# Interacting Rarita-Schwinger field on the light front

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The coupled spin-3/2 Rarita-Schwinger field is investigated in light-front coordinates. The class of interactions considered includes both minimal and anomalous magnetic-moment couplings to the electromagnetic field, coupling to a scalar field, and coupling to a Dirac and a scalar field. It is shown that the interacting theory suffers from a loss of constraints except when the external field satisfies certain noncovariant conditions. The anticommutators are obtained and are shown not to be positive-definite in the case that no constraints are lost. In the case of fewer constraints they are inconsistent with each other as well as singular in the free-field limit.

## I. INTRODUCTION

Recently there has been considerable interest in field theories on the "light front." This approach was motivated by the hope that one could find a more convenient treatment of the infinite-momentum limit of the usual space-time formulation. Two questions arise in this connection: Can field theories be consistently formulated on the light front? And, is the formalism equivalent to the usual one? Earlier works[1] involving electrodynamics of the Dirac field as well as some other interactions of fields with spins $\leq 1$ appeared to answer both questions in the affirmative. However, recent work[2] involving two-dimensional models and their generalizations has shown that such conclusions are not tenable. In particular it has been shown that a consistent theory is not generally possible on the light front, and even in the rare case when consistent, it may not be equivalent to the same theory in ordinary space-time.

In this connection it is of interest to consider interacting higher-spin fields on the light front. As is well known, not only are higher-spin fields more complex, but they are also plagued with curious inconsistencies such as indefinite metric[3] and noncausal propagation[4] once interactions are introduced.

In this paper an interacting Rarita-Schwinger (spin-$\frac{3}{2}$) field is considered. Whereas the free-field case exhibits four degrees of freedom (half the number encountered in the usual approach) and leads to consistent anticommutation relations, the introduction of interactions leads to serious inconsistencies as a result of a loss of constraints.

In Sec. II the free-field equations are considered, with various interactions being introduced in Sec. III. One finds that constraints are lost in the interacting case except when the external fields satisfy certain noncovariant conditions. Thus, for example the minimally coupled field has six degrees of freedom unless the electromagnetic field $F_{\mu\nu}$ satisfies $F_{0i}=F_{3i}$, $i=1,2$. In Sec. IV the anticommutators for the minimally coupled field are obtained using the action principle.[5] For values of the electromagnetic field leading to four degrees of freedom, the anticommutators are non-positive-definite even though their free-field limit is entirely consistent. On the the other hand, an attempt to quantize the minimally coupled field in the general case results in anticommutators that are singular in the free-field limit and are inconsistent among themselves. Section V concludes by summarizing the situation with some remarks concerning the light-front approach.

## II. THE FREE FIELD

Before proceeding to the discussion of the field equations a summary of notation is in order. The light-front coordinates are defined by $x^\mu \equiv (x^1, x^2, x^-, x^+)$ with $x^\pm \equiv (1/\sqrt{2})(x^0 \pm x^3)$. One studies the evolution of the field along the $x^+$ direction which plays a role analogous to the time. With the usual space-time metric taken to be $g_{\mu\nu} \equiv \mathrm{diag}(1,1,1,-1)$, the metric on the light front is seen to be $g_{11}=g_{22}=-g_{+-}=-g_{-+}=1$ with all other components vanishing. The contravariant and covariant components of a vector are thus related by

$$a_i = a^i\ , \quad a_\pm = -a^\mp\ ,$$

with the scalar product of two vectors given by[6]

$$a\cdot b \equiv a_\mu b^\mu = a_i b_i - a_- b_+ - a_+ b_-\ .$$

The Dirac matrices $\gamma^\mu$ are taken in a Majorana representation such that

$$\{\gamma^\mu,\gamma^\nu\}=-2g^{\mu\nu}, \quad \gamma^{\mu\dagger}=-\gamma_\mu .$$

Thus one has the useful formulas

$$(\gamma^+)^2=(\gamma^-)^2=0$$

and

$$\gamma^+\gamma^-=2\Lambda^{(-)}, \quad \gamma^-\gamma^+=2\Lambda^{(+)},$$

where $\Lambda^{(\pm)}$ are orthogonal projectors such that $\Lambda^{(+)}+\Lambda^{(-)}=1$. The latter observation allows one to write

$$\gamma^\pm=\sqrt{2}\,\beta\Lambda^{(\pm)}=\sqrt{2}\,\Lambda^{(\mp)}\beta ,$$

where $\beta\equiv\gamma^0$, and to define "upper" and "lower" components for a given spinor $\psi$. These are written $\psi^{(+)}$ and $\psi^{(-)}$, respectively, and are given by

