Is an electron's charge independent of its velocity?*

D. F. Bartlett
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540 and Department of Physics, University of Colorado, Boulder, Colorado 80309†

B. F. L. Ward
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540 and Department of Physics, Purdue University, West Lafayette, Indiana 47907†

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We hypothesize that a particle's charge varies weakly with its velocity. Specifically, suppose that the charge of a slow electron (or proton) varies as $q=e(1+kv^2/c^2)$, where $|k|<<1$. This hypothesis can be tested for conduction electrons by rotating a current-carrying solenoid inside a Faraday ice pail. Alternatively, a block of metal may be cooled inside an ice pail. Using either of these two techniques we find $|k|<10^{-5}$. For free electrons, we use electron beam measurements of others to determine that $|k|<0.2$. Finally a limit of $|k|<2\times10^{-15}$ is inferred for bound electrons from existing data on the neutrality of various atoms.

INTRODUCTION

A recent measurement suggests that a moving charge may be surrounded by a velocity-dependent electric field in addition to the normal Coulombic field. Specifically, Edwards proposed a second-order field of the form

$$\vec{E}_2 = k\frac{ev^2\hat{r}}{r^2c^2}. \qquad (1)$$

Here e is the customary electronic charge, v is the particle's velocity, r is the distance from particle to observer, and k is a constant of order unity.[1]

Since Edwards has also given a comprehensive review of the history of such proposals, we shall be content to examine the experimental consequences. To do so, it is convenient to integrate E_2 over a closed surface surrounding the moving charge. Applying Gauss's law, we find that

$$\oint \vec{E}_2\cdot d\vec{S} = 4\pi\frac{kev^2}{c^2}.$$

How should we interpret this integral? Purcell[2] has argued cogently that the only natural way to define the charge of a *moving* particle is by evaluating $\oint \vec{E}\cdot d\vec{S}$. Adopting this view, the effect of Eq. (1) is to give a velocity dependence to the charge of the moving particle

$$q=e(1+kv^2/c^2). \qquad (2)$$

In the following we show how a variety of techniques can be used to set limits on k.

SPINNING COIL

The experimental investigation of a v^2/c^2 term for conduction electrons is made difficult by the extreme smallness of the effect. Even a 10-A current flowing through a 1-mm-diam copper wire has a drift velocity of only $3\times10^{-12}\times c$. Edwards sought to enhance this small velocity by using superconducting wire. Here the density of conduction electrons is lower, and hence their drift velocity is higher than it is for a normal metal.

We choose rather to increase the electron's drift velocity by mechanically moving the current-carrying wire. Imagine a current of electrons which has a drift velocity v with respect to a wire which itself is moving with a velocity V as seen by a stationary observer. The observer records an electron velocity (squared) of $(V+v)^2=V^2+2Vv+v^2$; whereas the residual positive ions at the lattice sites have a velocity of only V. Thus, there is an unbalanced contribution of $2Vv+v^2$ to Eq. (2). Since V may be several meters per second, while v is only a fraction of a millimeter per second, the v^2 term may now be neglected, leaving as the apparent charge of an electron plus residual ion:

$$q=k2evV/c^2.$$

This velocity-dependent charge can be investigated by determining whether the potential of an isolated, hollow conductor may be changed by spinning a solenoid about its axis inside the cavity. Since $\vec{E}=0$ everywhere in the conducting shell, the spinning coil will induce an opposing charge on the inner wall of the shell. If the hollow conductor is uncharged when the solenoid is at rest, upon spinning the solenoid, the outside of the conductor will acquire a charge

