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Eikonal field theory production of interacting Pomerons

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Starting from an underlying field theory in eikonal approximation, interacting Pomerons are produced by retaining those Feynman graphs that correspond to self-energy and related radiative corrections. Multiple-Reggeon t-channel thresholds may be viewed in a simple s-channel field theory framework, while the degree of s-channel unitarity required depends upon the spin content of the underlying field theory and the classes of permitted processes. An approximate eikonal calculation suggests how restricted triple-Pomeron interactions can serve to remove the bare Pomeron, and substitute an alternate asymptotic expression for σ_{tot}.

I. INTRODUCTION

One of the most interesting theoretical programs in recent years has been the emergence of triple-Pomeron (TP) interactions as a useful phenomenological tool in the description of high-energy reactions.[1] More recently, renormalization-group arguments have been used to estimate the asymptotic behavior of Reggeon field theory (RFT) built out of interacting TP's.[2] While the results obtained are physically reasonable and even appealing, in spite of certain conceptual problems,[3] the question remains as to the nature of the underlying mechanism that generates the bare Reggeons, or Pomerons, which then interact with themselves and with physical particles. In particular, if one retains the canonical point of view that physical particles are the quanta of complicated but underlying fields, then one is led to ascribe a similar origin to Reggeons.

Historically, the work of Gribov and associates[4] originated in just this way, "abstracting" from a simple field theory such expected Reggeon and TP interactions. More recently, Abramovski, Gribov, and Kancheli[5] have written "cutting rules," allowing one to proceed from sets of simple field-theory ladder graphs to interacting Pomerons; while most recently, Guerin and Meunier[6] have extended these rules to include the effect of transverse-momentum distributions of inelastically emitted particles. The present paper is essentially a return to the original Gribov spirit, but with the analysis performed within the context of an eikonal field-theory model, thereby at once guaranteeing s-channel unitarity. With the adoption of a simple but representative hybrid field theory previously employed elsewhere,[7] and the use of a simplified but sufficiently general eikonal analysis, one is able to generate the typical interacting TP's of RFT. It turns out that it is precisely those "self-energy" insertions in mainly multiperipheral Feynman graphs, discarded in previous phenomenological treatments of field-theory eikonal models, which are responsible for the appearance of these interacting Reggeons, and that, in general, one has a compact and well-defined way of classifying those conventional field-theory graphs whose ordered-rapidity processes are important to RFT. One sees how multiple t-channel Reggeon thresholds appear naturally in terms of "self-energy" insertions in a simpler s-channel field-theory process, rather like a variant of duality. This analysis suggests that the spin character of the underlying field theory and the class of radiative corrections considered have a strong effect on the degree of s-channel unitarity required to satisfy the Froissart bound. Finally, a simple eikonal summation of leading rapidity dependence indicates just how TP interactions can act to remove the bare Pomeron and lead to a gently increasing total cross section, as suggested by RFT. It should be kept in mind, however, that other possibly important amplitudes found in the eikonal analysis, conventionally neglected because they could not be estimated, could serve to change the RFT results, qualitatively, if the bare Pomeron is actually a construct of some underlying field theory.

Arrangement of the present remarks is as follows. In the next section, a very brief review of phenomenological Pomeron construction is given, following from the most convenient hybrid field theory. The proper basis of this procedure is outlined in Sec. III, with a complete but compact (functional) statement of the eikonal analysis, including formal expressions for self-energy and related insertions in all relevant Feynman graphs. From this general form, one simple piece, or subset, of the radiative corrections is then extracted and all of its insertions are followed in the construction of TP's. In Sec. IV a simple eikonal model is developed for the numerical computation of these radiative insertions, with unitarity playing an important field-theory role, one which subsequently leads to an imaginary TP coupling in the equivalent RFT. Certain leading-logarithm (in rapidity) expansions then exponentiate, suggesting how the bare Pomeron can in this manner be removed, leaving a σ_{tot} increasing with rapidity. A final section contains a summary and a brief discussion of some relevant and open questions.

II. REVIEW OF POMERON BUILDING

The origin of the method is the effective eikonal reproduction in the asymptotic region of Pomeron poles and cuts. One begins with the representation

$$T(s,t)\sim is\int d^2b\, e^{i\vec{q}\cdot\vec{b}}[1-e^{i\chi(s,b)}],\qquad \sigma_{\text{tot}}\sim\frac{1}{s}\,\text{Im}\,T(s,0)\,,\tag{2.1}$$

where $t=-q^2$ and $-t\ll s$. In any field theory where eikonalization exists—where one sums over multiple exchanges of an appropriate singlet representation between distinct scattering particles—$i\chi$ may be formally exhibited in terms of a sum over all connected, t-channel Feynman graphs.[8] As a useful example, consider a hybrid φ^3 theory with couplings (g,λ) between nucleon N, scalar meson A, and scalar pion Π,

$$\mathcal{L}'=-g\bar{\psi}A\psi-\frac{\lambda}{2}A^2\Pi\,,\tag{2.2}$$

where the basic interactions corresponding to this interaction Lagrangian serve to generate the eikonal graphs of Fig. 1. Effectively, a Reggeon pole-plus-cut contribution to (2.1) is produced by the leading $\ln(s)$ sums of $i\chi_2$,

$$i\chi_2\simeq-2\left(\frac{g^2}{4\pi}\right)^2\int d^2q\, e^{i\vec{q}\cdot\vec{b}}\alpha_2(q)\left[\left(\frac{s}{s_0}\right)^{\lambda^2\alpha_2(q)/8\pi}-1\right],\tag{2.3}$$

where

$$\alpha_2(q)=\frac{1}{4\pi^2}\int d^2Q(Q^2+\mu^2)^{-1}[(Q+q)^2+m^2]^{-1}$$
$$=\frac{1}{4\pi}\int_0^1 dx[\mu^2+q^2x(1-x)]^{-1}.$$