$$\psi^{(\pm)}\equiv\Lambda^{(\pm)}\psi .$$

The Rarita-Schwinger field is described by the 16-component (Hermitian)vector-spinor $\psi_\mu$. The free-field Lagrangian is given by

$$\mathcal{L}=-\tfrac{1}{4}\psi^\mu\beta(g_{\mu\nu}\gamma^\alpha-\delta^\alpha_\mu\gamma_\nu-\delta^\alpha_\nu\gamma_\mu-\gamma_\mu\gamma^\alpha\gamma_\nu)p_\alpha\psi^\nu+\text{H.c.}$$
$$-\tfrac{1}{2}m\psi^\mu\beta(g_{\mu\nu}+\gamma_\mu\gamma_\nu)\psi^\nu , \quad (1)$$

where $p_\mu\equiv-i\partial_\mu$, and leads to the field equations

$$(\gamma\cdot p+m)\psi^\mu-p^\mu(\gamma\cdot\psi)-\gamma^\mu(p\cdot\psi)$$
$$-\gamma^\mu(\gamma\cdot p-m)(\gamma\cdot\psi)=0 . \quad (2)$$

The equations with $\mu=+$ are constraints, free of derivatives with respect to $x^+$

$$(\Lambda^{(+)}-\Lambda^{(-)})[(\gamma_i p_i+m)\psi^+ + p_-\zeta]$$
$$+\gamma^+[(m-\tfrac{1}{2}\gamma_i p_i)\zeta-p_i\eta_i]=0 ,$$

where the transverse components $\psi_i$ have been decomposed into a traceless part $\eta_i$ and a trace part $\zeta$ according to

$$\psi_i=\eta_i-\tfrac{1}{2}\gamma_i\zeta, \quad \gamma_i\eta_i=0, \quad \zeta=\gamma_i\psi_i . \quad (3)$$

Separating the upper and lower components, one gets

$$(\gamma_i p_i+m)\psi^{+(+)}+p_-\zeta^{(+)}=0 \quad (4)$$

and

$$(\gamma_i p_i+m)\psi^{+(-)}+p_-\zeta^{(-)}$$
$$+\gamma^+[(\tfrac{1}{2}\gamma_i p_i-m)\zeta^{(+)}+p_i\eta_i^{(+)}]=0 . \quad (5)$$

The upper component of the traceless part of the $\mu=i$ equations is another constraint,

$$p_-\eta_i^{(-)}+(p_i+\tfrac{1}{2}\gamma_i\gamma_j p_j)\psi^{+(-)}$$
$$+\tfrac{1}{2}\gamma^+[(\gamma_j p_j+m)\eta_i^{(+)}-\gamma_i p_j\eta_j^{(+)}]=0 , \quad (6)$$

while the lower component is an equation of motion,

$$p_+\eta_i^{(+)}+(p_i+\tfrac{1}{2}\gamma_i\gamma_j p_j)\psi^{-(+)}$$
$$+\tfrac{1}{2}\gamma^-[(\gamma_j p_j+m)\eta_i^{(-)}-\gamma_i p_j\eta_j^{(-)}]=0 . \quad (7)$$

The remaining eight field equations (the $\mu=i$ equations contracted with $\gamma_i$ and the $\mu=-$ equations) are all equations of motion. Separated into upper and lower components they read

$$p_+\psi^{+(+)}-p_-\psi^{-(+)}-\tfrac{1}{2}m\zeta^{(+)}$$
$$+\tfrac{1}{2}\gamma^-[p_-\zeta^{(-)}+(\gamma_i p_i+2m)\psi^{+(-)}]=0 , \quad (8)$$

$$p_+(\zeta^{(+)}-\gamma^-\psi^{+(-)})+(\gamma_i p_i+2m)\psi^{-(+)}$$
$$+\gamma^-(p_-\psi^{-(-)}-\tfrac{1}{2}m\zeta^{(-)})=0 , \quad (9)$$

$$p_+\zeta^{(-)}+(\gamma_i p_i+m)\psi^{-(-)}=0 , \quad (10)$$

and

$$p_+\zeta^{(+)}+(\gamma_i p_i+m)\psi^{-(+)}$$
$$+\gamma^-[p_i\eta_i^{(-)}+(\tfrac{1}{2}\gamma_i p_i-m)\zeta^{(-)}]=0 . \quad (11)$$

By eliminating $p_+\zeta^{(+)}$ from Eq. (9) with the help of Eq. (11) one obtains

$$p_+\psi^{+(-)}+p_i\eta_i^{(-)}-p_-\psi^{-(-)}+\tfrac{1}{2}(\gamma_i p_i-m)\zeta^{(-)}$$
$$-\tfrac{1}{2}m\gamma^+\psi^{-(+)}=0 . \quad (12)$$

Thus 10 of the 16 field components, namely $\eta_i^{(+)}$, $\psi^{+(\pm)}$, and $\zeta^{(\pm)}$ are dynamical components,[7] i.e., they satisfy equations of motion [Eqs. (7), (8), and (10)–(12)]. Not all of these dynamical components are independent as they are related by the constraints (4) and (5). Equation (6) determines $\eta_i^{(-)}$ while $\psi^{-(\pm)}$ are undetermined at this point.

