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Asymptotic chiral invariance. II. The consequences*

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Based upon the invariance of SU(2)⊗SU(2) and SU(3)⊗SU(3) for asymptotic momenta, we first derive constraints on certain strong and electromagnetic exclusive scattering processes, where all the hadrons are viewed as fundamental fields. Then we consider asymptotic SU(3)⊗SU(3) for meson-meson and baryon-baryon fixed-angle, elastic scattering in the Landshoff model, wherein the hadrons are viewed as bound states of quarks and antiquarks. We find that that part of the pseudoscalar-pseudoscalar scattering amplitude which is a function of the center-of-mass scattering angle only is just the average of the analogous functions of the scattering angle for the independent nonzero helicity amplitudes for vector-vector scattering. Finally, in the case of baryon-baryon scattering we find that total baryon helicity is conserved.

I. INTRODUCTION

Approximate chiral symmetry has been used extensively in previous years to study a wide class of hadronic processes. For a comprehensive discussion of departures from chiral symmetry at finite momenta we refer the reader to Ref. 1. In the present article we move into the realm of asymptotic momenta and explore the consequences of exact SU(2)⊗SU(2) and SU(3)⊗SU(3) invariance.

In article I[2] we used the homogeneous renormalization-group equations to discuss the conditions under which chiral symmetries become exact at asymptotic momenta. We considered as our prototype the chiral SU(2)⊗SU(2) σ model and allowed for both spontaneous symmetry breaking and explicit breaking of the symmetry by a term in the Lagrangian linear in the σ field. Further, we checked our results to two-loop order in perturbation theory.

The lesson which we learned is that if the experimentally observed hadrons are viewed as fundamental entities, then under suitable conditions at asymptotic momenta all mass parameters scale to zero and hence all chiral-symmetry breaking vanishes. We also noted that this does not necessarily hold for the observed hadrons viewed as bound states of more fundamental hadrons (perhaps quarks and antiquarks). Experimental data seem to favor the description of the observed hadrons as bound states as opposed to being fundamental; therefore, all subsequent discussions based upon the description of the hadrons by fundamental fields should be considered more instructive than a statement of physical law.

As we saw in article I, under certain conditions at asymptotic momenta, the one-particle-irreducible amplitudes of the σ model (aside from overall scaling factors) approach those of a finite-momentum, massless theory. We explicitly showed this for the case of SU(2)⊗SU(2) but the arguments should also hold for SU(3)⊗SU(3). Therefore, we must demand the existence of the massless limit of such amplitudes. As such, for two-body elastic scattering we shall limit ourselves to fixed-angle scattering. Hence, our procedure for determining the consequences of asymptotic chiral invariance shall be simply to study exact chiral invariance for a finite-momentum, massless theory.

In any discussion of massless theories one immediately thinks of exact γ_5 invariance which just reflects the zero massness of the theory. Using the γ_5 transformations on the massless SU(2) ⊗SU(2) σ model

$$\begin{aligned} &N \to \gamma_5 N\,, \\ &\bar{N} \to -\bar{N}\gamma_5\,, \\ &(\sigma, \vec{\pi}) \to G(\sigma, \vec{\pi})G^{-1} = -(\sigma, \vec{\pi}),^{3} \end{aligned} \tag{1.1}$$

where $N, \sigma, \vec{\pi}$ are the nucleon, σ, and pion fields, respectively, and G is the G-parity operator, one can show that nucleon helicity is conserved in elastic pion-nucleon scattering; also, in elastic electron-nucleon scattering,

$$\frac{F_2(q^2)}{F_1(q^2)} \to 0 \text{ as } |q^2| \to \infty\,, \tag{1.2}$$

where F_1 and F_2 are the Dirac and Pauli electromagnetic form factors, respectively, for the nucleon and q is the momentum transfer to the nucleon.[4] Though these are interesting observations, we are more interested in extracting information from the full chiral group structure of SU(2)⊗SU(2) and SU(3)⊗SU(3).

For the case in which all the hadrons are described by fundamental fields, we derive constraints for certain strong and electromagnetic exclusive scattering processes. These constraints are mostly statements about allowable helicities, and we are especially interested in noting any differences between the predictions of exact SU(2) $\otimes$ SU(2) and SU(3) $\otimes$ SU(3) on the π-N sector of the hadrons. *A priori* we would not expect to find such disagreements between the two chiral groups since SU(2) $\otimes$ SU(2) is a subgroup of SU(3) $\otimes$ SU(3), but as we shall see such disagreements can and do occur and we shall comment as to why this happens.

Then for the case in which the hadrons are viewed as bound states, we consider asymptotic SU(3) $\otimes$ SU(3) in the Landshoff model for meson-meson and baryon-baryon fixed-angle, elastic scattering. In particular, we show that that part of the pseudoscalar-psuedoscalar scattering amplitude which is a function of the center-of-mass (c.m.) scattering angle only is just the average of the analogous functions for the independent nonzero helicity amplitudes for vector-vector scattering. Also, we show that total baryon helicity is conserved in baryon-baryon scattering.

In Secs. II and III we discuss the consequences of exact SU(2) $\otimes$ SU(2) and SU(3) $\otimes$ SU(3), respectively, for the hadrons viewed as fundamental fields. In Sec. IV we turn our attention to asymptotic SU(3) $\otimes$ SU(3) for the hadrons viewed as bound states in the Landshoff model for meson-meson and baryon-baryon scattering. Finally, in Sec. V we make concluding remarks.

II. EXACT SU(2) $\otimes$ SU(2)

In this section we explore the consequences of exact SU(2) $\otimes$ SU(2) invariance treating the hadrons as fundamental in a finite-momentum, zero-mass theory (since this is the asymptotic limit under certain conditions of the standard hadronic theory[2]). Our consequences take the form of constraints as to what kinds of exclusive processes are allowed and what are the corresponding SU(2) $\otimes$ SU(2) structures of the allowed amplitudes.