For at least moderate impact parameters, $mb>1$, this becomes

$$i\chi_2\sim-\frac{G^2}{Y}\left(\frac{s}{s_0}\right)^{\Lambda-2}e^{-cb^2/Y}\,,$$

with $Y=\ln(s/s_0)$, $\Lambda=\lambda^2/32\pi^2m^2$, $G=g^2/4\pi$, $c\sim m^2/\Lambda$; in this model, effectively, $\Lambda=1+\alpha(0)$, where $\alpha(0)$ denotes the bare Reggeon intercept.

This eikonal model may most easily be constructed as the g^4 expansion of the formal, functional solution[7]

$$i\chi=\exp\left(-\frac{i}{2}\int\frac{\delta}{\delta\Pi}D_c\frac{\delta}{\delta\Pi}\right)\times\exp\left[ig^2\int\mathfrak{F}_1\bar{\Delta}_c(\Pi)\mathfrak{F}_2\right]\Bigg|_{\Pi\to0,\,\text{conn}}-1,\tag{2.4}$$

where D_c and Δ_c are respectively pion and A-meson propagators, and where $\mathfrak{F}_1(u)=\int_{-\infty}^{+\infty}d\xi\,\delta^{(4)}(u-z_i+\xi p_i)$ represents the classical current of the ith scattering nucleon, with position z_i and momentum p_i [here, $\vec{b}=(\vec{z}_1-\vec{z}_2)_T$]. The effect of ordered pion rapidities in the desired set of ladder graphs is quite simply reproduced by the functional replacement

$$\bar{\Delta}_c(x,y\,|\,\Pi)\to\int\frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-n)}}{q^2+m^2}e^{\lambda\int\Pi\sigma_q},\tag{2.5}$$

where

$$\int\Pi\sigma_q=\int d^4k\,\tilde{\Pi}(k)\gamma(k;y)[(q-k)_\perp^2+m^2]^{-1}.$$

together with the subsequent prescription

$$\int dk_{(+)}\int dk_{(-)}\gamma^2(k;y)(k^2+\mu^2-i\epsilon)^{-1}\to i\pi\int_{y_2}^{y_1}dy=i\pi Y\,.$$

Here, pion rapidities are summed over all possible values, with the necessary $(n!)^{-1}$ factor of the desired ordered rapidities produced by expansion of the exponential source dependence of (2.5). In terms of rapidity and impact-parameter variables, one could equivalently write

$$-\frac{i}{2}\int\frac{\delta}{\delta\Pi}D_c\frac{\delta}{\delta\Pi}\to\frac{\pi^2}{4}\int d^2b\int_{y_2}^{y_1}dy\left(\frac{\delta}{\delta\Pi(b,y)}\right)^2$$

together with

$$\int\Pi\sigma_q\to\int_{y_b}^{y_a}dy\int d^2b\,\Pi(b,y)K_0(mb)e^{-i\vec{b}\cdot\vec{q}}.$$

In each case, the rapidity range of the functional differentiation operator is that of the full range in the problem, y_1-y_2, while the rapidity range of the source dependence is to be specified by those "horizontal" pion rapidities associated with their vertical positions along any A line. In the laboratory frame computation of the Pomeron, for example, $y_a=Y=y_1-y_2$ and $y_b=0$, but over any finite "vertical" rapidity interval y_a-y_b this exchange of "horizontal" pions builds the corresponding eikonal-function contribution of a Reggeon exchanged across that rapidity interval. Pictorially, the sum of such leading-log ladder graphs may be replaced by a Reggeon between the same rapidity values, as in Fig. 2. The Pomeron may be defined, in this phenomenological field-theory manner, by the

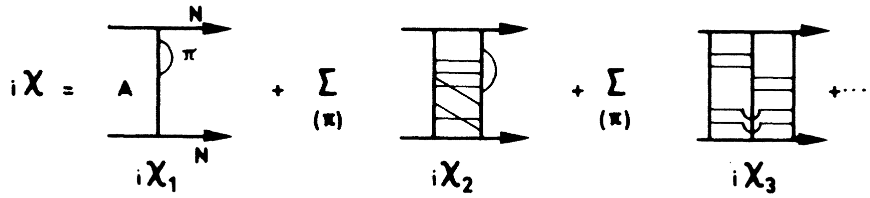

FIG. 1. Some graphs which enter into the complete eikonal expression.

choice $\Lambda=2$, so that $-i\chi_2=\rho\sim Y^{-1}\exp(-cb^2/Y)$. In the familiar way, this then generates

$$\sigma_{\text{tot}}\simeq 2\int d^2b(\rho-\tfrac{1}{2}\rho^2+\cdots)$$
$$=A-B/Y+\cdots, \tag{2.6}$$

with A and B positive constants, representing the Pomeron pole and cut contributions to the total cross section.