In order to get the secondary constraints,[7] one operates on the primary constraints involving the dynamical variables only, namely Eqs. (4) and (5), with $p_+$ and eliminates the "time derivatives" using the equations of motion. Thus the $x^+$ derivative of Eq. (5) with Eqs. (7) and (10)–(12) yields

$$\zeta^{(-)}-\gamma^+\psi^{-(+)}=0 , \quad (13)$$

thereby determining $\psi^{-(+)}$. Similarly the $x^+$ derivative of Eq. (4), upon using Eqs. (8) and (11), leads to

$$2\gamma^- p_- p_i\eta_i^{(-)}-m\gamma^- p_-\zeta^{(-)}-m(\gamma_i p_i+m)\zeta^{(+)}$$
$$+\gamma^-(m-\gamma_i p_i)(2m+\gamma_i p_i)\psi^{+(-)}=0 .$$

Upon eliminating the $\eta_i^{(-)}$ terms with the help of Eq. (6) and subsequently simplifying the result using Eq. (5), this finally reduces to

$$\zeta^{(+)}-\gamma^-\psi^{+(-)}=0 , \quad (14)$$

which again involves only the dynamical variables. As a result of Eqs. (4), (5), and (14), precisely four of the ten dynamical variables remain independent, in agreement with the fact that the number of degrees of freedom on the light-front must be half that of the space-time approach.

Equation (14), in conjunction with Eqs. (11), (12), and (13), further leads to the tertiary constraint

$$p_- \psi^{-(-)} - \tfrac{1}{2}(\gamma_i p_i - m)\zeta^{(-)} = 0\,, \qquad (15)$$

which determines the remaining components $\psi^{-(-)}$. This completes the proof that the free Rarita-Schwinger field on the light front has four degrees of freedom.

Before proceeding to the interacting case it is worthwhile to note that the secondary constraints [Eqs. (13) and (14)] are nothing but the lower and upper components of the equation $\gamma^\mu \psi_\mu = 0$, which is most conveniently derived by the covariant procedure of contracting the field equation with $\gamma_\mu$ and $p_\mu$, respectively, and subsequently eliminating the derivative terms between the resulting equations. This is the procedure to be used in obtaining the secondary constraints in the interacting case.

## III. THE INTERACTING FIELD

In the presence of interactions one has the same ten dynamical variables as in the free-field case. These satisfy equations of motion corresponding to Eqs. (7), (8), (10), (11), and (12), while the field equations corresponding to Eqs. (4)–(6) continue in their role as primary constraints. This indicates that three more (two-component) constraints are needed if one is to obtain the desired four degrees of freedom appropriate to a spin-$\frac{3}{2}$ field. For the free field the derivation of these is made possible by two things. First, two of the primary constraints involve the dynamical variables only [the third, namely Eqs. (6), determines $\eta_i^{(-)}$] thereby leading to two secondary constraints, and secondly, whereas one of the secondary constraints determines $\psi^{-(+)}$, the other again involves the dynamical variables alone and leads to the tertiary constraint. Inasmuch as the primary constraint corresponding to (6) still contains $\eta_i^{(-)}$, it is essential that the nature of the remaining constraints be left unchanged by the interaction. However, it will be seen below that even when the primary constraints remain essentially intact, the secondary constraints in general involve $\psi^{-(-)}$ in addition to $\psi^{-(+)}$. As a result the equation corresponding to (14) serves to determine $\psi^{-(-)}$, thereby precluding the existence of the tertiary constraint. One is thus left with six degrees of freedom rather than the required four.

