PHYSICAL REVIEW D VOLUME 16, NUMBER 12 15 DECEMBER 1977

Normal-product methods and the functional formalism, with applications to gauge theories

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(Received 6 June 1977)

We discuss the use of normal-product methods in dimensional regularization to effect the renormalization of quantum field theories expressed in the functional formalism. In particular, we discuss the renormalization of symmetry operations in this formalism, and the consequent renormalized Ward identities, and apply these to a discussion of the gauge invariance of general gauge theories, and the definition of renormalized gauge-invariant operator insertions.

INTRODUCTION

Normal-product methods in quantum field theories have, since their definition by Zimmermann,[1,2] been recognized as providing a very convenient and rigorous calculus for deriving finite relations between Green's functions without having to consider explicitly the divergences of the naive theory. In particular, Ward identities can be derived[3] without using canonical commutation relations with the consequent appearance and cancellation of "Schwinger" and "seagull" terms. The only other way of avoiding these complications of the canonical theory is to use functional methods to define the Green's functions of the theory and to derive Ward identities between them. In particular, the use of functional methods appears to be essential in the discussion of the quantization and renormalization of non-Abelian gauge theories.[4,5] For these reasons it would be useful to have at least a partial marriage between normal-product methods and the functional formalism. The actual renormalization would then be taken care of as usual by the definition of the normal products within the Lagrangian, and we would be able to discuss the invariances, and hence Ward identities, of the renormalized theory by the usual powerful functional techniques. It is the aim of this paper to show how at least some parts of this project can be carried through.

To start on the program it is essential to have a normal-product formalism, that is, a renormalization prescription, which satisfies two basic criteria. Firstly, with an eye to gauge theories, it should be compatible with the symmetries of the theory. Secondly, there should be a convenient Wilson expansion which can be inverted to express the normal products in terms of the naive field products of the theory in a useful way. For both these reasons we will use exclusively the dimensionally regularized normal products as defined by Collins.[6] The dimensional renormalization preserves the symmetries of the theory which are not explicitly dependent on the dimension of space-time[7] (modulo some difficulties with γ_5-type chiral symmetries), and, moreover, as we have shown previously,[8] it allows a Wilson expansion. In particular, we can express the Wilson coefficients in terms of dimensional singularities, that is, singularities in the dimension of space-time, and thus relate directly the Wilson expansion of operators with the counterterms in the renormalized Lagrangian.[9] Of course, everything relies very heavily on the fact that the dimensional renormalization scheme is a consistent local renormalization, and that a renormalized action principle holds in the renormalized theory. The former has been shown directly by Breitenlohner and Maison,[10] who also proved the latter statement which was also demonstrated by Collins.[6]

In Sec. I we will discuss normal products in a functional formalism beginning with a review of the definitions of the various types of normal products, and proceeding to use these definitions to discuss the properties of the normal products, and their use in the derivation of renormalized symmetry principles within a functional scheme. In Sec. II we use these methods to discuss the properties of gauge theories, and provide a normalized analog of the Becchi-Rouet-Stora (BRS) transformations.[11] With these we proceed to discuss the gauge invariance of the renormalized S matrix, and then the definition and properties of renormalized gauge-invariant operators within the theory. The main result of this is that the two criteria of gauge invariance proposed by Lowenstein and Schroer[12] within a Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) normal-product scheme are seen to be redundant, in the sense that it is shown quite simply that any naive field product which is invariant under the generic BRS transformation gives rise to a renormalized insertion product which is invariant under the renormalized BRS transformation, and whose Green's functions obey the same Ward identities as the uninserted Green's functions, and whose S-matrix elements are invariant under a change of gauge.

I. NORMAL PRODUCTS AND FUNCTIONAL METHODS

A. Introduction

Following closely the original definition of Green's functions of normal products by Zimmermann,[1,2] which employed the BPH subtraction scheme and the definition of the subtractions via the forest formula,[1] Collins[6] defined the Green's functions of normal products in dimensional renormalization. The BPH subtraction scheme is replaced by the 't Hooft and Veltman[7] dimensional subtraction scheme suitably expressed in terms of a forest formula. That the dimensional scheme produced a consistent local renormalization was proved indirectly by Speer,[13] and directly by Breitenlohner and Maison[10] in the course of showing that a renormalized action principle held in this scheme. We have shown in a previous paper[8] how Zimmermann's method can be extended to provide Wilson expansions in the dimensional method. These are expressible in two terms, with the singularities of the unrenormalized products expressed either in terms of a space-time cutoff as usual, or in terms of a dimensional cutoff, that is, as poles in the dimension of space-time. This latter representation is exactly that used for the counterterms which define the renormalized Lagrangian relative to the generic one, and it was this property which was exploited to express the various normal products in a "normalized" Lagrangian in terms of the naive field products.[9] It was found essential to distinguish between various types of normal products, specifically those appearing in the Lagrangian, of which we demand that they give finite Green's functions for the theory upon arbitrary insertion in the free theory via the Gell-Mann–Low formula and renormalized insertion products, in particular, those appearing in the equations of motion, of which we require only that they should have finite Green's functions upon single insertion in the already renormalized theory.

Once we have reorganized the renormalized Lagrangian into a normalized Lagrangian, where all the divergences are hidden in the definition of the normal products, and established the relationship between these products and the corresponding insertion products in the finite equations of motion, etc., we are free to use this normalized Lagrangian in our functional integral which generates Green's functions, and so long as we suitably restrict the class of differential operations on this generating functional we generate relations among finite Green's functions expressed in terms of normal and insertion products.

In the following we shall adopt the convention that the term "normal products" embraces both the Lagrangian normal products, N-products, and the insertion products, I-products, and also, if required, the intermediately renormalized products, R-products.[9]

B. A review of normal-product definitions and properties

Given a classical, or generic, Lagrangian,

$$\mathfrak{L}_G=\sum_{j\in J}\lambda_j\Phi_j\,, \tag{1}$$

with λ_j coupling constants and Φ_j monomials of fields, quantum field theory dictates that we must add formerly divergent counterterms to $\mathfrak{L}_G$ to form

$$\begin{aligned}\mathfrak{L}_R&=\mathfrak{L}_G+\mathfrak{L}_{\text{ct}}\\&=\sum_{j\in J}\lambda_j\Phi_j+\sum_{\substack{j\in J\\ k\in K}}\lambda_j\hat{L}_{jk}\Phi_k\,,\end{aligned} \tag{2}$$

with $\hat{L}$ some matrix with divergent entries, and K some set labeling field monomials which, without loss of generality, can be taken to include J, and is possibly strictly larger, or even of countably infinite size. The problem of renormalization theory is to choose J so that $K=J$ so that we can then define bare quantities

$$\lambda_{0j}=Z_{jk}\lambda_k,\quad \Phi_{0j}=\hat{Z}_{jk}\Phi_k \tag{3}$$

so as to reduce the renormalized Lagrangian to the bare Lagrangian:

$$\mathfrak{L}_R=\mathfrak{L}_B\equiv\sum_{j\in J}\lambda_{0j}\Phi_{0j}\,. \tag{4}$$

Throughout we are using the definitions and terminology of Ref. 9, in which the attitude adopted is explained in greater detail.

The problem of "normalization theory" is to provide a useful definition of a normal product such that the renormalized Lagrangian can be written as a normalized Lagrangian:

$$\mathfrak{L}_R=\mathfrak{L}_N\equiv\sum_{j\in J}\lambda_jN[\Phi_j]\,. \tag{5}$$

The way these steps are achieved is via perturbation theory and the Gell-Mann–Low formula; the N-products are chosen such that

$$\begin{aligned}\langle T\varphi_1(x_1)\cdots\varphi_r(x_r)\rangle&=\left\langle T\varphi_1^0(x_1)\cdots\varphi_r^0(x_r)\exp\left[i\int\mathfrak{L}^0_{N\,\text{int}}(z)dz\right]\right\rangle\\&=\sum_{n=0}^{\infty}\left\langle T\varphi_1^0(x_1)\cdots\varphi_r^0(x_r)\prod_i\left\{\int N[\Phi_i](z_i)dz_i\right\}^{n_i}\frac{1}{n_i!}\right\rangle\end{aligned} \tag{6}$$

is finite. In contrast to other formalisms, no claim to finiteness is made for the individual terms of the series, only that the sum of all terms up to order $\underline{n}$ in the N-products will be finite to order $\underline{n}$ in the coupling constants of the theory.[9]

The renormalized insertion products, I-products, are a generalization of the N-products and are defined so that $\{I_{\underline{n}}^{\underline{\lambda};\underline{\mu}}[\Psi_i]\}$ is a set of operators which give Green's functions finite to order $\underline{n}$ in $\underline{\mu}$, the coupling-constant vector associated with the $\underline{\Psi}$, when all possible simultaneous insertions up to the level of n_i insertions of $I[\Psi_i]$ are considered in a theory defined by $\underline{\lambda}$.

