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Asymptotic freedom and the baryon-quark phase transition

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We have calculated the ground-state properties of a quark gas to second order in the quark-gluon coupling constant. Asymptotic freedom has been taken into account by using the renormalized coupling constant of Politzer and Gross and Wilczek. We find that this asymptotically free perturbation theory leads to an equation of state for a quark gas which for pressure $P > 0$ is very similar to the equation of state obtained from the MIT bag model of hadrons. In particular, we can identify a "bag pressure" term in the perturbation theory expression for the pressure as a function of density. We obtain estimates for the baryon-quark transition pressure by comparing the perturbation theory results with the Gibbs energy per baryon of baryonic matter. Our calculations show that the baryon-qua›k transition takes place at densities on the order of 10–20 times that in ordinary nuclei. These transition densities are higher than the maximum central density calculated for a neutron star.

I. INTRODUCTION

The quark theory of hadrons has had great success in classifying the spectrum of baryon and meson states,[1] providing an interpretation of lepton-hadron deep-inelastic scattering,[2] and explaining e^+e^- annihilation data.[3] The fact that free quarks have never been observed in nature possibly means that quarks are permanently bound inside hadrons. Such a situation could result[4] if quarks interact via a massless non-Abelian gauge field. There are, indeed, theoretical reasons[5] for believing that the gluon field which carries the strong interactions between quarks is a non-Abelian SU(3) massless vector gauge field coupled to quark color. The permanent confinement of quarks in such a theory could be due to the fact that in a non-Abelian gauge theory renormalization efforts can become very large when the gauge field carries small momentum transfers. In particular, using renormalization group arguments it has been shown[6] that the effective quark-gluon coupling constant α_c ($\equiv g^2/16\pi$) is

$$\alpha_c = \frac{\pi}{11 - \frac{2}{3}K}\,\frac{1}{\ln(Q^2/\Lambda^2)}, \qquad (1.1)$$

where K is the number of quark flavors, Q is the momentum transfer, and Λ is the single parameter which characterizes the effective coupling. In principle, the value of Λ may be deduced from e^+e^- annihilation data[3] or from deep-inelastic lepton-nucleon scattering,[7] even though in practice the determination of the exact value of Λ is complicated because of threshold effects associated with heavy quarks and possible contributions to the data from heavy leptons.[3]

Although Eq. (1.1) is only rigorously valid for small values of α_c, it does suggest that the quark-gluon coupling constant can become very large for momentum transfers $Q \to \Lambda$. This large increase in quark-gluon coupling at small momentum transfers may result in quarks being confined to a finite region of space. However, this is only a conjecture at the present time since a rigorous theory of quark interactions when α_c is large has not yet been developed.

Another and more straightforward result of the renormalization of the quark-gluon coupling constant is the asymptotic-freedom property, which states that the quark-gluon coupling becomes very small at large momentum transfers if the number of quark flavors is not too large, as is evident from (1.1). If quarks are indeed asymptotically free at large momentum transfers then one interesting consequence would be that at superhigh densities where the quark Fermi energy is large matter should behave like a gas of free quarks.[8] Of course, the density where such a description becomes valid must be higher than the density of matter inside nuclei, because it is well known that the behavior of matter up to these densities can be described in terms of baryons (protons, neutrons, and hyperons). Evidently then, nuclear matter undergoes some kind of transition at densities above those occurring in ordinary nuclei from a state where the quarks are localized inside baryons to a state where the quarks are delocalized and approximately free.

This question of a phase transition between baryon matter and quark matter is not entirely of academic interest because nuclear densities higher than those occurring in ordinary nuclei occur inside neutron stars. Thus it is of interest in connection with problems such as the maximum mass of a neutron star to know the baryon densities where the baryon-quark phase transition takes place. High baryon densities also occur in the early universe, and the baryon-quark transition may have important cosmological implications.[9]

In order to investigate the baryon-quark phase transition one needs a theory of quark matter. In principle one should in fact be able to calculate where the baryon-quark phase transition takes place from the theory of quark-gluon interactions, i.e., quantum chromodynamics (QCD). At the present time, however, one cannot carry out such a calculation because of the lack of a calculational procedure for describing quark confinement, i.e., the regime when the quark-gluon coupling becomes large. Instead, one must adopt a more phenomenological procedure.