### A. Electromagnetic interactions

We begin with the minimal electromagnetic coupling achieved by the usual replacement

$$p_\mu \to \pi_\mu = p_\mu - eqA_\mu$$

in the field equations, where $q$ is the matrix $\left(\begin{smallmatrix} 0 & -i \\ i & 0 \end{smallmatrix}\right)$ in the two-dimensional charge space. Once again Eq. (6a) [Eq. (6) with minimal coupling[8]] determines $\eta_i^{(-)}$, whereas Eqs. (4a) and (5a) are constraints involving the dynamical variables only, and lead to secondary constraints. In order to obtain the latter, one first contracts Eq. (2a) with $\gamma^\mu$, thereby obtaining

$$(\gamma\cdot\pi)(\gamma\cdot\psi) + (\pi\cdot\psi) - \tfrac{3}{2}m(\gamma\cdot\psi) = 0\,, \qquad (16)$$

while contraction of Eq. (2a) with $\pi_\mu$ yields

$$m[(\gamma\cdot\pi)(\gamma\cdot\psi) + (\pi\cdot\psi)] - ieq\gamma^\mu F_{\mu\nu}\psi^\nu - \tfrac{1}{2}\, ieq(\gamma^\mu\gamma^\nu F_{\mu\nu})(\gamma\cdot\psi) = 0\,, \qquad (17)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The elimination of the derivative terms from Eqs. (16) and (17) results in the usual secondary constraint

$$(3m^2 - i\gamma^\mu\gamma^\nu F_{\mu\nu})(\gamma\cdot\psi) - 2ieq\gamma^\mu F_{\mu\nu}\psi^\nu = 0\,.$$

Separating the upper and lower components one obtains

$$(1+QF_{+-})\zeta^{(+)} - (1-Q\gamma^1\gamma^2F_{12})\gamma^-\psi^{+(-)} - Q(\gamma^-F_{-i}\eta_i^{(-)} + \gamma^iF_{-i}\psi^{-(+)} + \tfrac{1}{2}\gamma^-\gamma^iF_{-i}\zeta^{(-)} - \gamma^iF_{+i}\psi^{+(+)}) = 0\,, \qquad (18)$$

and

$$(1+QF_{+-})\zeta^{(+)} - (1-Q\gamma^1\gamma^2F_{12})\gamma^+\psi^{-(+)} - Q(\gamma^+F_{+i}\eta_i^{(+)} + \tfrac{1}{2}\gamma^+\gamma^iF_{+i}\zeta^{(+)} + \gamma^iF_{+i}\psi^{+(-)} - \gamma^iF_{-i}\psi^{-(-)}) = 0\,, \qquad (19)$$

where $Q \equiv \frac{2}{3}ieqm^{-2}$.

Equations (18) and (19) correspond to Eqs. (14) and (13), respectively, in the free-field case, the latter being recovered in the limit $e\to 0$. However, they differ drastically from the free equations in that (18) no longer involves the dynamical variables alone, but contains a term involving $\psi^{-(+)}$. The latter cannot be eliminated, as Eq. (19), which in the free case determines $\psi^{-(+)}$, now contains the hitherto undetermined components $\psi^{-(-)}$. Thus the only constraints among the ten dynamical variables are Eqs. (4a) and (5a), leading to the result that in general the minimally coupled theory has six degrees of freedom rather than four.

However, consistency is possible if $F_{-i} = 0$, for which case Eq. (19) becomes independent of $\psi^{-(-)}$ and a definition of $\psi^{-(+)}$ results. Equation (18) then involves the dynamical variables alone thereby leading to the desired four degrees of freedom. Finally, the use of Eq. (18) in conjunction with the equations of motion results in the tertiary constraint, corresponding to (15), which defines $\psi^{-(-)}$.

The condition $F_{-i}=0$, required for a consistent theory, can of course be satisfied only locally, and in a limited set of Lorentz frames. Thus one concludes that the minimally coupled theory is inconsistent on the light front even at the classical level.

It can also be shown that the introduction of anomalous moment terms does not improve the situation. In general, this involves an addition of

$$\mathcal{L}'=\tfrac{1}{2}e\psi^{\mu}\beta q[iaF_{\mu\nu}+a'\gamma_5\overline{F}_{\mu\nu}+bg_{\mu\nu}\sigma^{\alpha\beta}F_{\alpha\beta} + c(F_{\mu\alpha}\sigma^{\alpha}{}_{\nu}+F_{\nu\alpha}\sigma^{\alpha\mu})]\psi^{\nu}$$

to the minimally coupled Lagrangian. Here $a$, $a'$, $b$, $c$ are arbitrary real constants, and

$$\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=i\gamma^1\gamma^2(\Lambda^{(+)}-\Lambda^{(-)}),$$

$$\sigma_{\mu\nu}=\tfrac{1}{2}i[\gamma_\mu,\gamma_\nu],$$

and

$$\overline{F}_{\mu\nu}=\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta},$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor with $\epsilon^{+-12}=-1$. The requirement that the primary constraints retain their proper form, i.e., that the equations corresponding to (4) and (5) involve only the dynamical variables, results in the condition[9]

$$a=a',\quad c=0.$$

The secondary constraints now involve the derivatives of the electromagnetic field. Since the manipulation becomes extremely tedious, it will not be reproduced here. It is sufficient to note that the inconsistency persists even if the term involving the derivatives of the electromagnetic field are dropped, as their inclusion only makes the constraint problem more difficult.