Since the scalar σ meson has not been confirmed experimentally, we exclude it from the external legs of the amplitudes and take as our meson wave-function matrices the following:

$$\Pi = \Pi^\dagger = \frac{1}{\sqrt{2}}\vec{\pi}\cdot\vec{\tau} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}, \tag{2.1}$$

where π^i is the pion wave-function isotopic vector and τ^i are the Pauli matrices. The nucleon spinor wave functions are denoted

$$N_R = \begin{pmatrix} u_{pR} \\ u_{nR} \end{pmatrix} = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} u_p \\ u_n \end{pmatrix},$$
$$N_L = \begin{pmatrix} u_{pL} \\ u_{nL} \end{pmatrix} = \tfrac{1}{2}(1-\gamma_5)\begin{pmatrix} u_p \\ u_n \end{pmatrix}, \tag{2.2}$$

where the u's are the Dirac spinors. Under the SU(2) $\otimes$ SU(2) transformations we have

$$\begin{aligned} &\Pi \to U_L \Pi U_R^\dagger, \\ &\Pi^\dagger \to U_R \Pi^\dagger U_L^\dagger, \\ &N_R \to U_R N_R, \\ &N_L \to U_L N_L, \end{aligned} \tag{2.3}$$

(see Ref. 5) where

$$\begin{aligned} U_R &= e^{-i(\vec{\alpha}-\vec{\beta})\cdot\vec{\tau}/2} \\ U_L &= e^{-i(\vec{\alpha}+\vec{\beta})\cdot\vec{\tau}/2}, \end{aligned} \tag{2.4}$$

where $\vec{\alpha}$ and $\vec{\beta}$ are constant parameters. In zero-mass theories (such as the case here), fermions have only two helicity states: plus and minus, which correspond to N_R and N_L, respectively, in the above. Therefore, our constraints on the amplitudes will involve statements about nucleon helicities.

We write the chiral structure of the amplitude $\mathfrak{M}$ in the form

$$\mathfrak{M} \sim \sum_i A_i O_i, \tag{2.5}$$

where A_i are SU(2) $\otimes$ SU(2)-invariant amplitudes depending only upon kinematic variables and Dirac matrices, O_i for nonelectromagnetic processes are SU(2) $\otimes$ SU(2)-invariant products of the external particles' wave functions, and O_i for electromagnetic processes (such as pion photoproduction) are products of wave functions transforming as isotopic scalars or the third components of left-handed or right-handed vectors.

With this reasoning applied to pion-nucleon scattering we arrive at results no different from those obtained previously from γ_5 invariance, namely that nucleon helicity is conserved. For pion photoproduction we consider two cases: (i) an odd number of pions photoproduced and (ii) an even number photoproduced.

(i) Odd number of pions photoproduced. As an example consider $\gamma_h + N_{1L} \to \pi + N_{2R}$, where h is the photon helicity. We denote the amplitude by

$$\mathfrak{M}_{\gamma_h, L\to R} = \epsilon_\mu M_h^\mu, \tag{2.6}$$

where ϵ_μ is the photon polarization vector, and

$$\epsilon_\mu M_h^\mu \sim \langle \pi N_{2R} | \epsilon_\mu J^\mu | N_{1L}\rangle = \langle \pi N_{2R} | (\epsilon_\mu J^{S\mu} + \epsilon_\mu J^{V\mu}) | N_{1L}\rangle = \epsilon_\mu M_h^{S\mu} + \epsilon_\mu M_h^{V\mu}, \tag{2.7}$$

where π denotes a charged or uncharged pion and $J^{S\mu}$ and $J^{V\mu}$ are the decomposition of the electromagnetic current into its scalar and vector components, respectively, where $J^{V\mu}$ transforms under $SU(2) \otimes SU(2)$ according to $(1,0) \oplus (0,1)$ in the standard (left, right) notation. Consequently $\epsilon_\mu M_h^{S\mu}$ and $\epsilon_\mu M_h^{V\mu}$ must be constructed out of $\overline{N}_{2R}$, N_{1L}, and Π or $\Pi^\dagger$ and transform under $SU(2)\otimes SU(2)$ as a scalar and as the sum of the third components of right-handed and left-handed isotopic vectors, respectively. The results are

$$\begin{aligned} \epsilon_\mu M_h^{S\mu} &= \overline{N}_{2R} A^S \Pi^\dagger N_{1L}, \\ \epsilon_\mu M_h^{V\mu} &= \overline{N}_{2R} A_L^V \Pi^\dagger \tau_3 N_{1L} + \overline{N}_{2R} A_R^V \tau_3 \Pi^\dagger N_{1L}, \end{aligned} \tag{2.8}$$

so that we are led to

$$\begin{aligned} \mathfrak{M}_{\gamma_h, L\to R} &= \overline{N}_{2R} A^S \Pi^\dagger N_{1L} + \overline{N}_{2R} A_L^V \Pi^\dagger \tau_3 N_{1L} + \overline{N}_{2R} A_R^V \tau_3 \Pi^\dagger N_{1L}, \\ \mathfrak{M}_{\gamma_{-h}, R\to L} &= \overline{N}_{2L} A'^S \Pi N_{1R} + \overline{N}_{2L} A_R'^V \Pi \tau_3 N_{1R} + \overline{N}_{2L} A_L'^V \tau_3 \Pi N_{1R}, \end{aligned} \tag{2.9}$$

where parity conservation implies

$$\mathfrak{M}_{\gamma_h, L\to R} = \mathfrak{M}_{\gamma_{-h}, R\to L}, \tag{2.10}$$

or

$$A^S = A'^S, \qquad A_{L(R)}^V = A_{R(L)}'^V. \tag{2.11}$$

Now consider an amplitude that does not involve a helicity flip of the nucleon: $\gamma_h + N_{1R} \to \pi + N_{2R}$. Since for all such amplitudes we cannot form an $SU(2)\otimes SU(2)$-invariant combination of the available nucleon and pion wave-function matrices, nor can we construct something transforming as the sum of the third components of left and right isotopic vectors, we have

$$\mathfrak{M}_{\gamma_h, R\to R} = \mathfrak{M}_{\gamma_h, L\to L} = 0. \tag{2.12}$$

For three or more odd numbers of pions photoproduced the results are essentially the same, i.e., nonzero nucleon helicity-flip amplitudes and zero nucleon helicity-nonflip amplitudes.