$$Q_2=kn\frac{2evV}{c^2}, \qquad (3)$$

where n is the number of conduction electrons in

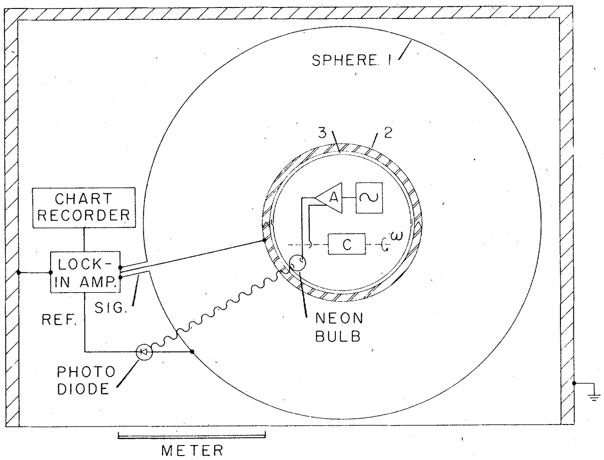

FIG. 1. Schematic of the apparatus. The signal from the oscillator is amplified (A) and applied by brushes to the rotating coil (C). A lock-in amplifier searches for an induced voltage in phase with the oscillator's signal.

the moving wire.

To apply this equation to a practical case, note that $nev = IL$, where I is the measured current passing through a wire of length L. In particular, for a solenoid of radius a and total turns N which rotates with a frequency $\nu = V/2\pi a$, we have

$$Q_2 = kN\frac{8\pi^2 a^2 I\nu}{c^2}. \tag{4}$$

This charge will elevate the potential of the hollow conductor by an amount $\Delta U = Q_2/C$, where C is the capacitance between the conductor and ground. Thus the dimensionless constant k can be determined from measured quantities by the formula

$$k = \frac{(\Delta U)Cc^2}{8\pi^2 a^2 N I \nu}. \tag{5}$$

The spinning-coil measurement was originally proposed and executed some time ago by Tate[3] who found $k \approx 1$. In 1971, using a similar technique, we made the following null measurement of k.

In our experiment, a 2000-A-turn, 5-cm-diam solenoid was spun about its axis at about 40 revolutions per second. The coil was inside a 100-pF spherical capacitor. A potential difference as small as 1 nV could have readily been detected. Using Eq. (5), which is equally valid in cgs and MKSA units, the expected sensitivity is then $k \approx 10^{-5}$.

Our ability to detect such small potentials is based on using an alternating current rather than a direct one.[4] If one uses a direct current, he is generally limited by thermal fluctuations in contact potential to detecting signals only as small

TABLE I. Dimensions of spheres.

Sphere number	Material	Mean diameter (m)	Thickness (cm)
1	Aluminum	2.96	0.06
2	Steel	1.10	5.0
3	Copper	1.00	0.19

as a millivolt.[5] By using a sinusoidally varying current and detecting the signal with a lock-in amplifier, however, one can approach the Johnson noise limit which, for our apparatus, is about a nanovolt. Fortunately most of the apparatus needed for this search as well as the general technique was already developed having been used previously in a test of Coulomb's law.[6]

The basic arrangement consists of three concentric spheres (see Fig. 1). A battery-powered dc motor spun the solenoid inside the innermost sphere; the induced signal between the two outermost spheres was measured. The spheres' dimensions are given in Table I.

The largest sphere, No. 1, was formed by bolting together two 10-ft-diam silo covers. This sphere served as a local ground. The remaining spheres were made from mating hemispheres which could be separated. Spheres Nos. 2 and 3 gave partial shielding against the solenoid's alternating magnetic fields which could induce an emf across stray inductances in the detector.

Such fringing fields were found to give a spurious signal of about 30 nV even when the solenoid was not spinning. This unwanted signal was reduced a decade by installing a stationary coil (S) coaxially with the rotating coil (R) (see Fig. 2). The stationary coil was wound so as to have a magnetic moment equal but opposite to that of the rotating coil when both were supplied with the same current. Table II lists parameters of these solenoids.