It has been pointed out[10,11] that one may obtain an estimate of the densities where the baryon-quark phase transition is likely to take place by comparing the Gibbs energy of a quark gas with the Gibbs energy of baryon matter calculated using phenomenological nucleon-nucleon potentials. The Gibbs energy of the quark gas should of course be calculated as accurately as possible, but since one does not have an exact theory of quark-gluon interactions one must make some sort of approximation to calculate the properties of the quark gas. One possibility is to make use of the phenomenological MIT bag model[12] of hadrons. This theory has been successful in explaining the observed properties of baryon states[13] and in addition provides a prescription for calculating the properties of a quark gas. In particular, it can be shown[10,11] that the energy density ϵ and pressure P of quark matter in the limit of zero quark masses is

$$\epsilon = An^{4/3}+B\,, \tag{1.2a}$$

$$P=\frac{A}{3}n^{4/3}-B\,, \tag{1.2b}$$

where A and B are constants. Note that (1.2a) and (1.2b) lead to an equation of state

$$P=\tfrac{1}{3}(\epsilon-4B)\,. \tag{1.3}$$

In second order in the quark-gluon coupling g, A gets a contribution from the single-gluon-exchange diagram for quark-quark scattering and is given by[10,11]

$$A=\frac{9}{4}\left(\frac{3\pi^2}{K}\right)^{1/3}\left(1+\frac{8\alpha_c}{3\pi}\right)\hbar c\,. \tag{1.4}$$

The coupling α_c and the bag pressure B do not depend on quark momentum in the MIT bag model, and they are determined by fitting to the observed N and Δ masses.[13] Using these bag parameters and some recent phenomenological equations of state for baryonic matter it was found[10,11] that the baryon-quark phase transition occurs at a density of 10–60 times that in nuclei.

Another approach to calculating the properties of a quark gas would be to use perturbation theory, including the higher-order effects which lead to the coupling-constant renormalization.[14,15] As a consequence of asymptotic freedom, perturbation theory will be valid at very high densities. Whether or not perturbation theory is valid at densities as low as the baryon-quark phase transition depends on the value of the quark-gluon coupling constant at the transition. We will show in the following that at the baryon-quark transition density the effective strength of quark-gluon interactions α_c probably lies somewhere in the range 0.25 to 0.45. Although these values are not so small that one can be confident that perturbation theory is a good approximation near the transition density, they are small enough so that one might hope that perturbation theory is not too bad an approximation near the transition density. In this paper we would like to reinvestigate the question of where the baryon-quark phase transition takes place, adopting this point of view.

In calculating the ground-state properties of a quark gas we will apply our previous result, (1.2a), for the energy density of a quark gas, but with $B=0$. Instead of using a fixed gluon coupling constant as in the MIT bag model, we will use a renormalized gluon coupling constant that depends on the Gibbs energy per quark and appears as the natural expansion parameter in a perturbation-theory treatment of many-quark systems.[14,15] In other words, we will assume that the energy density of a quark gas is given by

$$\epsilon = An^{4/3}\,, \tag{1.5}$$

where A is related to the quark-gluon coupling α_c as in (1.4), but now this coupling depends on the Gibbs energy per quark. The form of (1.1) suggests that an appropriate expression for α_c in a Fermi gas would be[16]

$$\alpha_c=\frac{\pi}{22-\frac{4}{3}K}\,\frac{1}{\ln(k_F/\Lambda_F)}\,, \tag{1.6}$$

where k_F is the quark Fermi momentum and Λ_F is a constant. This assumes that the medium screens the long-range quark-quark interaction[8] as in the corresponding case of an electron gas. In order that the value of α_c calculated from (1.6) be consistent with (1.1) the quantity Λ_F should be comparable to $\frac{1}{2}\Lambda$.

