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Weak neutral currents in electron-positron annihilation into three pions with polarized beams

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We present a detailed discussion of the possible angular asymmetries in $e^+e^- \to \pi^+\pi^-\pi^0$ and their relation with the presence of a weak neutral current for the case when the initial beams are either transversely or longitudinally polarized. We define several asymmetry parameters which provide information on the axial-vector and vector couplings of the weak neutral current. We also estimate the order of magnitude of these parameters for beam energies of the next generation of accelerators.

I. INTRODUCTION

Recently it has been proposed[1,2] to observe the weak neutral current in the annihilation process

$$e^+e^- \to \pi^+\pi^-\pi^0 . \tag{1.1}$$

The idea is to look for angular asymmetries which arise from the interference of the amplitude for annihilation via a neutral particle Z and the amplitude for annihilation via one photon. The origin of the asymmetries lies in the opposite charge-conjugation properties and in the opposite parities of the axial-vector neutral current and either the photon or the vector neutral current. In Refs. 1 and 2 the lepton beams were assumed to be unpolarized and it was indicated how to detect the coupling constants of the weak neutral current. It is the purpose of this note to analyze the case when the initial beams are polarized.

In Sec. II we derive the explicit form of the square of the matrix element for process (1.1). In Sec. III we assume that the polarization is transverse and isolate the polarization-dependent part of the differential cross section. The angular asymmetries allow us to define six asymmetry parameters, A_{p1}, A_{p2}, A_{p3}, A_{c1}, A_{c2}, and A_{c3}, which are proportional to different combinations of the coupling constants. We show also that two asymmetry parameters may have a zero for beam energies $< M_Z/2$, where M_Z is the mass of the Z particle. In Sec. IV we discuss the case of longitudinal polarization and define three additional asymmetry parameters proportional to the polarization. In Sec. V we present our conclusions.

II. GENERAL MATRIX ELEMENT

In this and the following sections we will often need to interchange constants in some expressions. To this end we find it convenient to introduce the symbol

$$\hat{R}[E; a \to a', b \to b', \ldots] \tag{2.1}$$

to denote the result of replacing in the expression for E the constants a, b,... by a', b',... .

As before[1] we assume that the neutral current is of the $V-A$ form and that the negative G parity piece of the hadronic neutral current has a vector isoscalar part and an axial-vector isovector one. We denote by q_-, q_+, p_-, p_+, and p_f the four-momenta of the electron, positron, pions, and total, respectively, in the center-of-mass frame of e^+e^-. Furthermore, let s_+ (s_-) be the polarization four-vector of the positron (electron).

The sum of the amplitude for annihilation via one photon and the amplitude for annihilation via a neutral particle Z is

$$M = \frac{l^\mu}{s} L_\mu + (v_\mu + a_\mu) \frac{g^{\mu\nu} - p_f^\mu p_f^\nu / M_Z^2}{s - M_Z^2} (V_\nu + A_\nu) , \tag{2.2}$$

where

$$s = p_f^2 , \tag{2.3}$$

$$l^\mu = e i \bar{v} \gamma^\mu u , \tag{2.4}$$

$$v_\mu = g_v i \bar{v} \gamma_\mu u , \tag{2.5}$$

$$a_\mu = g_a i \bar{v} \gamma_\mu \gamma_5 u . \tag{2.6}$$

e is the electron charge and g_v (g_a) is the coupling constant of the vector (axial-vector) neutral current to leptons. L_μ describes the γ-$\pi^+\pi^-\pi^0$ vertex and V_ν and A_ν describe the Z-$\pi^+\pi^-\pi^0$ one with vector and axial-vector coupling, respectively.

L_μ and V_μ are axial vectors antisymmetric in their dependence upon p_+, p_-, and p_0. That is,

$$L_\mu = i e \epsilon_{\mu\nu\rho\sigma} p_+^\nu p_-^\rho p_0^\sigma F_1 , \tag{2.7}$$

$$V_\mu = i g_V \epsilon_{\mu\nu\rho\sigma} p_+^\nu p_-^\rho p_0^\sigma F_2 , \tag{2.8}$$

where F_1 and F_2 are Lorentz scalars symmetric in their dependence on p_+, p_-, and p_0. On the other hand, A_μ is a vector symmetric in p_+ and p_-:

$$A_\mu = g_A[(p_+ + p_-)_\mu F_3 + (p_+ - p_-)_\mu F_4 + p_{f\mu} F_5] , \tag{2.9}$$

where F_3 and F_5 are symmetric in their dependence on p_+ and p_- and F_4 is antisymmetric. g_V and g_A are the coupling constants of the vector and axial vector neutral current to hadrons.

The square of the matrix element M can be written as the sum of an electromagnetic (e), a pure weak (w), and an interference (i) part:

$$|M|^2=\tau^{(e)}_{\mu\nu}T^{\mu\nu}_{(e)}+\tau^{(w)}_{\mu\nu}T^{\mu\nu}_{(w)}+(\tau^{(i)}_{\mu\nu}T^{\mu\nu}_{(i)}+\text{c.c.}) , \tag{2.10}$$

where each term is the product of a leptonic tensor, $\tau_{\mu\nu}$, and a hadronic one $T^{\mu\nu}$. Neglecting the lepton masses and defining

$$c_{\mu\nu}=(q^-_\mu q^+_\nu+q^-_\nu q^+_\mu-g_{\mu\nu}s/2)(1-s_+s_-)+(q^-_\nu s^+_\mu+q^-_\mu s^+_\nu)q_+s_-+(q^+_\nu s^-_\mu+q^+_\mu s^-_\nu)q_-s_+ + (s^+_\nu s^-_\mu+s^+_\mu s^-_\nu)q_-q_+-g_{\mu\nu}(q_-s_+)(q_+s_-) , \tag{2.11}$$