(ii) Even number of pions photoproduced. As an example consider $\gamma_h + N_{1L} \to \pi_1 + \pi_2 + N_{2R}$, where π_i refers to a charged or uncharged pion. The above line of reasoning leads us to immediately conclude that

$$\mathfrak{M}_{\gamma_h, L\to R} = \mathfrak{M}_{\gamma_h, R\to L} = 0. \tag{2.13}$$

Now consider the analogous helicity-nonflip amplitude: $\gamma_h + N_{1R} \to \pi_1 + \pi_2 + N_{2R}$. The amplitude for this process is given by

$$\begin{aligned} \mathfrak{M}_{\gamma_h, R\to R} = {} & \overline{N}_{2R} A_1^S N_{1R} \operatorname{tr}(\Pi_1 \Pi_2^\dagger) \\ & + \overline{N}_{2R} A_2^S N_{1R} \operatorname{tr}(\Pi_2 \Pi_1^\dagger) \\ & + \overline{N}_{2R} A_3^S \Pi_1^\dagger \Pi_2 N_{1R} + \overline{N}_{2R} A_4^S \Pi_2^\dagger \Pi_1 N_{1R} \\ & + \overline{N}_{2R} A_{1L}^V N_{1R} \operatorname{tr}(\Pi_1^\dagger \tau_3 \Pi_2) \\ & + \overline{N}_{2R} A_{2L}^V N_{1R} \operatorname{tr}(\Pi_2^\dagger \tau_3 \Pi_1) \\ & + \overline{N}_{2R} A_{3L}^V \Pi_1^\dagger \tau_3 \Pi_2 N_{1R} \\ & + \overline{N}_{2R} A_{4L}^V \Pi_2^\dagger \tau_3 \Pi_1 N_{1R} \\ & + \overline{N}_{2R} A_{1R}^V \tau_3 N_{1R} \operatorname{tr}(\Pi_1 \Pi_2^\dagger) \\ & + \overline{N}_{2R} A_{2R}^V \tau_3 N_{1R} \operatorname{tr}(\Pi_2 \Pi_1^\dagger) \\ & + \overline{N}_{2R} A_{3R}^V \Pi_1^\dagger \Pi_2 \tau_3 N_{1R} \\ & + \overline{N}_{2R} A_{4R}^V \Pi_2^\dagger \Pi_1 \tau_3 N_{1R}. \end{aligned} \tag{2.14}$$

In general for even numbers of pions photoproduced there are nonzero nucleon helicity-nonflip amplitudes and zero nucleon helicity-flip amplitudes. Thus, considering cases (i) and (ii) above we arrive at the following constraint for pion photoproduction:

Constraint II-1. (A) Nucleon helicity flips in $\gamma + N_1 \to N_2 + \pi_1 + \pi_2 + \cdots + \pi_{2n+1}$; (B) nucleon helicity is conserved in $\gamma + N_1 \to N_2 + \pi_1 + \pi_2 + \cdots + \pi_{2n}$, where n is a positive integer.

Since our program is clear, to avoid a lengthy discussion we simply list the other constraints which we have derived.

Constraint II-2. Nucleon helicity is conserved in nucleon Compton scattering ($\gamma + N \to \gamma + N$).

Constraint II-3. For off-mass-shell photons, the amplitudes for $\gamma \to \pi_1 + \pi_2 + \cdots + \pi_{2n+1}$ vanish while those for $\gamma \to \pi_1 + \pi_2 + \cdots + \pi_{2n}$ do not vanish, where n is a positive integer.

Constraint II-4. For nucleon-nucleon scattering, the number of right-handed nucleons (incoming plus outgoing) must be even and the number of left-handed nucleons must be even. Furthermore, the amplitudes for the following processes vanish:

$$p_{R(L)} + n_{L(R)} \to p_{L(R)} + n_{R(L)},$$

$$p_{R(L)} + p_{R(L)} \to p_{L(R)} + p_{L(R)},$$

$$n_{R(L)} + n_{R(L)} \to n_{L(R)} + n_{L(R)},$$

where we indicate parity conjugate processes by the subscripts in parentheses.

From constraint II-4 we see that the only $N_R + N'_R \to N''_L + N'''_L$ type process allowed is $p_R + n_R \to p_L + n_L$, which has zero total incoming and zero total outgoing right and left chiral quantum numbers.

This concludes our discussion of the consequences of exact $SU(2)\otimes SU(2)$ invariance for certain exclusive processes where the hadrons are described by fundamental fields. In the next section we discuss consequences of exact chiral

SU(3)$\otimes$SU(3) and see to what extent they agree with previously derived results.

III. EXACT SU(3)$\otimes$SU(3)

In this section we explore the implications of exact SU(3)$\otimes$SU(3) invariance, again assuming that the hadrons can be described by fundamental fields as opposed to bound states. The discussion here parallels that for the SU(2)$\otimes$SU(2) case.

Neglecting scalar mesons, we take the meson wave-function matrices M and $M^\dagger$ to be

$$M=M^\dagger=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -(\frac{2}{3})^{1/2}\eta^0 \end{pmatrix}, \tag{3.1}$$

with M and $M^\dagger$ transforming as the representations $(3,\overline{3})$ and $(\overline{3},3)$, respectively, under SU(3)$\otimes$SU(3). The baryon wave-function matrices are given by

$$B_{L(R)}=\begin{pmatrix} \frac{\Sigma^0_{L(R)}}{\sqrt{2}}+\frac{\Lambda^0_{L(R)}}{\sqrt{6}} & \Sigma^+_{L(R)} & p_{L(R)} \\ \Sigma^-_{L(R)} & -\frac{\Sigma^0_{L(R)}}{\sqrt{2}}+\frac{\Lambda^0_{L(R)}}{\sqrt{6}} & n_{L(R)} \\ -\Xi^-_{L(R)} & \Xi^0_{L(R)} & -(\frac{2}{3})^{1/2}\Lambda^0_{L(R)} \end{pmatrix}, \tag{3.2}$$

with

$$B_R=\tfrac{1}{2}(1+\gamma_5)B\,,\qquad B_L=\tfrac{1}{2}(1-\gamma_5)B\,, \tag{3.3}$$

where B is the SU(3) baryon octet. In the standard (L,R) notation, we let $B_L, \overline{B}_L, B_R, \overline{B}_R$ transform as $(\overline{3},3), (3,\overline{3}), (3,\overline{3}), (\overline{3},3)$, respectively. So our SU(3)$\otimes$SU(3) transformations are

$$\begin{aligned} B_R &\to U_L B_R U_R^\dagger\,,\\ B_L &\to U_R B_L U_L^\dagger\,,\\ M &\to U_L M U_R^\dagger\,,\\ M^\dagger &\to U_R M^\dagger U_L^\dagger\,, \end{aligned} \tag{3.4}$$

where

$$U_R=e^{-i(\vec\alpha-\vec\beta)\cdot\vec\lambda/2}\,,\qquad U_L=e^{-i(\vec\alpha+\vec\beta)\cdot\vec\lambda/2} \tag{3.5}$$

where $\vec\alpha$ and $\vec\beta$ are constant parameters. Also, we again note that B_R and B_L correspond to + and − helicities, respectively.