Alternatively, it has been suggested[17] that the MIT bag model applied to quark matter[10,11] might be improved by using a renormalized coupling constant α_c that depends on the Gibbs energy per quark. However, there are questions concerning the validity of such an approach. For example, if the coupling α_c depends on the quark Gibbs energy, why should the bag pressure B be kept constant? We will show in Sec. II that this question finds a natural resolution within the framework of renormalized perturbation theory. In particular, one can identify a term in the renormalized perturbation-theory expression for the pressure which plays the role of a bag pressure. This effective bag pressure does depend on quark momentum, but in the region of the baryon-quark phase transition it is approximately constant. Thus we find that as far as the equation of state for quark matter is concerned there is practically no difference between the bag-model theory and second-order perturbation theory using a renormalized quark-gluon coupling.

The question of exactly where the baryon-quark phase transition occurs is discussed in Sec. III. The density where the baryon-quark transition occurs depends on the parameter Λ_F and on the equation of state of baryonic matter, but for reasonable values of Λ_F the transition density probably lies in the region of 10–20 times the density of matter in nuclei.

II. PROPERTIES OF A QUARK GAS IN QCD

In this section we discuss the ground-state properties of a quark gas in second-order perturbation theory in QCD, including the effects of renormalization of the quark-gluon coupling constant α_c (1.6). Introducing the dimensionless parameter $\chi = k_F/\Lambda_F$, we can write α_c for $K=3$ in the form

$$\alpha_c = \frac{\pi}{18 \ln \chi}, \tag{2.1}$$

where χ is determined by the conserved baryon number density

$$n = \frac{\Lambda_F^3 \chi^3}{\pi^2}. \tag{2.2}$$

Substituting (2.1) and (2.2) into Eq. (1.5) we obtain for the energy density

$$\epsilon = \frac{9}{4\pi^2}\Lambda_F^4 \chi^4 \left(1 + \frac{4}{27 \ln \chi}\right). \tag{2.3}$$

Then the pressure $P \equiv n\, d\epsilon/dn - \epsilon$ and the Gibbs energy $\mu \equiv d\epsilon/dn$ derived from (2.2) and (2.3) are

$$P = \frac{3}{4\pi^2}\Lambda_F^4 \chi^4 \left[1 + \frac{4}{27 \ln \chi}\left(1 - \frac{1}{\ln \chi}\right)\right], \tag{2.4}$$

$$\mu = 3\Lambda_F \chi \left[1 + \frac{4}{27 \ln \chi}\left(1 - \frac{1}{4 \ln \chi}\right)\right]. \tag{2.5}$$

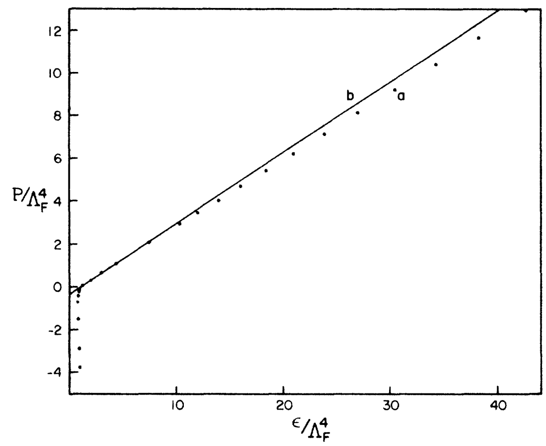

FIG. 1. The pressure P/Λ_F^4 as a universal function of energy density ϵ/Λ_F^4, shown for the case (a) QCD equation (2.4), and for (b) the MIT bag model, (1.3), where α_c and B are determined according to (2.7) and (2.13).

Note from (2.1) to (2.5) that α_c, n/Λ_F^4, ϵ/Λ_F^4, P/Λ_F^4, and μ/Λ are universal functions parametrized by χ independent of Λ_F. This is illustrated in curves (a) of Figs. 1 and 2 and the curve of Fig. 3, where we show P/Λ_F^4 as a function of ϵ/Λ_F^4, μ/Λ_F as a function of P/Λ_F^4, and α_c as a function of P/Λ_F^4. We have included only three quark flavors, $K=3$, in (2.1)–(2.5) because the charmed quark is expected to have a mass $m_c \sim 2$ GeV and, it turns

