least one unit of rapidity between the Λ^0 and its recoiling mass, inclusive Λ^0 production is consistent with one-kaon exchange. The average charge multiplicity, $\langle N_{CH}\rangle$, recoiling from the Δ^{++} and Λ^0 is studied as a function of massing mass M^2 for both π^+p and pp interactions.

INTRODUCTION

There has been increasing evidence[1-5] that inclusive $\Delta^{++}(1236)$ production in the hundred-GeV range is mediated by one-pion exchange. In this paper we show that this is also the case for both the pp and π^+p interactions at 100 GeV/c. The average multiplicity, $\langle N_{CH}\rangle$, of the mass recoiling off the Δ^{++} produced in pp interactions is shown to be consistent with that of π^-p scattering. Examining the mass recoiling against the Δ^{++} from π^+p interactions allows us to study $\langle N_{CH}\rangle$ for off-mass-shell $\pi^-\pi^+$ interactions.

The quantum numbers for a Λ^0 produced backward in the center of mass off a proton target are consistent with kaon exchange. Analysis of the mass recoiling against the Λ^0 for the reaction $\pi^+p\to\Lambda^0X$ allows us to study $\langle N_{CH}\rangle$ for off-mass-shell $K^+\pi^+$ scattering. A consistency check of $\langle N_{CH}\rangle$ is obtained by extracting $\langle N_{CH}\rangle$ for off-mass-shell K^+p from the process $pp\to\Lambda^0X$ at 100 GeV and comparing it with the on-mass-shell values.

SCANNING AND MEASUREMENT

The data consist of an approximately-100 000-picture exposure of the Fermi National Accelerator Laboratory's 30-in. hydrogen bubble chamber to a mixed, tagged positive beam at 100 GeV/c.[6] This exposure yielded 7821 pp events, 4304 π^+p events, and a combined total of 2418 events with at least one neutral-strange-particle decay or γ conversion. The tracks from the primary vertex were measured twice on a Lawrence Berkeley Laboratory (LBL) spiral reader.[7] Half of the neutral decays and conversions were measured on an LBL Franckenstein, the other half were measured on a spiral reader.[8] All remeasurements were made on a Franckenstein. The measurements were processed using the programs POOH and TVGP. The program SQUAW was used to fit the neutral decays and conversions that were unambiguously identified with a primary vertex and also to fit the four-constraint (4C) hypothesis for events with no V's. The overall scanning efficiency for V's was 96%, while the overall measurement efficiency for neutral decays and conversions was 89%. The scanning efficiency for four prongs and higher multiplicities was 99%, and for two prongs was 93%.

The efficiency for measuring positive tracks in the backward hemisphere in the c.m. system is about 96% for all topologies. Thus we have a high efficiency for measuring the two positive tracks decaying from a slow Δ^{++}. Protons were distinguished from pions or kaons both by automatic ionization information and by a visual check of bubble density for laboratory momentum up to 1.3 GeV/c.

Tracks from neutral decays were assigned an identity, when possible, by using ionization information. Ambiguous events were selected on the basis of the confidence level of the kinematic fit and the transverse momentum of the decay tracks with respect to the line of flight of the neutral. Strange-particle decays were rejected if they were within 2 cm of the production vertex. The number of decays satisfying the criteria of a Λ^0 were 135 (57) for pp (π^+p) interactions. The contamination of the Λ^0 by γ's is estimated to be less than 5%.

INCLUSIVE $\Delta^{++}(1236)$ PRODUCTION

Since we can distinguish protons from mesons for laboratory momentum up to 1.3 GeV/c, a cut of $|t_{p\Delta^{++}}| < 0.88$ GeV^2 is imposed in order to ensure that all decay angles of the Δ^{++} are covered. The invariant mass of all proton-π^+ combinations with $|t_{p\Delta^{++}}| < 0.88$ GeV^2 is shown in Fig. 1. A strong $\Delta^{++}(1236)$ signal is seen for both pion- and proton-induced events. After imposing the above-mentioned $|t|$ cut and correcting for background, the