$$c'_{\mu\nu}=\hat{R}[c_{\mu\nu}\,;s_+\to -s_+] , \tag{2.12}$$

$$d_{\mu\nu}=\epsilon_{\mu\nu\alpha\beta}q^\alpha_-q^\beta_+-g_{\mu\nu}\epsilon_{\alpha\beta\rho\sigma}s^\alpha_-q^\beta_-q^\rho_+s^\sigma_++q^+_\mu\epsilon_{\nu\alpha\beta\rho}q^\alpha_-s^\beta_+s^\rho_-+q^+_\nu\epsilon_{\mu\alpha\beta\rho}q^\alpha_-s^\beta_+s^\rho_- - s^+_\mu\epsilon_{\nu\alpha\beta\rho}q^\alpha_-q^\beta_+s^\rho_- - s^+_\nu\epsilon_{\mu\alpha\beta\rho}q^\alpha_-q^\beta_+s^\rho_- , \tag{2.13}$$

we have[3]

$$\tau^{(e)}_{\mu\nu}=e^2c_{\mu\nu} , \tag{2.14}$$

$$\tau^{(w)}_{\mu\nu}=g_v{}^2c_{\mu\nu}+g_a{}^2c'_{\mu\nu}+ig_vg_a(d_{\mu\nu}-d_{\nu\mu}) , \tag{2.15}$$

$$\tau^{(i)}_{\mu\nu}=eg_vc_{\mu\nu}+ieg_ad_{\mu\nu} . \tag{2.16}$$

Noticing that $d_{\mu\nu}$ depends on the polarization only through its symmetric part, we can immediately see from Eq. (2.15) that there will be no polarization-dependent part, in the differential cross section, proportional to g_vg_a.

Since[4]

$$c_{00}=c_{0i}=d_{00}=d_{0i}=d_{i0}=0 , \tag{2.17}$$

it is sufficient to specify the spatial-spatial parts of the hadronic tensors[5]:

$$T^{ij}_{(e)}=\frac{e^2}{s}(\vec{p}_+\times\vec{p}_-)^i(\vec{p}_+\times\vec{p}_-)^j|F_1|^2 , \tag{2.18}$$

$$T^{ij}_{(w)}=\frac{1}{(s-M_Z{}^2)^2}\left[g_A(p_++p_-)^iF_3+g_A(p_+-p_-)^iF_4-ig_V\sqrt{s}(\vec{p}_+\times\vec{p}_-)^iF_2\right]\times\left[g_A(p_++p_-)^jF^*_3+g_A(p_+-p_-)^jF^*_4+ig_V\sqrt{s}(\vec{p}_+\times\vec{p}_-)^jF^*_2\right] , \tag{2.19}$$

$$T^{ij}_{(i)}=\frac{-ie}{(s-M_Z{}^2)\sqrt{s}}(\vec{p}_+\times\vec{p}_-)^iF_1\left[g_A(p_++p_-)^jF^*_3+g_A(p_+-p_-)^jF^*_4+ig_V\sqrt{s}(\vec{p}_+\times\vec{p}_-)^jF^*_2\right] . \tag{2.20}$$

In this way Eq. (2.10) becomes

$$\begin{aligned}|M|^2={}&e^4\frac{1}{s}c_{ij}(\vec{p}_+\times\vec{p}_-)^i(\vec{p}_+\times\vec{p}_-)^j|F_1|^2\\&+\frac{1}{(s-M_Z{}^2)^2}[g_v{}^2c_{ij}+g_a{}^2c'_{ij}+ig_ag_v(d_{ij}-d_{ji})][g_A(p_++p_-)^iF_3+g_A(p_+-p_-)^iF_4-ig_V\sqrt{s}(\vec{p}_+\times\vec{p}_-)^iF_2]\\&\times[g_A(p_++p_-)^jF^*_3+g_A(p_+-p_-)^jF^*_4+ig_V\sqrt{s}(\vec{p}_+\times\vec{p}_-)^jF^*_2]\\&+\frac{e^2}{\sqrt{s}(s-M_Z{}^2)}\{(g_ad_{ij}-ig_vc_{ij})(\vec{p}_+\times\vec{p}_-)^iF_1[g_A(p_++p_-)^jF^*_3+g_A(p_+-p_-)^jF^*_4+ig_V\sqrt{s}(\vec{p}_+\times\vec{p}_-)^jF^*_2]+\text{c.c.}\} ,\end{aligned} \tag{2.21}$$

where

$$c_{ij}=-(g_{ij}s/2+2q_iq_j)(1-s_+s_-)+\sqrt{s}\,s^0_-(q_js^+_i+s^+_iq_j)-\sqrt{s}\,s^0_+(q_js^-_i+q_is^-_j)-\tfrac{1}{2}s(s^+_is^-_j+s^+_js^-_i)+ss^0_-s^0_+g_{ij} , \tag{2.22}$$

$$d_{ij} = -\sqrt{s}\epsilon_{ijk}q^k + \sqrt{s}g_{ij}[\vec{q}\cdot(\vec{s}_-\times\vec{s}_+)] - 2s_+^0[q_i(\vec{q}\times\vec{s}_-)_j + q_j(\vec{q}\times\vec{s}_-)_i]$$
$$-\sqrt{s}[s_i^+(\vec{q}\times\vec{s}_-)_j + s_j^+(\vec{q}\times\vec{s}_-)_i] - \frac{4}{\sqrt{s}}q_iq_j[\vec{q}\cdot(\vec{s}_+\times\vec{s}_-)] \ , \quad (2.23)$$

$$\vec{q} = \vec{q}_- = -\vec{q}_+ \ . \quad (2.24)$$

III. TRANSVERSE POLARIZATION

Let $s_+ = (0, -\vec{s})$, $s_- = (0, \vec{s})$. In this case the expressions (2.22) and (2.23) simplify to

$$c_{ij} = -(sg_{ij}/2 + 2q_iq_j)(1 - \vec{s}^2) + ss_is_j \ , \quad (3.1)$$

$$d_{ij} = -\sqrt{s}\epsilon_{ijk}q^k + \sqrt{s}[s_i(\vec{q}\times\vec{s})_j + s_j(\vec{q}\times\vec{s})_i] \ . \quad (3.2)$$

Choosing the z and x axes in the $\vec{q}$ and $\vec{s}$ directions, respectively, we have for any two vectors $\vec{a}$ and $\vec{b}$

$$c_{ij}a^ib^j = \tfrac{1}{2}s[a^1b^1 + a^2b^2 + \vec{s}^{\,2}(a^1b^1 - a^2b^2)]$$
$$= \tfrac{1}{2}s[\vec{a}\cdot\vec{b} - a^3b^3 + \vec{s}^{\,2}(a^1b^1 - a^2b^2)] \ , \quad (3.3)$$

$$d_{ij}a^ib^j = -\tfrac{1}{2}s[a^1b^2 - a^2b^1 + \vec{s}^{\,2}(a^1b^2 + a^2b^1)] \ , \quad (3.4)$$

which allows us to easily distinguish the polarization-dependent terms from the polarization-independent ones in $|M|^2$.