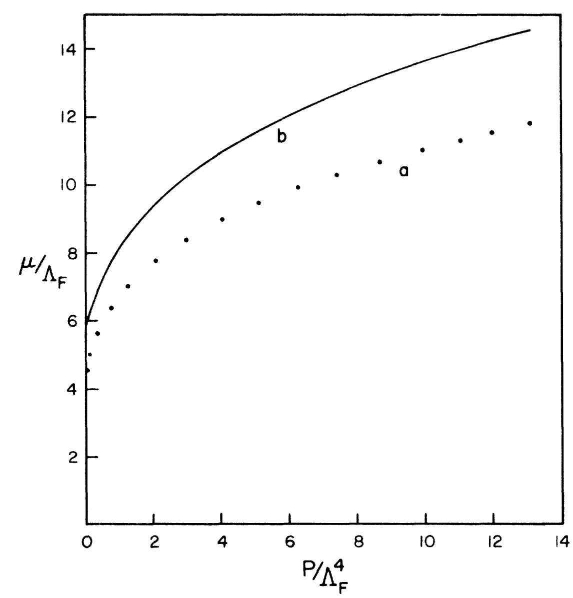

FIG. 2. The Gibbs energy per baryon, μ/Λ_F, as a universal function of the pressure P/Λ_F^4 for the case (a) QCD equation (2.5), and for (b) the MIT-bag-model result (Ref. 11), $\mu = 4(A/3)^{3/4}(P+B)^{1/4}$.

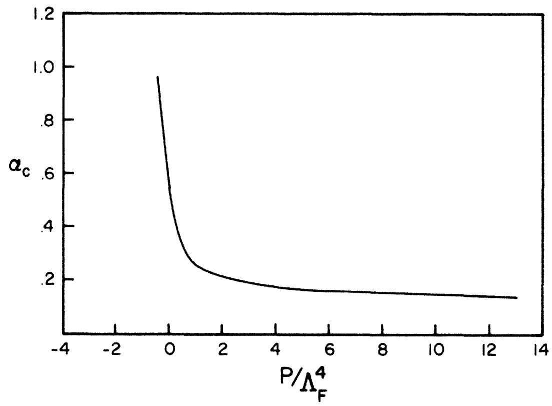

FIG. 3. The renormalized quark-gluon coupling constant α_c, (2.1), as a function of the pressure P/Λ^4, (2.4).

out, can be ignored for densities near the baryon-quark phase transition. Our neglect of the mass of the strange quark $m_s \sim 0.3$ GeV can also be justified *a posteriori* for these densities.

According to (2.2) and (2.3) the energy per baryon ϵ/n has a minimum as a function of χ which occurs when the pressure P vanishes. Solving (2.4) for $P=0$ we obtain

$$\ln\chi_0 = \tfrac{2}{27}(2\sqrt{7}-1) = 0.3178$$

or (2.6)

$$\chi_0 = 1.374\,.$$

The first interesting consequence of this result is the value of the effective quark-gluon coupling constant α_c, (2.1), at $P=0$,

$$\alpha_c = \frac{\pi}{18\ln\chi_0} = 0.549\,, \tag{2.7}$$

which is numerically equal to the value obtained by fitting the hadron spectrum in the MIT bag model.[13] The corresponding baryon density n (2.2) and energy density ϵ (2.3) at $P=0$ are then

$$n_0 = \frac{\Lambda_F^{\,3}}{\pi^2}\chi_0^{\,3} = 0.263\Lambda_F^{\,3}\,, \tag{2.8}$$

$$\epsilon_0 = \frac{\Lambda_F^{\,4}}{3\pi^2}\frac{\chi_0^{\,4}}{\ln^2\chi_0} = 1.192\Lambda_F^{\,4}\,, \tag{2.9}$$

giving an energy per baryon

$$\frac{\epsilon_0}{n_0} = \frac{\Lambda_F}{3}\frac{\chi_0}{\ln^2\chi_0} = 4.533\Lambda_F\,. \tag{2.10}$$

Of course we know that the physical ground state of the system at $P=0$ is not a quark gas but nuclear matter. This implies that a phase transition occurs at somewhat higher densities, below which quarks become confined inside individual baryons. We can apply (2.10) to determine a *lower bound* to Λ_F by the consistency requirement that this energy per baryon at $P=0$ be higher than the nucleon mass. For $\epsilon_0/n_0 > 940$ MeV we then obtain $\Lambda_F > 207$ MeV. Actually this lower bound Λ_F is only approximate because near $P=0$ for $\Lambda_F \lesssim 300$ MeV the mass of strange quarks is not negligible.