By far the most important I-products are the N-products,

$$N^{\underline{\lambda}}[\Phi_i] \equiv I_{\underline{\infty}}^{\underline{\lambda}_0;\underline{\lambda}-\underline{\lambda}_0}[\Phi_i], \quad \forall\, \Phi_i \in \mathfrak{L}_{G\,\mathrm{int}},$$

where $\underline{\lambda}_0$ defines $\mathfrak{L}_{G0}$, and the single insertion products,

$$I_{\underline{e}_\alpha}^{\underline{\lambda};\underline{\lambda}}[\Psi_\alpha],$$

which appear in the equations of motion with each Ψ_α a $\partial\Phi_i/\partial\varphi_j$ or a $\partial_\mu\partial\Phi_i/\partial(\partial_\mu\varphi_j)$.

We denote by $I^{\underline{\lambda}}[\Psi_1;\cdots;\Psi_r](x_1;\ldots;x_r)$ what is crudely

$$\frac{1}{i}\frac{\partial}{\partial\mu_1}\bigg|_0 \cdots \frac{1}{i}\frac{\partial}{\partial\mu_r}\bigg|_0 \exp(i\mathfrak{L}_R^{\underline{\lambda};\underline{\mu}}),$$

where $\mathfrak{L}_G^{\underline{\lambda};\underline{\mu}} \equiv \mathfrak{L}_G^{\underline{\lambda}} + \sum_i \mu_i \Psi_i$, or more correctly as

$$\frac{1}{i}\frac{\delta}{\delta\mu_1(x_1)}\bigg|_0 \cdots \frac{1}{i}\frac{\delta}{\delta\mu_r(x_r)}\bigg|_0 \exp(iS_R^{\underline{\lambda};\underline{\mu}}),$$

where

$$S_G^{\underline{\lambda};\underline{\mu}} \equiv \int \mathfrak{L}_G^{\underline{\lambda};\underline{\mu}}(z)\,dz$$
$$\equiv S_G^{\underline{\lambda}} + \int \sum_i \mu_i(z)\Psi_i(z)\,dz.$$

It should be noted that an insertion of

$$\int dx_1 \cdots dx_r\, I[\Psi_1;\ldots;\Psi_r](x_1;\ldots;x_r)$$

(often written $\prod_i \int N[\Psi_i](x_j)dx_i$ by many authors) will trivially have finite Green's functions. What is nontrivial is its relationship to $\prod_i \int I[\Psi_i](x_i)dx_i$, in our notation.[9] As above, all arguments of N- and I-products will be dropped when there is no risk of ambiguity.

In terms of these definitions we can now give the properties of the normal products. Given monomials M_i of the interacting fields of a theory defined by a generic Lagrangian $\mathfrak{L}_G$ then for deductive reasons we will define $I^{\underline{\lambda}}[M_1;\ldots;M_r]$ by its Green's functions in the form

$$\langle TI^{\underline{\lambda}}[M_1;\ldots;M_r](x_1;\ldots;x_r)\varphi_1(y_1)\cdots\varphi_n(y_n)\rangle$$
$$\equiv \mathrm{FP}\,\langle TM_1(x_1)\cdots M_r(x_r)\varphi_1(y_1)\cdots\varphi_n(y_n)\rangle. \quad (7)$$

"FP" denotes a finite-part operation which can be applied to the right-hand side via an extended Gell-Mann-Low expansion:

$$\langle TM_1(x_1)\cdots\varphi_n(y_n)\rangle$$
$$= \left\langle TM_1^0(x_1)\cdots\varphi_n^0(y_n)\exp\left[i\int \mathfrak{L}_{R\,\mathrm{int}}^0(z)\,dz\right]\right\rangle. \quad (8)$$

It will be seen that this definition of I corresponds to the form given above. The dimensionally regularized FP is essentially as given by Collins, but see Ref. 8 for an interpretive note.

In a sense these definitions are all we need to discuss the properties of operator insertions, etc., but in order to apply functional techniques we must know how the Lagrangian is cast into "normal" form. To do this we make use of the Wilson expansion and the equations of motion as in Ref. 9. Essentially from a knowledge of $\langle TI^{\underline{\lambda}}[M_1]\varphi_1\cdots\varphi_n\rangle$ and $\langle TM_1\varphi_1\cdots\varphi_n\rangle$ we obtain a Wilson expansion following Zimmermann's methods[1,2] as used in Ref. 8. Applying these to the products appearing in the equations of motion we demand that

$$\frac{\partial}{\partial\varphi}N[\Phi] = I\left[\frac{\partial\Phi}{\partial\varphi}\right],$$
$$\frac{\partial}{\partial(\partial_\mu\varphi)}N[\Phi] = I\left[\frac{\partial\Phi}{\partial(\partial_\mu\varphi)}\right], \quad (9)$$

so that the naive functional proof[4] of the equations of motion goes through as if the N- and I-symbols were absent.[9] The properties of the insertion product system as proved by Breitenlohner and Maison[10] and Collins[6] can be written as follows:

A. Linearity:

$$\langle TI[\alpha_1 M_1 + \alpha_1' M_1'; M_2;\ldots;M_r]\varphi_1\cdots\varphi_n\rangle$$
$$= \alpha_1\langle TI[M_1;\cdots;M_r]\varphi_1\cdots\varphi_n\rangle$$
$$+ \alpha_1'\langle TI[M_1';\ldots;M_r]\varphi_1\cdots\varphi_n\rangle, \quad (10)$$

where the α_i's are independent of ν, the dimension of space-time, and are Lorentz scalars. This, together with the trivial total symmetry of the i labels, gives complete linearity.

B. Derivative property:

$$\frac{\partial}{\partial x_i^\mu}\langle TI[M_1;\ldots;M_i;\ldots;M_r](x_1;\ldots;x_i;\ldots;x_r)\varphi_1\cdots\varphi_n\rangle$$
$$= \langle TI[M_1;\ldots;\partial_\mu M_i;\ldots;M_r]\,(x_1;\ldots;x_i;\ldots;x_r)\varphi_1\cdots\varphi_n\rangle. \quad (11)$$

C. The quantum action principle: Let us define the generating function of connected Green's functions with operator insertions by

$$W[\underline{M};\underline{a};\underline{\lambda}]=\left\langle T\exp\left[i\int \mathcal{L}'_{R\,\mathrm{int}}(\varphi(x);\underline{a}(x);\underline{\lambda})dx\right]\right\rangle_c . \tag{12}$$

$\underline{a}$ are unquantized external fields including sources for the φ fields, say $\underline{j}$, and for the monomials $\underline{M}$ of these fields whose insertion products we wish to discuss, say $\underline{\mu}$. Let us split the generic Lagrangian into

$$\mathcal{L}'_G\equiv\mathcal{L}^{\lambda}_G+\mathcal{L}^{\mu}_G+\mathcal{L}^{j}+\mathcal{L}^{a'}_G , \tag{13}$$

where

$\mathcal{L}^{\lambda}_G$ defines the theory we wish to discuss,

$\mathcal{L}^{\mu}_G\equiv\sum_i \mu_i M_i$, the monomial sources,

$\mathcal{L}^{j}\equiv\sum_i j_i\varphi_i$, the field sources,

$\mathcal{L}^{a'}_G$ contains the rest, say other composite operator sources, or pieces wholely dependent on the unquantized fields.

Now we can discuss three types of variation of W.

(*i*) *Variation of the external field*:

$$\frac{\delta W}{\delta a_i(x)}=\left\langle TI'\left[\frac{\delta\mathcal{L}'_G}{\delta a_i}\right](x)\exp\left[i\int\mathcal{L}'_{R\,\mathrm{int}}(z)dz\right]\right\rangle_c . \tag{14}$$

$\delta/\delta f(x)$ will denote a functional derivative, whereas $\delta/\delta f$ will denote the Euler derivative, $\partial/\partial f - \partial_\mu\partial/\partial(\partial_\mu f)+\cdots$.

There should be no possibility of confusion.

This property (i) is what makes the generating functional useful. It should be noted that we use I' to signify that the I-product is with respect to the whole of $\mathcal{L}'$. We have in particular that

$$\frac{1}{i}\frac{\delta W}{\delta\mu_i(x_i)}=\left\langle TI'[M_i](x_i)\exp\left[i\int\mathcal{L}_{R\,\mathrm{int}}(z)dz\right]\right\rangle_c , \tag{15}$$

so that taking another functional derivative, say

$$\frac{1}{i}\left.\frac{\partial}{\partial\mu_j(x_j)}\right|_{\mu=0} ,$$

we pick up two terms, one from I', the other from $\int\mathcal{L}'_R dz$, thus forming $I[M_i;M_j](x_i;x_j)$, and so on. Of course, $I'[M_0;M_1;\ldots;M_r]=M_0 I'[M_1;\ldots;M_r]$ for M_0 a linear function of the fields.

(ii) *Variations of the quantum field*: Suppose $\varphi\to\varphi'=\varphi+\epsilon Q$, then we will have

$$0=\frac{1}{i}\left.\frac{\delta W}{\delta\epsilon(x)}\right|_{\epsilon=0}$$
$$=\left\langle TI'\left[Q\frac{\delta\mathcal{L}'_G}{\delta\varphi}\right](x)\exp\left[i\int\mathcal{L}'_{R\,\mathrm{int}}(z)dz\right]\right\rangle_c . \tag{16}$$

These are the generalized equations of motion, and can be cast into the more usual form[6,14] by splitting $\mathcal{L}'_G$ as above.