There are two main questions requiring further comment if this method of Pomeron construction is to have a realistic field-theory content. The first point concerns the reasons underlying the special choice $\Lambda=2$ and its possible relation to the ladder-graph approximation used to extract those leading terms that act as the Pomeron. The second question relates to the neglect of all the other $i\chi_n$, $n\geq 3$, which terms contain nonplanar Feynman graphs that have never been properly evaluated.

Concerning the second point first, a very crude estimate has been given[7] in which the pion linkages are such that $i\chi_n$ is represented by Reggeon behavior between each pair of the n A-mesons exchanged, $i\chi_n\sim i^n s^{-m}s^{\Lambda n(n-1)/2}$, with $m=n$ for scalar A's and $m=0$ for vector A's. It was argued that such dependence—going far beyond the tower graphs of $i\chi_2$—can have a crucial effect, changing the saturation of the Froissary bound into a cross section that vanishes asymptotically. More precise evaluations are really needed, for if these higher eikonal terms are so important, one can have little confidence in any phenomenological method, such as the present one, which deals only with the tower graphs of $i\chi_2$. Perhaps a better

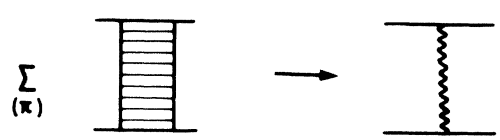

FIG. 2. Construction of a phenomenological Reggeon/Pomeron by the leading-ln(s) portion of ladder graphs.

graphical analysis will indicate that effective Reggeons are formed only between "nearest neighbor" A lines, rather than between each pair of A's exchanged; or perhaps strict attention must be paid to available energy restrictions in the construction of all the $i\chi_n$. Whatever the possible reason, suppression of the $i\chi_n$, $n\geq 3$, is an essential initial assumption to be made at present, here restricting attention to the tower graphs only, and to those modified eikonal quantities built out of them that generate interacting TP's.

Secondly, it is necessary to ask if there can exist a reason for the special choice $\Lambda=2$, which defines this phenomenological Pomeron. Remembering that in any exchange of m pions between the pair of A lines, one has selected only the ladder graphs containing the leading $s^{-2}(m!)^{-1}(\ln s)^m$ dependence (thereby neglecting $m!-1$ crossed-pion graphs, each of less importance in that mth order) it is not difficult to imagine that sums over at least a partial subset of the neglected crossed graphs could have the effect of generating cancellations which reduce the power s dependence to a final form essentially independent of s. At present it is not known how to estimate the relevant corrections due to the crossed graphs, and one can only guess at the result; but if such cancellations do take place, one can perhaps understand the phenomenological choice $\Lambda=2$ as simply a matter of correcting a too-enthusiastic first evaluation of the ladder graphs' leading $\ln s$ dependence.

III. APPROXIMATIONS TO THE COMPLETE EIKONAL

It is useful to first rewrite (2.4) in terms of a complete eikonal expression that, at least formally, contains all radiative corrections and insertions. For the present hybrid theory, one may write the generating functional and its formal solution as

$$\mathfrak{z}\{j,k,\bar{\eta},\eta\}=\left\langle 0\left|\left\{\exp\left[i\int(j\Pi+kA+\bar{\eta}\psi+\bar{\psi}\eta)\right]\right\}_+\right|0\right\rangle, \tag{3.1}$$

where $j,k,\bar{\eta},\eta$ are appropriate c-number sources for the operator fields Π, A, ψ, $\bar{\psi}$; and

$$N\mathfrak{z} = \exp\left(i\int \mathcal{L}'\left\{\frac{1}{i}\frac{\delta}{\delta j}, \frac{1}{i}\frac{\delta}{\delta k}, -\frac{1}{i}\frac{\delta}{\delta \eta}, \frac{1}{i}\frac{\delta}{\delta \bar{\eta}}\right\}\right) \times \exp\left(i\int \bar{\eta} S_c \eta + \frac{i}{2}\int k\Delta_c k + \frac{i}{2}\int \Pi D_c \Pi\right),$$

where S_c, Δ_c, and D_c represent the free nucleon, A-meson, and pion propagators, respectively. N is a normalization constant representing the vacuum-to vacuum S-matrix element. Using well-defined Gaussian techniques[9] this may be put into the form

$$N\mathfrak{z}\{0,0,\bar{\eta},\eta\} = \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta \Pi} D_c \frac{\delta}{\delta \pi}\right)\exp\left(-\frac{i}{2}\int \frac{\delta}{\delta A}\bar{\Delta}_c(\Pi)\frac{\delta}{\delta A}\right)\exp\left(i\int \bar{\eta} G_c(A)\eta + L(A) + L'(\Pi)\right)\Big|_{\Pi = A = 0}, \tag{3.2}$$

with $L(A) = \mathrm{Tr}\ln(1+gA\,S_c)$ and $L'(\Pi) = -\frac{1}{2}\mathrm{Tr}\ln(1+\lambda\Pi\Delta_c)$ given in terms of fictitious c-number fields A and Π; here $G_c(A) = S_c(1+gAS_c)^{-1}$ and $\bar{\Delta}_c(\Pi) = \Delta_c(1+\lambda\Pi\Delta_c)^{-1}$ denote respective propagators defined in the presence of the indicated source fields. In anticipation of constructing an elastic nucleon scattering amplitude, the j and k sources of $\mathfrak{z}$ have been set equal to zero. This derivation tacitly suggests that the "nucleon" is a fermion and the other pair of fields are spin-zero bosons, but after subsequent eikonal limits are taken, they may all be considered as spin-zero bosons.