Let $E_\pm = p_\pm^0$, $\theta_+(\theta_-)$, and $\psi_+(\psi_-)$ the polar coordinates of $\vec{p}_+(\vec{p}_-)$, and θ_{+-} the angle between $\vec{p}_+$ and $\vec{p}_-$ given by

$$\cos\theta_{+-} = \frac{s + M^2 + 2E_+E_- - 2\sqrt{s}\,(E_+ + E_-)}{2|\vec{p}_+||\vec{p}_-|} \ , \quad (3.5)$$

where M is the pion mass. Separating the differential cross section in its symmetric (s), charge-antisymmetric (ca), and parity-violating (pv) parts,

$$d\sigma = d\sigma^{s} + d\sigma^{ca} + d\sigma^{pv} \ , \quad (3.6)$$

the dependence on the polarization becomes

$$d\sigma^k(\vec{s}) = d\sigma^k(0) + \vec{s}^{\,2}d\Sigma^k \ , \quad k = \text{s, ca, pv}, \quad (3.7)$$

where the terms $d\Sigma^k$, which are specified below, do not depend on the polarization. Let $\hat{I}$ be an integral operator defined by

$$\hat{I} = \frac{1}{16s(2\pi)^5}\int_M^{\bar{E}} dE_+ \int_M^{\bar{E}} dE_-\delta(\cos\theta_{+-} - \cos\theta_+\cos\theta_- - \sin\theta_+\sin\theta_-\cos(\psi_+ - \psi_-)) \ , \quad (3.8)$$

where $\bar{E} = (s - 3M^2)/2s$, then

$$d\sigma^s(0) = d\Omega_+d\Omega_-\hat{I}\{C_s^{(1)}[1 + \cos(\theta_+ + \theta_-)\cos(\theta_+ - \theta_-) - 2\cos\theta_{+-}\cos\theta_+\cos\theta_-]$$
$$+ C_s^{(2)}(\sin^2\theta_+ + \sin^2\theta_-) + C_s^{(3)}(\cos\theta_{+-} - \cos\theta_+\cos\theta_-)\} \ , \quad (3.9)$$

$$d\Sigma^s = d\Omega_+d\Omega_-\hat{I}\{C_s^{(1)\prime}[2\sin\theta_+\sin\theta_-\cos\theta_+\cos\theta_-\cos(\psi_+ + \psi_-) - \sin^2\theta_+\cos^2\theta_-\cos2\psi_+ - \sin^2\theta_-\cos^2\theta_+\cos2\psi_-]$$
$$+ C_s^{(2)\prime}[\sin^2\theta_+\cos2\psi_+ + \sin^2\theta_-\cos2\psi_-] + C_s^{(3)\prime}\sin\theta_+\sin\theta_-\cos(\psi_+ + \psi_-)\} \ , \quad (3.10)$$

$$d\sigma^{ca}(0) = d\Omega_+d\Omega_-\hat{I}C_{ca}^{(1)}(\cos\theta_+ - \cos\theta_-) \ , \quad (3.11)$$

$$d\Sigma^{ca} = d\Omega_+d\Omega_-\hat{I}C_{ca}^{(2)}[\sin^2\theta_+\cos\theta_-\cos2\psi_+ - \sin^2\theta_-\cos\theta_+\cos2\psi_- + \sin\theta_+\sin\theta_-(\cos\theta_- - \cos\theta_+)\cos(\psi_+ + \psi_-)] \ , \quad (3.12)$$

$$d\sigma^{pv}(0) = d\Omega_+d\Omega_-\hat{I}C_{pv}^{(1)}(\cos\theta_+ + \cos\theta_-)\sin\theta_+\sin\theta_-\sin(\psi_+ - \psi_-) \ , \quad (3.13)$$

$$d\Sigma^{pv} = d\Omega_+d\Omega_-\hat{I}C_{pv}^{(1)\prime}[\sin^2\theta_+\cos\theta_-\sin2\psi_+ - \sin^2\theta_-\cos\theta_+\sin2\psi_- + \sin\theta_+\sin\theta_-(\cos\theta_- - \cos\theta_+)\sin(\psi_+ + \psi_-)] \ , \quad (3.14)$$

where

$$C_s^{(1)} = \vec{p}_+^{\,2}\vec{p}_-^{\,2}\left[8\pi^2\alpha^2 + \frac{s}{s - M_Z^2}4\pi\alpha g_v g_V + \frac{s^2}{2(s - M_Z^2)^2}(g_v^2 + g_a^2)g_V^2\right]|F_2|^2 \ , \quad (3.15)$$

$$C_s^{(2)} = \frac{s}{4(s-M_Z^2)^2}(g_v^2+g_a^2)g_A^2(|F_3+F_4|^2\vec{p}_+^2+|F_3-F_4|^2\vec{p}_-^2) , \tag{3.16}$$

$$C_s^{(3)} = \frac{s}{(s-M_Z^2)^2}(g_v^2+g_a^2)g_A^2|\vec{p}_+|\,|\vec{p}_-|(|F_3|^2-|F_4|^2) , \tag{3.17}$$

$$C_s^{(i)\prime} = \hat{R}[C_s^{(i)}; g_a^2 \to -g_a^2], \quad i=1,2,3 \tag{3.18}$$

$$C_{ca}^{(1)} = \frac{g_a g_A\sqrt{s}}{s-M_Z^2}(1+\cos\theta_{+-})|\vec{p}_+||\vec{p}_-|\left(2\pi\alpha + \frac{s g_v g_V}{s-M_Z^2}\right)\mathrm{Re}\{F_1^*[F_3(|\vec{p}_+|+|\vec{p}_-|)+F_4(|\vec{p}_+|-|\vec{p}_-|)]\} . \tag{3.19}$$