A battery-powered oscillator and amplifier supplied the current for the coils. To maximize the

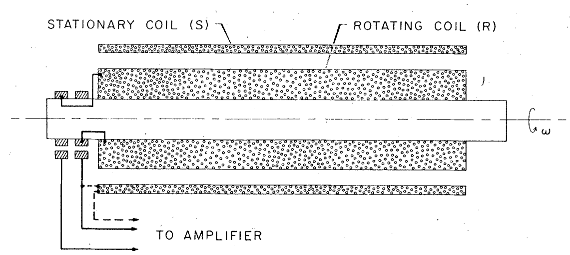

FIG. 2. Rotating coil (R) inside stationary coil (S) of equal but opposite magnetic moment used to reduce stray magnetic fields.

TABLE II. Parameters of solenoids.

Solenoid:	Rotating (R)	Stationary (S)
Length	23 cm	23 cm
Inner diameter	2.5 cm	8.4 cm
Outer diameter	6.0 cm	9.4 cm
rms diameter	4.4 cm	...
Number of turns	3000	750
Wire (Cu) diameter	1.0 mm (No. 18-AWG)	1.0 mm (No. 18-AWG)
Current (rms)	0.6 A	0.6 A
Frequency of current	320 Hz	320 Hz
Frequency of coil rotation	43 rev/sec	0

current at a convenient frequency, the coils were incorporated into a resonant $R\ L\ C$ circuit (see Fig. 3). A 4.8-μF capacitor permitted the circuit to resonate at 320 Hz. The small resistor (8.3Ω) provided needed damping and allowed a calibration of the equipment even when the spheres were closed.

To effect this calibration, an insulated, conducting plug was inserted flush with the outer surface of sphere No. 2. This plug was connected to the $R\ L\ C$ circuit by a socket as indicated on Fig. 3. Reflecting the potential difference across the resistor, the potential of the plug was elevated by about 5 V with respect to sphere No. 2. In the charge-free region between spheres 1 and 2, a mean-value theorem shows that the potential averaged over the surface of *any* sphere times the radius of that sphere is a constant. Since the surface area of the plug was approximately a millionth that of the sphere, we thus predict that an induced potential of 5 μV will be detected between spheres No. 1 and No. 2. This prediction was

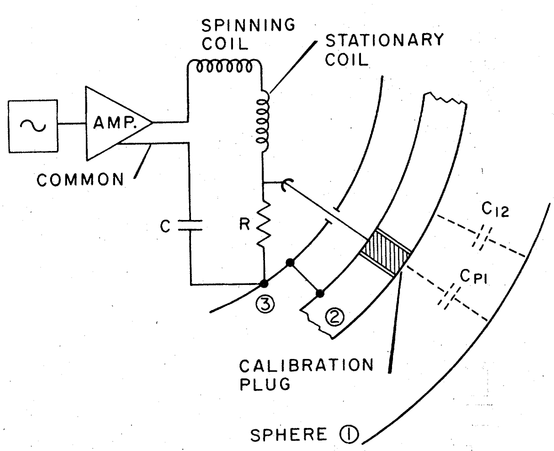

FIG. 3. RLC resonant circuit with calibration plug inserted. $V_2 - V_1 = -(V_p - V_2)C_{p1}/C_{12}$; C_{p1}/C_{12}= area of plug/area of sphere 2.

verified experimentally to within 25%.[7]

The calibrator plug also helped synchronize the phase of the lock-in amplifier[8] to that of the current in the coil. The basic synchronization was provided by a light beam whose intensity was modulated at the frequency of the alternating current. There was, however, an unavoidable small phase shift between the alternating current inside the sphere and the reference signal received by the lock-in amplifier. This phase shift was removed by adjusting the phase dial on the lock-in amplifier until a maximum signal was recorded with the calibration plug in place. If we let ϕ be the phase by which the reference signal leads the current in the coil, this procedure establishes the $\phi = 0°$ condition.

