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Super-Higgs effect in a new class of scalar models and a model of super QED

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We present results on the scalar supermultiplet coupled to supergravity. A locally supersymmetric theory with generalized kinetic and nonderivative interaction terms is found. We discuss a number of examples. Symmetry breakdown and the "super-Higgs" effect are studied, as is a consistent truncation of the SO(4) extended theory. Finally, we find the local extension of supersymmetric massless QED and discuss symmetry breakdown in this system.

I. INTRODUCTION

Supergravity[1] is the gauge theory of local supersymmetry transformations.[2] It has as gauge fields a spin-2 graviton and one (or more in extended theories[3,4]) spin-$\frac{3}{2}$ fields; recently it has been shown that supergravity is the theory of "spinning space,"[5] and as such it is as fundamental as general relativity itself. Pure supergravity can be coupled to various global supermultiplets in order to promote the global supersymmetry to a local one.[6] The resulting theories have many higher-order contact terms in the Lagrangian; gauge invariance of the S matrix and dimensional arguments restrict the contact terms to be at most quartic in the Fermi fields of the theory and allow only linear couplings of $F_{\mu\nu}$, the vector field, to the Fermi fields. The couplings of the scalar fields, however, are under no such restriction, as they are not gauge fields and κ (the Planck length) times a scalar field is dimensionless. Thus, in general, supersymmetric theories with scalar fields can show nonpolynomial structure. The massive scalar multiplet[7] does indeed have nonpolynomial terms, and there are strong indications that the SO(4)-symmetric extended theory does as well.[4]

In Sec. II we consider a very general locally supersymmetric theory of one scalar multiplet which contains as a special case the original scalar multiplet.[8] This theory contains one graviton $V_{a\mu}$, one massless spin-$\frac{3}{2}$ field ψ_μ, one Majorana spinor χ, one scalar A, and one pseudoscalar B (before symmetry breakdown). Further, we find a very general form of the scalar self-interaction term, which contains as a special case the theory of Ref. 7.

In Sec. III we present some details of the construction. In Sec. IV we consider a number of new special cases, including theories with spontaneous symmetry breakdown, and a restriction of the SO(4) extended theory to the gravitational and scalar multiplets only.[9] In Sec. V we present results on the global theory of a vector supermultiplet interacting with two massless scalar multiplets (super-QED),[10] and discuss symmetry breakdown, and finally, in Sec. VI, we present the locally supersymmetric version of this theory.[11]

II. GENERAL SCALAR THEORY

Using the functional method of Ref. 7, we have found the following locally supersymmetric Lagrangian[12]:

$$\mathfrak{L}=\mathfrak{L}_0+\mathfrak{L}_I\,,\quad \mathfrak{L}_0=\mathfrak{L}_{SG}+\mathfrak{L}_{KI}\,, \tag{2.1}$$

$$\mathfrak{L}_{SG}=-\frac{1}{4\kappa^2}VR-\tfrac{1}{2}\epsilon^{\lambda\mu\nu\rho}\bar\psi_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho-\frac{\kappa^2V}{16}\left[(\bar\psi^\lambda\gamma^\mu\psi^\rho)(\bar\psi_\lambda\gamma_\mu\psi_\rho+2\bar\psi_\mu\gamma_\lambda\psi_\rho)-4(\bar\psi\cdot\gamma\psi^\rho)^2\right], \tag{2.2}$$

$$\begin{aligned}\mathfrak{L}_{KI}=&\frac{iV}{2}\bar\chi\not{D}\chi+\frac{V}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2]\Theta(u)-\frac{\kappa V}{\sqrt2}[\bar\psi_\mu\partial_\nu(A+i\gamma_5B)\gamma^\nu\gamma^\mu\chi][\Theta(u)]^{1/2}+\frac{i\kappa^2}{4}\epsilon^{\lambda\mu\nu\rho}(\bar\psi_\lambda\gamma_\mu\psi_\nu)(A\overleftrightarrow{\partial}_\rho B)\Omega'(u)\\&+\frac{\kappa^2V}{8}(\bar\psi_\mu\gamma_5\gamma^a\psi^\mu)(\bar\chi\gamma_5\gamma^a\chi)+\frac{i\kappa^2}{16}\epsilon^{\lambda\mu\nu\rho}(\bar\psi_\lambda\gamma_\mu\psi_\nu)(\bar\chi\gamma_5\gamma_\rho\chi)-\frac{\kappa^2V}{4}(\bar\chi\gamma_5\gamma^\rho\chi)(A\overleftrightarrow{\partial}_\rho B)\left[2\frac{\Theta'(u)}{\Theta(u)}-\Omega'(u)\right]\\&+\frac{\kappa^2V}{32}(\bar\chi\gamma_5\gamma^\rho\chi)(\bar\chi\gamma_5\gamma_\rho\chi)\left[\frac{2}{\Theta(u)}\left(\frac{u\Theta'(u)}{\Theta(u)}\right)'-1\right],\end{aligned} \tag{2.3}$$

where

$$u\equiv\kappa^2(A^2+B^2),\quad \Omega(u)\equiv u\left(1+\sum_{j=1}^{\infty}\omega_j u^j\right),\quad \omega_j \text{ are arbitrary real dimensionless constants},$$

$$\Theta(u)\equiv[u\Omega'(u)]'=1+\sum_{j=1}^{\infty}\omega_j(j+1)^2u^j\,; \tag{2.4}$$

$$\mathcal{L}_I=-\frac{V}{2}\bar{\chi}M(z^*)\chi e^{\Omega(u)/2}-\frac{V}{2}[\Lambda(z)\Lambda(z^*)-3\kappa^2\Phi(z)\Phi(z^*)]e^{\Omega(u)}-\frac{i\kappa V}{\sqrt{2}}\bar{\psi}\cdot\gamma\Lambda(z^*)\chi e^{\Omega(u)/2}-\kappa^2V\bar{\psi}_\mu\sigma^{\mu\nu}\Phi(z)\psi_\nu e^{\Omega(u)/2}, \tag{2.5}$$

where

$$z\equiv A+i\gamma_5B,\quad z^*\equiv A-i\gamma_5B,\quad \Phi(z)\equiv\sum_{j=0}^{\infty}\varphi_j z^j,\quad \varphi_j \text{ are arbitrary real constants of dimension } \kappa^{j-3},$$

$$\Lambda(z)\equiv\frac{1}{[\Theta(u)]^{1/2}}[\Phi'(z)+\kappa^2z^*\Omega'(u)\Phi(z)],$$
$$M(z^*)\equiv\frac{1}{[\Theta(u)]^{1/2}}\left[\Lambda'(z^*)+\kappa^2z\Lambda(z^*)\left(\Omega'(u)-\frac{1}{2}\frac{\Theta'(u)}{\Theta(u)}\right)\right]. \tag{2.6}$$