Repeating the eikonal construction of Ref. 8, one obtains

$$e^{i\chi} = \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta \Pi} D_c \frac{\delta}{\delta \Pi}\right)\exp\left(-\frac{i}{2}\int \frac{\delta}{\delta A}\bar{\Delta}_c(\pi)\frac{\delta}{\delta A}\right)\exp\left(ig\int \mathfrak{F}_{12}A + L(A) + L'(\Pi)\right)\Big|_{A \to \Pi = 0} \tag{3.3}$$

or

$$e^{i\chi} = \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta \Pi} D_c \frac{\delta}{\delta \Pi}\right)\exp\left(\frac{i}{2}g^2\int \mathfrak{F}_{12}\bar{\Delta}_c(\Pi)\mathfrak{F}_{12}\right)\exp\left(-\frac{i}{2}\int \frac{\delta}{\delta A}\bar{\Delta}_c(\Pi)\frac{\delta}{\delta A}\right) \times \exp\left[L'(\Pi) + L\left(A + g\int \mathfrak{F}_{12}\bar{\Delta}_c(\Pi)\right)\right]\Big|_0, \tag{3.4}$$

where $\mathfrak{F}_{12} = \mathfrak{F}_1 + \mathfrak{F}_2$. Unfortunately, the simple eikonal limits applied to self-energy processes generate ambiguous, or ill-defined results, and further modeling will be required, below.

In previous derivations of eikonal representations tions, the closed-loop functionals $L(A)$ and $L'(\Pi)$ were, for the most part, neglected. Here, all pion insertions generated by closed A loops are again dropped, $L'(\Pi) \to 0$, but those radiative corrections representing the simplest self-energy bubble insertion in every virtual A line are retained. That is, $L(A)$ is approximated by its quadratic dependence only, $L \to (i/2)\int A(x)K(x-y)A(y)$, with $K(x-y) = ig^2\,\mathrm{tr}[S_c(x-y)S_c(y-x)]$. For scalar nucleons, S_c becomes an ordinary boson propagator, Δ_N, and $L(A) = -\frac{1}{2}\mathrm{Tr}\ln(1+gA\Delta_N)$ so that $K(x-y) = -(i/2)g^2\Delta_N(x-y)\Delta_N(y-x)$. Retention of more complicated dependence of $L(A)$ would lead, among other things, to quartic and higher Pomeron vertices in the equivalent RFT. While this relatively simple approximation must be improved in any serious field-theory calculation, it is at least an intuitive method of beginning to understand the interplay of radiative corrections and Pomeron construction. With these simplifications, (3.4) becomes

$$e^{i\chi} = \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta \Pi} D_c \frac{\delta}{\delta \Pi}\right) \times \exp\left(\frac{i}{2}g^2\int \mathfrak{F}_{12}\bar{\Delta}_c(\Pi)[1-K\bar{\Delta}_c(\Pi)]^{-1}\mathfrak{F}_{12} - \frac{1}{2}\mathrm{Tr}\ln[1-K\bar{\Delta}_c(\Pi)]\right)\Big|_{\Pi \to 0}. \tag{3.5}$$

The same quantity K enters into other processes defined within this and related field theories. In particular, one of subsequent interest is that expression of unitarity in the description of (scalar) nucleon pair production by a suitably time-dependent, external source $A(x)$. In this simplest of problems, where a nucleon field interacts only with an external c-number source A, corresponding to $\mathcal{L}' = -(g/2)\psi_N{}^2 A$, one has for the generating functional $\mathfrak{z}\{\eta\} = {}_{\mathrm{in}}\langle 0|[\exp(i\int \eta\psi]_+|0\rangle_{\mathrm{in}}$ the formal solution

$$N\mathfrak{z}\{\eta\} = \exp\left[\frac{i}{2}\int \eta\Delta_N(1+gA\Delta_N)^{-1}\eta - \frac{1}{2}\mathrm{Tr}\ln(1+gA\Delta_N)\right]. \tag{3.6}$$

where the zero-source limit of (3.6) provides the identification of $N = {}_{\text{in}}\langle 0|S[A]|0\rangle_{\text{in}}$. It follows that the probability of the vacuum to remain a vacuum, under the influence of the source A, will in general be less than unity,

$$P_0 = |{}_{\text{out}}\langle 0|0\rangle_{\text{in}}|^2 = |N|^2 = \exp[-\operatorname{Re}\operatorname{Tr}\ln(1+gA\Delta_N)]$$

or, in quadratic source approximation,

$$\ln P_0 \simeq \frac{g^2}{2}\operatorname{Re}\operatorname{Tr}[A\Delta_N A\Delta_N]$$

$$= \operatorname{Re}\left[i\int A(x)K(x-y)A(y)\right], \tag{3.7}$$

using the second (scalar nucleon) form of K. It is clear from the physical interpretation of P_0 that (3.7) must be negative, an easily verifiable situation, since

$$\operatorname{Re}[i\tilde{K}(q)] = -\frac{g^2}{16\pi}\theta(-q^2)\theta(-q^2-4m^2)(1+4m^2/q^2)^{1/2}$$

and $A(x)$ is real. By this computation, one sees that the absorptive part of K is fixed above threshold to be positive, and that, asymptotically, it is just a constant, $\operatorname{Im}K(q)\to +g^2/16\pi$. This requirement, dictating the phase of that part of K which corresponds to particle production, will be useful in constructing an eikonal model of self-energy insertions which respects this particular aspect of unitarity.