MEASUREMENT

The sensitivity of the experiment can be doubled if measurements are made both with the reference signal in phase with the current ($\phi = 0°$) and out of phase ($\phi = 180°$). One half of the difference between the former and latter measurements is the desired signal, $V_2 - V_1$ (the "I signal"). For completeness, we recorded data also with $\phi = 90°$ and 270°. These conditions are particularly sensitive to emf's induced by changing magnetic fields. Thus we have labeled as the "$\dot{I}$ signal" one half the difference between the $\phi = 90°$ signal and the 270° signal.

The sensitivity of the experiment can again be doubled by changing the direction of rotation of the coil. This operation could be performed without otherwise disturbing the system by means of a reversing switch mounted on sphere No. 2.

Data were taken on a chart recorder over several 3-hour periods. At the beginning of each period, the spheres were sealed and a calibration taken to establish the $\phi = 0°$ condition. The calibration plug was then replaced by a dummy plug and data taken in successive 10-min runs with the

TABLE III. Measurements with solenoidal coil (typical 3-h periods).

Arrangement	ω	$V_2 - V_1$ signal (rms, nV) for 10-min runs 0°	180°	I[a]	90°	270°	$\dot{I}$
Spinning coil alone[c]	0	8	−8	8	−33	29	−31
	+	10	−10	10	−29	33	−31
	−	4	−6	5	−30	30	−30
	−[b]	2[b]	2[b]	<1[b]	0[b]	0[b]	<1[b]
Spinning coil with stationary coil[d]	0	−3.0	0.0	−1.5	−3.0	0.0	−1.5
	+	0.0	3.2	−1.6	−0.5	−3.0	1.2
	−	−3.5	−4.0	0.3	0.0	−1.0	−0.5

[a]A positive value for k should produce a positive I signal, $V_2 - V_1$, when $\omega = +$, and a negative signal when $\omega = -$.

[b]These figures are for the case when the frequency of the lock-in detector was set at twice the oscillator frequency. (See text.)

[c]The measured voltages in the four other 3-h periods were about the same as or a little smaller than the one quoted.

[d]Voltages in the quoted 3-h period had an rms signal of 2.4 nV in each 10-min run. Voltages in the 3-h period which was not quoted had a 2.8-nV signal/10-min run.

coil not spinning ($\omega = 0$) and $\phi = 0°$, 90°, 180°, and 270°. This procedure was repeated for $\omega = +$ (angular velocity of the coil is directed away from the motor); then repeated again for $\omega = -$. Finally the calibration was checked and the spheres opened. Data were taken for two 3-hour periods with both spinning and stationary coils and for five 3-hour periods without the stationary coil. In addition, some data were taken using the first harmonic of the modulated light beam as a reference signal. This was done to check for effects that might vary as $e(vV)^2$. Results from a typical 3-hour period are shown in Table III.

The data for the $\dot{I}$ signal show the effectiveness of the bucking current in the stationary solenoid in reducing stray fields. This solenoid reduced the $\dot{I}$ signal by a factor of 20.

To isolate the effect of rotation, we define an over-all I signal as one half the difference between the signal $(V_2 - V_1)$ with $\omega = +$ and that with $\omega = -$. From this data we feel confident that this I signal is less than 3 nV, a value that is only a little larger than the measured noise of 1 nV.[9]

Substituting the parameters for this measurement into Eq. (5) we find

$$|k| < \frac{(3\ \mathrm{nV})(110\ \mathrm{pF})(3\times 10^8\ \mathrm{m/sec})^2}{(3000)(8\pi^2)(2.2\ \mathrm{cm})^2(0.6\ \mathrm{A})(43\ \mathrm{rev/sec})};$$

i.e.,

$$|k| < 1.0\times 10^{-5}.$$

Apparently this value is inconsistent with any measurements (such as those of Refs. 1 and 3) which give $k \approx 1$.

OTHER COIL CONFIGURATIONS

We have considered two other coil configurations. In these the velocity of the conduction electrons, v, is perpendicular rather than parallel to the velocity of the conducting material, $\vec{V}$.