[Note: $\Omega'(u)\equiv\partial\Omega(u)/\partial u$, $\Phi'(z)\equiv\partial\Phi(z)/\partial z$, $\Lambda'(z^*)\equiv\partial\Lambda(z^*)/\partial z^*$, etc.] $\mathcal{L}_{SG}+\mathcal{L}_{KI}$ is invariant under the following local supersymmetry transformations:

$$\delta V_{a\mu}=-i\kappa\bar{\epsilon}(x)\gamma_a\psi_\mu,\quad \delta A=\frac{1}{\sqrt{2}}\frac{\bar{\epsilon}(x)\chi}{[\Theta(u)]^{1/2}},\quad \delta B=\frac{i}{\sqrt{2}}\frac{\bar{\epsilon}(x)\gamma_5\chi}{[\Theta(u)]^{1/2}},$$

$$\begin{aligned}\delta_{KI}\bar{\chi}=&\frac{i}{\sqrt{2}}\bar{\epsilon}(x)(A+i\gamma_5B)\overleftarrow{\not\partial}[\Theta(u)]^{1/2}-\frac{i\kappa}{2}[(\bar{\psi}_\rho\chi)\bar{\epsilon}(x)\gamma^\rho+(\bar{\psi}_\rho\gamma_5\chi)\bar{\epsilon}(x)\gamma^\rho\gamma_5]\\&-\frac{\kappa^2}{2\sqrt{2}}[\bar{\epsilon}(x)\gamma_5(A+i\gamma_5B)\chi]\left[2\frac{\Theta'(u)}{\Theta(u)}-\Omega'(u)\right]\frac{1}{[\Theta(u)]^{1/2}}\bar{\chi}\gamma_5,\\ \delta_{KI}\bar{\psi}_\lambda=&\kappa^{-1}\bar{\epsilon}(x)\overleftarrow{D}_\lambda+\frac{i\kappa}{4}(2\bar{\psi}_\lambda\gamma_\mu\psi_\nu+\bar{\psi}_\mu\gamma_\lambda\psi_\nu)\bar{\epsilon}(x)\sigma^{\mu\nu}-\frac{i\kappa}{2}\bar{\epsilon}(x)\gamma_5(A\overleftrightarrow{\partial}_\lambda B)\Omega'(u)+\frac{i\kappa}{4}(\bar{\chi}\gamma_5\gamma^\rho\chi)\bar{\epsilon}(x)\gamma_5\sigma_{\rho\lambda}\\&-\frac{\kappa^2}{2\sqrt{2}}[\bar{\epsilon}(x)\gamma_5(A+i\gamma_5B)\chi]\bar{\psi}_\lambda\gamma_5\frac{\Omega'(u)}{[\Theta(u)]^{1/2}}.\end{aligned} \tag{2.7}$$

$\mathcal{L}=\mathcal{L}_{SG}+\mathcal{L}_{KI}+\mathcal{L}_I$ is invariant under the above transformations with the following additional terms:

$$\delta_I\bar{\chi}=\frac{\bar{\epsilon}(x)}{\sqrt{2}}\Lambda(z)e^{\Omega(u)/2},\quad \delta_I\bar{\psi}_\lambda=-\frac{i\kappa}{2}\bar{\epsilon}(x)\Phi(z)\gamma_\lambda e^{\Omega(u)/2}. \tag{2.8}$$

We observe that $\Omega(u)=u$ and $\Phi(z)=0$ yields the original scalar multiplet of Ref. 8, while $\Omega(u)=u$ and $\Phi(z)=\frac{1}{2}mz^2+\frac{1}{3}gz^3$ yields the massive self-interacting theory of Ref. 7.

If we require the coefficient function $\theta(u)$ to have leading term 1, we insure that theories with different θ's cannot be made equivalent by a redefinition of A and B in terms of each other, as any such transformation that preserves the form of the scalar kinetic term $[(\partial_\mu A)^2+(\partial_\mu B)^2]f(u)$ can only multiply the term by a power of $u\equiv\kappa^2(A^2+B^2)$.

In the global limit $V_{a\mu}\to\eta_{a\mu}$, $\epsilon(x)\to\epsilon$, $\psi_\mu\to0$, $\kappa\to0$, we find $\mathcal{L}\to\mathcal{L}_{\text{global}}$:

$$\mathcal{L}_{\text{global}}=\frac{i}{\sqrt{2}}\bar{\chi}\not\partial\chi+\tfrac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2]-\tfrac{1}{2}\bar{\chi}V''(A-i\gamma_5B)\chi-\tfrac{1}{2}V'(A+iB)V'(A-iB), \tag{2.9}$$

where $V(A+i\gamma_5B)\equiv\Phi(z)$ above. This Lagrangian is invariant under the following global transformations ($\bar{\epsilon}$ is a constant spinor):

$$\delta A_{\text{global}}=\frac{1}{\sqrt{2}}\bar{\epsilon}\chi,\quad \delta B_{\text{global}}=\frac{i}{\sqrt{2}}\bar{\epsilon}\gamma_5\chi,\quad \delta\bar{\chi}_{\text{global}}=\frac{i}{\sqrt{2}}\bar{\epsilon}(A+i\gamma_5B)\overleftarrow{\not\partial}+\frac{\bar{\epsilon}}{\sqrt{2}}V'(A+i\gamma_5B). \tag{2.10}$$

This globally supersymmetric theory was first presented by Salam and Strathdee[13] where the Lagrangian is also written in superfield notation:

$$\mathcal{L}_{\text{global}} = \tfrac{1}{8}(\bar{D}D)^2(\phi_+\phi_-) - \tfrac{1}{2}\bar{D}D(V(\phi_+) + V(\phi_-)) . \tag{2.11}$$

We can also take the global limit by introducing dimensional constants $\bar{\omega}_j = \omega_j \kappa^{2j}$, so that, although $\Omega(u) \to 0$, $\theta(u) \to Q(A^2+B^2)$ and $\kappa^2\theta'(u) \to Q'(A^2+B^2)$ as $\kappa \to 0$. Then $\mathcal{L}_{KI} \to \mathcal{L}_{KI}^{\text{global}}$,

$$\begin{aligned}\mathcal{L}_{KI}^{\text{global}} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi + \tfrac{1}{2}[(\partial_\mu A)^2 + (\partial_\mu B)^2]Q(A^2+B^2) - \tfrac{1}{2}(\bar{\chi}\gamma_5\gamma^\rho\chi)(A\overleftrightarrow{\partial}_\rho B)\frac{Q'(A^2+B^2)}{Q(A^2+B^2)} \\ &\quad + \frac{1}{16}(\bar{\chi}\gamma_5\gamma^\rho\chi)(\bar{\chi}\gamma_5\gamma_\rho\chi)\left[\frac{(A^2+B^2)Q'(A^2+B^2)}{Q(A^2+B^2)}\right]'\frac{1}{Q(A^2+B^2)} ,\end{aligned} \tag{2.12}$$

is invariant under

$$\begin{aligned}&\delta A_{KI}^{\text{global}} = \frac{1}{\sqrt{2}}\frac{\bar{\epsilon}\chi}{[Q(A^2+B^2)]^{1/2}} , \quad \delta B_{KI}^{\text{global}} = \frac{i}{\sqrt{2}}\frac{\bar{\epsilon}\gamma_5\chi}{[Q(A^2+B^2)]^{1/2}} , \\ &\delta\bar{\chi}_{KI}^{\text{global}} = \frac{i}{\sqrt{2}}\bar{\epsilon}(A+i\gamma_5 B)\overleftarrow{\partial\!\!\!/}[Q(A^2+B^2)]^{1/2} - \frac{1}{\sqrt{2}}[\bar{\epsilon}\gamma_5(A+i\gamma_5 B)\chi]\bar{\chi}\gamma_5\frac{Q'(A^2+B^2)}{[Q(A^2+B^2)]^{3/2}} .\end{aligned} \tag{2.13}$$