(*iii*) *Variations of a parameter*:

$$\frac{1}{i}\frac{\partial W}{\partial\lambda_i}=\left\langle TI'\left[\frac{\partial S'_G}{\partial\lambda_i}\right]\exp\left[i\int\mathcal{L}'_{R\,\mathrm{int}}(z)dz\right]\right\rangle_c , \tag{17}$$

where $S'_G=\int\mathcal{L}'_G(z)dz$. This is merely a degenerate, but useful, case of (i) when $\lambda_i\in\mathcal{L}'_{G\mathrm{int}}$, but also holds for $\lambda_i\in\mathcal{L}'_{G0}$.

When we pass from the I-products to the N-products we would like to maintain relationships A and B. A is clear from the definition of N-products, and B is almost as easy:

$$\frac{\delta}{\delta\varphi}\left(N[\partial_\mu\Phi]-\partial_\mu N[\Phi]\right)$$
$$=\frac{\delta}{\delta\varphi}N[\partial_\mu\Phi]-\partial_\mu\frac{\delta}{\delta\varphi}N[\Phi]-\left[\frac{\delta}{\delta\varphi},\partial_\mu\right]N[\Phi]$$
$$=I\left[\frac{\delta}{\delta\varphi}(\partial_\mu\Phi)\right]-\partial_\mu I\left[\frac{\delta\Phi}{\delta\varphi}\right]-\left[\frac{\delta}{\delta\varphi},\partial_\mu\right]N[\Phi]$$
$$=I\left[\partial_\mu\frac{\delta\Phi}{\delta\varphi}\right]+I\left[\left[\frac{\delta}{\delta\varphi},\partial_\mu\right]\Phi\right]-\partial_\mu I\left[\frac{\delta\Phi}{\delta\varphi}\right]$$
$$-\left[\frac{\delta}{\delta\varphi},\partial_\mu\right]N[\Phi] , \tag{18}$$

but $[\delta/\delta\varphi,\partial_\mu]=\partial_\mu\partial/\partial\varphi$ on Lagrangian products Φ since they contain no second-order derivatives; so we get

$$=I\left[\partial_\mu\frac{\delta\Phi}{\delta\varphi}\right]+I\left[\partial_\mu\frac{\partial\Phi}{\partial\varphi}\right]-\partial_\mu I\left[\frac{\delta\Phi}{\delta\varphi}\right]-\partial_\mu I\left[\frac{\partial\Phi}{\partial\varphi}\right].$$

So we see that $I[\partial_\mu\Psi]=\partial_\mu I[\Psi]\ \forall\,\Psi$ is a sufficient condition for $N[\partial_\mu\Psi]=\partial_\mu N[\Psi]$.

C. Functional methods

We are now ready to consider the functional formalism per se. The essential step is just to express $W[\underline{M};\underline{a};\underline{\lambda}]$ as a functional integral with respect to the quantum fields in the usual way: $W=-i\ln Z$, where

$$Z[\underline{j}]=\int d[\varphi]\exp\{iS[\varphi]+ij\varphi\} . \tag{19}$$

Z is the generating functional of full Green's functions, and we have suppressed all dependences on external fields except for the field sources $\underline{j}$. S is the action, the space-time integral of the relevant Lagrangian density $\mathcal{L}$, and we have used a notation in which $j\varphi$ means $j_\alpha\varphi_\alpha$ with the indices suppressed, and we include integration in our summation convention. This compact notation of Lee[4] and Zinn-Justin[5] will be extensively used in Sec. II. The generic, renormalized, and normalized generating functionals Z_G, Z_R, Z_N, are defined using the corresponding S's.

The most important part of the action principle, namely (i) the equations of motion, are derived in the generic theory by considering shifts in the fields φ, giving just a change of integration variables in the functional integral. Thus under $\varphi \to \varphi' = \varphi + \epsilon Q$,

$$Z_G[j] \to Z_G[j] + \int \epsilon(x) \int d[\varphi]\, dx \left[i\,\frac{\delta S_G}{\delta\varphi} Q + ijQ + \delta(0)\,\frac{\delta Q}{\delta\varphi}\right](x) \times e^{iS_G[\varphi]+ij\varphi} + O(\epsilon^2)\,, \tag{20}$$

where the $\delta(0)\delta Q/\delta\varphi$ term comes from the Jacobian of the transformation. So we get to the generic equations of motion:

$$0 = \int d[\varphi]\left[\frac{\delta S_G}{\delta\varphi}\,Q + j\,Q + \frac{1}{i}\,\delta(0)\,\frac{\delta Q}{\delta\varphi}\right] e^{iS_G[\varphi]+ij\varphi}\,. \tag{21}$$

In diagrammatic terms the $\delta(0)$ term corresponds to the contraction of a single-line loop to a point if the $\delta S_{G0}/\delta\varphi$ operator acts on a line terminating in Q. We have not made a Wick ordering of our original Lagrangian, which would remove these terms. If the transformation $\varphi \to \varphi'$ is measure-preserving this term disappears since then $\delta Q/\delta\varphi = 0$.

We can now ask how to derive the equivalent result in the renormalized or normalized theories. We know from the renormalized action principle[6,10] that the equations of motion will be

$$0 = \int d[\varphi]\left\{ I\left[\frac{\delta S_G}{\delta\varphi}\,Q\right] + jI[Q]\right\} e^{iS_N[\varphi]+ij\varphi}. \tag{22}$$

However, we would like to be able to derive these from a trivial change of variables to keep us wholely within the functional scheme. To do this consider a change of variables: $\varphi \to \varphi^* = \varphi + \epsilon I[Q]$. The result of this will be

$$\int d[\varphi]\left\{\frac{\delta S_N}{\delta\varphi}\,I[Q] + jI[Q] - i\delta(0)\,\frac{\delta I[Q]}{\delta\varphi}\right\} e^{iS_N[\varphi]+ij\varphi} = 0 \tag{23}$$

or

$$\int d[\varphi]\left\{ I\left[\frac{\delta S_G}{\delta\varphi}\right] I[Q] + jI[Q] - i\delta(0)\,\frac{\delta I[Q]}{\delta\varphi}\right\} e^{iS_N[\varphi]+ij\varphi} = 0 \tag{24}$$

since we know[9] that $\delta S_N/\delta\varphi(x) = I[\delta S_G/\delta\varphi]$. This is an interesting result which can be viewed several ways. It first of all tells us that $I[\delta S_G/\delta\varphi]I[Q] - i\delta(0)\delta I[Q]/\delta\varphi$ is a finite operator, since $jI[Q]$ is by definition. This is far from *apriori* obvious. If we think of the subtractions needed to render finite a Green's function containing $(\delta S_G/\delta\varphi)Q$ as an insertion, then we would expect to include subtractions for all kinds of subdiagrams involving lines in both Q and $\delta S_G/\delta\varphi$. What the above tells us is that we only need subtractions involving a single contracted loop. In particular, we will have formed a finite operator by applying a set of minimal subtractions to $(\delta S_G/\delta\varphi)Q$ which must therefore define $I[(\delta S_G/\delta\varphi)Q]$.[15] Hence we have arguments, independent of the detailed proof of the action principle, that the equations of motion are (22) as above. Now this argument[15] relies heavily on the minimal nature of the subtractions in dimensional regularization (DR) and, in fact, we can go one stage further. In DR we should perform our subtractions in momentum space before coming back to coordinate space, whereas above by parametrizing the functional integral in coordinate space we have effectively been working in coordinate space. So suppose we make a change of variables in our functional integral from $\{\varphi(x)\}$ to $\{\tilde\varphi(p)\}$ where

$$\tilde\varphi(p) = \frac{1}{(2\pi)^{\nu/2}} \int d^\nu x\, e^{ip\cdot x}\varphi(x)\,,$$

the ν-dimensional Fourier transform. Since this is a linear transformation we preserve the functional measure to within a constant, and we can consider the transformation equivalent to $\varphi \to \varphi' = \varphi + \epsilon Q$. Without loss of generality, consider $Q = \varphi^n$. Then

$$\tilde\varphi \to \tilde\varphi' = \tilde\varphi + (\epsilon\varphi^n)^{\sim} = \tilde\varphi + (\epsilon\varphi^{n-1})^{\sim} * \tilde\varphi\,.$$

Therefore

$$\frac{\delta\tilde\varphi'(p)}{\delta\tilde\varphi(r)} = \delta(p-r) + n(\epsilon\varphi^{n-1})^{\sim}(p-r)\,,$$

so we get a Jacobian to order ϵ of $1 + n\,\mathrm{tr}\,(\epsilon\varphi^{n-1})^{\sim} = 1 + n(\epsilon\varphi^{n-1})^{\sim}(0)\int dp$, which is the Fourier-transformed analog of $1 + n\int \epsilon\varphi^{n-1}(x)dx\,\delta(0)$. Now, however, we have to interpret $\int dp$ in a ν-dimensional continuation [in terms of Feynman diagrams this is just a term $\int (p^2 - m^2)dp/(p^2 - m^2 + i\epsilon)$ coming from the contraction of the single-line loop] which, as usual, is zero.[7,16] Hence we get the elimination of the measure-induced term in an analogous fashion to the elimination of the Feynman diagrams with a single-line loop in the proof of Collins[6] with respect to equations of motion. Hence, miraculously, we no longer have to worry about the measure-preserving properties of our transformations $\varphi \to \varphi'$ or $\varphi \to \varphi^*$, and in both the generic and renormalized theories can proceed in a completely naive way.