### B. Other interactions

There exist two other interactions of the Rarita-Schwinger field which have received some attention in the literature. These involve the coupling to a scalar field $\phi(x)$ through the substitution[10]

$$m\to M(x)\equiv m+g\phi(x) \tag{20}$$

in the free Lagrangian, and an interaction with a scalar field $\phi(x)$ and a Dirac field $\psi(x)$ according to the interaction Lagrangian[11]

$$\mathcal{L}'=-\tfrac{1}{2}g[\psi^{\mu}\beta(g_{\mu\nu}+\gamma_\mu\gamma_\nu)\psi\partial^{\nu}\phi+\text{H.c.}]. \tag{21}$$

In the case where the scalar field is taken to be external, the discussion follows along the same lines as in the case of electromagnetic coupling. We briefly present the results of the calculation.

For the interaction given by (20), two of the primary constraints, obtained by ths substitution $m\to M$ in Eqs. (4) and (5), connect the dynamical variables. A third constraint, corresponding to Eq. (6), determines $\eta_i^{(-)}$. The secondary constraints are easily found to be

$$(1+\gamma^{\mu}a_{\mu})(\gamma\cdot\psi)+(a\cdot\psi)=0, \tag{22}$$

where $a_\mu\equiv-\frac{2}{3}iM^{-2}\partial_\mu M$. The upper and lower components of Eq. (22) are

$$(1+\tfrac{1}{2}\gamma_i a_i)\zeta^{(+)}-\gamma^-(1-\gamma_i a_i)\psi^{+(-)}+a_i\eta_i^{(+)} +\gamma^-a_-\zeta^{(-)}+a_+\psi^{+(+)}-a_-\psi^{-(+)}=0 \tag{23}$$

and

$$(1+\tfrac{1}{2}\gamma_i a_i)\zeta^{(-)}-\gamma^+(1-\gamma_i a_i)\psi^{-(+)} +a_i\eta_i^{(-)}+\gamma^+a_+\zeta^{(+)}-a_+\psi^{+(-)}-a_-\psi^{-(-)}=0. \tag{24}$$

Once again both $\psi^{-(+)}$ and $\psi^{-(-)}$ appear in the secondary constraints unless $a_-=0$. In the latter case Eq. (23) involves the dynamical variables alone, leading to four degrees of freedom. The tertiary constraint following from Eq. (23) and the equations of motion lead in the usual way to a determination of $\psi^{-(-)}$.

Next, consider the interaction given by Eq. (21). Since the coupling is linear in $\psi_\mu$, the inconsistency appears only when the equations for the Dirac field are also included in the analysis. The Lagrange equations for the fermion fields are

$$(\gamma\cdot p+M)\psi+gA^{\nu}(g_{\mu\nu}+\gamma_\mu\gamma_\nu)\psi^{\mu}=0 \tag{25}$$

and

$$(\gamma\cdot p+m)\psi_\mu-p_\mu(\gamma\cdot\psi)-\gamma_\mu(p\cdot\psi) -\gamma_\mu(\gamma\cdot p-m)(\gamma\cdot\psi)+g(g_{\mu\nu}+\gamma_\mu\gamma_\nu)A^{\nu}\psi=0, \tag{26}$$

where $A_\mu\equiv\partial_\mu\phi$. The upper and lower components of Eq. (25) are

$$p_-\psi^{(-)}+\tfrac{1}{2}\gamma^+(\gamma_i p_i+M)\psi^{(+)}+gA_-(\zeta^{(-)}-\tfrac{1}{2}\gamma^+\psi^{-(+)})+\tfrac{1}{2}g\gamma^+[(\gamma_i A_i)(\tfrac{1}{2}\zeta^{(+)}-\gamma^-\psi^{+(-)})+A_i\eta_i^{(+)}+A_+\psi^{+(+)}]=0. \tag{27}$$

and

$$p_+\psi^{(+)}+\tfrac{1}{2}\gamma^-(\gamma_i p_i+M)\psi^{(-)}+gA_+(\zeta^{(+)}-\tfrac{1}{2}\gamma^-\psi^{+(-)})+\tfrac{1}{2}g\gamma^-[(\gamma_i A_i)(\tfrac{1}{2}\zeta^{(-)}-\gamma^+\psi^{-(+)}+A_i\eta_i^{(-)}+A_-\psi^{-(-)}]=0. \tag{28}$$

Equation (27) determines $\psi^{(-)}$ while Eq. (28) is an equation of motion for $\psi^{(+)}$. As expected there are two degrees of freedom associated with the Dirac field.