$$C_{ca}^{(2)} = \hat{R}[C_{ca}^{(1)}; g_v g_V \to 0] , \tag{3.20}$$

$$C_{pv}^{(1)} = \frac{\sqrt{s}}{s-M_Z^2}|\vec{p}_+||\vec{p}_-|\left[2\pi\alpha g_v g_A + \frac{s(g_v^2+g_a^2)g_V g_A}{2(s-M_Z^2)}\right]\mathrm{Im}\{F_1^*[F_3(|\vec{p}_+|+|\vec{p}_-|)+F_4(|\vec{p}_+|-|\vec{p}_-|)]\} , \tag{3.21}$$

$$C_{pv}^{(1)\prime} = \hat{R}[C_{pv}^{(1)}; g_a^2 \to -g_a^2] . \tag{3.22}$$

In the Eqs. (3.15) and (3.18)–(3.22) we have assumed the universality-type relation

$$F_1 = F_2 . \tag{3.23}$$

If $F_1 \neq F_2$ then in $d\Sigma^{pv}$ there is an additional term proportional to $\mathrm{Im}F_1F_2^*$.

A. Asymmetries

We will now isolate $d\sigma^{ca}(0)$, $d\Sigma^{ca}$, $d\sigma^{pv}(0)$, and $d\Sigma^{pv}$ from each other and from $d\sigma^s(\vec{s})$. Since both $d\sigma^{ca}(\vec{s})$ and $d\sigma^{pv}(\vec{s})$ are antisymmetric under the transformation $\theta_\pm \to \pi-\theta_\pm$,

$$\int_{\theta_0}^{\pi-\theta_0} d\theta_+ \int_{\theta_0}^{\pi-\theta_0} d\theta_- \frac{d\sigma^{ca}(\vec{s})}{d\theta_+ d\theta_- d\psi_+ d\psi_-} = \int_{\theta_0}^{\pi-\theta_0} d\theta_+ \int_{\theta_0}^{\pi-\theta_0} d\theta_- \frac{d\sigma^{pv}(\vec{s})}{d\theta_+ d\theta_- d\psi_+ d\psi_-} = 0 , \tag{3.24}$$

where θ_0 is some given angular cutoff. Thus we conclude that there are no pure-azimuthal asymmetries.

On the other hand, since $d\sigma^{pv}(0)$ is the only part of $d\sigma(\vec{s})$ that is proportional to $\sin(\psi_+ - \psi_-)$, we can isolate the polarization-independent part of the parity-violating effects in an asymmetry parameter defined by

$$A_{p1} = \frac{(\sigma_{NE}-\sigma_{NW})-(\sigma_{SE}-\sigma_{SW})}{\sigma_{NE}+\sigma_{NW}+\sigma_{SE}+\sigma_{SW}} , \tag{3.25}$$

where

$$\sigma_{NE} = \int_{\theta_0}^{\pi/2} d\theta_+ \int_{\theta_0}^{\pi/2} d\theta_- \int_0^{2\pi} d\psi_- \times \int_{\psi_-}^{\psi_-+\pi} d\psi_+ \frac{d\sigma}{d\theta_+ d\theta_- d\psi_+ d\psi_-} , \tag{3.26}$$

$$\sigma_{SE} = \int_{\pi/2}^{\pi-\theta_0} d\theta_+ \int_{\pi/2}^{\pi-\theta_0} d\theta_- \int_0^{2\pi} d\psi_- \times \int_{\psi_-}^{\psi_-+\pi} d\psi_+ \frac{d\sigma}{d\theta_+ d\theta_- d\psi_+ d\psi_-} , \tag{3.27}$$

and where σ_{NW} and σ_{SW} are defined by an integration analogous to that in Eqs. (3.26) and (3.27), except that ψ_+ is integrated from $\psi_- + \pi$ to $\psi_- + 2\pi$. Using the transformation properties mentioned above and performing the azimuthal integration we conclude that

$$A_{p1} = \frac{\sigma_{NE}^{pv}(0)}{\sigma_{NE}^{s}(0)} . \tag{3.28}$$

That is, A_{p1} is given by the ratio of the parity-violating polarization-independent contribution to Eq. (3.26) to the polarization-independent part of the symmetric contribution. This parameter was already analyzed and estimated in Ref. 1.

Now, $d\sigma^{pv}(\vec{s})$ is the only part of $d\sigma$ that is antisymmetric under the transformation $\psi_\pm \to -\psi_\pm$. Furthermore, $d\sigma^{pv}(0)$ is antisymmetric under the interchange of ψ_+ and ψ_-. Thus, to isolate the information on the polarization-dependent part of the parity-violating effects we may define

$$A_{p2} = \frac{(\sigma_{NEE}-\sigma_{NWW})-(\sigma_{SEE}-\sigma_{SWW})}{\sigma_{NEE}+\sigma_{NWW}+\sigma_{SEE}+\sigma_{SWW}} , \tag{3.29}$$

where

$$\sigma_{NEE} = \int_{\theta_0}^{\pi/2} d\theta_+ \int_{\theta_0}^{\pi/2} d\theta_- \int_0^{\pi/2} d\psi_+ \times \int_0^{\pi/2} d\psi_- \frac{d\sigma}{d\theta_+ d\theta_- d\psi_+ d\psi_-} , \tag{3.30}$$

$$\sigma_{NWW} = \int_{\theta_0}^{\pi/2} d\theta_+ \int_{\theta_0}^{\pi/2} d\theta_- \int_{-\pi/2}^{0} d\psi_+ \times \int_{-\pi/2}^{0} d\psi_- \frac{d\sigma}{d\theta_+ d\theta_- d\psi_+ d\psi_-} , \tag{3.31}$$

and where σ_{SEE} and σ_{SWW} are defined by an integration analogous to that in Eqs. (3.30), (3.31), except that θ_+ and θ_- are both integrated from $\pi/2$ to $\pi-\theta_0$. It should be obvious that

$$A_{p2}=\tilde{s}^2\frac{\Sigma^{pv}_{NEE}}{\sigma^s_{NEE}(0)} , \tag{3.32}$$

where $\tilde{s}^2\Sigma^{pv}_{NEE}$ is the polarization-dependent parity-violating contribution to (3.30).