To compare the equation of state from QCD with previous results using the MIT bag model[10,11] we note that (2.3) and (2.4) imply that

$$P = \tfrac{1}{3}[\epsilon - 4B(\chi)]\,, \tag{2.11}$$

where

$$B(\chi) = \frac{\Lambda_F^{\,4}}{12\pi^2}\frac{\chi^4}{\ln^2\chi}\,. \tag{2.12}$$

Comparing (2.11) with the corresponding equation of state in the MIT model (1.3) we interpret $B(\chi)$ as a density-dependent bag pressure.

At $P=0$ we have

$$B(\chi_0) = \frac{\chi_0^{\,4}}{12\pi^2\ln^2\chi_0}\Lambda_F^{\,4} = 0.298\Lambda_F^{\,4}\,, \tag{2.13}$$

which for $\Lambda_F = 207$ MeV gives $B(\chi_0) = 71.4$ MeV fm^{-3}, a value somewhat higher than the value of the constant MIT bag parameter $B = 59.2$ MeV fm^{-3}. Note that $B(\chi)$ (2.12) has a minimum value at

$$\ln\chi_1 = \tfrac{1}{2}$$

or

$$\chi_1 = 1.649\,, \tag{2.14}$$

which gives

$$B(\chi_1) = 0.2495\Lambda_F^{\,4}\,. \tag{2.15}$$

For $\Lambda_F = 207$ MeV we find $B(\chi_1) = 59.8$ MeV fm^{-3}. We expect therefore that the QCD equation of state, (2.11), will be in close agreement for $P>0$ with the corresponding equation for the MIT bag model (1.3). This is indeed the case as is shown in Fig. 1. In particular note that to a very good approximation the pressure P depends linearly on the energy density for $P>0$, and $dP/d\epsilon \cong \frac{1}{3}$. This implies that the critical values of P/ϵ and the adiabatic index $\gamma \equiv d\ln P/d\ln(n)$ for the maximum mass of a quark star should be nearly the same as those previously calculated in the MIT model.[11]

If one fixes α_c and B according to (2.7) and (2.13), respectively, then one can plot the MIT-bag-model equation of state and Gibbs energy as universal functions of $\epsilon/\Lambda_F^{\,4}$ and $P/\Lambda_F^{\,4}$, respectively. Such plots are shown as curves (b) in Figs. 1 and 2. It can be seen as expected that there is very good agreement between the bag-model equation of state and the equation of state from QCD for $P>0$. On the other hand, for a given P the value of μ calculated from QCD is smaller than the MIT-bag-model result[11] $\mu = 4(A/3)^{3/4}(P+B)^{1/4}$. This can also be