Whenever the measure-induced term vanishes, we have the relationship

$$I\left[\frac{\delta S_G}{\delta\varphi(x)}Q(x)\right]=I\left[\frac{\delta S_G}{\delta\varphi(x)}\right]I[Q(x)] \tag{25}$$

for each x, which, we now see in the dimensional scheme, holds for all Q and can, in effect, be taken to be the content of the equations of motion, and is a nontrivial statement. The elimination of the measure term quite trivially will have pleasant simplifying effects when we consider renormalized transformations, for then it is by no means obvious that the measure is preserved in the trivial sense, namely $\delta I[Q]/\delta\varphi=0$, even if $\delta Q/\delta\varphi=0$. Since we are now free from all restrictions as to the functions Q we can use, we can choose them to be the infinitesimal generators of symmetry (or broken symmetry) transformations of the generic theory. It is clear that if $\{Q\}$ generate a symmetry of the generic theory, then $\{I[Q]\}$ generate a corresponding symmetry for the renormalized or normalized theory since the infinitesimal change in S_N is

$$\frac{\delta S_N}{\delta\varphi}I[Q]=I\left[\frac{\delta S_G}{\delta\varphi}\right]I[Q]=I\left[\frac{\delta S_G}{\delta\varphi}\right]=0$$

if and only if $(\delta S_G/\delta\varphi)Q=0$, i.e., $\{Q\}$ generates a symmetry of the generic theory.

In the general case we easily derive the Ward identity

$$\int d[Q]\{I[\Delta]-\partial_\mu I[J^\mu]+jI[Q]\}e^{iS_N[\varphi]+ij\varphi}=0\,, \tag{26}$$

where Δ is the change of $\mathcal{L}_G$ under $\varphi\to\varphi'=\varphi+\epsilon Q$, and J^μ is the associated Noether current.

II. APPLICATIONS TO GAUGE THEORIES

A. Introduction and notation

We shall now apply the formalism and results of the first section to gauge theories. The object is to show how the analysis of gauge theories in their renormalized form can be carried out in the normal-product formalism. In particular, we discuss how the gauge invariance of the renormalized S matrix [assumed to exist, if necessary by spontaneously breaking the gauge symmetry down to U(1)] is demonstrated, and find considerable simplifications while discussing the definition and properties of gauge-invariant renormalized operator insertions. We shall not be discussing the renormalizability of such theories, which we assume has been done, presumably using functional techniques and dimensional renormalization along the lines of the work of Zinn-Justin,[4] Lee,[5] and Lee and Joglekar.[17] We shall assume throughout that a renormalized action S_R exists in the usual sense.

Since to a large extent we shall be repeating the analysis of Lee and Zinn-Justin in a normalized theory, we will rely heavily on their methods and notation, in particular, the summation convention over all indices, discrete and continuous. To distinguish the two we shall throughout use Latin indices $\{i,j,\ldots\}$ to represent discrete, e.g., internal-symmetry, labels, Latin labels $\{a,b,\ldots\}$ to denote pairs of internal and Lorentz indices $\{(i,\mu),(j,\nu)\cdots\}$, and Greek indices to represent space-time as well: $\{\alpha,\beta,\ldots\}$ corresponding to $\{(i,\mu,x),(j,\nu,y),\ldots\}$. Thus

$$A_\alpha B_\alpha\equiv\int dx\sum_a A_a(x)B^a(x)$$
$$\equiv\int dx\sum_{i,\mu}A_i^\mu(x)B_\mu^i(x)\,. \tag{27}$$

We shall denote the collection of all physical, i.e., nonghost, fields by $\{A_\alpha\}$ which allows for both gauge and matter fields. Their sources will be denoted $\{J_\alpha\}$. $\{\varphi_\alpha\}$ will represent all fields, including ghosts, with sources $\{j_\alpha\}$, and the collection of all composite fields introduced into the Lagrangian at various stages will be $\{B_\alpha\}$ with sources $\{k_\alpha\}$.

B. Generic and renormalized gauge symmetries

Following the analysis of Zinn-Justin,[4] we consider the generating functional in a gauge specified by a function of the fields F_α:

$$Z_G'^F[J;\eta,\bar\eta;K,L,R]\equiv\int d[A;\bar C,C]\exp(i\{S_G[A;\bar C,C]+J_\alpha A_\alpha+\eta_\alpha\bar C_\alpha+\bar\eta_\alpha C_\alpha+K_\alpha D_\alpha^\beta C_\beta-\tfrac{1}{2}L_\alpha f_{\alpha\beta\gamma}C_\beta C_\gamma-R_\alpha F_\alpha\}) \tag{28}$$

or

$$Z_G'^F[j;k]\equiv\int d[\varphi]\exp(i\{S_G[\varphi]+j\varphi+kB\})\equiv\int d[\varphi]\exp(i\{S_G'[\varphi]\})\,. \tag{29}$$

In principle for full generality F can be any quadratic function of dimension two and ghost number zero of all the fields, including the ghosts $\overline{C}, C$. If this possibility is allowed then, of course, we lose the usual train of argument leading to the effective Lagrangian, and instead just impose Slavnov invariance (i.e., the invariance under BRS transformations[11]) on our effective action S_G:

$$S_G \equiv \int \mathcal{L}_{G\,\mathrm{eff}}(x)dx\,,$$
$$\mathcal{L}_{G\,\mathrm{eff}}(\varphi) \equiv \mathcal{L}_{G\,\mathrm{inv}}(A) - \tfrac{1}{2}F^2 + \mathcal{L}_{G\,\mathrm{FP}}\,, \tag{30}$$

where the Faddeev-Popov term $\mathcal{L}_{G\,\mathrm{FP}}$ is chosen just to restore Slavnov invariance. Curci and Ferrari[18,19] give a simple example of this, and a discussion of the connection between Slavnov invariance and the unitarity of the S matrix. The motivation for this is that since a quadratic F will mix under renormalization with $\overline{C}C$ terms, there appears to be no *a priori* reason for excluding them from the original Lagrangian. However, we shall have more to say about this later.

The sources of composite fields are introduced since the composite operators appear in the BRS[11] transformation $\varphi \to \varphi' = \varphi + \delta'\varphi$:

$$\delta' A_\alpha = D_\alpha^\beta C_\beta \delta\lambda'\,,$$
$$\delta' C_\alpha = -\tfrac{1}{2} f_{\alpha\beta\gamma} C_\beta C_\gamma \delta\lambda'\,,$$
$$\delta' \overline{C}_\alpha = -F_\alpha \delta\lambda'\,, \tag{31}$$

or

$$\delta'\varphi_\beta = B_\beta \delta\lambda'\,,$$

where $\delta\lambda'$ is an infinitesimal anticommuting parameter. The naive Ward identities (WI's) and equations of motion (EM) are[4]

$$\left(j_\alpha \frac{\delta}{\delta k_\alpha} + k_\beta \frac{\delta B_\beta}{\delta\varphi_\alpha}\frac{\delta}{\delta k_\alpha}\right) Z_G'^F = 0 \quad \text{(GWI)}\,,$$
$$\left(\frac{\delta S_G}{\delta\varphi_\alpha} + j_\alpha + k_\beta \frac{\delta B_\beta}{\delta\varphi_\alpha}\right) Z_G'^F = 0 \quad \text{(GEM)}\,. \tag{32}$$

It is clear, by differentiation with respect to k_α, that these equations are mutually compatible only if $\delta B_\beta/\delta\varphi_\beta = 0$, which is precisely our previous condition for the preservation of the measure under the BRS transformation. For this generic case $\delta B_b(x)/\delta\varphi_b(y)$ is clearly zero for F linear, and for F nonlinear but globally symmetric in any theory with only totally antisymmetric three-index symbols. However, for other theories this is by no means clear even in the generic case, let alone when we come to the renormalized case, and we are only saved by the inclusion of the zero-valued $\int d^\nu p$ in dimensional regularization, as explained above.