Since we consider all the observed hadrons to be fundamental, their respective fields must interact via SU(3)$\otimes$SU(3)-invariant couplings among themselves in the Lagrangian. This is why we choose the baryons to lie in $(3,\overline{3})\oplus(\overline{3},3)$ instead of $(1,8)\oplus(8,1)$ representation so that they can couple to the mesons in an SU(3)$\otimes$SU(3)-invariant manner. If by $(B_R)^\beta_{\tilde\alpha}$ we mean that the right-handed baryon matrix transforms on the left index $\tilde\alpha$ (with ~ denoting left-handedness) according to 3_L and on the right index β according to $\overline{3}_R$, then the baryon-meson couplings in the Lagrangian are given by

$$\mathfrak{L}_I\sim g\,(\epsilon^{\tilde\alpha\tilde\alpha'\tilde\alpha''}\epsilon_{\beta\beta'\beta''}\overline{B}^{\beta}_{L\tilde\alpha}M^{\beta'}_{\tilde\alpha'}B^{\beta''}_{R\tilde\alpha''} + \epsilon_{\tilde\alpha\tilde\alpha'\tilde\alpha''}\epsilon^{\beta\beta'\beta''}\overline{B}^{\tilde\alpha}_{R\beta}M^{\dagger\tilde\alpha'}_{\beta'}B^{\tilde\alpha''}_{L\beta''})\,. \tag{3.6}$$

Now we turn our attention to constraints on certain exclusive scattering processes which result from exact SU(3)$\otimes$SU(3) in a zero-mass, finite-momentum theory. Just as was done for SU(2)$\otimes$SU(2), we select which nonelectromagnetic processes are allowed according to whether we can construct appropriate SU(3)$\otimes$SU(3)-invariant products of the external particles' wave-function matrices. For electromagnetic processes such as meson photoproduction, the product of wave-function matrices must transform as the third or eighth members of left-handed or right-handed octets. We shall be particularly interested in seeing to what extent chiral SU(3)$\otimes$SU(3) yields entirely new and different results from those obtained from SU(2)$\otimes$SU(2) as a consequence of opening up the "strange" hadronic sector.

The first process which we consider is baryon-meson scattering: $B_1+m_1\to B_2+m_2$. For this process there are two cases: (i) helicity flip and (ii) helicity nonflip.

The amplitude for $B_{1L}+m_1\to B_{2R}+m_2$ is given by

$$\begin{aligned}\mathfrak{M}_{L\to R}=&\operatorname{tr}(M_2\overline{B}_{2R})A_1\operatorname{tr}(M_1B_{1L})\\ &+\operatorname{tr}(M_2\lambda_a\overline{B}_{2R})A_2\operatorname{tr}(M_1\lambda_aB_{1L})\\ &+\operatorname{tr}(\overline{B}_{2R}\lambda_aM_2)A_3\operatorname{tr}(B_{1L}\lambda_aM_1)\\ &+\operatorname{tr}(\lambda_a\overline{B}_{2R}\lambda_bM_2)A_4\operatorname{tr}(\lambda_aB_{1L}\lambda_bM_1)\,,\end{aligned} \tag{3.7}$$

and that for its parity-conjugate process $\mathfrak{M}_{R\to L}$ is gotten by the following substitutions into the above:

$$B_{1L} \to B_{1R}, \quad M_1 \to M_1^\dagger,$$

$$\overline{B}_{2R} \to \overline{B}_{2L}, \quad M_2 \to M_2^\dagger.$$

In the π-N sector of the theory, we get nucleon helicity conservation because there $\mathfrak{M}_{L\to R}=0$. An example of a nonzero helicity-flip amplitude is $\Sigma^0_{L(R)}+\pi^0 \to \Sigma^0_{R(L)}+\pi^0$. So we see that in the full SU(3) $\otimes$ SU(3) theory there are usually more allowable possibilities for amplitudes than in the π-N sector. After a similar analysis for the helicity-nonflip case we arrive at the following constraint for baryon-meson scattering:

Constraint III-1. For the process $B_1+m_1 \to B_2 + m_2$, there are both helicity-flip and helicity-nonflip amplitudes; however, in the π-N sector of the theory nucleon helicity is conserved.

To avoid a lengthy discussion we do as we did for the SU(2) $\otimes$ SU(2) results and simply list the other constraints which we have derived.

Constraint III-2. (A) Baryon helicity flips in $\gamma_h + B_1 \to m + B_2$; (B) for $\gamma_h + B_1 \to m_1 + m_2 + B_2$, there are nonzero amplitudes involving baryon helicity flip and helicity nonflip; however, in the π-N sector of the theory nucleon helicity is conserved. An example of a nonvanishing baryon helicity-flip amplitude for two mesons photoproduced is $\gamma_h + p_L \to \eta^0 + \eta^0 + p_R$. We also note that constraint III-2 is consistent with constraint II-1.

Constraint III-3. Baryon helicity is conserved in baryon Compton scattering $(\gamma + B_1 \to \gamma + B_2)$. This is consistent with constraint II-2.

Constraint III-4. For off-mass-shell photons, the amplitudes for $\gamma_h \to m$ and $\gamma_h \to m_1 + m_2 + m_3$ vanish while those for $\gamma_h \to m_1 + m_2 + \cdots + m_{2n}$ do not vanish, where n is a positive integer. This is consistent with constraint II-3. Also, for five or more odd numbers of mesons produced we get nonvanishing amplitudes except if all the mesons are pions in which case they vanish, again in agreement with the SU(2) $\otimes$ SU(2) prediction.