Since in the above analysis the π^0 and the energy of the charged pions are not observed, there are only three independent angular variables. If we choose ψ_- as dependent variable, then integrating it from 0 to 2π we obtain

$$d\sigma^s(0)=d\theta_+d\theta_-d\psi_+\hat{J}\{C_s^{(1)}[1+\cos(\theta_++\theta_-)\cos(\theta_+-\theta_-)-2\cos\theta_{+-}\cos\theta_+\cos\theta_-] + C_s^{(2)}(\sin^2\theta_++\sin^2\theta_-)+C_s^{(3)}(\cos\theta_{+-}-\cos\theta_+\cos\theta_-)\} , \tag{3.33}$$

$$d\Sigma^s=\cos2\psi_+d\theta_+d\theta_-d\psi_+\hat{J}\{C_s^{(1)\prime}[2\cos^2\theta_{+-}-\sin^2\theta_+\cos^2\theta_-+\cos^2\theta_+\sin^2\theta_- -2(\cos_{+-}-\cos\theta_+\cos\theta_-)^2/\sin^2\theta_+] + C_s^{(2)\prime}[\sin^2\theta_++2(\cos\theta_{+-}-\cos\theta_+\cos\theta_-)^2/\sin^2\theta_+-\sin^2\theta_-] + C_s^{(3)\prime}(\cos\theta_{+-}-\cos\theta_+\cos\theta_-)\} , \tag{3.34}$$

$$d\sigma^{ca}(0)=d\theta_+d\theta_-d\psi_+\hat{J}\,C_{ca}^{(1)}(\cos\theta_+-\cos\theta_-) , \tag{3.35}$$

$$d\Sigma^{ca}=\cos2\psi_+\hat{J}C_{ca}^{(2)}[(\cos\theta_--\cos\theta_+-2/\sin^2\theta_+)(\cos\theta_{+-}-\cos\theta_+\cos\theta_-)+\sin^2\theta_+\cos\theta_-+\sin^2\theta_-\cos\theta_+] , \tag{3.36}$$

$$d\sigma^{pv}(0)=0 , \tag{3.37}$$

$$d\Sigma^{pv}=\sin2\psi_+d\theta_+d\theta_-d\psi_+\hat{J}C_{pv}^{(1)\prime}[\cos\theta_-(1+\cos\theta_{+-})-\cos\theta_+(\cos2\theta_-+\cos\theta_{+-}) -2\cos\theta_+(\cos\theta_{+-}-\cos\theta_+\cos\theta_-)^2/\sin^2\theta_+] , \tag{3.38}$$

where $\hat{J}$ is an integral operator defined by

$$\hat{J}=\frac{\sin\theta_+\sin\theta_-}{8(2\pi)^5 s}\int_M^{\bar{E}}dE_+\int_M^{\bar{E}}dE_-\theta(1-\cos^2\theta_{+-})\theta[\cos\theta_{+-}-\cos(\theta_++\theta_-)] \times\theta[\cos(\theta_+-\theta_-)-\cos\theta_{+-}][\cos\theta_{+-}-\cos(\theta_++\theta_-)]^{-1/2}[\cos(\theta_+-\theta_-)-\cos\theta_{+-}]^{-1/2} . \tag{3.39}$$

Thus, to isolate the polarization-independent part of the charge-antisymmetric contribution to $d\sigma$ we may define the asymmetry parameters

$$A_{c1}=\frac{\sigma_{+-}-\sigma_{-+}}{\sigma_{+-}+\sigma_{-+}} , \quad A_{c2}=\frac{\sigma_+-\sigma_-}{\sigma_++\sigma_-} , \tag{3.40}$$

where

$$\sigma_{\pm\mp}=\int_{\theta_0}^{\pi/2}d\theta_\pm\int_{\pi/2}^{\pi-\theta_0}d\theta_\mp\int_0^{2\pi}d\psi_+\frac{d\sigma}{d\theta_+d\theta_-d\psi_+} , \tag{3.41}$$

$$\sigma_+=\int_{\theta_0}^{\pi/2}d\theta_+\int_{\theta_0}^{\pi-\theta_0}d\theta_-\int_{\theta}^{2\pi}d\psi_+\frac{d\sigma}{d\theta_+d\theta_-d\psi_+} , \tag{3.42}$$

$$\sigma_-=\int_{\pi/2}^{\pi-\theta_0}d\theta_+\int_{\theta_0}^{\pi-\theta_0}d\theta_-\int_0^{2\pi}d\psi_+\frac{d\sigma}{d\theta_+d\theta_-d\psi_+} . \tag{3.43}$$

It is evident that

$$A_{c1}=\frac{\sigma^{ca}_{+-}(0)}{\sigma^s_{+-}(0)} , \quad A_{c2}=\frac{\sigma^{ca}_+(0)}{\sigma^s_+(0)} , \tag{3.44}$$

in an obvious notation. These parameters, A_{c1} and A_{c2}, were already analyzed and estimated in Ref. 1.

Finally, to isolate Σ^{ca} it is expedient to study the parameter defined by

$$A_{c3}=\frac{\sigma_A-\sigma_B}{\sigma_A+\sigma_B} , \tag{3.45}$$

where

$$\sigma_A=\int_0^{2\pi}d\psi_+\theta(\cos2\psi_+) \times\int_{\theta_0}^{\pi-\theta_0}d\theta_+\int_{\theta_0}^{\pi-\theta_0}\epsilon(\cos\theta_-)\,d\theta_-\frac{d\sigma}{d\theta_+d\theta_-d\psi_+} , \tag{3.46}$$

and where σ_B is defined analogously, except that $\theta(\cos2\psi_+)$ should be replaced by $\theta(-\cos2\psi_+)$. In this way

$$A_{c3}=\tilde{s}^2\frac{\Sigma^{ca}_A}{\sigma^s_A(0)} . \tag{3.47}$$

The above equations suggest that we can define another parity-violating asymmetry parameter given by

$$A_{p3}=\vec{s}^{\,2}\frac{\sigma_C-\sigma_D}{\sigma_C+\sigma_D}\ , \tag{3.48}$$

where σ_C (σ_D) is defined by an integration, analogous to that in Eq. (3.46), except that $\theta(\cos 2\psi_+)$ $[\theta(-\cos 2\psi_+)]$ has to be replaced by $\theta(\sin 2\psi_+)$ $[\theta(-\sin 2\psi_+)]$. Thus

$$A_{p3}=\vec{s}^{\,2}\frac{\Sigma_c^{\rm pv}}{\sigma_c^s(0)}\ . \tag{3.49}$$

B. Estimate of the asymmetry parameters

In Ref. 1 it was found that A_{p1}, A_{c1}, and A_{c2} are of the order of magnitude of 3 to 4% at beam energies ≈ 20 GeV.