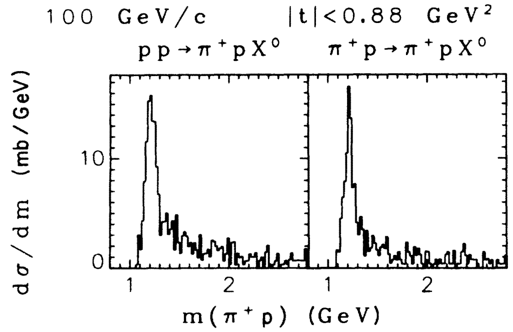

FIG. 1. Invariant mass $m(\pi^+p)$ for all πp combinations. There is a $|t_{p\to\pi^+p}|$ cut of 0.88 GeV^2 made on the data. The above figure corresponds to 809 (711) weighted pp (π^+p) events.

inclusive production cross sections of Δ^{++} produced in the backward hemisphere are 1.24 ± 0.12 and 1.04 ± 0.10 mb for pp and π^+p interactions.[9] The pp result is in agreement with the result of Ref. 3, 1.36 ± 0.14 mb. For the rest of this paper a Δ^{++} is defined to be any proton-π^+ combination with $1.16\leq m(\pi^+p)\leq1.32$ GeV and is limited to $|t|<0.88$ GeV^2. For those cases (24 events for pp, 22 events for π^+p) where more than one π^+p combination satisfies our selection criteria, we chose the combination which has the mass closest to 1.236 GeV.

To test for one-pion exchange we study the spin alignment of the π^+p pair in terms of the Treiman-Yang angle and the Jackson angle. The decay angles of the proton are computed in the rest frame

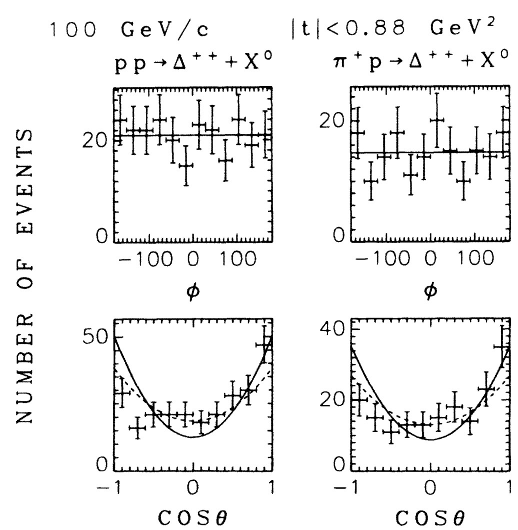

FIG. 2. The t-channel decay angles of the Δ^{++}. The ϕ distributions are compared with a normalized constant. The $\cos\theta$ distributions are compared with both the one-pion exchange ($1+3\cos^2\theta$ in solid line) and one-pion exchange with abosrbtion ($0.37\pm0.39\cos^2\theta$ in dotted line).

of the Δ^{++}, the t-channel frame. In this frame, the z axis is the direction of the target-particle momentum ($\vec{p}$) and the y axis is defined by $\vec{q}\times\vec{p}$, where $\vec{q}$ = momentum of the beam particle. The distribution in the Treiman-Yang angle is shown in Fig. 2. For both $pp\to\Delta^{++}X^0$ and $\pi^+p\to\Delta^{++}X^0$ this distribution is flat, consistent with one-pion exchange. The t-channel decay density matrix element $\mathrm{Re}\rho_{3-1}$ is determined to be 0.00 ± 0.03 for $pp\to\Delta^{++}X^0$ and -0.03 ± 0.04 for $\pi^+p\to\Delta^{++}X^0$. These values are in agreement with zero, as expected from one-pion exchange. Additional evidence of one-pion exchange is provided by the Jackson angle ($\cos\theta$) distribution, also shown in Fig. 2. The $\cos\theta$ distributions are found to be consistent with predictions of either the one-pion-exchange model $[dN/d(\cos\theta)=1+3\cos^2\theta]$ or the one-pion-exchange absorptive model $[dN/d(\cos\theta)=0.37+0.39\cos^2\theta]$. We find (not shown) that for $M(\pi^+p)>1.32$ GeV the Jackson angle distribution is strongly peaked in the forward ($\cos\theta=1$) direction. This difference in the distribution is additional evidence that we have a strong Δ^{++} signal. In the Δ^{++} region ρ_{33} is determined to be 0.13 ± 0.04 for both p-induced and π^+-induced reactions. Knowing that background is surely present and recalling the large difference between the Jackson angle distributions in the Δ^{++} region and the region of higher π^+p masses, we do not feel that our measurement of ρ_{33} distinguishes between the one-pion-exchange ($\rho_{33}=0$) and the one-pion-exchange absorptive model ($\rho_{33}=0.12$). In summary mary, the spin-alignment study here shows that inclusive Δ^{++} production at 100 GeV/c is consistent with one-pion exchange in both π^+p and pp interactions.