Constraint III-5. For baryon-baryon scattering, the number of right-handed baryons (incoming plus outgoing) must be even and the number of left-handed baryons must be even. Furthermore, the amplitude for $B_{1R(L)} + B_{2R(L)} \to B_{3L(R)} + B_{4L(R)}$ vanishes, and for the nucleons the amplitude for $p_{R(L)} + n_{L(R)} \to p_{L(R)} + n_{R(L)}$ vanishes.

Unlike what we found for SU(2) $\otimes$ SU(2) invariance for nucleon-nucleon scattering, for the baryons it is not necessary that the incoming and outgoing left- (right-) handed baryons be the same for $B_{1L} + B_{2R} \to B_{3L} + B_{4R}$, e.g., $\mathfrak{M}(p_L + \Xi^-_R \to n_L + \Xi^0_R) \neq 0$. In the nucleon sector, however, we still have $\mathfrak{M}(p_{R(L)} + n_{L(R)} \to p_{R(L)} + n_{L(R)}) \neq 0$ while $\mathfrak{M}(p_{R(L)} + n_{L(R)} \to p_{L(R)} + n_{R(L)}) = 0$ in agreement with our SU(2) $\otimes$ SU(2) results. On the other hand, the vanishing of $B_{1R(L)} + B_{2R(L)} \to B_{3L(R)} + B_{4L(R)}$ is in direct disagreement with the SU(2) $\otimes$ SU(2) prediction for the nucleons. This seeming paradox can be understood if one considers that in the case of SU(2) $\otimes$ SU(2) the nucleons transform as a fundamental representation of the chiral group, whereas in the case of SU(3) $\otimes$ SU(3) they do not, but transform as members of a product representation. It is precisely these different transformation properties which give rise to different predictions for the nucleons in the two chiral groups. We believe the SU(3) $\otimes$ SU(3) results are more compatible with reality since this chiral group with its inherent strangeness sector is an experimental reality.

This concludes our discussion of the consequences of exact SU(3) $\otimes$ SU(3) invariance if the hadrons are described by fundamental fields. In the next section we take a brief look at what to expect if the hadrons are described as bound states of more fundamental constituents.

IV. HADRONS AS BOUND STATES

So far we have confined ourselves to a description of the physically observed hadrons as being actual quanta of fundamental fields. As mentioned earlier, the physical evidence seems to favor their description as bound states of more fundamental field quanta. What happens to chiral-symmetry breaking as we go to asymptotic momenta for hadrons viewed as bound states is quite different from what we have so far considered. In this section we consider the bound-state case within the context of a specific model for meson-meson (both pseudoscalar-pseudoscalar and vector-vector) and baryon-baryon high-energy, fixed-angle, elastic scattering. The model we choose is due to Landshoff.[6]

To begin, let us consider the elastic scattering of two pseudoscalar or vector mesons. In the fixed-angle, asymptotic momentum limit, we adopt the Landshoff notation. The dominant diagram is shown in Fig. 1, where the momenta are labeled so that

$$\begin{aligned} &p\cdot q = p'\cdot q = 0\,, \\ &p^2 = p'^2 = -q^2 + m^2 = \tau\,, \\ &p\cdot p' = \lambda\tau\,, \\ &s \sim 2\tau(1+\lambda)\,, \\ &t \sim -4\tau\,, \\ &u \sim 2\tau(1-\lambda)\,, \end{aligned} \tag{4.1}$$

where m is the meson mass and $\tau \to \infty$ with λ fixed. Figure 1 is the diagram for both the elastic scattering of two pseudoscalar mesons m_1 and m_2, for example $\pi^+ + \pi^- \to \pi^+ + \pi^-$, and of two vector mesons v_1 and v_2, for example $\rho^+ + \rho^- \to \rho^+ + \rho^-$.

In the Landshoff model each meson is pictured as being composed of a number of quark constituents whose momenta are all essentially parallel to that of their parent meson. As such, in our discussion we consider the spin of the parent meson as due solely to the spins of the quark constituents.

The eight momenta k_i, k_i' are parametrized in terms of 16 scalar parameters as follows:

$$\begin{aligned}
\begin{pmatrix} k_1 \\ k_1' \end{pmatrix} &= \begin{pmatrix} x_1 \\ 1-x_1 \end{pmatrix}(p+q) \pm y_1 \frac{(p'+\lambda q)}{[\tau(\lambda^2-1)]^{1/2}} \pm \kappa_1 n \pm \frac{z_1}{2\tau} q\,, \\
\begin{pmatrix} k_2 \\ k_2' \end{pmatrix} &= \begin{pmatrix} x_2 \\ 1-x_2 \end{pmatrix}(p-q) \pm y_2 \frac{(p'-\lambda q)}{[\tau(\lambda^2-1)]^{1/2}} \pm \kappa_2 n \pm \frac{z_2}{2\tau} q\,, \\
\begin{pmatrix} k_3 \\ k_3' \end{pmatrix} &= \begin{pmatrix} x_3 \\ 1-x_3 \end{pmatrix}(p'+q) \pm y_3 \frac{(p+\lambda q)}{[\tau(\lambda^2-1)]^{1/2}} \pm \kappa_3 n \pm \frac{z_3}{2\tau} q\,, \\
\begin{pmatrix} k_4 \\ k_4' \end{pmatrix} &= \begin{pmatrix} x_4 \\ 1-x_4 \end{pmatrix}(p'-q) \pm y_4 \frac{(p-\lambda q)}{[\tau(\lambda^2-1)]^{1/2}} \pm \kappa_4 n \pm \frac{z_4}{2\tau} q\,,
\end{aligned} \tag{4.2}$$

where n is a spacelike unit vector orthogonal to p, p', and q. In the limit of large s, the masses of the k_i and k_i' are as follows:

$$\begin{aligned}
k_i^2 &= -x_i z_i + m^2 x_i^2 - y_i^2 - \kappa_i^2\,, \\
k_i'^2 &= (1-x_i) z_i + m^2(1-x_i)^2 - y_i^2 - \kappa_i^2\,.
\end{aligned} \tag{4.3}$$