This theory appears not to have been considered before. If we redefine $\chi(x) \to [Q(A^2+B^2)]^{1/2}\chi(x)$, we can write the Lagrangian in the superfield notation of Ref. 13. Let $Q(y) \equiv \sum_{j=1}^{\infty} q_j j^2 y^{j-1}$, then

$$\mathcal{L}_{KI}^{\text{global}} = \frac{1}{8}\sum_{j=1}^{\infty} q_j(\bar{D}D)^2(\phi_+)^j(\phi_-)^j . \tag{2.14}$$

If the self-interaction terms are kept, then this second global limit is taken and we find a trivial modification of (1.9):

$$\begin{aligned}\mathcal{L}_I^{\text{global}} &= -\tfrac{1}{2}\bar{\chi}\left[V''(A-i\gamma_5 B) - \tfrac{1}{2}(A+i\gamma_5 B)V'(A-i\gamma_5 B)\frac{Q'(A^2+B^2)}{Q(A^2+B^2)}\right]\chi\frac{1}{[Q(A^2+B^2)]^{1/2}} \\ &\quad - \tfrac{1}{2}V'(A+iB)V'(A-iB)\frac{1}{Q(A^2+B^2)} .\end{aligned} \tag{2.15}$$

III. CONSTRUCTION

We were motivated to search for a more general locally supersymmetric theory of the scalar multiplet than that of Ref. 8 by the observation that a consistent truncation of the $O(\kappa^2)$ results of the SO(4) extended theory yielded a new theory, locally supersymmetric to $O(\kappa^2)$, with precisely the same particle content as in Ref. 8. We used the functional technique[7] (the rest of this section is for the technically minded).

Our ansatz was suggested by the results of Ref. 4:

$$\mathcal{L}_0 = \mathcal{L}_{SG} + \mathcal{L}_{KI} , \quad \mathcal{L}_{SG} \text{ as in (1.2)} , \tag{3.1}$$

$$\begin{aligned}\mathcal{L}_{KI} &= \frac{i}{2}V\bar{\chi}D\!\!\!\!/\,\chi F_1(u) + \frac{V}{2}[(\partial_\mu A)^2 + (\partial_\mu B)^2]F_2(u) - \frac{\kappa V}{\sqrt{2}}\bar{\psi}_\mu\partial_\nu(A+i\gamma_5 B)\gamma^\nu\gamma^\mu\chi F_3(u) + \frac{i\kappa^2}{4}\epsilon^{\lambda\mu\nu\rho}(\bar{\psi}_\lambda\gamma_\mu\psi_\nu)(A\overleftrightarrow{\partial}_\rho B)F_4(u) \\ &\quad + \frac{\kappa^2}{8}V(\bar{\psi}_\mu\gamma_5\gamma^a\psi^\mu)(\bar{\chi}\gamma_5\gamma^a\chi)F_5(u) + \frac{i\kappa^2}{16}\epsilon^{\lambda\mu\nu\rho}(\bar{\psi}_\lambda\gamma_\mu\psi_\nu)(\bar{\chi}\gamma_5\gamma_\rho\chi)F_6(u) + \frac{n}{4}\kappa^2 V(\bar{\chi}\gamma_5\gamma^\rho\chi)(A\overleftrightarrow{\partial}_\rho B)F_7(u) \\ &\quad + \frac{n}{32}\kappa^2 V(\bar{\chi}\gamma_5\gamma^\rho\chi)(\bar{\chi}\gamma_5\gamma_\rho\chi)F_8(u) .\end{aligned} \tag{3.2}$$

Our ansatz for the transformation laws was

$$\begin{aligned}&\delta V_{a\mu} = -i\kappa\bar{\epsilon}(x)\gamma_a\psi_\mu , \quad \delta A = \frac{\bar{\epsilon}(x)\chi}{\sqrt{2}}F_9(u) , \quad \delta B = \frac{i\bar{\epsilon}(x)\gamma_5\chi}{\sqrt{2}}F_9(u) , \\ &\delta\bar{\chi} = \frac{i}{\sqrt{2}}\bar{\epsilon}(x)(A+i\gamma_5 B)\overleftarrow{\partial\!\!\!/}F_{10}(u) - \frac{i\kappa}{2}[(\bar{\psi}_\rho\chi)\bar{\epsilon}(x)\gamma^\rho F_{11}(u) + (\psi_\rho\gamma_5\chi)\bar{\epsilon}(x)\gamma_5\gamma^\rho F_{12}(u)] - \frac{n\kappa^2}{2\sqrt{2}}[\bar{\epsilon}(x)\gamma_5(A+i\gamma_5 B)\chi]\bar{\chi}\gamma_5 F_{13}(u) , \\ &\delta\bar{\psi}_\lambda = \kappa^{-1}\bar{\epsilon}(x)\overleftarrow{D}_\lambda + \frac{i\kappa}{4}(2\bar{\psi}_\lambda\gamma_\mu\psi_\nu + \bar{\psi}_\mu\gamma_\lambda\psi_\nu)\bar{\epsilon}(x)\sigma^{\mu\nu} + \frac{i\kappa}{2}\bar{\epsilon}(x)\gamma_5(A\overleftrightarrow{\partial}_\lambda B)F_{14}(u) \\ &\qquad + \frac{i\kappa}{4}(\bar{\chi}\gamma_5\gamma^\rho\chi)\bar{\epsilon}(x)\gamma_5\sigma_{\rho\lambda}F_{15}(u) - \frac{\kappa^2}{2\sqrt{2}}[\bar{\epsilon}(x)\gamma_5(A+i\gamma_5 B)\chi]\bar{\psi}_\lambda\gamma_5 F_{16}(u) .\end{aligned} \tag{3.3}$$

The $F_i(u)$ are unknown functions; our result is unaltered by introducing further arbitrary functions of $u=\kappa^2(A^2+B^2)$. The parameter n is any real number.

Using this ansatz, we found nine kinds of variations: (1) ψ^3; (2) $\chi^2\psi^3$; (3) $\psi^2\chi^3$; (4) ψ; (5) χ; (6) χ^3; (7) $\chi^4\psi$; (8) $\psi^2\chi$; (9) $\psi\chi^2$. In principle, there are also χ^5 terms, but these vanish automatically because χ is a Majorana spinor. We present the fate of each type of variation in tabular form in Table I. Only 15 of these relations are independent:

$$\begin{aligned}
&F_1=F_5=F_6=F_{11}=F_{12}=F_{15}=1\ ,\quad F_3=F_{10}\ ,\\
&F_4=F_{14}\ ,\quad F_2=(F_3)^2\ ,\quad F_{13}=F_7F_9\ ,\\
&F_{16}=F_4F_9\ ,\quad F_3F_9=1\ ,\quad F_2=(uF_4)'\ ,\\
&F_8F_2=(uF_7)'\ ,\quad (F_4+nF_7)F_3=4F_3'\ .
\end{aligned} \tag{3.4}$$

These are related to $\Theta(u)$ and $\Omega(u)$ in Sec. I by $\Theta(u)=F_2(u)$, $\Omega'(u)=F_4(u)$, etc.