The eikonal of (3.5) is still somewhat complicated for these purposes, and further simplifications are useful. In the absence of the self-energy bubble K, (3.5) reduces to (2.4) plus additional radiative corrections corresponding to A linkages along each nucleon line. The latter are now neglected, following the procedure of examining only those radiative insertions defined by the simple bubble K relative to the form (2.4). Further, we retain only the K dependence corresponding to a single-bubble insertion in any A meson, but neglect all K dependence in A linkages between nucleons 1 and 2,

$$\exp\left(\frac{i}{2}g^2\int \mathfrak{F}_{12}\bar{\Delta}_c(\Pi)[1-K\bar{\Delta}_c(\Pi)]^{-1}\mathfrak{F}_{12}\right)\to\exp\left[ig^2\int \mathfrak{F}_1\bar{\Delta}_c(\Pi)\mathfrak{F}_2+\frac{i}{2}g^2\int \mathfrak{F}_1\bar{\Delta}_cK\bar{\Delta}_c\mathfrak{F}_1+\frac{i}{2}g^2\int \mathfrak{F}_2\bar{\Delta}_cK\bar{\Delta}_c\mathfrak{F}_2\right]. \tag{3.8}$$

This is perhaps the simplest way in which TP's are generated, and it will become clear, later on, that the neglected terms correspond to the modification of TP vertices and to virtual Pomeron-particle interaction. The remaining closed-loop part of (3.5), which in the absence of pion linkages would just correspond to vacuum fluctuations, will also be approximated following the spirit of the steps leading to (3.8) by retaining only an exponential of quadratic K dependence,

$$\exp[-\tfrac{1}{2}\operatorname{Tr}\ln(1-K\bar{\Delta}_c)]\to\exp[\tfrac{1}{4}\operatorname{Tr}(K\bar{\Delta}_cK\bar{\Delta}_c)]. \tag{3.9}$$

The terms of (3.9) upon expansion will correspond to the sources of Pomeron self-energy graphs in RFT, with the terms omitted again corresponding to nonconventional Pomeron interactions, as described in the sentence following (3.8).

The simplified eikonal function to be examined is then given by

$$i\chi = \exp\left(-\frac{i}{2}\int\frac{\delta}{\delta\Pi}D_c\frac{\delta}{\delta\Pi}\right)$$

$$\times\exp\left[ig^2\int \mathfrak{F}_1\bar{\Delta}_c\mathfrak{F}_2+\frac{i}{2}g^2\int \mathfrak{F}_1\bar{\Delta}_cK\bar{\Delta}_c\mathfrak{F}_1+\frac{i}{2}g^2\int \mathfrak{F}_2\bar{\Delta}_cK\bar{\Delta}_c\mathfrak{F}_2+\tfrac{1}{4}\operatorname{Tr}(K\bar{\Delta}_cK\bar{\Delta}_c)\right]_{\Pi\to 0,\text{conn}}-1, \tag{3.10}$$

with the understanding that each $\bar{\Delta}_c(\Pi)$ is to be represented by the form (2.5), and suitable care is to be taken in the ordering of all relative rapidities. Finally, the desired modification of the g^4 expansion of (2.4) is obtained by performing the same expansion here:

$$i\chi_2' = \exp\left[-\frac{i}{2}\int\frac{\delta}{\delta\Pi}D_c\frac{\delta}{\delta\Pi}\right]\frac{(ig^2)^2}{2!}\left(\int \mathfrak{F}_1\bar{\Delta}_c\mathfrak{F}_2\right)^2\exp\left[\frac{i}{2}g^2\int \mathfrak{F}_1\bar{\Delta}_cK\bar{\Delta}_c\mathfrak{F}_1+(1\to 2)+\tfrac{1}{4}\operatorname{Tr}(K\bar{\Delta}_cK\bar{\Delta}_c)\right]_{\Pi\to 0,\text{conn}}, \tag{3.11}$$

where $i\chi_2'$ represents the eikonal function containing the stated K-bubble insertions into the simpler functional representation of $i\chi_2$. It is clear that many classes of graphs have intentionally been omitted in the passage from (3.5) to (3.11); but it is also true that (3.11) represents a simple description of all the interations of the two basic processes which have been retained.

IV. EIKONAL MODEL OF THE RADIATIVE INSERTIONS

Evaluation of these forms is not without difficulty, because the adoption of the naive eikonal limit for self-energy processes quite generally leads to spurious divergences and associated zeroes. The first of these arises because of the incompatibility of translational invariance with the eikonal limit. For example, in the momentum-space integrand of the quantity $\int \mathfrak{F}_1\bar{\Delta}_c K\bar{\Delta}_c\mathfrak{F}_1$, there will appear two factors of $\delta(qp_1)$, one from each $\bar{\Delta}_c$ propagator, with the same argument following from the translational invariance of $K(x-y)$. That this occurs is really no surprise, for the eikonal limit treats every A line emitted or absorbed by a nucleon as independent, while every such self-energy graph requires their momenta to be the same. One can avoid the divergence by taking the eikonal limit only at a later stage; but the combinatoric structure then seems to become prohibitively complicated. What one would really like to have is a simple method for the construction of such radiative corrections that respects both unitarity and the permutation sums always associated with the eikonal model, and one such method proceeds as follows.