The upper and lower components of Eq. (26) for $\mu \equiv +$ yield, respectively,

$$(\gamma_i p_i + m)\psi^{+(+)} + p_- \zeta^{(+)} - gA_- \psi^{(+)} = 0 \tag{29}$$

and

$$(\gamma_i p_i + m)\psi^{+(-)} + p_- \zeta^{(-)} - gA_- \psi^{(-)} - \gamma^+[(m - \tfrac{1}{2}\gamma_i p_i)\zeta^{(+)} - p_i \eta_i^{(+)} + g\gamma_i A_i \psi^{(+)}] = 0 . \tag{30}$$

Upon elimination of $\psi^{(-)}$ from Eq. (30) using Eq. (27) there results a constraint containing $\psi^{-(+)}$. Equation (29) is then the only relation among the dynamical variables. As the secondary constraint resulting from the $x^+$ derivative of Eq. (29) contains $\psi^{-(-)}$ due to the term $p_+\psi^{(+)}$, no tertiary constraint can be obtained. Thus one is left with only 3 two-component constraints [namely Eqs. (29), (30), and the secondary constraint resulting from (29)] for the Rarita-Schwinger field, implying ten degrees of freedom. It can easily be verified that the situation is remedied if and only if one imposes the noncovariant condition $A_- = 0$.

## IV. THE ANTICOMMUTATION RELATIONS

It is of considerable interest to determine whether the interacting field can be consistently quantized on the light front, at least in the case for which the required number of constraints exists. Using the minimally coupled field as an example, we present here the results of such calculations.

The anticommutators follow in the usual way from the action principle.[8] Thus one has

$$i[G, \chi(x)] = \tfrac{1}{2}\, \delta\chi(x) \tag{31}$$

for any field variable $\chi$, where the generator for the infinitesimal field variations is given by the surface terms in the corresponding variation of the action integral

$$\delta S = G(\sigma_1) - G(\sigma_2) .$$

The generator on the surface $x^+ =$ constant is readily determined from the Lagrangian (1) to be

$$G(x^+) = \frac{i}{2} \int d^3x \left( \sqrt{2}\, \eta_i^{(+)} \delta\eta_i^{(+)} + \frac{1}{\sqrt{2}}\, \zeta^{(+)}\delta\zeta^{(+)} + \zeta^{(-)}\beta\delta\psi^{+(+)} - \psi^{+(+)}\beta\delta\zeta^{(-)} + \psi^{+(-)}\beta\delta\zeta^{(+)} - \zeta^{(+)}\beta\delta\psi^{+(-)} \right) ,$$

where $d^3x = dx_1 dx_2 dx^-$. The variations of various field components must, of couse, be compatible with the constraints (4a), (5a), and (18), namely,

$$(\gamma_i \pi_i + m)\psi^{+(+)} + \pi_- \zeta^{(+)} = 0 , \tag{4a}$$

$$(\gamma_i \pi_i + m)\psi^{+(-)} + \pi_- \zeta^{(-)} + \tfrac{1}{2}\gamma^+ (\gamma_i \pi_i - 2m)\zeta^{(+)} + \gamma^+ \pi_i \eta_i^{(+)} = 0 , \tag{5a}$$

and

$$(1 + QF_{+-})\zeta^{(+)} - (1 - Q\gamma^1\gamma^2 F_{12})\gamma^- \psi^{+(-)} + Q\gamma^i F_{+i}\psi^{+(+)} = 0 . \tag{18}$$

Note that $F_{-i}$ has been set equal to zero in order to obtain the correct form of (18).

The constraints are most conveniently handled by the method of Lagrange multipliers. Thus Eq. (31) is written as