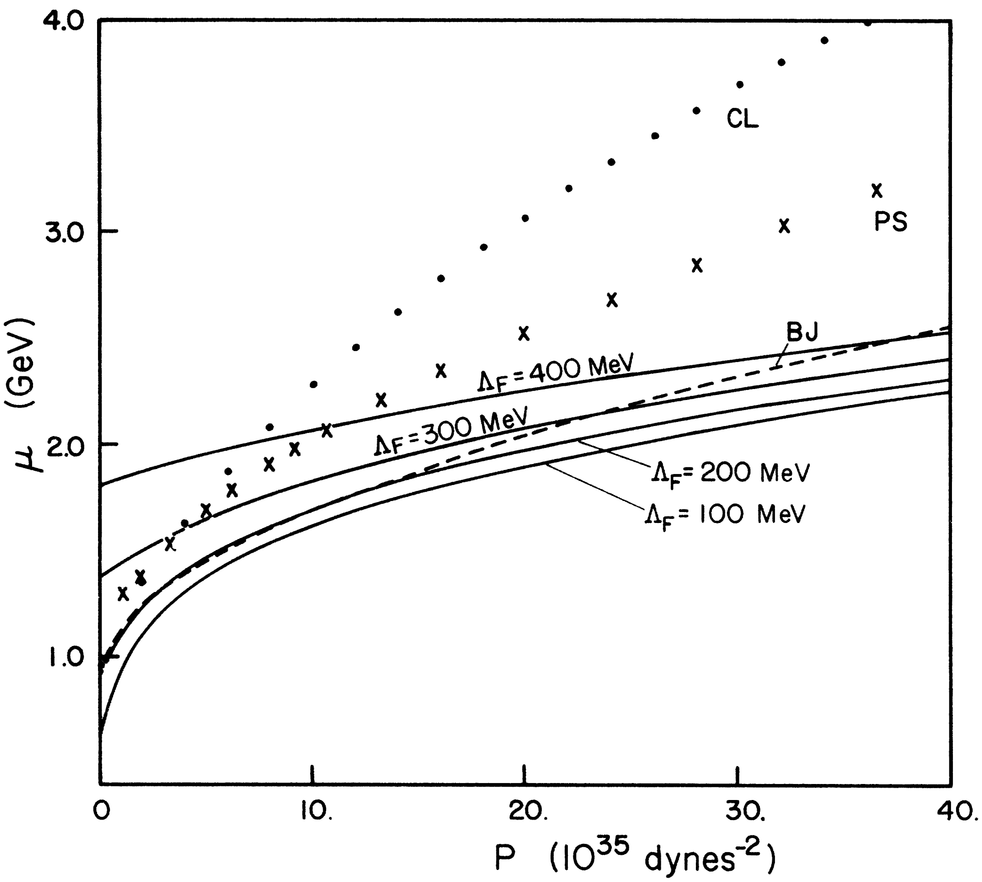

FIG. 4. The Gibbs energy per baryon μ as a function of pressure P calculated using QCD, (2.5), for $\Lambda_F = 100$, 200, 300, and 400 MeV. Also shown are the Gibbs energy per baryon for baryonic matter calculated using the Bethe-Johnson (– – –), Pandharipande-Smith (× × ×), and causality-limit (• • •) equations of state.

easily seen analytically; for example, at $P=0$, we obtain from (2.5) and (2.6)

$$\mu_0 = \frac{9}{27}\frac{\chi_0}{\ln^2\chi_0}\Lambda_F = 4.533\Lambda_F\,, \qquad (2.16)$$

while using the corresponding MIT-bag-model result for μ_0 we have

$$\mu_0 = \frac{4^{3/4}}{3^{7/4}}\frac{\chi_0}{\ln^2\chi_0}\Lambda_F = 5.6246\Lambda_F\,. \qquad (2.17)$$

Likewise at $P\to\infty$ we obtain in QCD

$$\mu \to 4(\tfrac{3}{4})^{3/4}\pi^{1/2}P^{1/4} = 5.714\,P^{1/4}\,, \qquad (2.18)$$

while the MIT bag yields[10]

$$\mu \to 4(\tfrac{3}{4})^{3/4}\pi^{1/2}\left(1+\frac{4}{27\ln\chi_0}\right)P^{1/4} = 7.613\,P^{1/4}\,. \qquad (2.19)$$

Finally, in Fig. 3 we show the dependence of α_c on $P/\Lambda_F{}^4$.

III. RESULTS FOR THE BARYON-QUARK PHASE TRANSITION

Given the QCD result for the Gibbs energy per baryon of quark matter, (2.5), we can now proceed to determine where the baryon-quark phase transition takes place by comparing this Gibbs energy with calculations of the Gibbs energy per baryon of baryonic matter. We show in Fig. 4 the Gibbs energy per baryon, μ, for quark matter as a function of pressure calculated using (2.5) for $\Lambda_F = 100$, 200, 300, and 400 MeV. We also show on the same graph the Gibbs energy per baryon for three possible equations of state of baryon matter: the phenomenological Bethe-Johnson[18] (their model labeled "VH"), Pandharipande-Smith–solid,[19] and "causality-limit"[20] equations of state. The causality-limit equation of state assumes that the speed of sound in baryonic matter equals the speed of light for $n > 0.3$ fm^{-3} and is matched to the Bethe-Johnson equation of state at $n = 0.3$ fm^{-3}. This equation of state is the stiffest one possible in the region $n > 0.3$ fm^{-3}.