OFF-MASS SHELL π^-p and $\pi^-\pi^+$ SCATTERING

The diagram in Fig. 3(a) represents inclusive Δ^{++} production via one-pion exchange. If, however,

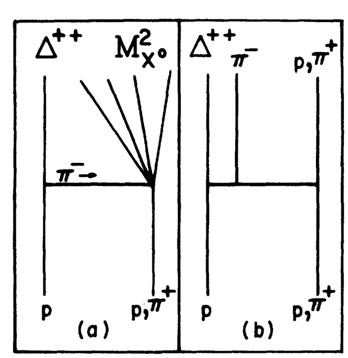

FIG. 3. (a) One-pion-exchange diagram representing the production process of Δ^{++}. (b) Diagram for Δ^{++} production in 4C four-prong events. To isolate off-mass-shell π^- inelastic scattering we remove events represented by diagram (b).

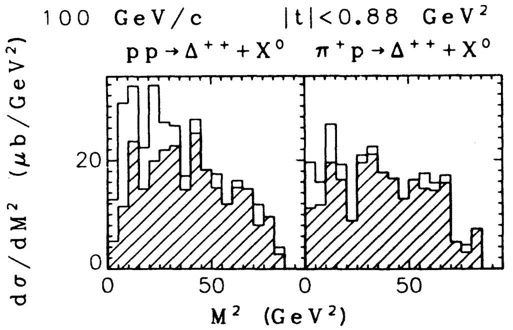

FIG. 4. Distribution of recoiling mass squared M^2 of the X^0. The shaded distributions do not include 4C four-prong events. See Ref. 9 for background.

one wants to study the inelastic multiplicity of the mass recoiling against the Δ^{++}, events due to processes shown in Fig. 3(b) must be removed (e.g. $N^{*+} \rightarrow \Delta^{++}\pi^-$). These events are four prongs that are consistent with no π^0's or neutral strange particles being produced (4C four-prong).[10] In terms of Fig. 3(b) a 4C four-prong event is an event of elastic π^-p or $\pi^-\pi^+$ scattering. Therefore, in order to study the inelastic production of X^0 via π^- exchange, these 4C four-prong events must be removed.

Figure 4 shows the square of the recoiling mass of all events and events with the 4C four-prong event removed.

After removing the 4C four-prong events in $pp \rightarrow \Delta^{++}X^0$ the multiplicity of X^0 should agree with that of the known multiplicity of inelastic π^-p interactions. We parameterized the on-mass-shell π^-p inclusive multiplicity by Eq. (1),

$$\langle N_{CH}\rangle = -4.5 + 10/\sqrt{s} + 2\ln s.^{11} \quad (1)$$

The left-hand side of Fig. 5 shows excellent agreement between the average multiplicity of X^0 from $pp \rightarrow \Delta^{++}X^0$ without the 4C-four-prong events and Eq. (1) (compared at the same M^2 as s for the on-mass-shell reactions).