In one case a torus having an inside diameter of 10 cm, an outside diameter of 18 cm and axial length of 5 cm was wound with 3000 turns of 0.64-mm-diam copper wire (see Fig. 4). The torus carried a current of 0.4 A at a frequency of 320

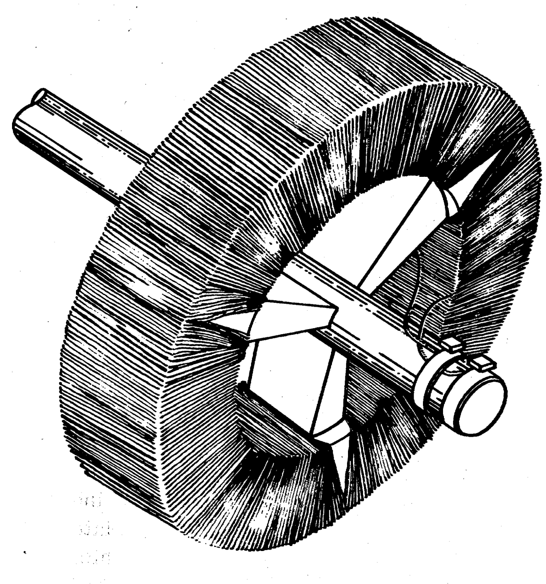

FIG. 4. Toroidally wound coil testing effect of $|\vec{v}\times\vec{V}| \neq 0$.

Hz and was spun at a frequency of 30 rev/sec. The observed I signal, $V_2 - V_1$, was less than 6 nV.

In another case, a copper cylinder 11 cm in diameter and 46 cm long was rotated at 30 rev/sec about its axis. A 3-A current was sent along this axis between two carbon brushes placed at opposite ends of the cylinder. The I signal was again less than 6 nV.

It is difficult to relate these experiments to a hypothesized breakdown of Coulomb's law as was done for our original spinning solenoid, where an anomalous v^2 term appears as a $\vec{v}\cdot\vec{V}$ term. In the case of the toroid or cylinder, however, the scalar product $\vec{v}\cdot\vec{V}$ vanishes when the two vectors are perpendicular. By using the magnetic field vector one could construct a nonvanishing scalar product, $\vec{V}\cdot\vec{H}$, but this unfortunately is a pseudoscalar.

OTHER TESTS

If a velocity-dependent potential existed, should not it be possible to "charge" a block of metal simply by heating it? In this manner the mean speed of the conduction electrons can be raised.[10] The molar specific heat C_V of the conduction electrons in brass at room temperature is about $0.02R$, where R is the gas constant.[11] Admittedly, the specific heat associated with the translational motion of the residual ions is approximately $\frac{3}{2}R$, but by virtue of their much lower mass, the consequent gain in rms velocity of the conduction electrons upon heating is much greater than that of the ions. From Eq. (1), we predict a change in charge upon heating of

$$Q_2 = kN_A e \Delta(v^2/c^2) = \frac{k2eC_V \Delta T}{mc^2}$$
$$= k \times 10^{-6} \text{ Coulombs/deg mol}. \quad (6)$$

We have tested this hypothesis by allowing a 500-g sphere of brass to cool from 100 °C to 25 °C inside a Faraday cup. The capacitance of the cup was 30 pF; hence, we expect a change in potential of $(k \times 10^{-6}) \times (8 \text{ moles}) \times 75°/30$ pF or $(10^7 \times k)$ V. In fact, a well-isolated electrometer sensitive to changes as small as 100 V detected no change in voltage. Hence, we find that $|k|$ must be less than 10^{-5}. Ours was only a rough investigation which could readily be improved.

A direct test of the variation of charge with velocity can be made by seeing whether the apparent charge of a beam of particles changes after it has been accelerated. In several accelerators the passing beam is monitored by the charge it induces on isolated conducting plates placed immediately above and below the beam. By comparing this induced charge with that measured when the beam is subsequently stopped in a Faraday cup, one can check the dependence of charge on velocity.