The nonderivative self-interaction term $\mathcal{L}_I$ was found by noting that the results of Ref. 7 suggested the following generalization:

$$\begin{aligned}
\mathcal{L}_j=&-V\varphi_j\bar{\chi}(A-i\gamma_5B)^{j-2}\bar{\chi}P_1(u)\\
&-\frac{V}{2}(\varphi_j)^2(A^2+B^2)^{j-1}P_2(u)\\
&-\frac{i\kappa}{\sqrt{2}}V\varphi_j\bar{\psi}\cdot\gamma(A-i\gamma_5B)^{j-1}\chi P_3(u)\\
&-\kappa^2V\varphi_j\bar{\psi}_\mu\sigma^{\mu\nu}(A+i\gamma_5B)^j\psi_\nu P_4(u)
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
\delta_j\bar{\chi}&=-\frac{\varphi_j}{\sqrt{2}}\bar{\epsilon}(x)(A+i\gamma_5B)^{j-1}P_3(u)\ ,\\
\delta_j\bar{\psi}_\lambda&=-\frac{i\kappa}{2}\varphi_j\bar{\epsilon}(x)(A+i\gamma_5B)^j\gamma_\lambda P_4(u)\ .
\end{aligned} \tag{3.6}$$

Term for term we followed the procedure of Ref. 7 and found relations that uniquely specified the p_i's in terms of j and the F_i's. We then considered $\mathcal{L}_{j+l}=\mathcal{L}_j+\mathcal{L}_l-V\phi_j\phi_lQ(A,B)$, and found a unique expression for Q in terms of j, l, F_i, and P_i. Finally, we observed that the solutions could be summed over j to yield arbitrary functions, e.g.,

TABLE I. Variations of the Lagrangian and the functional relations. Some results appearing later in the table have been simplified by substitution of earlier results.

	Variation	Result	Comment
(1)	ψ^3	$F_4=F_{14}$	This variation is proportional to a variation in Ref. 8 known to vanish
(2)	$\chi^2\psi^3$	$F_{15}=F_5=F_{11}$ $=F_{12}=1$	This variation is proportional to a variation in Ref. 8 known to vanish
(3)	$\psi^2\chi^3$	$F_5=F_6=1$	
(4)	ψ	$(uF_4)'=F_3F_{10}$ $F_2=F_3F_{10}$	After integrating by parts to remove all derivatives on $\bar{\epsilon}(x)$, this can be subdivided into variations with or without $D\psi_\mu$
(5)	χ	$F_3=F_1F_{10}$ $F_3=F_2F_9$ $2F_2'F_9=F_3F_{14}+nF_7F_{10}$ $F_2F_9'+F_1'F_{10}=2F_3'$	As above with χ instead of ψ_μ
(6)	χ^3	$F_1=1$ $F_1F_{13}=F_7F_9$ $F_9(F_7)'=F_8F_{10}$	
(7)	$\chi^4\psi$	$F_{16}=F_9F_4$	We use $F_5=F_6$, $F_{11}=F_{12}=1$
(8)	$\psi^2\chi$	$F_3=F_9(uF_4)'$	This is not independent of earlier results; much time can be saved by noting which terms are identical to variations in Ref. 8
(9)	$\psi\chi^2$	$F_3F_9=F_1=F_5$ $=F_6=1$	These are very tedious and are equivalent to earlier relations

$\Phi(A+i\gamma_5 B)\equiv\sum_{j=1}^{\infty}\phi_j(A+i\gamma_5 B)^j$. This completes the derivation of the results of Sec. II.

IV. EXAMPLES AND SYMMETRY BREAKDOWN

We now consider several special cases of interest. For the original scalar supermultiplet,[8] as observed above, $\Omega(u)=u$; we can add any number of interaction terms. In particular,

$$\Phi(z)=\lambda z+\tfrac{1}{2}mz^2+\tfrac{1}{3}gz^3 \tag{4.1}$$

gives the local version of the original Wess and Zumino Lagrangian with a λF term.[14] In the global case, the λ term could be eliminated by a shift in the fields, but due to the presence of nonpolynomial interactions in the local theory, this is no longer the case. This theory shows symmetry breakdown similar to that in Ref. 7, but the more complicated potential is rich in false vacuums which for some values of the parameters have a zero cosmological constant. The true vacuum always has a nonzero cosmological constant, and, for m not too large, a term quadratic in the spin-$\frac{3}{2}$ field which can be absorbed into de Sitter covariant derivatives.[15] The $m=g=0, \lambda\neq 0$ case is similar to the axial gauge theory[17] in that there is a cosmological term but no term quadratic in the spin-$\frac{3}{2}$ field.

For $\Phi(z)=\mu$, we find the analog of Ref. 14 for the scalar field:

$$\begin{aligned}\mathcal{L}_I=&-\mu\frac{\kappa^2}{2}V\bar{\chi}(A+i\gamma_5 B)^2\chi e^{u/2}+\frac{\mu^2}{2\kappa^2}V(3-u)e^u\\&-\frac{i\kappa}{\sqrt{2}}\mu V\bar{\psi}\cdot\gamma(A+i\gamma_5 B)\chi e^{u/2}\\&-\mu V\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu e^{u/2}.\end{aligned} \tag{4.2}$$

If we let $\Phi(z)=\mu+\lambda z$, we can cancel the cosmological term, but symmetry breakdown restores it.

The question arises whether the cosmological constant can ever be canceled in a theory with $m_\psi\neq 0$; there exist partial results[16] that indicate an affirmative answer, but until now there has not been a complete theory with a "super-Higgs" mechanism and no cosmological constant. We present two such theories, neither entirely satisfactory. For

$$\Phi(z)=\kappa^{-3}\exp(\sqrt{3}\kappa z-\kappa^2z^2/2) \tag{4.3}$$

we have a potential

$$2\kappa^{-2}B^2\exp(2\sqrt{3}\kappa A+3\kappa^2B^2) \tag{4.4}$$

which has a minimum at $\langle B\rangle=0$ for any $\langle A\rangle$; thus

$$\begin{aligned}&m_\psi=\kappa^{-1}\exp(\kappa\sqrt{3}\langle A\rangle-\tfrac{1}{2}\kappa^2\langle A\rangle^2),\\&m_B{}^2=4\kappa^{-2}\exp(2\kappa\sqrt{3}\langle A\rangle),\quad m_A{}^2=0.\end{aligned} \tag{4.5}$$

Global supersymmetries of both gravitational and scalar multiplets are broken. Since the vacuum is flat, interpretation and quantization of the theory is not a problem. The spin-$\frac{3}{2}$ field is massive and the unphysical Goldstone spinor χ still remains in the theory. But our theory still possesses a gauge symmetry and we are free to choose a gauge. In fact, we can choose a gauge in which the χ field disappears from the theory,[16] namely, the analog of the Higgs gauge $\chi=0$. This restores our degrees of freedom, and the spin-$\frac{3}{2}$ propagator is the usual propagator for a massive spin-$\frac{3}{2}$ field, viz.,

$$D_{\mu\nu}(P)=\frac{\not{P}-m}{P^2-m^2}\left[g_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{1}{3m}(\gamma_\mu P_\nu-\gamma_\nu P_\mu)-\frac{2}{3m^2}P_\mu P_\nu\right]. \tag{4.6}$$

The only unsatisfactory feature of this model, aside from its probable nonrenormalizability, is the strange dependence of the potential on A: For any $\langle B\rangle\neq 0$, away from the vacuum, the minimum of the potential occurs at $\langle A\rangle\to-\infty$.