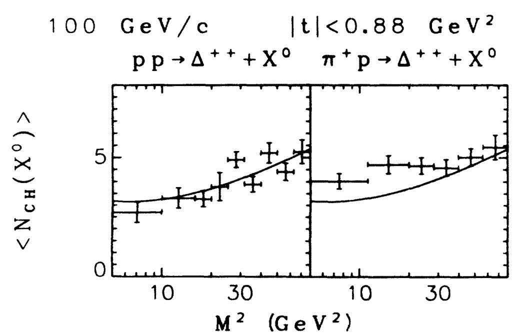

FIG. 5. The average charge multiplicity of X^0 as a function of M^2, not including 4C four-prong events. The curve shown is a fit of the π^-p on-mass-shell inelastic charge multiplicity with substitution of $M^2 = s$ (Ref. 11).

In order to extract the average $\pi^-\pi^+$ inealstic multiplicity (right side of Fig. 5) we remove the 4C four-prong events from $\pi^+p \rightarrow \Delta^{++}X^0$. The average $\pi^-\pi^+$ inelastic multiplicity that we extract is an average of 0.6 units higher than for π^-p. Although the $\pi^-\pi^+$ multiplicity appears to be qualitatively flatter than the π^-p multiplicity, it is not inconsistent with the same functional form shifted upward. It is well known that the π^+p average multiplicity is higher than the pp average multiplicity by about 0.3 units.[5,11] We have now observed that the $\pi^-\pi^+$ average multiplicity is even higher (by about 0.6 units) than that of π^+p interactions.

INCLUSIVE Λ^0 PRODUCTION

The total inclusive cross section for Λ^0 production is 1.1 ± 0.2 mb for $\pi^+p \rightarrow \Lambda^0X^{++}$ and 3.5 ± 0.4 mb for $pp \rightarrow \Lambda^0X^{++}$.[8] To calculate the cross section for $pp \rightarrow \Lambda^0X$ the Λ^0's that were going backward in the center of mass and the symmetry of the reaction was used. The cross section for $\pi^+p \rightarrow \Lambda^0X^{++}$ uses Λ^0's from both center-of-mass hemispheres. For the rest of this paper only Λ^0's produced backward in the center of mass are used.

The distribution of the mass recoiling from the Λ^0 is shown in Fig. 6. This distribution is peaked toward higher M^2 values. To be consistent with the analysis in the Δ^{++} case, we should remove events that are candidates for elastic Kp or $K\pi$ scattering. There are no events consistent with $\pi^+p \rightarrow \Lambda K^+p$, giving an upper limit of 0.07 mb at the 90% confidence level. For the proton case there is one event consistent with $pp \rightarrow \Lambda K^+p$ corresponding to a cross section of 0.06 mb.

Our large bins in M^2 are not merely due to limited statistics. Even with good statistics, the M^2 distribution will be smeared by an unknown amount of Σ^0 production. It is reasonable to assume that Λ^0 and Σ^0 are produced in the same way, since they would involve exchange of the same quantum num-

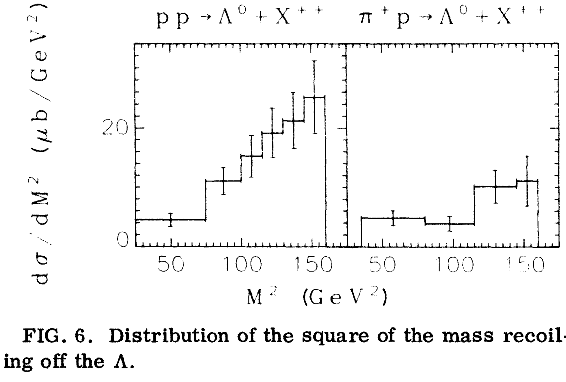

FIG. 6. Distribution of the square of the mass recoiling off the Λ.

bers. However, the square of the missing mass recoiling from the Λ^0, $M_{\Lambda^0}{}^2$, from $\Sigma^0 \to \Lambda^0\gamma$, is not always a good measure of the square of the missing mass from the parent Σ^0, $M_{\Sigma^0}{}^2$. In the extreme cases where the γ makes a large angle with the recoiling mass as viewed in the rest system of the Σ^0, $M_{\Lambda^0}{}^2$ is an overestimate of $M_{\Sigma^0}{}^2$. The bounds of M^2 are given in Eq. (2):