Consider the exchange of a pair of A lines between nucleon 1 and a third nucleon, say nucleon 3; to this process the eikonal limits, involving the desired permutation sums, may immediately be performed. In the absence of connecting pion linkages, this corresponds to the skeleton graph of Fig. 3(a), and its contribution can be written down immediately, as proportional to the factor $(ig^2\int \mathfrak{F}_2\bar{\Delta}_c\mathfrak{F}_3)^2$. One then associates with nucleon 3 an extra, virtual, scalar nucleon propagator, as represented by Fig. 3(b); only the mass-shell part of this virtual propagator is employed. Pions with ordered rapidities are then inserted, as in Fig. 3(c), with care taken to distinguish pions with rapidity larger or smaller than y_3; insertion of the quantity $1\equiv\int dy_3\delta(y_3-\frac{1}{2}\ln(p_3^{(+)}p_3^{(-)}))$ under the $\int dp_3^{(+)}\int dp_3^{(-)}$ integrals of this closed loop serves to define y_3 as the rapidity of nucleon 3. Integration is then carried out over all relevant transverse variables, and finally over all y_3, with $y_1>y_3>y_2$. The overall phase of the loop is determined by the requirement that the same construction must contribute an appropriately absorptive term when applied to the decay of a sufficiently massive A meson into a pair of scalar nucleons; it is essentially this requirement that subsequently fixes the TP coupling as pure imaginary.

As in the discussion following (3.7), unitarity requires that whatever construction is used, it must result in a positive imaginary part for asymptotic $\tilde{K}$, and this property is here achieved in an *ad hoc* way, by simply adjusting the phase of the result to correspond to such absorption. This is really a model calculation of $\mathrm{Im}\tilde{K}$, for both its constituent nucleon lines are on their mass shells; and it leaves open the estimate of $\mathrm{Re}\tilde{K}$, a quantity that could conceivably become important when many TP's are involved.

It should be emphasized that this assignment of phase is an extra model assumption, performed only to circumvent the awkwardness of a proper eikonal–plus–self-energy calculation. For example, it is obvious from the expansion of the K dependence of (3.11) that TP absorption arises when $\tilde{K}$ has a positive imaginary part, a requirement satisfied by the unitarity discussion following (3.7). Upon closer examination, however, one notes that $\mathrm{Im}\tilde{K}(q)$ vanishes below threshold, and, indeed, for spacelike q^2, just the range of q values that enter into any eikonal computation [where $q \ll p_3$, $q(\pm)\to 0$]. But this zero then multiplies an infinity [the second $\delta(qp_1)$], giving a nicely indeterminate result and one which is here *defined* to have just the phase given by the known asymptotic value of $\mathrm{Im}\tilde{K}(q)$, and with a magnitude proportional to the third-nucleon construction above. Obviously, this method of self-energy calculation is even more heuristic than the eikonal model itself; but the result is surely correct, for it contains the two essential physical requirements for these insertions: all (eikonal) permutation sums plus unitarity of the simple bubble.

We first fix the definition of K. Relative to the eikonal of $i\chi_2$, and before pion linkages are inserted, the third-nucleon construction generates an extra dependence of amount

$$\zeta i\pi\int d^4p_3\delta(p_3^{\,2}+M^2)\left[ig^2\int \mathfrak{F}_1\bar{\Delta}_c(\Pi)\mathfrak{F}_3\right]^2, \qquad (4.1)$$

where ζ is some constant whose phase is determined by comparison with the formal self-energy insertion factors

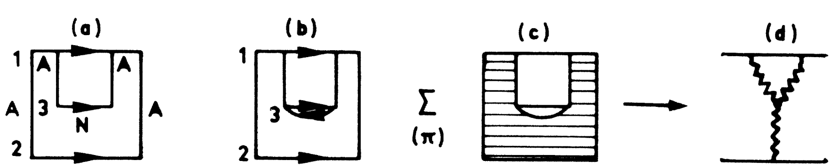

FIG. 3. Three steps in the phenomenological construction of a triple-Pomeron graph.

$$\frac{i}{2}g^2\int \mathfrak{F}_1\bar{\Delta}_c K\bar{\Delta}_c\mathfrak{F}_1 .$$

Thus, in effect,

$$K(u-v)\to -2\pi g^2\zeta\int d^4p_3\delta(p_3{}^2+M^2)\mathfrak{F}_3(u)\mathfrak{F}_3(v)\,, \tag{4.2}$$

and in order for $\tilde{K}$ to have a positive imaginary part, we must in (4.1) choose ζ as $-i|\zeta|$.

Once this identification has been made, the same construction can now be applied to every term in the K expansion of the formal (3.11), replacing each K there by the form (4.2). Thus, for example, the closed-loop expansions of (3.11) are obtained by connecting each pair of "extra" nucleons (say, p_3 and p_4) by eikonal exchanges of A lines, illustrated in the skeleton graphs of Fig. 4(a); all eikonal A-meson permutations are automatically included here. Then, mass-shell propagators are associated with each such extra nucleon, converting to the graphs of Fig. 4(b); and finally, pions with ordered rapidities are inserted, as in Fig. 4(c), to generate an effective Pomeron self-energy graph, Fig. 4(d).