If we let

$$\Phi(z)=\left(\frac{\sqrt{3}}{2\kappa}+z\right)^3e^{-\kappa\sqrt{3}z/2} \tag{4.7}$$

we find a model free of this flaw. The potential is

$$\tfrac{1}{2}[(A+\sqrt{3}/2\kappa)^2+B^2]^2\{\kappa^2B^2(\tfrac{9}{2}+\kappa^2B^2+2\kappa^2A^2)+(\kappa A-\tfrac{1}{2}\sqrt{3})^2[3+(\kappa A+\tfrac{1}{2}\sqrt{3})^2]\}\exp[-\kappa\sqrt{3}A+\kappa^2(A^2+B^2)]. \tag{4.8}$$

This has two minima. Both occur at $\langle B\rangle=0$, one at $\langle A\rangle=+\sqrt{3}/2\kappa$, the other at $\langle A\rangle=-\sqrt{3}/2\kappa$. The former gives

$$m_\psi=\kappa^{-1}3\sqrt{3}e^{-3/8},\quad m_A{}^2=m_B{}^2=54\kappa^{-2}e^{-3/4} \tag{4.9}$$

and hence corresponds to a vacuum where the global supersymmetry of the gravitational multiplet is broken, while the latter gives $m_\psi=m_A{}^2=m_B{}^2=0$, and hence corresponds to no symmetry breaking. For the first case, we can once again apply the analysis of Freedman and Das[18,19] and find that we have evidence for a Higgs mechanism; again there is a constant part in the χ transformation law. For the second case there is no constant in the χ transformation, and χ is clearly dynamical. Normally, we would expect the degeneracy of the vacuums to be broken by quantum corrections, but since the theory is unlikely to be renormalizable, we can only make sense of the theory at the tree level.

The last particular case we consider is the SO(4) scalar reduction. In a theory with only one spin-$\frac{1}{2}$ field χ, any χ^5 terms vanish. Since the full SO(4) theory has four χ_i's and nonvanishing χ^5 terms in the variation of $\mathcal{L}$, we expect further restrictions on the Lagrangian when we require these terms to vanish. Comparison of our Lagrangian and the Lagrangian of Ref. 4 imposes the condition that

$$F_8(u) = \frac{2}{\Theta(u)}\left[\frac{u\Theta'(u)}{\Theta(u)}\right]' - 1 = 3 \tag{4.10}$$

which leads to

$$\Theta(u) = \frac{1}{(1-u)^2}, \quad \Omega(u) = \ln\left(\frac{1}{1-u}\right). \tag{4.11}$$

Finally, we observe that when the SO(4) internal symmetry is gauged, we expect in analogy with SO(3) and SO(2)[18] to find a cosmological term. Letting $\Phi(z) = \mu = e\kappa^{-1}$, we find

$$\mathcal{L}_I = \frac{\mu^2}{2\kappa^2} V \frac{3-u}{1-u} - \frac{i\kappa}{\sqrt{2}} \mu V \psi \cdot \gamma (A + i\gamma_5 B)\chi \frac{1}{(1-u)^{1/2}} - \mu V \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \frac{1}{(1-u)^{1/2}}. \tag{4.12}$$

In particular, the scalar kinetic term becomes $\frac{1}{2}V[(\partial_\mu A)^2 + (\partial_\mu B)^2]/(1-u)^2$, which bears a resemblance to the σ-model Lagrangian; we do not understand the physical consequences of a Lagrangian that diverges for some finite field strength. Note that for $u \gtrsim 1$ the theory has severe problems of interpretation, as the energy is not bounded below and the Lagrangian is not even Hermitian. The additional terms in the transformation laws are

$$\delta_I \bar{\chi} = \mu \bar{\epsilon}(x)(A - i\gamma_5 B)\frac{1}{(1-u)^{1/2}},$$
$$\delta_I \bar{\psi}_\lambda = -\frac{i\kappa}{2} \mu \bar{\epsilon}(x)\gamma_\lambda \frac{1}{(1-u)^{1/2}}. \tag{4.13}$$

Since the full SO(4) theory is expected to have one-loop finite physical amplitudes,[20] one should be able to calculate the effective potential for A and B; it will be interesting to see if the cosmological term above is completely or partially canceled.

V. GLOBAL MASSLESS SUPER-QED

In this section we study the Fayet-Iliopoulos model[10] in a particular limit, namely, we consider a vector multiplet interacting with a complex scalar multiplet where all the fields have zero mass. The Lagrangian contains the fields $(A_\mu, \lambda, \chi^i, A^i, B^i)$ and does not conserve parity. All boson fields are Hermitian and all spinors are Majorana fields. When the auxiliary fields are eliminated the Lagrangian has the form

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}i\bar{\lambda}\not{\partial}\lambda + \tfrac{1}{2}i\bar{\chi}^i\not{\partial}\chi^i + \tfrac{1}{2}[(\partial_\mu A)^2 + (\partial_\mu B)^2] - eA_\mu\epsilon^{ij}(A^i\partial^\mu A^j + B^i\partial^\mu B^j - \tfrac{1}{2}i\bar{\chi}^i\gamma^\mu\chi^j)$$
$$- e\xi\epsilon^{ij}A^iB^j - e\epsilon^{ij}\bar{\chi}^i(A^j + i\gamma_5 B^j)\lambda + \tfrac{1}{2}e^2A_\mu(A^{i2} + B^{i2}) - \tfrac{1}{2}e^2(\epsilon^{ij}A^iB^j)^2 - \tfrac{1}{2}\xi^2, \tag{5.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, e is the electromagnetic coupling, and ξ is the parity violating parameter.