$$\begin{aligned}\int d^3x' \Big( & \delta\chi(x')\delta^{(3)}(x'-x) - \sqrt{2}\, \{\eta_i^{(+)}(x'), \chi(x)\}\, \delta\eta_i^{(+)}(x') - \frac{1}{\sqrt{2}}\, \{\zeta^{(+)}(x'), \chi(x)\}\, \delta\zeta^{(+)}(x') - \{\psi^{+(-)}(x'), \chi(x)\}\, \beta\delta\zeta^{(+)}(x') \\ & + \{\zeta^{(+)}(x'), \chi(x)\}\, \beta\delta\psi^{+(-)}(x') + \{\psi^{+(+)}(x'), \chi(x)\}\, \beta\delta\zeta^{(-)}(x') - \{\zeta^{(-)}(x'), \chi(x)\}\, \beta\delta\psi^{+(+)}(x') \\ & + [(m - \gamma_i \pi_i')\Sigma^{(1)}(x', x)]\, \delta\psi^{+(+)}(x') - [\pi_-' \Sigma^{(1)}(x', x)]\, \delta\zeta^{(+)}(x') - [\pi_-' \Sigma^{(2)}(x', x)]\, \delta\zeta^{(-)}(x') \\ & + [(m - \gamma_i \pi_i')\Sigma^{(2)}(x', x)]\, \delta\psi^{+(-)}(x') + [(\pi_i' + \tfrac{1}{2}\gamma_i\gamma_j\pi_j')\gamma^-\Sigma^{(2)}(x', x)]\, \delta\eta_i^{(+)}(x') + \tfrac{1}{2}[(\gamma_i\pi_i' + 2m)\gamma^-\Sigma^{(2)}(x', x)]\, \delta\zeta^{(+)}(x') \\ & + [(1 - QF_{+-})\Sigma^{(3)}(x', x)]\, \delta\zeta^{(+)}(x') + [Q\gamma^i F_{i+}\Sigma^{(3)}(x', x)]\, \delta\psi^{+(+)}(x') + [(1 - Q\gamma^1\gamma^2 F_{12})\gamma^+\Sigma^{(3)}(x', x)]\, \delta\psi^{+(-)}(x') \Big) = 0 ,\end{aligned} \tag{32}$$

where all variations are now treated as independent. Here $\delta^{(3)}(x - x') \equiv \delta(x_1 - x_1')\delta(x_2 - x_2')\delta(x^- - x'^-)$, and $\Sigma^{(i)}(x', x)$, $i = 1, 2, 3$, are undetermined multipliers, to be evaluated from the requirement that the resulting anticommutators be compatible with the constraints (4a), (5a), and (18).

For $\chi \equiv \eta_j^{(+)}$, Eq. (32) yields

$$\{\eta_i^{(+)}(x'),\eta_j^{(+)}(x)\}=\frac{1}{\sqrt{2}}\Lambda^{(+)}P_{ij}\delta^{(3)}(x-x')+\frac{1}{\sqrt{2}}P_{ik}\pi_k'\gamma^-\Sigma^{(2)},$$
$$\{\zeta^{(+)}(x'),\eta_j^{(+)}(x)\}=\beta(m-\gamma_i\pi_i')\Sigma^{(2)}+\sqrt{2}\,(1-Q\gamma^1\gamma^2F_{12})\Sigma^{(3)},$$
$$\{\psi^{+(-)}(x'),\eta_j^{(+)}(x)\}=\beta\pi_-'\Sigma^{(1)}-\frac{1}{\sqrt{2}}m\Sigma^{(2)}+\beta Q(F_{+-}-\gamma^1\gamma^2F_{12})\Sigma^{(3)},\tag{33}$$
$$\{\psi^{+(+)}(x'),\eta_j^{(+)}(x)\}=-\beta\pi_-'\Sigma^{(2)},$$

and

$$\{\zeta^{(-)}(x'),\eta_j^{(+)}(x)\}=\beta(\gamma_i\pi_i'-m)\Sigma^{(1)}-\beta Q\gamma^iF_{i+}\Sigma^{(3)},$$

where

$$P_{ij}=\delta_{ij}+\tfrac{1}{2}\gamma_i\gamma_j$$

and

$$\pi_\mu'=-i\frac{\partial}{\partial x'^\mu}-eqA_\mu(x').$$

On substituting thse in Eqs. (4a) and (5a), one gets

$$\Sigma^{(3)}=0$$

and

$$\Sigma^{(2)}=\frac{\sqrt{2}}{3m^2}\beta(1-Q\gamma^1\gamma^2F_{12})^{-1}\pi_i'P_{ij}\Lambda^{(+)}\delta^{(3)}(x-x'),$$

whereas Eq. (18) leads to the rather involved relation

$$\sqrt{2}\,(1-Q\gamma^1\gamma^2F_{12})\pi_-'\Sigma^{(1)}=\beta[(1+QF_{+-})(m-\gamma_i\pi_i')+Q\gamma^iF_{+i}\pi_-'+m(1-Q\gamma^1\gamma^2F_{12})]\Sigma^{(2)}.$$

These determine the $\Sigma^{(i)}$.