One final simplification will always be made: Rather than performing integration over all momentum components of the subsidiary nucleons introduced to compute these eikonal self-energies, it is more convenient to integrate over their third and fourth momentum components (or $p_{(-)}$ and $p_{(-)}$, with $p_\pm = E\pm p_3$) together with an integration over their spatial transverse coordinates $z_{1,2}$; and we shall everywhere replace $\int d^4p\,\delta(p^2+M^2)$ by $\frac{1}{2}\int d^2z\int dp_{(+)}\int dp_{(-)}\delta(M^2-p_{(+)}p_{(-)})$ a form appropriate to the ordinary Pomeron assumption of limited but conserved transverse momentum.

For simplicity, we calculate only the contributions of these terms to $\sigma_{\rm tot}$, now given by

$$\sigma_{\rm tot}=2\int d^2b[-i\chi_2'(b,s)]\,; \tag{4.3}$$

that is, it is assumed, as with the ordinary Pomeron computations, that complete s-channel unitarity—the actual exponentiation of $i\chi_2'(b,s)$—is not necessary. Those situations where such exponentiation is necessary—when $\alpha(0)>1$, or including nonplanar pion linkages, or (at least in this model) when fermion rather than boson fields are used in the construction of K—will become apparent shortly. We work out the first computation in detail, and then merely quote the results for the other, similar calculations.

After the parametric eikonal integrations have been performed, (4.1) may be put into the form

$$\sim -|\zeta|\int dp_3^{(+)}\int dp_3^{(-)}\delta(p_3^{(+)}p_3^{(-)}-M^2)(p_1^{(+)}p_3^{(-)}-p_1^{(-)}p_3^{(+)})^{-2}$$
$$\times\int d^2z_3\int\frac{d^2q}{q^2+m^2}\int\frac{d^2q'}{q'^2+m^2}\exp\left[i\vec{q}\cdot(\vec{z}_1-\vec{z}_3)+i\vec{q}\,'\cdot(\vec{z}_1-\vec{z}_3)+\lambda\int\Pi\sigma_q+\lambda\int\Pi\sigma_{q'}\right]. \tag{4.4}$$

Fixing the definition of y_3 by introducing $1=\int dy_3\delta(y_3-\frac{1}{2}\ln(p_3^{(+)}/p_3^{(-)}))$ into the p_3 integrands and then interchanging integrals, one finds that (4.4) becomes

$$\sim -\int\frac{d^2q}{(q^2+m^2)^2}\int dy_3[e^{y_1-y_3}-e^{y_3-y_1}]^{-2}$$
$$\times\exp\left[\lambda\int_{y_3}^{y_1}\Pi\sigma_q+\lambda\int_{y_3}^{y_1}\Pi\sigma_{-q}\right], \tag{4.5}$$

with $p_i^{(\pm)}=Me^{\pm y_i}$. The pion source dependence of (4.5) contains implicit dependence upon y_3, for the pions are to be inserted with ordered rapidities; according to the prescriptions of Sec. II the ordered pion graphs of Fig. 3(c) generate a contribution to $\sigma_{\rm tot}$ of amount

$$\delta\sigma_{\rm tot}'=-\frac{|\zeta|g^8}{\pi^2M^4\mu^2}\int\frac{dy_3e^{[1+\alpha(0)](y_3-y_2)}}{(e^{y_1-y_3}-e^{y_3-y_1})^2}$$
$$\times\int\frac{d^2q'}{(q'^2+m^2)^2}e^{2(y_1-y_3)[1+\alpha(0)-|\alpha'|q'^2]}\,, \tag{4.6}$$

where it has been assumed that $(y_1-y_3)|\alpha'|m^2>1$, and the definition $1+\alpha(Q^2)=\lambda^2\alpha_2(Q)/8\pi$ has been used; because the q' integrand effectively cuts off at $[|\alpha'|(y_1-y_3)]^{-1/2}$, some simplifications have been employed in reaching (4.6). For simplicity, we assume a unit rapidity range is required before any such distributions become valid, which leads to

$$\frac{\delta\sigma_{\rm tot}'}{\sigma_{\rm tot}^{(\rm Pom)}}=-q\int_1^{Y-1}\frac{dy}{y}e^{y[\alpha(0)-1]}\,, \tag{4.7}$$

where q is a positive constant.

When the bare Pomeron intercept is unity, this construction of the simplest TP contribution to the total cross section, Fig. 3(d), generates an amount $-q\ln Y\sigma_{\rm tot}^{(\rm Pom)}$. The negative sign here is indicative of expected strong absorptive effects, just as for the Pomeron cut, and has been obtained from a comparison of the formal eikonal solution with the unitarity requirement of the simple bubble, K. Were the Pomeron to have an intercept greater than unity, this contribution would carry the s dependence $-s^{2(\alpha-1)}/Y$. Similarly,

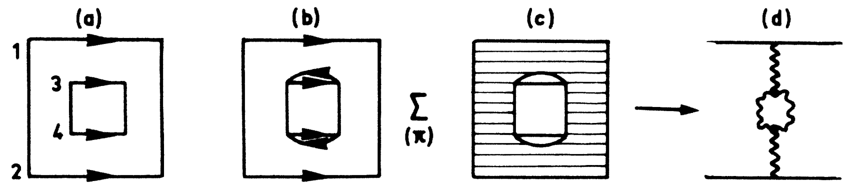

FIG. 4. Three steps in the construction of a Pomeron self-energy graph.

were pion linkages permitted between all the A lines of these graphs, as in Fig. 5, or, had spin-$\frac{1}{2}$ fermions been used in the construction of the bubble, then this contribution to σ_{tot} would have been proportional to $-s^2/Y$. In these latter cases, one would expect that the full exponentiation of the complete $i\chi_2'(s,\rho)$ would be necessary to avoid both exceeding the Froissart bound or finding a negative cross section.