This Lagrangian is invariant under the following supersymmetry transformation:

$$\delta A^i = \frac{1}{\sqrt{2}}\bar{\epsilon}\chi^i, \quad \delta B^i = \frac{i}{\sqrt{2}}\bar{\epsilon}\gamma_5\chi^i,$$

$$\delta\bar{\chi}^i = \frac{i}{\sqrt{2}}\bar{\epsilon}(A^i + i\gamma_5 B^i)\overleftarrow{\not{\partial}} + \frac{ie}{\sqrt{2}}\epsilon^{ij}\bar{\epsilon}(A^j + i\gamma_5 B^j)\not{A},$$

$$\delta\bar{\lambda} = -\frac{1}{\sqrt{2}}\bar{\epsilon}\sigma^{\mu\nu}F_{\mu\nu} + \frac{i}{\sqrt{2}}(\xi + e\epsilon^{ij}A^iB^j)\bar{\epsilon}\gamma_5,$$

$$\delta A_\mu = \frac{i}{\sqrt{2}}\bar{\epsilon}\gamma_\mu\lambda. \tag{5.2}$$

ϵ is a constant spinor parameter and the conserved Noether current is

$$j_\mu^N = \frac{i}{\sqrt{2}}\sigma^{\alpha\beta}F_{\alpha\beta}\gamma_\mu\lambda + \frac{1}{\sqrt{2}}\partial_\nu(A^i + i\gamma_5 B^i)\gamma^\nu\gamma_\mu\chi^i + \frac{1}{\sqrt{2}}(\xi + e\epsilon^{ij}A^iB^j)\gamma_5\gamma_\mu\lambda + \frac{e}{\sqrt{2}}\epsilon^{ij}(A^i + i\gamma_5 B^i)\not{A}\gamma_\mu\chi^j. \tag{5.3}$$

The theory is also invariant under the following local gauge transformations:

$$\delta A_\mu = \frac{1}{e}\partial_\mu\alpha(x),$$
$$\delta\lambda = 0,$$
$$\delta\bar{\chi}^i = -\epsilon^{ij}\alpha(x)\chi^j, \tag{5.4}$$
$$\delta A^i = -\epsilon^{ij}\alpha(x)A^j,$$
$$\delta B^i = -\epsilon^{ij}\alpha(x)B^j.$$

We now diagonalize the scalar mass matrix. Let us define

$$\tilde{A}_1=\frac{1}{\sqrt{2}}(A_1+B_2)\,,\quad \tilde{A}_2=\frac{1}{\sqrt{2}}(A_2+B_1)\,,$$
$$\tilde{B}_1=\frac{1}{\sqrt{2}}(B_1-A_2)\,,\quad \tilde{B}_2=\frac{1}{\sqrt{2}}(B_2-A_1)\,. \tag{5.5}$$

Then the scalar mass matrix and the four-point interaction terms become

$$V(\tilde{A}^i,\tilde{B}^i)=\frac{e\xi}{2}(\tilde{A}_1{}^2+\tilde{B}_1{}^2-\tilde{A}_2{}^2-\tilde{B}_2{}^2)+\frac{e^2}{8}(\tilde{A}_1{}^2+\tilde{B}_1{}^2-\tilde{A}_2{}^2-\tilde{B}_2{}^2)^2\,. \tag{5.6}$$

This potential has a minimum at

$$\tilde{A}_1=\tilde{B}_1=0\,,\qquad \tilde{A}_2{}^2+\tilde{B}_2{}^2=\frac{2\xi}{e}. \tag{5.7}$$

However, we can choose our orientation in the $\tilde{A}_2$-$\tilde{B}_2$ plane such that

$$\langle\tilde{A}_2\rangle=\left(\frac{2\xi}{e}\right)^{1/2}=v\,,\qquad \langle\tilde{A}_1\rangle=\langle\tilde{B}_1\rangle=\langle\tilde{B}_2\rangle=0 \tag{5.8}$$

represents the minimum. If we now shift fields, i.e.,

$$\tilde{A}_2\to\tilde{A}_2+v\,, \tag{5.9}$$

then the interaction Lagrangian becomes

$$\begin{aligned}\mathcal{L}'=\;&eA^\mu\left(\tilde{A}_1\overleftrightarrow{\partial}_\mu\tilde{B}_1-\tilde{A}_2\overleftrightarrow{\partial}_\mu\tilde{B}_2+\frac{i}{2}\epsilon^{ij}\bar{\chi}^i\gamma_\mu\chi^j\right)-e\xi\tilde{A}_2{}^2+\frac{e^2v}{2}\tilde{A}_2(\tilde{A}_1{}^2+\tilde{B}_1{}^2-\tilde{A}_2{}^2-\tilde{B}_2{}^2)-\frac{e^2}{8}(\tilde{A}_1{}^2+\tilde{B}_1{}^2-\tilde{A}_2{}^2-\tilde{B}_2{}^2)^2\\&+e\xi A_\mu{}^2+\frac{e^2}{2}A_\mu{}^2(A_1{}^2+B_1{}^2+A_2{}^2+B_2{}^2)-\frac{ev}{\sqrt{2}}(\bar{\chi}^1-i\bar{\chi}^2\gamma_5)\lambda-\frac{e}{\sqrt{2}}(\bar{\chi}^1-i\bar{\chi}^2\gamma_5)(\tilde{A}_2+i\gamma_5\tilde{B}_2)\lambda\\&-\frac{ie}{\sqrt{2}}(\bar{\chi}^1+i\bar{\chi}^2\gamma_5)\gamma_5(\tilde{A}_1+i\gamma_5\tilde{B}_1)\lambda\,.\end{aligned} \tag{5.10}$$

Since there is mixing between different spinors, the spinor mass matrix needs diagonalization. Let us define a new basis by

$$\eta_1=\tfrac{1}{2}(\chi^1-i\gamma_5\chi^2+\sqrt{2}\lambda)\,,\quad \eta_2=\frac{i}{2}[\gamma_5(\chi^1-i\gamma_5\chi^2)+\sqrt{2}\gamma_5\lambda]\,,\quad \zeta=\frac{1}{\sqrt{2}}(\chi^1+i\gamma_5\chi^2)\,. \tag{5.11}$$

Then the interaction Lagrangian becomes

$$\begin{aligned}\mathcal{L}'=&-\frac{m^2}{2}A_2{}^2+\frac{m^2}{2}A_\mu{}^2-\frac{m}{2}(\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2)+eA^\mu\left(\tilde{A}_1\overleftrightarrow{\partial}_\mu\tilde{B}_1-\tilde{A}_2\partial_\mu\tilde{B}_2+\frac{i}{2}\bar{\eta}_1\gamma_\mu\eta_2\right)\\&+\frac{e}{4}A_\mu(\bar{\eta}_1\gamma_5\gamma^\mu\eta_1+\bar{\eta}_2\gamma_5\gamma^\mu\eta_2-2\bar{\zeta}\gamma_5\gamma^\mu\zeta)+\frac{e^2v}{2}(\tilde{A}_1{}^2+\tilde{B}_1{}^2-\tilde{A}_2{}^2-\tilde{B}_2{}^2)-\frac{e^2}{8}(\tilde{A}_1{}^2+\tilde{B}_1{}^2-\tilde{A}_2{}^2-\tilde{B}_2{}^2)^2\\&-\frac{e}{2}\bar{\eta}_1(\tilde{A}_2+i\gamma_5\tilde{B}_2)\eta_1-\frac{e}{2}\bar{\eta}_2(\tilde{A}_2+i\gamma_5\tilde{B}_2)\eta_1-\frac{e}{2}\bar{\eta}_2(\tilde{A}_2+i\gamma_5\tilde{B}_2)\eta_2-\frac{ie}{\sqrt{2}}\bar{\eta}_1(\tilde{A}_1+i\gamma_5\tilde{B}_1)\zeta\\&+\frac{e}{\sqrt{2}}\bar{\eta}_2(\tilde{A}_1+i\gamma_5\tilde{B}_1)\zeta+\frac{e^2}{2}A_\mu{}^2(A_1{}^2+B_1{}^2+A_2{}^2+B_2{}^2)\,,\end{aligned} \tag{5.12}$$

where $m^2\equiv 2e\xi$.