This parametrization was chosen so that these masses are finite in the high-s limit for finite values of the parameters x_i, y_i, z_i, κ_i. This is needed because in the Landshoff model, the external blobs in Fig. 1 which represent the meson-bound state functions go to zero sufficiently rapidly when either k_i^2 or $k_i'^2$ is large that the dominant contribution to the diagram comes from small k_i^2 and $k_i'^2$. In this sense we approximate the Γ and Γ' as zero-mass quark-quark, antiquark-antiquark, or quark-antiquark scattering amplitudes obeying exact SU(3)$\otimes$SU(3). On the other hand, the external lines carry asymptotically large momenta $p+q$, $p'-q$, etc., while at least one—and not necessarily both—of the attached quark four-

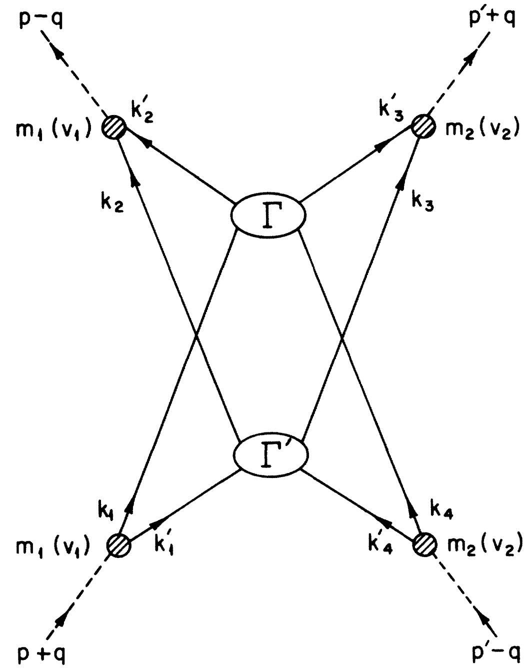

FIG. 1. Landshoff diagram for meson-meson scattering. $m(v)$ denotes pseudoscalar (vector) meson.

momenta is asymptotically large. In this sense we are dealing with a low-momentum problem, so we retain at most only SU(3) invariance for the bound-state wave functions; even this is not absolutely necessary for our final results. Finally, the quark momenta k_i satisfy the constraint

$$\sum_{i=1}^{4} k_i + p + p' = 0\,, \tag{4.4}$$

and in the limit of large s one may integrate over the 16 parameters x_i, y_i, z_i, κ_i subject to the constraint

$$\frac{1}{(stu)^{1/2}}\,\delta(x_1+x_2-1)\delta(x_3+x_4-1) \times \delta(x_1-x_2+x_3-x_4)\delta(\kappa_1+\kappa_2+\kappa_3+\kappa_4)\,.$$

Callan and Gross[7] have discussed the asymptotic behavior of the Landshoff graph using renormalization-group techniques. The three x_i δ functions leave only one x to be integrated over, which they call ξ, so that in the high-s limit the two four-quark amplitudes become $\Gamma^{(4)}(\xi p_1, \ldots, \xi p_4)$ and $\Gamma^{(4)}((1-\xi)p_1, \ldots, (1-\xi)p_4)$, which are just fixed-angle, high-energy, zero-mass quark-quark scattering amplitudes. They find in the high-s limit these amplitudes scale as

$$\Gamma^{(4)}(\xi p_i) \to (\xi\sqrt{s})^{4(1-d)} f(\theta)\,, \tag{4.5}$$

where d is the dimension (canonical + anomalous) at the fixed point g_0 of the quark field, θ is the center-of-mass (c.m.) scattering angle of the problem, and $f(\theta)$ describes the angular dependence of the zero-mass quark-quark or quark-antiquark scattering amplitude at the fixed point. As for the meson bound-state wave functions χ_m, Callan and Gross show that they scale as

$$\chi_m \sim \sqrt{s}\, g_m(k_i^2, k_i'^2)\,, \tag{4.6}$$

where $g_m(k_i^2, k_i'^2)$ are determined by the constraint of conformal invariance and provide the needed Landshoff damping.

First we consider pseudoscalar-pseudoscalar scattering and we turn our attention to the Γ's in Landshoff diagram. Either one Γ is a quark-quark and the other an antiquark-antiquark amplitude or they are both quark-antiquark amplitudes. For the first case, where one Γ is a quark-quark and the other an antiquark-antiquark amplitude, we list all possible independent helicity amplitudes subject to the constraint of exact $SU(3)\otimes SU(3)$ invariance. Equation (4.5) tells us that instead of talking in terms of $\Gamma(s,\theta)$, we should talk in terms of $f(\theta)$. Also, we need only consider the quark-quark amplitudes since they and the antiquark-antiquark amplitudes are equivalent by charge conjugation invariance. Goldberger *et al.*, have shown that the independent helicity amplitudes are as follows[8]:

$$\begin{aligned} f_1(\theta) &= ++\to++ = RR\to RR\,,\\ f_2(\theta) &= ++\to-- = RR\to LL\,,\\ f_3(\theta) &= +-\to+- = RL\to RL\,,\\ f_4(\theta) &= +-\to-+ = RL\to LR\,,\\ f_5(\theta) &= ++\to+- = RR\to RL\,, \end{aligned} \tag{4.7}$$

where + and − quark helicities correspond to their respective right (R) and left (L) representations of $SU(3)_R\otimes SU(3)_L$. To exploit the chiral group symmetry, we decompose the f_i into their respective chiral invariant subamplitudes:

$$\begin{aligned} f_1(\theta) &= (1,3)\otimes(1,3)\to(1,3)\otimes(1,3)\\ &= a(i,j)[(1,\bar 3)\to(1,\bar 3)] + b(i,j)[(1,6)\to(1,6)]\\ &\equiv a(i,j)f_{1,\bar 3\to\bar 3}(\theta) + b(i,j)f_{1,6\to 6}(\theta)\,, \end{aligned} \tag{4.8a}$$

$$f_2(\theta) = (1,3)\otimes(1,3)\to(3,1)\otimes(3,1) = 0\,, \tag{4.8b}$$

$$\begin{aligned} f_3(\theta) &= (1,3)\otimes(3,1)\to(1,3)\otimes(3,1)\\ &= (3,3)\to(3,3)\,, \end{aligned} \tag{4.8c}$$

$$\begin{aligned} f_4(\theta) &= (1,3)\otimes(3,1)\to(3,1)\otimes(1,3)\\ &= \delta_{q_iq_j}[(3,3)\to(3,3)]\\ &= \delta_{q_iq_j} f_3(\theta)\,, \end{aligned} \tag{4.8d}$$

$$f_5(\theta) = (1,3)\otimes(1,3)\to(1,3)\otimes(3,1) = 0\,, \tag{4.8e}$$

where $a(i,j)$ and $b(i,j)$ are squares of Clebsch-Gordan coefficients depending upon the particular quarks q_i and q_j involved and $f_4(\theta)$ vanishes unless both quarks are the same, in which case $f_4(\theta) = f_3(\theta)$.