Since all of the composite sources except for F are BRS invariant the WI reduces to the more usual

$$\left(j_\alpha \frac{\delta}{\delta k_\alpha} - R_\beta \frac{\delta F_\beta}{\delta\varphi_\alpha}\frac{\delta}{\delta k_\alpha}\right) Z_G'^F = 0\,. \tag{33}$$

Following the discussion of the preceding section, the renormalized version of the BRS transformations, and hence the symmetry of the renormalized theory, will be $\varphi_\alpha \to \varphi_\alpha^* = \varphi_\alpha + \delta^*\varphi_\alpha$,

$$\delta^* A_\alpha = I'[D_\alpha^\beta C_\beta]\delta\lambda^*\,,$$
$$\delta^* C_\alpha = I'[-\tfrac{1}{2} f_{\alpha\beta\gamma} C_\beta C_\gamma]\delta\lambda^*\,, \tag{34}$$
$$\delta^* \overline{C}_\alpha = I[-F_\alpha]\delta\lambda^*\,,$$

or

$$\delta^*\varphi_\beta = I'[B_\beta]\delta\lambda^*\,,$$

where, as usual, the I-products are known not only in principle in terms of the generic fields, by use of the Wilson expansion,[9] but also explicitly since the products B_β already appear in the Lagrangian. The I-product $I'[B_\beta]$ is, therefore, the renormalized operator for an insertion of B_β into a theory with arbitrarily many B_α's already, via the $k_\alpha B_\alpha$ terms in the Lagrangian. Hence

$$I'[B_\alpha] = I^{k;\,\epsilon_\alpha}_{\varrho\alpha}[B_\alpha] = \frac{\delta}{\delta\epsilon_\alpha}\bigg|_0 S_N'^\epsilon \tag{35}$$

with $S_G'^\epsilon = S_G' + \epsilon_\alpha B_\alpha$, and so

$$I'[B_\alpha] = \frac{\delta}{\delta k_\alpha} S_N' \neq \frac{\delta}{\delta k_\alpha}\bigg|_0 S_N = I[B_\alpha]\,. \tag{36}$$

In fact, by conservation of ghost number only the term R^2 could appear quadratic in the sources, and hence only $I'[F_\alpha]$ can be effected, by a constant linear in R. Thus we get renormalized WI's and EM:

$$\left(j_\alpha \frac{\delta}{\delta k_\alpha} + k_\beta I'\left[\frac{\delta B_\beta}{\delta\varphi_\alpha} B_\alpha\right]\right) Z_N'^F = 0 \text{ (RWI)}\,,$$
$$\left(I'\left[\frac{\delta S_G}{\delta\varphi_\alpha}\right] + j_\alpha + k_\beta I'\left[\frac{\delta B_\beta}{\delta\varphi_\alpha}\right]\right) Z_N'^F = 0 \text{ (REM)}\,. \tag{37}$$

When we come to use these WI's it must be remembered that although they are identities, which can therefore be arbitrarily differentiated, it is only when there are no summations (i.e., integrations) implied by the differential operator that we can guarantee the finiteness of the individual terms. In other cases we just derive WI's for partially renormalized operator insertions into a renormalized theory.

Now we should reconsider the question of the form of F_α, since it is only F_α of the $\{B_\beta\}$ which is not itself invariant under the generic BRS transformation. We can consider the most general form for F_α

$$F_\alpha(A;\bar{C},C)=F_{0\alpha}(A)+\bar{C}_\beta F_{1\alpha\beta}(A,C) \tag{38}$$

and the consequent most general Faddeev-Popov term

$$\mathcal{L}_{G\,\mathrm{FP}}=\bar{C}_\alpha L_{0\alpha}(A,C)+\bar{C}_\alpha\bar{C}_\beta L_{1\alpha\beta}(C)\,. \tag{39}$$

If we now introduce a BRS transformation defined by just $F_{0\alpha}$, i.e., $\delta'\bar{C}_\alpha=-F_{0\alpha}\delta\lambda'$, and demand invariance of $-\frac{1}{2}F_\alpha{}^2+\mathcal{L}_{G\,\mathrm{FP}}$ under this transformation, then it is an easy, but tedious, task to prove that the only solution for $\mathcal{L}_{G\,\mathrm{FP}}$ is just the one which reduces the terms in the Lagrangian to $-\frac{1}{2}F_{0\alpha}{}^2+\bar{C}_\alpha\partial F_{0\alpha}/\partial\lambda$, that is, the terms required if F were just taken independent of $\bar{C}$ and C. Therefore, if we start with only an F_0 term, the other terms will *not* appear upon renormalization. Following general arguments we know that we can renormalize the Lagrangian respecting the symmetry which will be expressed by $\delta^*C_\alpha=-I'[F_{0\alpha}]\delta\lambda^*$, and the beauty of the normal-product formulation is that exactly the same algebra as was used to prove the generic statement above will prove the corresponding renormalized statement that we have no need for terms of the form $N'[\bar{C}_\beta F_{1\alpha\beta}(A,C)]$. This is because we can use

$$\frac{\delta}{\delta\bar{C}_\beta}N'[\tfrac{1}{2}F_\alpha{}^2]=I'\left[F_\alpha\frac{\delta F_\alpha}{\delta\bar{C}_\beta}\right],$$

etc. That is, the most general form of the renormalized Lagrangian symmetric under the $I'[F_0]$ BRS transformation will just have a gauge-fixing term: $N'[-\frac{1}{2}F_{0\alpha}{}^2]$. Moreover,

$$\frac{\delta}{\delta\bar{C}_\beta}N'[\tfrac{1}{2}F_{0\alpha}{}^2]=I'\left[F_{0\alpha}\frac{\delta F_{0\alpha}}{\delta\bar{C}_\beta}\right]=0,$$

so no $\bar{C}C$ terms enter in that way. In particular, the Lagrangian will *not* contain $\bar{C}C\bar{C}C$ terms, and all the $\bar{C}CA^2$ and $\bar{C}CA$ terms will be contained within $N'[\bar{C}_\alpha(\delta F_{0\alpha}/\delta A_\beta)D^\gamma_\beta C_\gamma]$ which will be the normalized Faddeev-Popov term. That is not to say, as we shall see later, that $F_{0\alpha}$ does not mix with $\bar{C}C$ in the sense of $I'[F_{0\alpha}]$ containing a $\bar{C}C$ term. Normalization will have destroyed the close connection of $N'[\frac{1}{2}F_{0\alpha}{}^2]$ and $(I'[F_{0\alpha}])^2$.

Hence throughout we shall treat F_α as the most general quadratic function of A fields, and just indicate the simplification that occurs if we restrict to the usual case of F_α linear in A.

It is clear that now we can use the EM to simplify our WI since the only I-product that appears in the WI is $I'[(\delta F_\beta/\delta\varphi_\alpha)_{B_\alpha}]=I'[(\delta F_\beta/\delta A_\alpha)D^\gamma_\alpha C_\gamma]=I'[\delta S_G/\delta\bar{C}_\beta]$. Hence we have[4]

$$\left(j_\alpha\frac{\delta}{\delta k_\alpha}-R_\beta\eta_\beta\right)Z'^F_N=0\,. \tag{40}$$

From this form we can easily follow the standard steps[4] to derive the WI's for the generating functionals, W, of connected and, Γ, of one-particle irreducible (1PI) Green's functions where $W=-i\ln Z$ and $W+\Gamma+j\varphi=0$. We have finally

$$\frac{\delta\Gamma^F_N}{\delta\varphi_\alpha}\frac{\delta\Gamma^F_N}{\delta k_\alpha}-R_\beta\frac{\delta\Gamma^F_N}{\delta\bar{C}_\beta}=0\,. \tag{41}$$

It is customary in the generic theory with linear F_α to define a new functional $\tilde{\Gamma}^F_G$ by $\tilde{\Gamma}^F_G=\Gamma^F_G+\frac{1}{2}F_\alpha{}^2$, which has the effect of removing the gauge-fixing term at the tree level since $\Gamma^F_G=S^F_G+O(\hbar)$. The result is that we get

$$\frac{\delta\tilde{\Gamma}^F_G}{\delta A_\alpha}\frac{\delta\tilde{\Gamma}^F_G}{\delta K_\alpha}+\frac{\delta\tilde{\Gamma}^F_G}{\delta C_\alpha}\frac{\delta\tilde{\Gamma}^F_G}{\delta L_\alpha}-R_\beta\frac{\delta\tilde{\Gamma}^F_G}{\delta\bar{C}_\beta}=0\,. \tag{42}$$

If we now go to the renormalized theory with linear F_α the same trick goes through. The natural definition is

$$\tilde{\Gamma}^F_N\equiv\Gamma^F_N+N'[\tfrac{1}{2}F_\alpha{}^2]\,, \tag{43}$$

but here that F_α is linear implies that $N'[\frac{1}{2}F_\alpha{}^2]$ is quadratic in fields and hence trivial; N-products are defined from I-products and linear I-products are trivial.[9] Hence $N'[\frac{1}{2}F_\alpha{}^2]=:\frac{1}{2}F_\alpha{}^2:$.