Thus we see that because of the symmetry breaking the vector field becomes massive and the Goldstone boson $\tilde{B}_2$ can be eliminated by the Higgs mechanism. However, it also looks like the masses of particles within the multiplets are split badly. Thus one expects that supersymmetry is spontaneously broken. However, when one looks at the spinor transformations one does not find a constant anywhere suggesting that supersymmetry is still maintained.

To see more closely what exactly is happening, let us choose the gauge condition

$$\tilde{B}_2=0\,,\quad A_\mu\to B_\mu=A_\mu-(1/e)\partial_\mu\tilde{B}_2\,. \tag{5.13}$$

Then the Lagrangian becomes (omitting tildes)

$$\mathcal{L} = -\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{m^2}{2}B_\mu{}^2 + \frac{i}{2}(\bar\eta_1\not\partial\eta_1 + \bar\eta_2\not\partial\eta_2) - \frac{m}{2}(\bar\eta_1\eta_1 + \bar\eta_2\eta_2) + \frac{i}{2}\bar\zeta\not\partial\zeta + \tfrac{1}{2}(\partial_\mu A)^2 - \frac{m}{2}A^2 + \tfrac{1}{2}[(\partial_\mu A_1)^2 + (\partial_\mu B_1)^2]$$
$$+ eA_\mu(A_1\overleftrightarrow{\partial}^\mu B_1 + \frac{i}{2}\bar\eta_1\gamma^\mu\eta_2) + \frac{e}{4}A_\mu(\bar\eta_1\gamma_5\gamma^\mu\eta_1 + \bar\eta_2\gamma_5\gamma^\mu\eta_2 - 2\bar\zeta\gamma_5\gamma^\mu\zeta) + \frac{e^2 v}{2}A(A_1{}^2 + B_1{}^2 - A^2)$$
$$- \frac{e^2}{8}(A_1{}^2 + B_1{}^2 - A^2)^2 - \frac{e}{2}A(\bar\eta_1\eta_1 + \bar\eta_2\eta_2) - \frac{ie}{\sqrt2}\bar\eta_1(A_1 + i\gamma_5 B_1)\gamma_5\zeta$$
$$+ \frac{e}{\sqrt2}\bar\eta_2(A_1 + i\gamma_5 B_1)\zeta + \frac{e^2}{2}A_\mu{}^2(A_1{}^2 + B_1{}^2 - A^2)\,, \qquad (5.14)$$

where we have used

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{and } \tilde{A}_2 \to A\,. \qquad (5.15)$$

It is now clear that, as a consequence of the breaking of gauge invariance and the Higgs mechanism, the theory has rearranged itself into a supersymmetric theory of a massive vector multiplet[21] (B_μ, η^i, A) interacting with a scalar multiplet (ζ, A_1, B_1). The degrees of freedom are maintained and no constant term and, hence, no zero-point energy are generated.

VI. COUPLING TO SUPERGRAVITY

In order to couple the massless super-QED to supergravity, we follow the usual procedure of order-by-order coupling.[6] That is, we start with the Lagrangian for supergravity and the covariantized Lagrangian for the matter fields and add to it the Noether current terms. Then at each order in κ we add new terms to the Lagrangian and to transformation laws to maintain supersymmetry. The complete Lagrangian derived this way can be written as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'\,, \qquad (6.1)$$

where

$$\mathcal{L}_0 = \mathcal{L}_{\rm SG} - \frac{V}{4}F_{\mu\nu}F_{\mu\nu} + \frac{iV}{2}\bar\lambda\not{D}\lambda + \frac{V}{2}[(\partial_\mu A^i)^2 + (\partial_\mu B^i)^2] + \frac{iV}{2}\bar\chi^i\not{D}\chi^i - \frac{i\kappa V}{\sqrt2}\bar\psi_\rho\sigma^{\mu\nu}F_{\mu\nu}\gamma^\rho\lambda - \frac{\kappa V}{\sqrt2}\bar\psi_\mu\partial_\nu(A^i + i\gamma_5 B^i)\gamma^\mu\gamma^\nu\chi^i$$
$$- \frac{\kappa^2 V}{2}(\bar\psi_\rho\sigma^{\mu\nu}\gamma^\rho\lambda)(\bar\psi_\mu\gamma_\nu\lambda) - \frac{i\kappa^2}{16}\epsilon^{\mu\nu\rho\alpha}(\bar\psi_\mu\gamma_\alpha\psi_\nu)(\bar\lambda\gamma_5\gamma_\rho\lambda) + \frac{\kappa^2 V}{8}(\bar\psi_\mu\gamma_5\gamma^a\psi^\mu)(\bar\chi^i\gamma_5\gamma_a\chi^i)$$
$$+ \frac{i\kappa^2}{16}\epsilon^{\mu\nu\rho\alpha}(\bar\psi_\mu\gamma_\alpha\psi_\nu)(\bar\chi^i\gamma_5\gamma_\rho\chi^i) + \frac{i\kappa^2}{4}\epsilon^{\alpha\beta\rho\mu}(\bar\psi_\alpha\gamma_\rho\psi_\mu)(A^i\overleftrightarrow{\partial}_\beta B^i) + \frac{\kappa^2 V}{4}(\bar\chi^i\gamma_5\gamma^a\chi_i)(A^i\overleftrightarrow{\partial}_a B_j)$$
$$- \frac{\kappa^2 V}{4}(\bar\lambda\gamma_5\gamma_a\lambda)(A^i\overleftrightarrow{\partial}_a B^i) - \frac{\kappa^2 V}{32}(\bar\chi^i\gamma_5\gamma^a\chi_i)(\bar\chi^j\gamma_5\gamma_a\chi_j) - \frac{3\kappa^2 V}{32}(\bar\lambda\gamma_5\gamma^a\lambda)^2 - \frac{\kappa^2 V}{16}(\bar\chi^i\gamma_5\gamma^a\chi_i)(\bar\lambda\gamma_5\gamma_a\lambda)\,, \qquad (6.2)$$