TABLE I. Properties of the baryon-quark phase transition. P_T is the transition pressure, n_1 is the baryon number density on the baryon side of the transition, n_2 is the baryon number density on the quark side of the transition, ρ_1 is the matter density on the baryon side of the transition, and ρ_c is the maximum central density for a neutron star calculated using the indicated equation of state.

	Bethe-Johnson VH		Pandharipande-Smith		Causality limit	
Λ_F (MeV)	300	400	300	400	300	400
P_T (10^{35} dyn cm^{-2})	24.0	36.8	4.1	10.9	3.6	7.3
n_1 (fm^{-3})	2.15	2.58	0.7	1.1	0.47	0.59
n_2 (fm^{-3})	3.61	5.07	1.3	3.0	1.26	2.68
ρ_1 (10^{15} g cm^{-3})	5.60	7.39	1.8	3.0	0.90	1.30
ρ_c (10^{15} g cm^{-3})	3.31		1.1		1.6	

Using Fig. 4 one can read off the transition pressure, i.e., the pressure where μ (baryon matter) $= \mu$ (quark gas). One of the striking results evident from Fig. 4 is that there is no baryon-quark phase transition for the baryonic equations of state being considered for $\Lambda_F < 200$ MeV. This presumably means either that none of the baryonic equations of state being considered are realistic or that in reality $\Lambda_F > 200$ MeV. In fact we argued in Sec. II that Λ_F should be greater than 207 MeV on the basis that the Gibbs energy per baryon in the quark gas should be larger than the nucleon mass at zero pressure. More realistically, one expects that the energy per baryon in a quark gas at zero pressure will be greater than the spin-isospin weighted average of the N and Δ masses (1180 MeV) which corresponds to $\Lambda_F > 260$ MeV.

Another argument which supports the idea that Λ_F should be greater than 260 MeV is that one should expect that the value of $\alpha_c(\chi)$ will be close to the bag-model value $\alpha_c = 0.55$ when the average quark momentum in the Fermi gas, $\frac{3}{4}k_F$, is equal to the average quark momentum inside the stable nucleon. For the case of light quarks the bag model[13] gives about 400 MeV/c for the quark momenta inside the nucleon. Since $\alpha_c(\chi) = 0.55$ when $\chi = 1.373$, we obtain $\Lambda_F \approx \frac{4}{3}$ (400)/(1.373) = 388 MeV.

It should be noted that Λ_F in the range 300 to 400 MeV is not inconsistent with current interpretations[3] of e^+e^- annihilation data for center-of-mass energies in the range 4 to 8 GeV, which suggests that if there is a heavy quark with a mass ~2 GeV then Λ probably lies in the range 300 MeV to 1 GeV. Recent work on the breakdown of scaling in electroproduction[7] suggests that $\Lambda \approx 500$ MeV.

From the transition pressure one can obtain the matter densities and baryon densities on both sides of the phase transition by making use of the quark-gas and baryonic-matter equations of state. The results for $\Lambda_F = 300$ and 400 MeV and the different baryonic equations of state are shown in Table I. It can be seen that in the case of the Pandharipande-Smith–baryonic-solid equations of state the matter density ρ_1 on the baryon side of the transition is going to be very close to the result obtained earlier[10,11] using the MIT-bag-model theory if Λ_F lies between 300 and 400 MeV. On the other hand, if one uses the Bethe-Johnson baryonic equation of state then the baryon matter density at the transition will be about $\frac{1}{2}$ as large as the bag-model result.

The maximum central density ρ_c for a neutron star calculated using the Oppenheimer-Volkoff equation and the indicated equations of state[18,21] is also shown for comparison in Table I. It can be seen that with the possible exception of an equation of state lying near the causality limit the calculated baryon-matter transition densities lie above the neutron-star maximum central densities. Thus, although the possibility of having free quarks inside neutron stars is not entirely ruled out, it does not appear likely.

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