Following this procedure all the anticommutators can be obtained. The simpler ones are listed below.

$$\{\eta_i^{(+)}(x'),\eta_j^{(+)}(x)\}=\frac{1}{\sqrt{2}}P_{ik}\left\{\delta_{kl}+\frac{2}{3m^2}\pi_k'[1-Q\gamma^1\gamma^2F_{12}(x')]^{-1}\pi_l'\right\}P_{lj}\Lambda^{(+)}\delta^{(3)}(x-x'),$$
$$\{\zeta^{(+)}(x'),\zeta^{(+)}(x)\}=\frac{\sqrt{2}}{3m^2}(m+\gamma_i\pi_i')[1-Q\gamma^1\gamma^2F_{12}(x')]^{-1}(\gamma_i\pi_i'-m)\Lambda^{(+)}\delta^{(3)}(x-x'),$$
$$\{\psi^{+(+)}(x'),\psi^{+(+)}(x)\}=\frac{\sqrt{2}}{3m^2}\pi_-'[1-Q\gamma^1\gamma^2F_{12}(x')]^{-1}\pi_-'\Lambda^{(+)}\delta^{(3)}(x-x'),$$
$$\{\zeta^{(+)}(x'),\psi^{+(+)}(x)\}=-\frac{\sqrt{2}}{3m^2}(m+\gamma_i\pi_i')[1-Q\gamma^1\gamma^2F_{12}(x')]^{-1}\pi_-'\Lambda^{(+)}\delta^{(3)}(x-x'),$$
$$\{\psi^{+(+)}(x'),\eta_j^{(+)}(x)\}=-\frac{\sqrt{2}}{3m^2}\pi_-'[1-Q\gamma^1\gamma^1F_{12}(x')]^{-1}\pi_i'P_{ij}\Lambda^{(+)}\delta^{(3)}(x-x'),$$
$$\{\zeta^{(+)}(x'),\eta_j^{(+)}(x)\}=\frac{\sqrt{2}}{3m^2}(m+\gamma_i\pi_i')[1-Q\gamma^1\gamma^2F_{12}(x')]^{-1}\pi_i'P_{ij}\Lambda^{(+)}\delta^{(3)}(x-x').$$

One notices that the free-limit ($e\to 0$) leads to commutation relations which are quite consistent, and indeed one hardly expects any trouble (at this level anyway) for free fields.

In the interacting case, one can easily see from the first three anticommutators that an indefinite metric is required whenever $|Q\gamma^1\gamma^2F_{12}|>1$, i.e., when $2e|F_{12}|>3m^2$.

One may equally well try to quantize the interacting field for the case $F_{-i}\neq 0$, for which Eqs. (4a) and (5a) are the only constraints among the dynamical variables. Some of the resulting anticommutators are

$$\{\eta_i^{(+)}(x'),\eta_j^{(+)}(x)\}=\frac{1}{\sqrt{2}}P_{ij}\Lambda^{(+)}\delta^{(3)}(x-x'),$$
$$\{\zeta^{(+)}(x'),\zeta^{(+)}(x)\}=0=\{\psi^{+(+)}(x'),\psi^{+(+)}(x)\}=\{\psi^{+(+)}(x'),\eta_j^{(+)}(x)\}=\{\psi^{+(+)}(x'),\zeta^{(+)}(x)\}=\{\zeta^{(+)}(x'),\eta_j^{(+)}(x)\},\tag{34}$$
$$\{\zeta^{(+)}(x'),\zeta^{(-)}(x)\}=\beta(m-\gamma_i\pi_i')\frac{\gamma_jF_{j-}}{ieqF_{-k}F_{-k}}(m-\gamma_i\pi_i')\Lambda^{(-)}\delta^{(3)}(x-x').$$

The last of Eq. (34) clearly displays the singular nature of the theory in the $e \to 0$ limit, while its incompatibility with the second anticommutator follows from the observation that the latter implies $\xi^{(+)} = 0$.

## V. CONCLUDING REMARKS

In view of the results presented here it does not appear possible that the light-front approach can offer a convenient framework for the treatment of higher-spin fields. As evidenced by the free-field equations, the convenience of dealing with half as many degrees of freedom is amply compensated by the added complexity of the constraint structure. The interacting cases, on the other hand, are inconsistent at a much more primitive level, namely the loss of constraints.

As is well known, the coupled spin-$\frac{3}{2}$ field in the space-time approach is plagued with problems such as noncausal propagation at the classical level, and the appearance of indefinite metric in the quantized theory. The question naturally arises as to whether the problems found in the light-front approach might not be related to these. It has indeed been verified that all known models suffering from noncausal modes of propagation also suffer from a loss of constraints on the light front.[12] If the converse can be established, counting the constraints on the light front may offer a convenient check on the consistency of the space-time version and at the same time allow one to avoid the tedious calculations required in the Velo-Zwanziger approach to noncausality.

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[6]Greek tensor indices range over the values +, −, 1, 2; the latin ones take the values 1, 2.

[7]The terminology is that of Johnson and Sudarshan, Ref. 3.

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[9]We note that in the space-time approach, the consistency of the primary constraints requires $b = c = 0$.

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