In the present eikonal model, the sum of leading $(\ln Y)^N$ contributions, corresponding to N TP vertices illustrated (for $N=2$) in Fig. 6, can be shown to exponentiate, adding a contribution of $\exp(-q\ln Y)-1$ relative to the $+1$ of the bare Pomeron [assuming $\alpha(0)=1$]. In this highly approximate but suggestive way, one can see how such leading $\ln Y$ dependence could serve to suppress the bare Pomeron in the asymptotic limit. It is also interesting to see how, in this simple example, multiple-Pomeron exchanges corresponding to higher t-channel thresholds, are obtained by "dropping down" more and more radiative corrections to the original s-channel process of $i\chi_2$. Summation over *all* t-channel thresholds is included in the formalism, as is the availability of full s-channel unitarity.

Incidentally, the self-energy insertions in the graphs of Fig. 6, and their generalizations to higher N, can be attached to either nucleon line, with a resulting doubling of this contribution to σ_{tot}. The only proviso is that bubbles arising from the different nucleons cannot overlap in rapidity space, for then those graphs at fixed N_1 and N_2 do not generate the leading $(\ln Y)^{N_1+N_2}$ dependence.

More important contributions for large rapidity are associated with the graphs of Fig. 4(d). In the present model, and with $\alpha(0)=1$, these come out with a somewhat stronger Y dependence, generating a contribution to $\sigma_{\text{tot}}/\sigma_{\text{tot}}^{(\text{Pom})}$ of the form[10] $-pY(\ln Y-1)$, $p>0$. In this eikonal model, the leading behavior of the term with N such Pomeron self-energy bubbles, and no other TP radiative corrections, exponentiates to yield $e^{-pY\ln Y}-1$, again suggesting how these Pomeron self-energies can act to remove the bare Pomeron. The remaining contributions then generate a $\sigma_{\text{tot}}/\sigma_{\text{tot}}^{(\text{Pom})} \sim +pY(1+\cdots)$, where the next-to-leading-term of the contribution of Fig. 4(d) has been retained, plus the other "..." terms, not given by any such small-coupling analysis. For the sum of all contributions, one clearly must resort to renormalization-group arguments, or to special kinematical models.

V. SUMMARY AND OPEN QUESTIONS

In the present special and simplified eikonal model, it has been shown how the application of s-channel radiative corrections generates effective, multiple t-channel interacting TP's whose summed asymptotic contributions can readily act to produce a Y-dependent σ_{tot}, such as that suggested by RFT. The main advantages of the procedure is that it contains automatic s-channel unitarity along with an appropriate expression of all t-channel Pomeron thresholds, thereby approximating t-channel unitarity as well. The main qualifications to the model arise from the topics discussed in Sec. II: the necessity for excluding, in any equivalent field theory, those higher eikonal terms presently neglected, and the necessity for understanding the reasons behind the phenomenological choice $\alpha(0)=1$, which here serves to define the bare Pomeron.

For $\alpha(0)>1$, and in certain other situations, one is forced to employ the full s-channel unitarity provided by the eikonal model; and it will be no great surprise if the sum of all such radiative

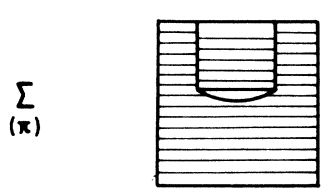

FIG. 5. Example of graphs leading to modified TP's for which full S-channel unitarity must be employed.

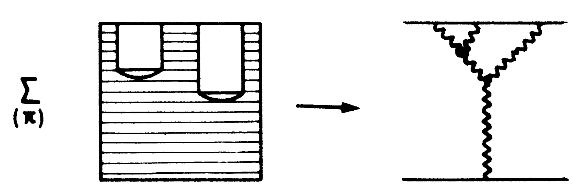

FIG. 6. Phenomenological construction of a graph with two TP vertices.

corrections lead to an expanding disk eikonal. Results suggestive of this form have been obtained in other recent $\alpha(0)>1$ model calculations.[12]

The contributions, modified by self-energy effects, of the higher nonplanar $i\chi_n$, $n>3$, remain to be properly estimated; and it is always possible that they could completely change all other qualitative results. If the simple, asymptotic arguments of Ref. 7 were even partially correct, then all RFT estimates require drastic and probably essential modifications. One wonders if the bound $\sigma_{tot} \leq \text{const} \times s^{-2\gamma(g_\infty)}$, recently suggested by Khuri[13] (in ϕ^4 theory, with the aid of arguments made plausible by their frequent usage in renormalization-group computations) could be understood in this way.

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