Since A_{p2} and A_{p3} are proportional to $C_{\rm pv}^{(1)\prime}$, they have a zero at

$$s=s_0=\frac{4\pi M_Z^2\alpha g_v}{(g_v^{\,2}-g_a^{\,2})g_V+4\pi\alpha g_v}\ . \tag{3.50}$$

Let E be the beam energy, $4E^2=s$, and let $4E_0^2=s_0$, then

$$\mathrm{sgn}(E_0-M_Z/2)=\mathrm{sgn}[(g_a^{\,2}-g_v^{\,2})g_Vg_v]\ . \tag{3.51}$$

In the following we will assume[6]

$$g_v=-(G/2\sqrt{2})^{1/2}M_Z(1-4\sin^2\theta_W)\ , \tag{3.52}$$

$$g_a=-(G/2\sqrt{2})^{1/2}M_Z\ , \tag{3.53}$$

$$g_V=-(8G/\sqrt{2})^{1/2}M_Z\sin^2\theta_W\ , \tag{3.54}$$

$$g_A=-(\sqrt{2}\,G)^{1/2}M_Z\ , \tag{3.55}$$

with $\sin^2\theta_W=0.35$ and $M_Z=75$ GeV. In this case, the zero of $C_{\rm pv}^{(1)\prime}$ occurs at $E\approx 28$ GeV. But for $E\approx 20$ GeV, $C_{\rm pv}^{(1)\prime}$ is of the order of $C_{\rm pv}^{(1)}/2$, so we expect A_{p2}, $A_{p3}\approx 0.5\,A_{p1}$ for such energies. On the other hand $A_{c3}/A_{c2}\approx C_{\rm ca}^{(2)}/C_{\rm ca}^{(1)}$ and for $E<30$ GeV we have $C_{\rm ca}^{(2)}\approx C_{\rm ca}^{(1)}$ and thus $A_{c3}\approx A_{c2}$.

IV. LONGITUDINAL POLARIZATION

Let $\lambda_\pm=(\mp q,0,0,E)h_\pm/m$ be the polarization four-vector of the positron and electron, respectively. E, q, and m denote energy, momentum, and mass of the electron. The mass term in the denominator of $\lambda_\pm$ forces to retain terms which were dropped in Eqs. (2.11)–(2.13).

This fact is more easily taken into account noticing that for $E\gg m$ we may write

$$\lambda_{-\mu}\gamma^\mu u\approx h_-u,\quad \lambda_{+\mu}\gamma^\mu v\approx h_+v\ , \tag{4.1}$$

where u and v are the electron and positron spinors, respectively. Thus, to obtain the matrix element in this case it is sufficient to insert in the appropriate place the projection operators $\Sigma(\lambda_\pm)=(1+h_\pm\gamma_5)/2$. In this way we obtain

$$c_{\mu\nu}=(1-h_+h_-)(q_\mu^-q_\nu^++q_\nu^-q_\mu^+-g_{\mu\nu}s/2)$$
$$+i(h_--h_+)\epsilon_{\mu\nu\alpha\beta}q_-^\alpha q_+^\beta\ , \tag{4.2}$$

$$c'_{\mu\nu}=c_{\mu\nu}\ , \tag{4.3}$$

$$d_{\mu\nu}=-(1-h_+h_-)\epsilon_{\mu\nu\alpha\beta}q_-^\alpha q_+^\beta$$
$$-i(h_--h_+)(q_\mu^-q_\nu^++q_\nu^-q_\mu^+-g_{\mu\nu}s/2)\ . \tag{4.4}$$

Thus $c_{00}=c_{0i}=c_{i0}=d_{00}=d_{0i}=d_{i0}=0$ and

$$c_{ij}=-(1-h_+h_-)(2q_iq_j+\tfrac{1}{2}sg_{ij})\ ,$$
$$-i(h_--h_+)\sqrt{s}\,\epsilon_{ijk}q^k\ , \tag{4.5}$$

$$d_{ij}=-(1-h_+h_-)\sqrt{s}\,\epsilon_{ijk}q^k$$
$$+i(h_--h_+)(2q_iq_j+\tfrac{1}{2}sg_{ij})\ . \tag{4.6}$$

Substituting Eqs. (4.4)–(4.6) into Eq. (2.21) and calling $\overline{M}$ the matrix element for the longitudinally polarized case, we immediately obtain

$$|\overline{M}|^2=(1-h_+h_-)\hat{R}[|M|^2;s_\pm\to 0]$$
$$+(h_--h_+)\hat{R}[|M|^2;s_\pm\to 0,c_{ij}\leftrightarrow id_{ij}]\ , \tag{4.7}$$

which leads to

$$d\sigma(h_+,h_-)=(1-h_+h_-)d\sigma(0,0)$$
$$+(h_--h_+)d\overline{\Sigma}\ , \tag{4.8}$$

where

$$d\sigma(0,0)=d\sigma(0)\ , \tag{4.9}$$

$$d\overline{\Sigma}=\hat{R}[d\sigma(0);\ \alpha^2\to 0, g_v^{\,2}+g_a^{\,2}\leftrightarrow 2g_ag_v\ ,$$
$$g_v\,e\leftrightarrow g_a e]\ . \tag{4.10}$$