The independent quark-antiquark helicity amplitudes are similarly evaluated giving

$$\begin{aligned} \bar f_1(\theta) &= (1,3)\otimes(1,\bar 3)\to(1,3)\otimes(1,\bar 3)\\ &= \bar a(i,j)[(1,1)\to(1,1)] + \bar b(i,j)[(1,8)\to(1,8)]\\ &\equiv \bar a(i,j)\bar f_{1,1\to 1}(\theta) + \bar b(i,j)\bar f_{1,8\to 8}(\theta)\,, \end{aligned} \tag{4.9a}$$

$$\begin{aligned} \bar f_2(\theta) &= \bar a(i,j)[(1,1)\to(1,1)]\\ &= \bar a(i,j)\bar f_{1,1\to 1}(\theta)\,, \end{aligned} \tag{4.9b}$$

$$\bar f_3(\theta) = (\bar 3,3)\to(\bar 3,3) = f_3(\theta)\,, \tag{4.9c}$$

$$\bar f_4(\theta) = (\bar 3,3)\to(3,\bar 3) = 0 \tag{4.9d}$$

$$f_5(\theta) = (1,1\oplus 8)\to(\bar 3,3) = 0\,, \tag{4.9e}$$

where again $\bar a(i,j)$ and $\bar b(i,j)$ are squares of Clebsch-Gordan coefficients.

So the constraint of chiral invariance implies there are only six surviving helicity amplitudes: $f_1(\theta), f_3(\theta), f_4(\theta), \bar f_1(\theta), \bar f_2(\theta), \bar f_3(\theta)$. There is an additional symmetry if we confine ourselves to theories such as the σ model wherein the quarks are bound via couplings to scalar and pseudoscalar mesons. The additional symmetry can be observed in the perturbation expansions and states

$$f_1(\theta) = \bar f_1(\theta)\,, \tag{4.10a}$$

$$f_3(\theta) = \bar f_3(\theta)\,. \tag{4.10b}$$

Equation (4.10a) is the only new result since Eq. (4.10b) is equivalent to Eq. (4.9c).

Using Eqs. (4.8), (4.9), and (4.10) together with the results of Callan and Gross[7] we get the following expression for the pseudoscalar-pseudoscalar scattering amplitude:

$$\begin{aligned} \mathfrak{M}(m_1+m_2\to m_1+m_2) = {}& \frac{2s^{4(1-d)+2}}{(stu)^{1/2}}\,F(\theta)\\ &\times\int d\xi\,[\xi(1-\xi)]^{4(1-d)}\int\prod_{i=1}^{4} dy_i dz_i d\kappa_i\,\delta(\textstyle\sum\kappa_j)\kappa_i\, g_{m(i)}(k_i^2, k_i'^2)\,, \end{aligned} \tag{4.11}$$

where for pseudoscalar-pseudoscalar scattering, if the quark q_i and antiquark $\bar q_j$ compositions of the mesons are

$$m_1 = \frac{1}{\sqrt{2}}\,(\bar{q}_{1+}q_{2-} - \bar{q}_{1-}q_{2+})\,,$$
$$m_2 = \frac{1}{\sqrt{2}}\,(\bar{q}_{3+}q_{4-} - \bar{q}_{3-}q_{4+})\,, \tag{4.12}$$

where i and j are as shown and do not refer to tensor indices and with the $\pm$ subscripts denoting helicities, then the above discussion gives the following for the function $F(\theta)$ of c.m. scattering angle:

$$F(\theta) = \tfrac{1}{4}\{[\bar{a}(2,4)\bar{a}(1,3) + 2\bar{a}(2,3)\bar{a}(1,4)]\bar{f}_{1,1\to 1}{}^2(\theta) + [\bar{b}(2,4)\bar{b}(1,3) + \bar{b}(2,3)\bar{b}(1,4)]\bar{f}_{1,8\to 8}{}^2(\theta)$$
$$+ [\bar{a}(2,4)\bar{b}(1,3) + \bar{a}(1,3)\bar{b}(2,4) + \bar{a}(2,3)\bar{b}(1,4) + \bar{a}(1,4)\bar{b}(2,3)]\bar{f}_{1,1\to 1}(\theta)\bar{f}_{1,8\to 8}(\theta)$$
$$+ (2 + \delta_{q_2 q_4}\delta_{\bar{q}_1\bar{q}_3}) f_3{}^2(\theta)\}\,. \tag{4.13}$$

We can find out more about these functions of scattering angle by considering high-energy fixed-angle elastic vector-meson scattering: $v_1 + v_2 \to v_1 + v_2$. In the asymptotic limit the vector mesons have negligible mass and thus only + and − helicities are allowed. So we examine the various helicity amplitudes. As mentioned earlier, this process proceeds through the same Landshoff diagram shown in Fig. 1 as does the pseudoscalar amplitude.