$$\mathcal{L}' = -VeA_\mu\epsilon^{ij}\ \ A^i\partial^\mu A^j + B^i\partial^\mu B^j - \frac{i}{2}\bar\chi^i\gamma^\mu\chi^j) - Ve\xi\epsilon^{ij}A^iB^j - Ve\epsilon^{ij}\bar\chi^i(A^j + i\gamma_5 B^j)\lambda - \frac{Ve^2}{2}A_\mu{}^2[(A^i)^2 + (B^i)^2]$$
$$- \frac{Ve^2}{2}(\epsilon^{ij}A^iB^j)^2 - \frac{V}{2}\xi^2 - \frac{\kappa V}{\sqrt2}(\xi + e\epsilon^{ij}A^iB^j)\bar\psi_\mu\gamma_5\gamma^\mu\lambda - \frac{\kappa e V}{\sqrt2}\epsilon^{ij}\bar\psi_\mu(A^j + i\gamma_5 B^j)\not{A}\gamma^\mu\chi^i$$
$$- \frac{i\kappa^2}{2}\epsilon^{\mu\nu\rho\alpha}(\xi + e\epsilon^{ij}A^iB^j)\bar\psi_\mu\gamma_\rho\psi_\alpha A_\nu - \frac{\kappa^2 V}{2}(\xi + e\epsilon^{ij}A^iB^j)\bar\lambda\gamma_5\not{A}\lambda + \frac{\kappa^2 V}{2}(\xi + e\epsilon^{ij}A^iB^j)\bar\chi^k\gamma_5\not{A}\chi^k\,. \qquad (6.3)$$

This Lagrangian is invariant under the following supersymmetry transformations:

$$\delta A^i = \frac{1}{\sqrt2}\bar\epsilon(x)\chi^i\,,\quad \delta B^i = \frac{i}{\sqrt2}\bar\epsilon(x)\gamma_5\chi^i\,,\quad \delta V_{a\mu} = -i\kappa\bar\epsilon(x)\gamma_a\Psi_\mu\,,\quad \delta A_\mu = \frac{i}{\sqrt2}\bar\epsilon(x)\gamma_\mu\lambda\,,$$
$$\delta\bar\lambda = -\frac{1}{\sqrt2}\bar\epsilon(x)\sigma^{\mu\nu}F_{\mu\nu} + \frac{i}{\sqrt2}(\xi + e\epsilon^{ij}A^iB^j)\bar\epsilon(x)\gamma_5 + i\kappa(\bar\Psi_\mu\gamma_\nu\lambda)\bar\epsilon(x)\sigma^{\mu\nu} - \frac{\kappa^2}{2\sqrt2}\bar\epsilon(x)\gamma_5(A^i + i\gamma_5 B^i)\chi^i\bar\lambda\gamma_5\,,$$
$$\delta\bar\chi^i = \frac{i}{\sqrt2}\bar\epsilon(x)(A^i + i\gamma_5 B^i)\overleftarrow{\not\partial} + \frac{ie}{\sqrt2}\epsilon^{ij}\bar\epsilon(x)(A^j + i\gamma_5 B^j)\not{A} - \frac{i\kappa}{2}(\bar\chi^i\psi_\mu)\bar\epsilon(x)\gamma^\mu - \frac{i\kappa}{2}(\bar\chi^i\gamma_5\psi_\mu)\bar\epsilon(x)\gamma^\mu\gamma^5 \qquad (6.4)$$
$$+ \frac{\kappa^2}{2\sqrt2}\bar\epsilon(x)\gamma_5(A^j + i\gamma_5 B^j)\chi^j\bar\chi^i\gamma_5\,,$$
$$\delta\psi_\alpha = \kappa^{-1}\bar\epsilon(x)\overleftarrow{\rm D}_\alpha + \frac{i\kappa}{4}(2\bar\psi_\alpha\gamma_a\psi_b + \bar\psi_a\gamma_\alpha\psi_b)\bar\epsilon(x)\sigma^{ab} + \frac{i\kappa}{4}(\bar\chi^i\gamma_5\gamma^a\chi^i)\bar\epsilon(x)\gamma_5\sigma_{a\alpha} - \frac{i\kappa}{2}(A^i\overleftrightarrow{\partial}_\alpha B^i)\bar\epsilon(x)\gamma_5$$
$$+ \frac{i\kappa}{8}(\bar\lambda\gamma_5\gamma^a\lambda)\bar\epsilon(x)\gamma_5(\gamma_\alpha\gamma_a + g_{\alpha a}) - i\kappa(\xi + e\epsilon^{ij}A^iB^j)\bar\epsilon(x)\gamma_5 A_\alpha - \frac{\kappa^2}{2\sqrt2}\bar\epsilon(x)\gamma_5(A^i + ii\gamma_5 B^i)\chi^i\bar\psi_\alpha\gamma_5\,.$$

Here $\epsilon(x)$ is a local spinorial parameter. The Lagrangian has two global invariances: It is unchanged under a rotation

$$\delta\chi^i = -\epsilon^{ij}\alpha\chi^j, \quad \delta A^i = -\epsilon^{ij}\alpha A^j, \quad \delta B^i = -\epsilon^{ij}\alpha B^j, \tag{6.5}$$

where α is a constant parameter. It is also invariant under the global chiral rotation

$$\delta\psi_\mu = i\theta\gamma_5\psi_\mu, \quad \delta\lambda = i\theta\gamma_5\lambda, \quad \delta\chi^i = -i\theta\gamma_5\chi^i. \tag{6.6}$$

In addition to these global invariances the theory also has an Abelian gauge symmetry: $\mathcal{L}$ is invariant under

$$\begin{aligned} &\delta A^i = -e\epsilon^{ij}\alpha(x)A^j, \\ &\delta B^i = -e\epsilon^{ij}\alpha(x)B^j, \\ &\delta A_\mu = \partial_\mu\alpha(x), \\ &\delta\lambda = -i\xi\kappa^2\alpha(x)\gamma_5\lambda, \\ &\delta\psi_\mu = -i\xi\kappa^2\alpha(x)\gamma_5\psi_\mu, \\ &\delta\chi^i = -e\epsilon^{ij}\alpha(x)\chi^j + i\xi\kappa^2\alpha(x)\gamma_5\chi^i, \\ &\delta V_{a\mu} = 0. \end{aligned} \tag{6.7}$$

The transformation looks very unusual in the sense that it mixes the usual vector-type transformations and the axial transformations of Ref. 17. This is simply a reflection of the fact that our starting theory does not conserve parity.

If we look at the interaction potential and diagonalize the mass matrix following Sec. V, then we observe that all the analysis of Sec. V goes through. In particular, the cosmological term disappears, and hence interpretation of the theory is easy. There is no spin-$\frac{3}{2}$–spin-$\frac{1}{2}$ mixing. Thus the spin-$\frac{3}{2}$ field still remains massless. In other words, although we coupled massless super-QED to supergravity, because of the gauge symmetry breaking we have obtained a supersymmetric theory of an interacting massive vector multiplet and a massless scalar multiplet coupled to supergravity. It is worth pointing out here that although the starting theory does not conserve parity, after the shift we can assign a set of parities to the new fields such that the Lagrangian is parity invariant. In such a scheme the spin-1 boson has an axial parity and if one expresses the transformation (6.7) in terms of the new fields, one finds it is a pure axial transformation. We have also tried to couple the massive super-QED to supergravity, and we have found no inconsistency up to order κ^4. However, beyond this order construction becomes extremely complicated.

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