In the case of F_α nonlinear the situation is less simple, even in the generic case, since the equations of motion multiplied by F_α do not take the usual simple form in the 1PI language. We can, however, follow the procedure of Zinn-Justin[4] in spirit. The renormalized action S^F_N will be the most general local polynomial of degree four satisfying

$$\frac{\delta S^F_N}{\delta A_\alpha}\frac{\delta S^F_N}{\delta K_\alpha}+\frac{\delta S^F_N}{\delta C_\alpha}\frac{\delta S^F_N}{\delta L_\alpha}+\frac{\delta S^F_N}{\delta\bar{C}_\alpha}\left(\frac{\delta S^F_N}{\delta R_\alpha}-R_\alpha\right)=0\,. \tag{44}$$

We now expand S^F_N in terms of R_α and subtract the gauge-fixing term on this level:

$$S^F_N=\tilde{S}^F_N-R_\alpha I'_0[F_\alpha]+N'_0[\tfrac{1}{2}F_\alpha{}^2]+\tfrac{1}{2}a_{\alpha\beta}R_\alpha R_\beta\,, \tag{45}$$

where $\tilde{S}^F_N$ is now independent of R, and the subscript 0 refers to setting $R_\alpha=0$ in the normal products. From this we derive

$$\frac{\delta\tilde{S}^F_N}{\delta A_\alpha}\frac{\delta\tilde{S}^F_N}{\delta K_\alpha}+\frac{\delta\tilde{S}^F_N}{\delta C_\alpha}\frac{\delta\tilde{S}^F_N}{\delta L_\alpha}=0\,, \tag{46a}$$

$$\frac{\delta I'_0[F_\beta]}{\delta A_\alpha}\frac{\delta\tilde{S}^F_N}{\delta K_\alpha}+\frac{\delta I'_0[F_\beta]}{\delta C_\alpha}\frac{\delta\tilde{S}^F_N}{\delta L_\alpha}=\frac{\delta\tilde{S}^F_N}{\delta\bar{C}_\beta}\,, \tag{46b}$$

$$\frac{\delta I'_0[F_\beta]}{\delta\bar{C}_\alpha}+\frac{\delta I'_0[F_\alpha]}{\delta\bar{C}_\beta}=0 \tag{46c}$$

after we have used the equations of motion. From (46b) we see that $I'_0[F_\alpha]$, which is $I'[F_\alpha]$ modulo a constant linear in R, can have some $\bar{C},C$ dependence, although we have seen that $N'[\frac{1}{2}F_\alpha{}^2]$ is independent of $\overline{C}$ and C. The relationship between the two statements is no longer at all direct, and there is no contradiction.

For the situation of F_α linear we can, of course, follow Lee's method for the generic theory[5] to derive

$$\frac{\delta\tilde{\Gamma}^F_{0N}}{\delta A_\alpha}[A]\,\tilde{\Gamma}^F_{2N\alpha\beta}[A]=0\,,\qquad \frac{\delta F_\alpha}{\delta A_\beta}\tilde{\Gamma}^F_{2N\beta\gamma}[A]=\tilde{\Gamma}^F_{2N\alpha\gamma}[A]\,, \tag{47}$$

where we have expanded $\tilde{\Gamma}^F_N$ as

$$\tilde{\Gamma}^F_N[\varphi,k]\equiv\tilde{\Gamma}^F_{0N}[A]+\overline{C}_\alpha\tilde{\Gamma}^F_{1N\alpha\beta}[A]C_\beta - K_\alpha\tilde{\Gamma}_{2N\alpha\beta}[A]C_\beta+\cdots\,. \tag{48}$$

We also know that $\delta\tilde{\Gamma}^F_N/\delta K_\alpha=-\,\delta W^F_N/\delta K_\alpha$ is the generating functional for Green's renormalized functions with an $I'[D^\beta_\alpha C_\beta]$ insertion, so we can identify $-\tilde{\Gamma}_{2N\alpha\beta}[A]C_\beta$ as this functional, and similarly $\tilde{\Gamma}_{1N\alpha\beta}[A]$ as the generating functional for renormalized Green's functions with exactly one ghost and one antighost external lines.

C. Gauge invariance of the renormalized S-matrix

If we consider a change of gauge-fixing term $F\to F+\Delta F\equiv F+\epsilon\Delta' F$, then we have invariance of the S matrix if we can prove that the change in Z^F in the physical sector is a homogeneous linear function of the physical sources.[5] This is equivalent to the proof of 't Hooft and Veltman[20] using a diagrammatically proved WI for a change in F_α. We would like to show how this transfers to the normalized theory.

The proof of Lee in the generic theory with linear F proceeds by transforming the obvious statement

$$\frac{1}{i}\,\epsilon\,\frac{\partial}{\partial\epsilon}\bigg|_0 Z^F_G=\int d[\varphi]\,\epsilon\,\frac{\delta S^F_G}{\delta\epsilon}\bigg|_0 e^{iS^F_G[\varphi]+ij\varphi} =\int d[\varphi]\left(F_\alpha\Delta F_\alpha-\overline{C}\,\frac{\delta\Delta F_\alpha}{\delta A_\beta}D^\gamma_\beta C_\gamma\right)\times e^{iS^F_G[\varphi]+ij\varphi} \tag{49}$$

by means of a WI for $F_\alpha\Delta F_\alpha$ derived from the usual WI involving F_α by applying $\Delta F_\alpha[(1/i)(\delta/\delta J)]$. In a normalized theory we cannot use this and must proceed from

$$\frac{1}{i}\,\epsilon\,\frac{\partial}{\partial\epsilon}\bigg|_0 Z^F_N=\int d[\varphi]\left\{I[F_\alpha\Delta F_\alpha]-I\left[\overline{C}_\alpha\,\frac{\delta\Delta F_\alpha}{\delta A_\beta}D^\gamma_\beta C_\gamma\right]\right\}\times e^{iS^F_N[\varphi]+ij\varphi} \tag{50}$$

by obtaining a WI involving $I[F_\alpha\Delta F_\alpha]$. This will be obtained by considering the normalized theory corresponding to $\overline{S}_G=S'_G+N\overline{C}_\alpha\Delta F_\alpha$ with N a new, constant, source. From this we have

$$0=\int d[\varphi]\left\{j_\alpha\,\frac{\delta}{\delta k_\alpha}-i\eta_\alpha R_\alpha-iNR_\alpha\bar{I}[\Delta F_\alpha]+N\left(\bar{I}[F_\alpha\Delta F_\alpha]-\bar{I}\left[\overline{C}_\alpha\,\frac{\delta\Delta F_\alpha}{\delta A_\beta}D^\gamma_\beta C_\gamma\right]\right)\right\}e^{i\bar{S}^F_N[\varphi]+ij\varphi}\,, \tag{51}$$

where we have used the ghost EM to simplify the WI term. Taking terms to $O(N)$ we obtain the usual WI plus

$$0=\int d[\varphi]\left\{I'[F_\alpha\Delta F_\alpha]-I'\left[\overline{C}_\alpha\,\frac{\delta\Delta F_\alpha}{\delta A_\beta}D^\gamma_\beta C_\gamma\right]+j_\beta I'[\overline{C}_\alpha\Delta F_\alpha;B_\beta]-iR_\alpha I'[\Delta F_\alpha]-i\eta_\beta R_\beta I'[\overline{C}_\alpha\Delta F_\alpha]\right\}e^{iS'^F_N[\varphi]+ij\varphi}\,, \tag{52}$$

which, when we set $k_\alpha=0$, becomes the renormalized generalization of 't Hooft and Veltman's WI.[20] Thus we derive

$$\frac{1}{i}\,\epsilon\,\frac{\partial}{\partial\epsilon}\bigg|_0 Z^N_F=-\int d[\varphi]\{j_\beta I[\overline{C}_\alpha\Delta F_\alpha;B_\beta]-i\eta_\beta R_\beta I[\overline{C}_\alpha\Delta F_\alpha]-iR_\alpha I[\Delta F_\alpha]\}\,e^{iS^F_N[\varphi]+ij\varphi}\,, \tag{53}$$

which in the sector with no external ghost fields gives

$$\frac{1}{i}\,\epsilon\,\frac{\partial}{\partial\epsilon}\bigg|_0 Z^N_F=-\epsilon\,J_iX_i \tag{54}$$

for some X_i which gives the desired result of the gauge invariance of the renormalized S matrix.[5]

In fact, by analogy with the treatment of an Abelian gauge theory by Lowenstein and Schroer[12] and Collins,[21] we can go further than this. In essence we have replaced the F_α of the $F_\alpha\Delta F_\alpha$ insertion with a ghost field $\overline{C}_\alpha$ (whose equation of motion we know readily) along with insertions of composite operators B'_α at external sources. In QED this is useful because the ghost $\overline{C}_\alpha$ is essentially free so the use of the ghost equation of motion greatly simplifies the structure of the Green's function. The next logical step would be to attempt to express the ΔF_α part in terms of a ghost field. In the Abelian case this is achieved for the Lorentz gauges in Ref. 20 by using a trick of Slavnov,[22] which corresponds to introducing by hand the free ghost particles. However, for general ΔF_α and a non-Abelian theory the situation is more complicated. Firstly, ΔF_α need not be proportional to F_α, and secondly, the ghosts are now no longer free. Hence to proceed we must introduce new sources for $\overline{C}C$ and A^2 (corresponding to mass terms for the ghost and gauge field) say $\mu^2\overline{C}C$ and $\frac{1}{2}m^2A^2$. Now we use the result of Curci and Ferrari[18] that the Slavnov-invariant piece of the action is also invariant under the transformations

$$\begin{aligned}&\overline{\delta}A_\alpha = D^\beta_\alpha \overline{C}_\beta \delta\overline{\lambda}\,,\\ &\overline{\delta}C_\alpha = (F_\alpha - f_{\alpha\beta\gamma}\overline{C}_\beta C_\gamma)\delta\overline{\lambda}, \quad \text{or} \quad \overline{\delta}\varphi_\beta = \overline{B}_\beta\,\delta\overline{\lambda}\,,\\ &\overline{\delta}\overline{C}_\alpha = -\tfrac{1}{2}f_{\alpha\beta\gamma}\overline{C}_\beta\overline{C}_\gamma\delta\overline{\lambda}\,,\end{aligned} \tag{55}$$