First we consider

$$v_{1+} + v_{2+} \to v_{1+} + v_{2+}\,,$$

where

$$v_{1+} = \bar{q}_{1+}q_{2+}\,, \qquad v_{2+} = \bar{q}_{3+}q_{4+} \tag{4.14}$$

for the quark compositions. The function of the scattering angle analogous to Eq. (4.13) is

$$F_{++\to++}(\theta) = [\bar{a}(2,4)\bar{a}(1,3) + \bar{a}(2,3)\bar{a}(1,4)]\bar{f}_{1,1\to 1}{}^2(\theta) + [\bar{b}(2,4)\bar{b}(1,3) + \bar{b}(2,3)\bar{b}(1,4)]\bar{f}_{1,8\to 8}{}^2(\theta)$$
$$+ [\bar{a}(2,4)\bar{b}(1,3) + \bar{a}(1,3)\bar{b}(2,4) + \bar{a}(2,3)\bar{b}(1,4) + \bar{a}(1,4)\bar{b}(2,3)]\bar{f}_{1,1\to 1}(\theta)\bar{f}_{1,8\to 8}(\theta)\,. \tag{4.15a}$$

Similar analyses lead us to the following results for the other helicity amplitudes:

$$F_{++\to--}(\theta) = \bar{a}(2,3)\bar{a}(1,4)\bar{f}_{1,1\to 1}{}^2(\theta)\,, \tag{4.15b}$$
$$F_{+-\to+-} = 2f_3{}^2(\theta)\,, \tag{4.15c}$$
$$F_{+-\to-+} = \delta_{q_2 q_4}\delta_{\bar{q}_1\bar{q}_3} f_3{}^2(\theta)\,, \tag{4.15d}$$
$$F_{++\to+-}(\theta) = 0\,, \tag{4.15e}$$

where again the δ functions mean that $F_{+-\to-+}(\theta)$ vanishes unless $q_2 = q_4$ and $\bar{q}_1 = \bar{q}_3$. The overall amplitude for any one of the above vector-vector processes is given by Eq. (4.11) without the factor of 2 and with the appropriate

$$F_{h_1 h_2 \to h'_1 h'_2}(\theta)$$

substituted for $F(\theta)$.

If we choose pseudoscalars m_a and vector mesons v_a ($a = 1, 2$) with the same quark content q_i^a and $\bar{q}_j^a$, then we arrive at the relation

$$F(\theta) = \tfrac{1}{4}[F_{++\to++}(\theta) + F_{++\to--}(\theta) + F_{+-\to+-}(\theta) + F_{+-\to-+}(\theta)]\,, \tag{4.16}$$

so that the angle-dependent part of the amplitude for pseudoscalar scattering is just the average of the analogous functions for the independent nonzero helicity amplitudes for vector scattering. Furthermore, there are only three independent chiral-invariant functions of θ: $\bar{f}_{1,1\to 1}(\theta)$, $\bar{f}_{1,8\to 8}(\theta)$, and $f_3(\theta)$, and they are readily obtained from $F_{++\to++}(\theta)$, $F_{++\to--}(\theta)$, and $F_{+-\to+-}(\theta)$ via Eqs. (4.15).

Landshoff's model readily gives information about the helicity amplitudes for baryon-baryon elastic, fixed-angle scattering at asymptotic energies. The baryons B_1 and B_2 are considered to be bound states of three quarks which scatter as shown in Fig. 2. The internal momenta and the Γ's are handled analogously as for meson-meson scattering. Even though the baryon bound-state wave functions are more difficult to handle than those for the mesons, we can exploit the exact chiral SU(3)⊗SU(3) invariance for the Γ's to make statements about helicity amplitudes. Simply note that for each Γ the only allowable helicity amplitudes are $f_1(\theta)$, $f_3(\theta)$, and $f_4(\theta)$ of Eq. (4.7), and for these amplitudes total quark helicity is conserved. This implies

$$\mathfrak{M}(B_{1+} + B_{2+} \to B_{3+} + B_{4-}) = 0\,,$$
$$\mathfrak{M}(B_{1+} + B_{2+} \to B_{3-} + B_{4-}) = 0\,, \tag{4.17}$$

so that total baryon helicity must be conserved. We find that this is in agreement with constraint III-5, which was derived in the last section by considering the baryons as fundamental particles.

Even though we have worked in this section with a specific model for the scattering of hadrons

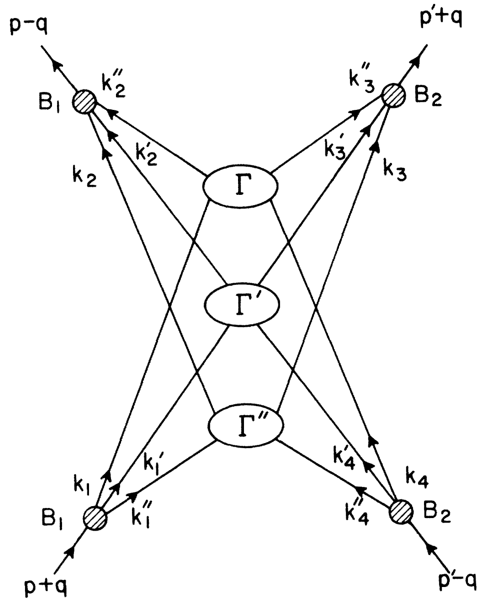

FIG. 2. Landshoff diagram for baryon-baryon scattering.

viewed as bound states, we hope that our procedure for exploiting chiral invariance to extract useful information about the scattering amplitudes has been sufficiently broad and instructive to be readily applied to other such models.

V. CONCLUSION AND DISCUSSION

Article I was a discussion of the conditions under which certain field theories become exactly chiral invariant at asymptotic momenta. In this article we went further and discussed what the consequences are for certain strong and electromagnetic scattering amplitudes in the asymptotic, zero-mass limit of such theories. First we considered the hadrons as quanta of fundamental fields and found general agreement between the results obtained from exact $SU(2)\otimes SU(2)$ and exact $SU(3)\otimes SU(3)$; however, there was an exception which arose because of the unanalogous transformation properties of the nucleon doublet in $SU(2)\otimes SU(2)$ and the nucleons as part of an octet in $SU(3)\otimes SU(3)$. Finally, we looked at what must be done to extract information about scattering processes if the hadrons are viewed as bound states of more fundamental constituents.

Experimental data suggest that among the results we have discussed in previous sections, the most reliable are probably those based upon the description of hadrons as bound states and based upon chiral $SU(3)\otimes SU(3)$ as the underlying symmetry. However, we must bear in mind that recent experimental breakthroughs surrounding the discovery of ψ (Ref. 9) and ψ' (Ref. 10) seem to indicate even larger asymptotic symmetry groups such as $SU(4)\otimes SU(4)$, but these questions have not yet been settled.

Finally, we note that at present there are no experimental data to check the results obtained in this paper. Most of our results involve statements about asymptotic helicity amplitudes which at this time are experimentally out of reach owing to the difficulties involved in producing high-energy polarized beams. But we remain hopeful that the future will unfold the technical expertise needed to test these ideas in the laboratory.

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