Such a check can be inferred from the measurements of Simanton.[12] During a calibration of the induction plates used at the Argonne Zero-Gradient Synchrotron, the plates were exposed to a beam of electrons having energies of about 30 keV $(v^2/c^2 = 0.1)$. Since the capacitance of the induction plates was known, the value of the moving charge could be determined by measuring the potential to which the passing beam elevated the plates. Subsequently the value of the stationary charge was measured by stopping the same beam in a Faraday cup of known geometry. Simanton found that the induction plates gave a correct measure of the charge if the plates were assumed to be 98% as long as they actually were. This slight discrepancy could have arisen from instrumental effects, but if we assume it to be entirely owing to a variation of the electron's charge we find that

$$|k| < 0.02/0.1 = 0.2 .$$

By extending this technique to higher energies this limit could probably be improved by at least a decade.[13] Further improvement might be difficult since induction plates, being sensitive to all charged particles, no matter how slow, are vulnerable to noise from collisions of the beam with residual atoms of gas.[14] Alternative beam monitors such as toroids, ionization chambers, quantameters, and calorimeters have been cross calibrated against Faraday cups up to electron energies as high as 20 GeV.[15] The precision of these checks is often better than 1%, but their interpretation in terms of a velocity-dependent charge is not direct.

A much lower limit on k may be set by the observed neutrality of atoms.[16] Recent experiments[17] have shown that a variety of atoms and molecules are extremely neutral $[q(\text{atom})/Z(\text{atom}) < 10^{-21}e]$. Since the mean speed of the bound electrons varies from atom to atom this observed neutrality may be used to set a lower limit on k. A particularly good test is provided by the experiment of King who observed that a container of gas did not become charged as its contents were exhausted.[18] In this manner he showed that the charges of both He and H_2 are less than $1 \times 10^{-20}e$. Since these are both two-electron systems, the mean velocity of the electrons may be estimated directly from the first ionization potentials. Using the virial theorem we find that $v^2/c^2 = 0.7 \times 10^{-4}$ for the electrons in H_2 and $v^2/c^2 = 1.0 \times 10^{-4}$ for He. Thus, from these two-electron states one finds that $|k|$ is less than

$$10^{-20}/2 \times (1 \times 10^{-4} - 0.7 \times 10^{-4}) \text{ or } 2 \times 10^{-16} .$$

It should be emphasized that this limit applies only to electrons *bound* by Coulombic forces in an atom. It is conceivable that in this binding a renormalization of charge can occur which would cancel the effect of a v^2/c^2 term. Therefore, the experiments with free or conduction electrons although less precise than those with atoms are more sensitive to any arbitrary mechanism which might produce a nonvanishing value for k.

In summary, we find no evidence for a change in electric charge with velocity. The most direct technique, that of monitoring the electron's charge in a beam gives a limit $|k|<0.2$. More indirect tests such as are provided by rotating a current-carrying coil or heating a block of metal give limits of $|k|<10^{-5}$. The most precise, but most restricted test, that of neutrality of atoms, gives a limit $|k|<2\times10^{-16}$.

These disparate experimental tests all have a common feature: They use Gauss's law to relate an apparent change in electric charge to an anomalous term in the expression for the electric field of a moving charge.

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†Present address.

[1]W. F. Edwards, *Measurements of an Electric Current Due to Conduction Currents* (Utah State Univ. Press, Logan, Utah, 1974), Eq. (3). An extension of this work has recently been published [W. F. Edwards, C. S. Kenyon, and D. K. Lemon, Phys. Rev. D 14, 922 (1976)]. Here it is stated that the general expression for the electric field of a moving charge to order $1/c^2$ might require as many as five anomalous terms. In all the measurements we discuss, the symmetry of the apparatus makes them insensitive to the two acceleration-dependent terms. In the notation of Edwards, Kenyon, and Lemon we measure $k=\gamma_1+\frac{1}{3}\gamma_2+\frac{1}{3}\gamma_3$.