$$M_{\Lambda^0}{}^2=\frac{1}{2m_{\Sigma^0}{}^2}\{(m_{\Sigma^0}{}^2+m_{\Lambda^0}{}^2)M_{\Sigma^0}{}^2+(m_{\Sigma^0}{}^2-m_{\Lambda^0}{}^2)[s-m_{\Sigma^0}{}^2\pm\lambda^{1/2}(s,M_{\Sigma^0}{}^2,M_{\Lambda^0}{}^2)]\}, \quad (2)$$

where

$$\lambda(s,M_{\Sigma^0}{}^2,M_{\Lambda^0}{}^2)=[s-(M_{\Sigma^0}+M_{\Lambda^0})^2]\times[s-(M_{\Sigma^0}-M_{\Lambda^0})^2],$$

m_{Σ^0} = rest mass of Σ^0,

m_{Λ^0} = rest mass of Λ^0.

For the energy of our experiment, Fig. 7 shows the bounds of the uncertainty, $M_{\Lambda^0}{}^2-M_{\Sigma^0}{}^2$, as a function of $M_{\Sigma^0}{}^2$ when the Λ is actually the product of $\Sigma^0 \to \Lambda^0\gamma$. It is not meaningful to have a bin size smaller than the smear. Fortunately the region with the best statistics (large M^2) is also the place where the smearing due to Σ^0 production is the smallest.

OFF-MASS-SHELL K^+p and $K^+\pi^+$ SCATTERING

The inclusive production of Λ^0 can be mediated by, among other things, kaon, K^*, or K^{**} exchange. Recent work[12] of triple-Regge analysis shows that *inclusive* Λ^0 production in pp interactions is dominated by kaon exchange. For the reaction $pp \to \Lambda^0 X^{++}$, $\langle N_{\rm CH}(X^{++})\rangle$ is, in the kaon-exchange picture, the average multiplicity of the off-mass-shell K^+p scattering. The average multiplicity of the X^{++} system as a function of M^2 is shown in Fig. 8. We compare these values to the average multiplicity of the on-mass-shell K^+p scattering of $s=M^2$. Relatively few data are available for on-mass-shell K^+p inelastic scattering. We find that Eq. (1), which represents the on-mass π^-p inelastic multiplicity, can represent those (K^+p) data with an uncertainty of ±0.4 units.

Figure 8 (left side) shows the inelastic multiplicity associated with the Λ^0 in pp interactions compared with Eq. (1). Except at very large M^2 the off-mass-shell multiplicity (dashed points) agrees with that for on-mass-shell K^+p and π^-p scattering. As s/M^2 becomes small, so does the rapidity gap, Δy, between the Λ^0 and its missing mass. The one-particle-exchange picture is not expected to be valid when Δy falls below one unit. *Demanding a rapidity gap of at least one unit* between the Λ^0 and its missing mass (solid data points), one obtains good agreement with on-mass-shell K^+p and π^-p scattering. With Δy less than 1, Λ^0's are presumably produced in some other process, the average multiplicity of which seems to be higher.

The average multiplicity associated with the Λ^0 in π^+p interactions is shown in Fig. 8 (right side). At large M^2, the solid and dashed points are those with and without at least one unit of rapidity between the Λ^0 and its missing mass. The solid points are the extracted average multiplicities of $K^+\pi^+$ scattering. The values of the average multiplicities for $K^+\pi^+$ are close to one unit above that of K^+p for all M^2.

CONCLUSION

We conclude that the effect which we observed in pion exchange from inclusive Δ^{++} production is also present in kaon exchange. The average multiplicity in off-mass-shell meson proton scattering agrees with the on-mass-shell values, and meson-

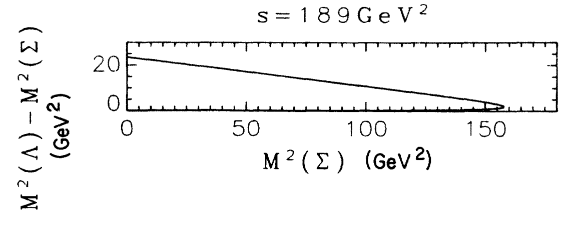

FIG. 7. For Λ's as product of $\Sigma^0 \to \Lambda\gamma$, the bounds of the overestimate of $M_\Sigma{}^2$ when $M_\Lambda{}^2$ is used.