$d\sigma(0)$ in Eqs. (4.9), (4.10) is given by

$$d\sigma(0)=d\sigma^s(0)+d\sigma^{\rm ca}(0)+d\sigma^{\rm pv}(0)\ , \tag{4.11}$$

where $d\sigma^s(0)$, $d\sigma^{\rm ca}(0)$, and $d\sigma^{\rm pv}(0)$ are given by Eqs. (3.9), (3.11), and (3.13), respectively. From (4.10) it follows that

$$d\overline{\Sigma}=d\overline{\Sigma}^{\rm pv}+d\overline{\Sigma}^{\rm ca}\ , \tag{4.12}$$

where

$$d\overline{\Sigma}^{\rm pv}=\hat{R}[d\sigma^s(0)+d\sigma^{\rm ca}(0);\ \alpha^2\to 0, g_a^{\,2}+g_v^{\,2}\leftrightarrow 2g_ag_v\ ,$$
$$g_a e\leftrightarrow g_v e]\ , \tag{4.13}$$

$$d\overline{\Sigma}^{\rm ca}=\hat{R}[d\sigma^{\rm pv}(0);\ g_ae\leftrightarrow g_ve,\ g_a^{\,2}+g_v^{\,2}\to 2g_ag_v]\ , \tag{4.14}$$

or, which is the same,

$$d\overline{\Sigma}^{\rm pv}=\hat{R}[d\sigma^s(0)+d\sigma^{\rm ca}(0);\ C_{\rm ca}^{(1)}\to\overline{C}_{\rm pv}^{(4)},\ C_s^{(i)}\to\overline{C}_{\rm pv}^{(i)}$$
$$\text{for } i=1,2,3]\ , \tag{4.15}$$

$$d\overline{\Sigma}^{\rm ca}=\hat{R}[d\sigma^{\rm pv}(0);\ C_{\rm pv}^{(1)}\to\overline{C}_{\rm ca}^{(1)}]\ , \tag{4.16}$$

where

$$\bar{C}_{\rm pv}^{(1)} = \vec{p}_+^{\,2}\vec{p}_-^{\,2}\left[\frac{s}{s-M_Z^2}\,4\pi\alpha g_a g_v + \frac{s^2}{(s-M_Z^2)^2}\,g_a g_v g_V^{\,2}\right]|F_1|^2\ , \tag{4.17}$$

$$\bar{C}_{\rm pv}^{(2)} = \frac{s g_a g_v g_A^{\,2}}{2(s-M_Z^2)^2}\left(|F_3+F_4|^2\vec{p}_+^{\,2} + |F_3-F_4|\vec{p}_-^{\,2}\right)\ , \tag{4.18}$$

$$\bar{C}_{\rm pv}^{(3)} = \frac{2 s g_a g_v g_A}{(s-M_Z^2)^2}\,|\vec{p}_+||\vec{p}_-|\left(|F_3|^2 - |F_4|^2\right)\ , \tag{4.19}$$

$$\bar{C}_{\rm pv}^{(4)} = \frac{g_A\sqrt{s}}{s-M_Z^2}(1+\cos\theta_{+-})|\vec{p}_+||\vec{p}_-|\left[2\pi\alpha g_v + \frac{s(g_v^{\,2}+g_a^{\,2})g_V}{2(s-M_Z^2)}\right]\mathrm{Re}\{F_1^*[F_3(|\vec{p}_+|+|\vec{p}_-|)+F_4(|\vec{p}_+|-|\vec{p}_-|)]\}\ , \tag{4.20}$$

$$\bar{C}_{\rm ca}^{(1)} = \frac{g_A\sqrt{s}|\vec{p}_+||\vec{p}_-|}{s-M_Z^2}\left(2\pi\alpha g_a + \frac{s g_a g_v g_V}{s-M_Z^2}\right)\mathrm{Im}\{F_1^*[F_3(|\vec{p}_+|+|\vec{p}_-|)+F_4(|\vec{p}_+|-|\vec{p}_-|)]\}\ . \tag{4.21}$$

From Eqs. (4.8), (4.12), and (4.13) it follows that

$$\sigma(h_-,h_+) = (1-h_+h_-)\sigma^s(0) + (h_- - h_+)\bar{\sigma}^{\rm pv}\ , \tag{4.22}$$

where

$$\bar{\sigma}^{\rm pv} = \hat{R}[\sigma^s(0);\ C_s^{(i)} \to \bar{C}_{\rm pv}^{(i)} \text{ for } i=1,2,3]\ . \tag{4.23}$$

Since $\bar{C}_{\rm pv}^{(3)} \approx -\frac{2}{3}C_s^{(3)}$ and $\bar{C}_{\rm pv}^{(2)} \approx -\frac{2}{3}C_s^{(2)}$ for any energy and $-\bar{C}_{\rm pv}^{(1)} \lesssim C_s^{(1)}$ for 18 GeV$\lesssim E$, we have $-\bar{\sigma}^{\rm pv} \lesssim \sigma^s(0)$ at least for $E \gtrsim 18$ GeV.

If we now define three asymmetry parameters $\bar{A}_{c1}$, $\bar{A}_{p1}$, and $\bar{A}_{p2}$ according to Eqs. (3.25) and (3.40), respectively, we will obtain

$$\bar{A}_{c1} = \frac{(h_- - h_+)\bar{\Sigma}_{\rm NE}^{\rm ca}}{(1-h_+h_-)\sigma_{\rm NE}^s(0) + (h_- - h_+)\bar{\sigma}_{\rm NE}^{\rm pv}}\ , \tag{4.24}$$

$$\bar{A}_{p1} = \frac{(h_- - h_+)\bar{\Sigma}_{+-}^{\rm pv}}{(1-h_+h_-)\sigma_{+-}^s(0) + (h_- - h_+)\bar{\sigma}_{+-}^{\rm pv}}\ , \tag{4.25}$$

$$\bar{A}_{p2} = \frac{(h_- - h_+)\bar{\Sigma}_{+}^{\rm pv}}{(1-h_+h_-)\sigma_{+}^s(0) + (h_- - h_+)\bar{\sigma}_{+}^{\rm pv}}\ , \tag{4.26}$$

in obvious notation. Since $\bar{\Sigma}_{\rm NE}^{\rm ca}/\sigma_{\rm NE}^{\rm pv}(0) \approx \bar{C}_{\rm ca}^{(1)}/C_{\rm pv}^{(1)} \approx -1$ for $E > 15$ GeV, then $|\bar{A}_{c1}| \approx |A_{p1}|$ for $E>15$ GeV. Similarly, from $\bar{\Sigma}_{+-}^{\rm pv}/\sigma_{+-}^{\rm ca}(0) \approx \bar{C}_{\rm pv}^{(4)}/C_{\rm ca}^{(1)} \approx \bar{\Sigma}_{+}^{\rm pv}/\sigma_{+}^{\rm ca}(0)$ and from $\bar{C}_{\rm pv}^{(4)}/C_{\rm ca}^{(1)} \approx 1$ for $E > 15$ GeV, we conclude that $|\bar{A}_{p1}| \approx |A_{c1}|$ and $|\bar{A}_{p2}| \approx |A_{c2}|$ for $E>15$ GeV.

