Reply to Hagen's comment

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We reply to Hagen's comments on our discussion of physical amplitudes in the two-dimensional Yang-Mills theory.

In the preceding comment,[1] Hagen makes four complaints about the two-dimensional non-Abelian gauge theory first studied by 't Hooft[2] and elaborated on by ourselves.[3] In our opinion, the answers to his objections are found in our paper, but we welcome the opportunity to reiterate some of our basic remarks.

Let us first discuss Hagen's point (d). He correctly remarks that the two cutoffs (really gauge choices) for the gluon propagator lead to different results for the quark propagator and that the "principal value" cutoff leads to what looks like a finite quark mass. But this finite quark mass is just as spurious as the infinite quark mass of the singular cutoff since the quark does not appear in the space of *physical* states, namely those generated by gauge-invariant operators acting on the vacuum. No matter what cutoff (gauge choice) one uses, one gets the same result when computing gauge-invariant (i.e., physical) quantities. Given the wild difference between the quark propagator in the two gauges this is perhaps surprising, but general principle says it must be so and explicit calculation verifies that it is so. We repeat—even though quantum numbers and masses would permit it in the nonsingular gauge, gauge-invariant operators do not produce quark-antiquark states when acting on the vacuum.

In regard to Hagen's points (a) and (b) one must remember that one may choose gauge conditions which do not commute with space-time symmetries. It is then difficult to write down simple operators which realize these symmetries on the *physical* state space and one will not in general be able to realize them on the unphysical space of non-gauge-invariant states. In our case we have no trouble with Lorentz invariance, and our amplitudes are explicitly covariant. Parity is nontrivial, however, which is why we went to the trouble of showing explicitly that the *physical* meson states *do* have a conserved parity eigenvalue. Hagen's remarks do not seem to bear on the relevant physical questions of symmetry properties of the gauge-invariant sector of the theory.

Hagen's point (c) seems to arise from a misreading of what we actually say in our paper. When we refer to an anomaly, we are referring to the anomaly of the external *flavor* current. This has nothing to do with the dynamics of the gluon field and is dealt with in the conventional way.

We would not claim that the model we have studied is without problems. In particular, the limit $m \to 0$ is problematic, and it may be true that the large-N expansion becomes nonsensical for sufficiently small m. This is a difficult dynamical question which we do not fully understand. When the large-N expansion makes sense, however, the issues raised by Hagen are all clarified by a correct understanding of the role of gauge invariance, and we continue to hold that the model is an interesting and revealing paradigm for confinement.

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[1] C. R. Hagen, preceding paper, Phys. Rev. D 16, 3612 (1977).
[2] G. 't Hooft, Nucl. Phys. B75, 461 (1974).
[3] C. G. Callan, N. Coote, and D. J. Gross, Phys. Rev. D 13, 1649 (1976).