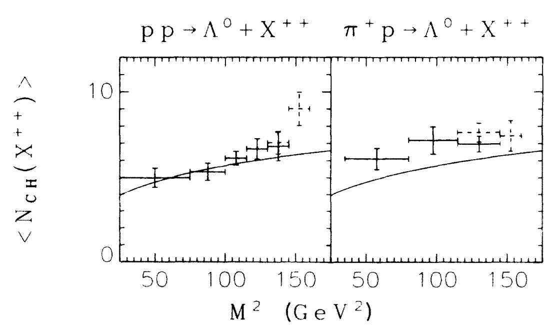

FIG. 8. The average charge multiplicity of X^{++} versus M^2. At large M^2, the solid and dashed points are those with and without demanding at least one unit of rapidity gap between the Λ and its missing mass. The curve represents the average multiplicity in on-mass-shell K^+p scattering (same as π^-p scattering), with the substitution $M^2=s$.

meson scattering has a higher average multiplicity than baryon-meson scattering.

ACKNOWLEDGMENTS

We thank the staffs of the 30-in. Bubble Chamber and Neutrino Laboratory of the Fermi National Accelerator Laboratory for their assistance. The aid of the Proportional Hybrid System Consortium is gratefully acknowledged. We also thank the scanning and measuring staffs at the University of California, Davis, and the Lawrence Berkeley Laboratory.

---

*Work supported in part by the U. S. Energy Research and Development Administration.

[1]F. T. Dao, D. Gordon, J. Lach, E. Malamud, T. Meyer, R. Poster, P. E. Schlein, and W. E. Slater, Phys. Rev. Lett. 30, 34 (1973).
[2]E. Gotsman, Phys. Rev. D 9, 9 (1974).
[3]J. P. DeBrion, C. Bromberg, T. Ferbel, T. Jensen, R. Schindler, P. Slattery, A. A. Seidl, and J. C. Vander Velde, Phys. Rev. Lett. 34, 910 (1975).
[4]S. Barish, M. Derrick, B. Musgrave, P. Schultz, J. Whitmore, R. Engelmann, T. Kafka, M. Pratap, and R. D. Field, Phys. Rev. D 12, 1260 (1975).
[5]J. Erwin, R. D. Kass, J. H. Klems, Winston Ko, R. L. Lander, D. E. Pellett, P. M. Yager, and M. Alston-Garnjost, Phys. Rev. Lett. 35, 980 (1975).
[6]J. Erwin, J. H. Klems, W. Ko, R. L. Lander, D. E. Pellett, P. M. Yager, and M. Alston-Garnjost, Phys. Rev. Lett. 32, 254 (1974).
[7]J. Erwin, W. Ko, R. L. Lander, D. E. Pellett, P. M. Yager, and M. Alston-Garnjost, Phys. Rev. Lett. 33, 1352 (1974).
[8]Margaret Alston-Garnjost, J. Erwin, J. H. Klems, W. Ko, R. L. Lander, D. E. Pellett, and P. M. Yager, Phys. Rev. Lett. 35, 142 (1975).
[9]The background under the Δ^{++} signal was estimated to be 36% for pp and 31% for π^+p events.
[10]D. Brick, B. Haber, M. Hodous, R. Hulsizer, V. Kistiakowsky, A. Levy, A. Nakkasyan, I. Pless, V. Simak, P. Trepagnier, J. Wolfson, and R. Yamamoto, Phys. Rev. Lett. 31, 488 (1973).
[11]J. Whitmore, Phys. Rep. 10C, 273 (1974). The curve shown in Fig. 6 is from Tow, Phys. Rev. D 7, 3535 (1973), parameterized $\langle N_{CH}\rangle$ to $a+b/\sqrt{s}+c\ln s$. We normalized the curve to fit the π^-p data with $a=-4.5$, $b=10$, and $c=2$.
[12]K. Alpgård *et al.*, Nucl. Phys. B105, 349 (1976).