Finally, permuting the helicities we may measure the parameter

$$A = \frac{\sigma(h,-h) - \sigma(-h,h)}{\sigma(h,-h) + \sigma(-h,h)} = \frac{h\bar{\sigma}^{\rm pv}}{(1+h^2)\sigma^s(0)}\ , \tag{4.27}$$

which is of order 1 for $E \gtrsim 18$ GeV.

V. CONCLUSIONS

We have assumed that the leptonic weak neutral current has only vector and axial-vector components and that the negative G parity piece of the hadronic weak neutral current has a vector isoscalar part and an axial-vector isovector one.

For transversely polarized beams the differential cross section contains polarization-independent terms and terms quadratic in the polarization

$$d\sigma(\vec{s}) = d\sigma(0) + \vec{s}^{\,2}d\Sigma\ ,$$

where $\vec{s}\,(-\vec{s})$ is the polarization vector of the electron (positron) and where $d\Sigma$ is polarization independent. In particular, if we integrate over the π^0 variables, over the energies of the charged pions, and over one azimuthal angle, ψ_- say, then

$$d\sigma^s(\vec{s}) = d\sigma^s(0) + \vec{s}^{\,2}\cos 2\psi_+\,d\Sigma^s\ ,$$

$$d\sigma^{\rm ca}(\vec{s}) = d\sigma^{\rm ca}(0) + \vec{s}^{\,2}\cos 2\psi_+\,d\Sigma^{\rm ca}\ ,$$

$$d\sigma^{\rm pv}(\vec{s}) = \vec{s}^{\,2}\sin 2\psi_+\,d\Sigma^{\rm pv}\ ,$$

where s, ca, and pv denote symmetric, charge-antisymmetric, and parity-violating parts, respectively.

In the case when the beams are longitudinally polarized we obtain a dependence on the polarizations h_- and h_+ of the electron and positron, respectively, of the form

$$d\sigma(h_-,h_+) = d\sigma(0,0)(1-h_+h_-) + (h_- - h_+)d\bar{\Sigma}\ ,$$

where $d\bar{\Sigma}$ is polarization independent.

By an appropriate choice of angular cutoffs we have defined the asymmetry parameters A_{c1}, A_{c2}, A_{c3}, A_{p1}, A_{p2}, A_{p3}, $\bar{A}_{c1}$, $\bar{A}_{p1}$, and $\bar{A}_{p2}$, where the subscript c (p) indicates that they are nonzero when charge asymmetries (parity-violating effects) are present. $\bar{A}_{c1}$, $\bar{A}_{p1}$, and $\bar{A}_{p2}$ are measurable only when the beams are longitudinally polarized. A_{c3}, A_{p2}, and A_{p3} are nonzero only when the beams are transversely polarized. In any case, it is always possible to measure the parameters A_{c1}, A_{c2}, and A_{p1} which are independent of the transverse polarization and proportional to $1-h_+h_-$.

We have also defined, in the case of longitudinally polarized beams, a helicity-asymmetry parameter A. We have shown that

$$A_{c1}, A_{c2}, \bar{A}_{c1} \propto \frac{2\pi\alpha g_a g_A s}{s-M_Z^2} + \frac{s^2 g_a g_v g_V g_A}{2(s-M_Z^2)^2}\ ,$$

$$A_{c3} \propto \frac{2\pi\alpha g_a g_A s}{s - M_Z^2},$$

$$A_{p1}, \bar{A}_{p1}, \bar{A}_{p2} \propto \frac{2\pi\alpha g_v g_A s}{s - M_Z^2} + \frac{s^2(g_v^2 + g_a^2) g_V g_A}{2(s - M_Z^2)^2},$$

$$A_{p2}, A_{p3} \propto \frac{2\pi\alpha g_v g_A s}{s - M_Z^2} + \frac{s^2(g_v^2 - g_a^2) g_V g_A}{2(s - M_Z^2)^2},$$

and that A is a sum of terms proportional to $\alpha g_a g_V$, $g_a g_v g_V^2$, and $g_a g_v g_A^2$. From these relations it is possible to obtain information on the four coupling constants of the weak neutral current. For example, (1) if $A \neq 0$ then $g_a \neq 0$, (2) if some angular-asymmetry parameter is nonzero then $g_A \neq 0$, (2a) if a charge-asymmetry parameter is nonzero then $g_A \neq 0$ and $g_a \neq 0$, (2b) if a parity-violation parameter is nonzero then $g_A \neq 0$ and g_v or $g_a \neq 0$.

If the values of g_a, g_v, g_A, and g_V are those given by Eqs. (3.52)–(3.55), then, according to Ref. 1, A_{p1}, A_{c1}, and A_{c2} are of the order of 3–4% at beam energies ≈20 GeV. We have argued that (1) A_{p2} and A_{p3} have a zero at a beam energy $E \approx 28$ GeV if $\sin^2\theta_W = 0.35$ and $M_Z = 75$ GeV, (2) A_{p2}, A_{p3} $\approx A_{p1}/2$ for $E \approx 20$ GeV, (3) $A_{c3} \approx A_{c2}$ for $E \lesssim 30$ GeV, (4) $\bar{A}_{c1} \approx A_{p1}$ for $E > 15$ GeV, (5) $\bar{A}_{p1} \approx A_{c1}$, $\bar{A}_{p2} \approx A_{c2}$ for $E > 15$ GeV. Finally, we have concluded that $A \approx 1$ for $E \gtrsim 18$ GeV.

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