[2]E. M. Purcell, *Electricity and Magnetism* (McGraw-Hill, New York, 1963), pp. 151–154.

[3]J. Tate, Master's thesis, University of Houston, 1968 (unpublished).

[4]There are two liabilities in using an alternating current. A spurious effect which is consistent with the conventional Maxwell's equations, but is present only because of time-varying fields may be confused with the desired dc effect. Such an occurrence did, in fact, happen. Time-varying magnetic fields gave an unwanted signal. The problem was alleviated by installing a stationary solenoid (see text). Alternatively, an anomalous term which is both quadratic in v and time dependent may just cancel the dc effect. Our measurement, however, is quasistatic in that the wavelength of radiation associated with the alternating current is much longer than the dimensions of the apparatus. Thus, we feel that the possibility of such a cancellation, although real, is remote.

[5]S. J. Plimpton and P. E. Lawton, Phys. Rev. 50, 1066 (1936).

[6]D. F. Bartlett, P. E. Goldhagen, and E. A. Phillips, Phys. Rev. D 2, 483 (1970).

[7]Unfortunately, this calibration was not done directly to the nanovolt level to which our equipment was sensitive. This measurement however, was done immediately following a test of Coulomb's law in which a direct measurement of sensitivity to 70 nV was made (Ref. 6). In addition, the response of the lock-in amplifier to 20-nV signals was regularly checked during the spinning-coil experiment by using this amplifier's internal calibration signal.

[8]Princeton Applied Research Co. Model HR-8 with preamplifier A modified by increasing the input impedance to ground from 10 MΩ to 100 MΩ.

[9]The Johnson noise of our detector for one 10-min run is given by $V=[(4kT)\ (\text{bandwidth})\ (\text{Re}Z)]^{1/2}=2$ nV. Here bandwidth $=1/(2)$ (600 sec) and $\text{Re}Z=R/(1+4\pi^2\nu^2C_{12}{}^2R^2)=3\times10^5\Omega$, where R is the input resistance of the lock-in amplifier (100 MΩ), C_{12} is the capacitance between spheres 1 and 2 (100 pF), and ν is the frequency of the driving circuit (320 Hz). The ability of the apparatus to approach the Johnson noise limit was confirmed experimentally in a mock 3-h period where conditions were left unchanged and the data analyzed into successive 10-min runs. The expected noise in our "over-all" I signal is that associated with a 40-min run or 2 nV/$\sqrt{4}$ = 1 nV.

[10]The speed of conduction electrons is about $kT_F/m=10^{-2}\times c$. Since this speed is so much greater than either the drift velocity v or the mechanical motion V, in the spinning-coil experiment, it may seem implausible to neglect thermal velocities in this discussion leading to Eq. (2). However, since thermal velocities have no preferred direction, they cannot contribute to a cross term like vV.

[11]Cu—65%; Zn—35%. See, for instance, C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1971), 4th edition, Table 2, p. 254.

[12]J. R. Simanton, IEEE Trans. Nucl. Sci. NS-16, 932 (1969). We thank R. L. Martin for telling us about this work.

[13]The motivation for Eq. (2) becomes questionable when $v^2/c^2\approx v^4/c^4$. A possible alternative is $q=e\gamma^{2k}$, where $\gamma=(1-v^2/c^2)^{-1/2}$.

[14]L. Holcomb, D. I. Porat, and K. Robinson, Nucl. Instrum. Methods 24, 399 (1963).

[15]D. Yount, in *Symposium of Beam Intensity Measurements*, edited by V. W. Hatton and S. A. Lowndes (Daresbury, United Kingdom, 1968), p. 75.

[16]See Ref. 2. We thank A. D. Franklin for bringing this point to our attention.

[17]H. F. Dylla and J. G. King, Phys. Rev. A 7, 1224 (1973), and references cited therein.

[18]J. G. King, Phys. Rev. Lett. 5, 562 (1960) with improved experimental result as quoted in Ref. 17.