which give

$$\begin{aligned}\overline{\delta}(\overline{C}C) &= \overline{C}_\alpha(F_\alpha - f_{\alpha\beta\gamma}\overline{C}_\beta C_\gamma)\delta\overline{\lambda} + \tfrac{1}{2}f_{\alpha\beta\gamma}\overline{C}_\beta\overline{C}_\gamma C_\alpha\delta\overline{\lambda}\\ &= (\overline{C}_\alpha F_\alpha - \tfrac{1}{2}f_{\alpha\beta\gamma}\overline{C}_\alpha\overline{C}_\beta C_\gamma)\delta\overline{\lambda}\\ &= (\overline{C}\cdot F - \tfrac{1}{2}\overline{C}\cdot\overline{C}\wedge C)\delta\overline{\lambda}\,,\end{aligned} \tag{56}$$

$$\begin{aligned}\overline{\delta}(\tfrac{1}{2}A^2) &= A^{a\mu}D^\beta_\mu\overline{C}_\beta\delta\overline{\lambda}\\ &= A^{a\mu}\partial_\mu\overline{C}_a\delta\overline{\lambda}\\ &= -(\partial A)\cdot\overline{C}\delta\overline{\lambda}\,.\end{aligned} \tag{57}$$

The terms which can appear in F_α, and hence ΔF_α, are precisely $\partial\cdot A_\alpha$ and $\frac{1}{2}d_{\alpha\beta\gamma}A_\beta A_\gamma$ if we restrict to globally symmetric F^2 terms (which we will do henceforth). Hence modulo the $\overline{C}\cdot\overline{C}\wedge C$ terms any admissible $\overline{C}_\alpha\Delta F_\alpha$ can be expressed as a linear combination of $\overline{\delta}(\overline{C}C)$ and $\overline{\delta}(\frac{1}{2}A^2)$. So we use a source $m(\lambda\overline{C}C+\mu\frac{1}{2}A^2)$:

$$\overline{\delta}(\lambda\overline{C}C+\mu\,\tfrac{1}{2}A^2) = (\overline{C}\cdot\Delta F - \tfrac{1}{2}\lambda\overline{C}\cdot\overline{C}\wedge C)\delta\overline{\lambda}\,. \tag{58}$$

This will produce a WI:

$$\begin{aligned}0=\int d[\varphi]\Big\{&I'[\overline{C}\Delta F] - \tfrac{1}{2}\lambda I'[\overline{C}\cdot\overline{C}\wedge C] + k_\alpha I'\Big[\lambda\overline{C}C+\tfrac{1}{2}\mu A^2;\ \frac{\overline{\delta}B}{\delta\overline{\lambda}}\Big]\\ &+ j_\alpha I'[\lambda\overline{C}C+\tfrac{1}{2}\mu A^2;\overline{B}_\alpha] - i\eta_\beta R_\beta I'[\lambda\overline{C}C+\tfrac{1}{2}\mu A^2]\Big\}e^{iS'^F_N[\varphi]+ij\varphi}\,,\end{aligned} \tag{59}$$

from which we can derive expressions for $I'[\overline{C}\Delta F;B\alpha]$ and $I[\overline{C}\Delta F]$ as required above. The result is

$$\begin{aligned}\frac{1}{i}\epsilon\frac{\partial}{\partial\epsilon}\Big|_0 Z^N_F = -\int d[\varphi]\Big\{&j_\alpha\Big(-I\Big[\lambda\overline{C}C+\mu\tfrac{1}{2}A^2;\ \frac{\overline{\delta}B_\alpha}{\delta\overline{\lambda}}\Big] + \tfrac{1}{2}\lambda I[\overline{C}\cdot\overline{C}\wedge C;B_\alpha] - j_\beta I[\lambda\overline{C}C+\tfrac{1}{2}\mu A^2;\overline{B}_\beta;B_\alpha]\Big)\\ &-i\eta_\beta R_\beta(\tfrac{1}{2}\lambda I[\overline{C}\cdot\overline{C}\wedge C] - j_\alpha I[\lambda\overline{C}C+\tfrac{1}{2}\mu A^2;\overline{B}_\alpha]) - iR_\alpha I[\Delta F_\alpha]\Big\}e^{iS^F_N[\varphi]+ij\varphi}\,.\end{aligned} \tag{60}$$

The term $I[\lambda\overline{C}C+\frac{1}{2}\mu A^2;\overline{\delta}(B_\alpha)/\delta\overline{\lambda}]$ might appear surprising especially since it is nonzero in the Abelian and linear F_α limit. As is clear from the derivation above it is present only because of the appearance of $I[\overline{C}\Delta F;B_\alpha]$, which necessitates keeping the k_α terms. Its effect is to cancel precisely those diagrams of $I[\lambda\overline{C}C;\overline{B}_{\alpha''};B_{\alpha'}]$ in which $\overline{C}^\bullet C^\bullet$ and $\overline{B}^\bullet_{\alpha''}B^\bullet_{\alpha'}$ are contracted as shown. This, and the most important structure of the above equation, is best seen by specializing to the Abelian case with $F=f\partial\cdot A$ and $\Delta F=(\Delta f)\partial\cdot A$. Then we can set $\mu=0=R$, and look at only the sector with physical external fields, i.e., with no ghost sources. Then

$$\frac{1}{i}\epsilon\frac{\partial}{\partial\epsilon}\Big|_0 Z^N_F = -\int d[\varphi]J_{\alpha'}\Big\{-I\Big[(\Delta f)\overline{C}C;\ \frac{\overline{\delta}B_{\alpha'}}{\delta\overline{\lambda}}\Big] - J_{\alpha''}I[(\Delta f)\overline{C}C;B_{\alpha'};\overline{B}_{\alpha''}]\Big\}e^{iS^F_N[\varphi]+ij\varphi}\,. \tag{61}$$

Let us now denote the gauge field alone by A with source J, and differentiate the matter fields as $\overline{\psi},\psi$ with sources $\zeta,\overline{\zeta}$. Then:

$$\begin{aligned}\frac{1}{i}\epsilon\frac{\partial}{\partial\epsilon}\Big|_0 Z^N_F = \int d[\varphi]\{&J^\mu I[(\Delta f)\overline{C}C;\partial_\mu F] + \zeta I[(\Delta f)\overline{C}C;\overline{\psi}\overline{C}C+\overline{\psi}F] + \zeta I[(\Delta f)\overline{C}C;-\overline{C}C\psi+F\psi]\\ &+J_{\alpha'}J_{\alpha''}I[(\Delta f)\overline{C}C;\overline{B}_{\alpha''};B_{\alpha'}]\}e^{iS^F_N[\varphi]+ij\varphi}\,.\end{aligned} \tag{62}$$

If we specialize to a Green's function with N external fermion and antifermion lines and arbitrary external gauge-field lines, then we will derive

$$\frac{1}{i}\,\epsilon\,\frac{\partial}{\partial\epsilon}\langle TY^{(N)}\rangle \equiv \Delta\langle TY^{(N)}\rangle$$

$$= (\Delta f)\sum_{i,j}\langle TI[\overline{C}C;\partial_{\mu_i}\overline{C};\partial_{\nu_j}\overline{C}]\,Y^{(N)}\backslash A_{\mu_i}\backslash A_{\nu_j}\rangle$$

$$+(\Delta f)\sum_{i}\langle TI[\overline{C}C;\partial_{\mu_i}F]Y^{(N)}\backslash A_{\mu_i}\rangle$$

$$+(\Delta f)\sum_{i,j=1}^{N}\{\langle TI[\overline{C}C;\overline{\psi}_i\overline{C};\psi_jC]\,Y^{(N)}\backslash\overline{\psi}_i\backslash\psi_j\rangle+\langle TI[\overline{C}C;\overline{\psi}_i\overline{C};\overline{\psi}_jC]\,Y^{(N)}\backslash\overline{\psi}_i\backslash\overline{\psi}_j\rangle$$

$$+\langle TI[\overline{C}C;\overline{\psi}_iC;\psi_j\overline{C}]\,Y^{(N)}\backslash\overline{\psi}_i\backslash\psi_j\rangle+\langle TI[\overline{C}C;\psi_iC;\psi_j\overline{C}]Y^{(N)}\backslash\psi_i\backslash\psi_j\rangle\}$$

$$+(\Delta f)\sum_{i=1}^{N}\{\langle TI[\overline{C}C;\overline{\psi}_i\overline{C}C+\overline{\psi}_iF]Y^{(N)}\backslash\overline{\psi}_i]+\langle TI[\overline{C}C;-\overline{C}C\psi_i+F\psi_i]Y^{(N)}\backslash\psi_i]\rangle\}\,. \tag{63}$$

$Y\backslash A$ means the expression for Y with the field A omitted. As before we can use the Ward identity of the BRS transformation to express F in the presence of other insertions, giving (essentially) the replacement of F by $\overline{C}$ and the insertion of a B_α on a j_α external line. In this manner the first two terms combine to give just the contributions with connected ghost lines, i.e., from diagrams of the form in Fig. 1. In a similar, but necessarily more complicated, way the last six terms conspire to eliminate the vacuum ghost loops and also to cancel the terms which appear with $Y^{(N)}\backslash\overline{\psi}_i\backslash A_j$, etc. to leave only the contributions in Fig. 2 (compare Ref. 21).

We have, of course, used that the ghost is free (i.e., the ghost equation of motion) and the diagrams should be understood as the generic versions of inserted diagrams. It is clear that the only divergences which need subtracting to define the renormalized operator are those from the first two sets of diagrams, and we proceed to produce the usual formula of Ref. 21 for $\Delta\langle TY^{(N)}\rangle$.

In general, of course, with interacting ghosts the situation is much less simple, although an interesting case to treat would be an Abelian theory in a nonstandard gauge which gives interacting ghosts but no extra terms.

D. Renormalized gauge-invariant operators

We now turn to a study of "gauge-invariant operators." The literature on this subject is plagued by two problems. Firstly, that formally gauge-invariant operators (namely operators which are functions of A invariant under the generic BRS transformation) mix under renormalization with non-gauge-invariant operators. In other words, the Wilson expansion for the insertion of a generically gauge-invariant operator can contain the renormalized insertion products of generically noninvariant operators. Secondly, the study of the gauge properties of the renormalized Green's functions with operator insertions looks highly complicated in the BPHZ approach, and led Lowenstein and Schroer[12] to define two criteria of gauge invariance of those renormalized insertions. In the case of massive vector-meson theory Collins[21] has shown that in a DR normal-product formalism the second criterion is redundant. We would like to show that they are in general both redundant, that is, that a generically gauge-invariant operator gives a renormalized insertion product whose Green's functions obey WI's strictly analogous to those for the uninserted Green's functions, and whose S-matrix elements are invariant under a change of gauge. In general, we will adopt the label renormalized gauge-invariant operator insertion (RGIOI) for an operator with finite Green's functions whose S-matrix elements are invariant under a change of gauge, and our statement is just that the I-product of a generically gauge-invariant operator is an RGIOI.

Firstly, let us investigate the renormalized WI satisfied by Green's functions with a single operator insertion Φ. To do this we define

$$\tilde{S}'_G \equiv S'_G + i\mu\Phi\,, \tag{64}$$

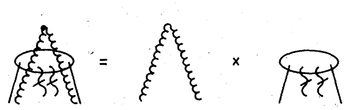

FIG. 1. Diagrams with connected ghost lines corresponding to the first two terms of Eq. (63).

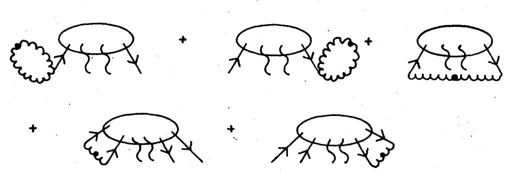

FIG. 2. Diagrams corresponding to the last six terms of Eq. (63).

consider

$$\tilde{Z}_N^F \equiv \int d[\varphi] e^{i\tilde{S}_N[\varphi]+ij\varphi}, \qquad (65)$$

and then consider a renormalized BRS transformation on this. This will be defined using $\tilde{I}[B_\alpha]$'s which are defined using $\tilde{S}_N$, and therefore involve μ. So we get as usual

$$0=\int d[\varphi]\left\{j_\alpha \tilde{I}[B_\alpha]+\frac{\delta\tilde{S}_N}{\delta\varphi_\alpha}\tilde{I}[B_\alpha]\right\}e^{i\tilde{S}_N+ij\varphi}$$
$$=\int d[\varphi]\left\{j_\alpha \tilde{I}[B_\alpha]+\tilde{I}\left[\frac{\delta\tilde{S}_G}{\delta\varphi_\alpha}B_\alpha\right]\right\}e^{i\tilde{S}_N+ij\varphi}$$
$$=\int d[\varphi]\left\{j_\alpha \tilde{I}[B_\alpha]+k_\alpha\tilde{I}\left[\frac{\delta B_\alpha}{\delta\varphi_\beta}B_\beta\right]+\mu\tilde{I}\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha\right]\right\}e^{i\tilde{S}_N+ij\varphi} \qquad (66)$$

since S_G is assumed invariant under the generic BRS transformation. Now taking $\partial/\partial\mu$ we get

$$0=\int d[\varphi]\left\{j_\alpha \tilde{I}[B_\alpha;\Phi]+k_\alpha\tilde{I}\left[\frac{\delta B_\alpha}{\delta\varphi_\beta}B_\beta;\Phi\right]+\tilde{I}\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha\right]+\mu\tilde{I}\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha;\Phi\right]\right\}e^{i\tilde{S}_N+ij\varphi}. \qquad (67)$$

It is clear that if we had introduced many Φ's we could go on differentiating *ad infinitum* to consider the WI for many operator insertions. However, considering just one and setting $\mu=0$ we arrive at

$$0=\int d[\varphi]\left\{j_\alpha I'[B_\alpha;\Phi]+k_\alpha I'\left[\frac{\delta B_\alpha}{\delta\varphi_\beta}B_\beta;\Phi\right]+I'\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha\right]\right\}e^{iS'_N+ij\varphi}, \qquad (68)$$

which is strictly analogous to the WI without Φ insertion if and only if $(\delta\Phi/\delta\varphi_\alpha)B_\alpha=0$, i.e., Φ is invariant under the generic BRS transformation. Similarly if $\delta\Phi/\delta\bar{C}=0$ then the ghost EM with a Φ insertion will also be analogous.

Now it is obvious that we can proceed to consider a change of gauge-fixing term in the usual way:

$$\frac{1}{i}\epsilon\frac{\partial}{\partial\epsilon}\bigg|_0\frac{1}{i}\frac{\partial}{\partial\mu}\bigg|_0\tilde{Z}_N^F=\int d[\varphi]\left\{I'[F_\alpha\Delta F_\alpha;\Phi]-I'\left[\bar{C}_\alpha\frac{\delta\Delta F_\alpha}{\delta\varphi_\beta}D_\beta^\gamma C_\gamma;\Phi\right]\right\}e^{iS'_N+ij\varphi}, \qquad (69)$$

and we require a WI for $I'[F_\alpha\Delta F_\alpha;\Phi]$ obtained starting from

$$\bar{\tilde{S}}_G=\tilde{S}_G+N\bar{C}_\alpha\Delta F_\alpha, \qquad (70)$$

which gives

$$0=\int d[\varphi]\left\{I'[F_\alpha\Delta F_\alpha;\Phi]-I'\left[\bar{C}_\alpha\frac{\delta\Delta F_\alpha}{\delta\varphi_\beta}D_\beta^\gamma C_\gamma;\Phi\right]+I'\left[\bar{C}_\alpha\Delta F_\alpha;\frac{\delta\Phi}{\delta\varphi_\beta}B_\beta\right]+j_\beta I'[\bar{C}_\alpha\Delta F_\alpha;B_\beta;\Phi]\right\}e^{iS'_N+ij\varphi}. \qquad (71)$$

So again with $(\delta\Phi/\delta\varphi_\beta)B_\beta=0$ we get invariance of the S matrix by the old argument. Notice, however, that the two statements relating to WI and variations of gauge are logically independent, each requiring an application of the generic condition on Φ. But we have, in fact, shown that the generic condition is sufficient to make $I'[\Phi]$ on RGIOI, and thus the two criteria of Lowenstein and Schroer are redundant in this formulation.

What we have effectively shown is that

$$\frac{\delta I[\Phi]}{\delta\varphi_\alpha}I[B_\alpha]=I\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha\right]$$

or (72)

$$\frac{\delta I[\Phi]}{\delta\lambda^*}=I\left[\frac{\delta\Phi}{\delta\lambda'}\right].$$

We would also ask about the analogous statements for N-products. We know that

$$\frac{\delta S_N}{\delta\varphi_\alpha}I[B_\alpha]=I\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha\right],$$

and with each gauge-invariant piece of S_G we could associate a different scale without fear of losing this property. Therefore we must have

$$\frac{\delta N[\Phi]}{\delta\lambda^*}=\frac{\delta N[\Phi]}{\delta\varphi_\alpha}I[B_\alpha]=I\left[\frac{\delta\Phi}{\delta\varphi_\alpha}B_\alpha\right]=0 \qquad (73)$$

for Φ generically gauge invariant.

It should be noticed, however, that we have made no statement about $\delta I[\Phi]/\delta\lambda'$, i.e., the change of the renormalized insertion product under the unrenormalized transformation. If Φ were part of a set closed under renormalization and containing only generically gauge-invariant operators, then using the Wilson expansion we could derive that $\delta I[\Phi]/\delta\lambda'=0$ However, in general (see the work of Lee and Joglekar[17]) this will not be true.

ACKNOWLEDGMENTS

I am grateful to the Royal Society for the provision of a fellowship under the European Science Exchange Program, and to Professor W. Zimmermann and Professor H. P. Dürr for hospitality at the Max-Planck-Institut für Physik in Munich during the tenure of that fellowship.

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