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Stress-tensor conformal anomaly for scalar, spinor, and vector fields

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The conformal trace anomalies for massless scalar, "neutrino," and photon fields propagating in an arbitrary Riemannian space-time are derived. They are seen to be a consequence of the subtraction, during renormalization, of a finite term, $\sim \ln(m^2L^2)$, which violates the scale invariance of the massless theory. A general derivation of the scalar anomaly is given based on the ζ-function regularization developed earlier.

I. INTRODUCTION

In this paper we wish to give an account of our own derivation of the so-called trace anomaly in the vacuum expectation value of the energy-momentum tensor of a quantum field propagating in a background space-time. The approach was used by us some time ago for the scalar field; but in view of current interest in this topic (references will be given as they become relevant) it was thought that it might be useful to indicate the method and to extend it to the spin-$\frac{1}{2}$ and spin-1 fields.

II. CALCULATION

For convenience we use the notation of a previous paper[1] and begin with the vacuum-averaged stress tensor as a functional derivative (e.g., see Ref. 2)

$$\langle \hat{T}_{\mu\nu}\rangle = 2(-g)^{-1/2}\frac{\delta W^{(1)}}{\delta g^{\mu\nu}}, \tag{1}$$

where $W^{(1)}$ is the one-loop effective gravitational action,

$$W^{(1)} = \int \mathcal{L}^{(1)}(x)(-g)^{1/2}\,d^4x .$$

From the explicit expression for the improved scalar $\hat{T}_{\mu\nu}$ [see Ref. 2 or Eq. (22) of Ref. 1] it is easily shown that the trace of $\langle \hat{T}_{\mu\nu}\rangle$ is given by the coincidence limit,

$$\langle \hat{T}_\mu{}^\mu\rangle = im^2 \lim_{x'\to x} G_\infty(x,x') \tag{2}$$

$$= -2\,\frac{\partial \mathcal{L}^{(1)}}{\partial \ln m^2}, \tag{3}$$

where $G_\infty(x,x')$ is the Feynman Green's function. (The corresponding equations for the spinor field will be given later.)

In Ref. 1 we gave the asymptotic perturbation expansion for $\mathcal{L}^{(1)}$ in the form

$$\mathcal{L}^{(1)} = (32\pi^2)^{-1}\lim_{\nu\to 1}\left\{\frac{\frac{1}{2}a_0m^4 - a_1m^2 + a_2}{\nu-1} + \sum_{n=0}^{2}\frac{(-m^2)^{2-n}}{(2-n)!}\,a_n[\psi(3-n)+\gamma-\ln(m^2L^2)] + \sum_{n=3}^{\infty}a_n(m^2)^{2-n}\,\Gamma(n-2)\right\} \tag{4}$$

$[\gamma \equiv -\psi(1)]$, where the a_n are the coincidence limits of the coefficients in the asymptotic expansion of the Fock-Schwinger-DeWitt quantum-mechanical propagator. We have also introduced the arbitrary scale length L.

The regularization employed in deriving (4) is explained in Ref. 1 where we referred to it as ζ-function regularization. It is virtually equivalent to a dimensional regularization. Further details can be found in Sec. IV.

A conventional renormalization (see Ref. 2) consists of removing both the divergent pole term and the first sum in (4), i.e., *all* the a_0, a_1, and a_2 terms. This leaves $\mathcal{L}^{(1)}_{\text{fin}}$ as

$$\mathcal{L}^{(1)}_{\text{fin}} \equiv \mathcal{L}^{(1)} - (32\pi^2)^{-1}\lim_{\nu\to 1}\left\{\frac{\frac{1}{2}a_0m^4 - a_1m^2 + a_2}{\nu-1} + \sum_{n=0}^{2}\frac{(-m^2)^{2-n}}{(2-n)!}\,a_n[\psi(3-n)+\gamma-\ln(m^2L^2)]\right\}. \tag{5}$$

Prior to renormalization we have the formal result that $\langle \hat{T}_\mu{}^\mu\rangle$ tends to zero as m^2 vanishes. Thus

$$\frac{\partial \mathcal{L}^{(1)}}{\partial \ln m^2} \to 0 \text{ as } m^2 \to 0 ,$$

which, together with (3) and (5), yields the conformal trace "anomaly,"

$$\lim_{m^2\to 0}\langle \hat{T}_\mu{}^\mu\rangle_{\text{fin}} = -\frac{1}{16\pi^2}a_2 . \tag{6}$$

This result is seen to be a consequence of the renormalization, in particular of the subtraction of the finite a_2 term. If desired we could leave part of the a_2 term, viz., $(4\pi)^{-2}\ a_2\ln(L'/L)$, with

$\mathcal{L}^{(1)}_{\text{fin}}$. This would simply alter L to L' in (5). However, it seems to be essential for the appearance of the anomaly that *all* of the $\ln(m^2L^2)$ term is removed. This raises problems of infrared divergences in the full $\langle \hat{T}_{\mu\nu}\rangle$ but not in the trace.

Before comparing with other results we deal with the spin-$\frac{1}{2}$ and spin-1 cases. The energy-momentum tensor for the Dirac field is

$$T_{\mu\nu}=\frac{i}{4}\{[\overline{\psi},\gamma_{(\mu}\nabla_{\nu)}\psi]-[\nabla_{(\nu}\overline{\psi}\gamma_{\mu)},\psi]\}\,, \qquad (7)$$

where the square brackets and comma indicate antisymmetrization when ψ is interpreted as an operator field.

The equation which corresponds to (2) is

$$\langle \hat{T}_\mu{}^\mu\rangle=-im\lim_{x'\to x}\operatorname{tr}S_\infty(x,x')\,, \qquad (8)$$

where $S_\infty(x,x')$ is the Feynman Green's function satisfying

$$(i\gamma^\mu\nabla_\mu-m)S_\infty(x,x')=1\delta(x,x')\,.$$

Note that since we imagine all divergent, or potentially divergent, expressions to have been regularized we do not include any parallel propagators in the definition of $\langle \hat{T}_{\mu\nu}\rangle$.

In place of S_∞ it is more convenient to use the solution of the iterated Dirac equation and write

$$S_\infty=-(i\gamma^\mu\nabla_\mu+m)G_\infty$$

so that G_∞ obeys the equation

$$(m-i\gamma^\mu\nabla_\mu)(m+i\gamma^\nu\nabla_\nu)G_\infty(x,x')=1\delta(x,x')\,.$$

Then (8) becomes closer to (2),

$$\langle \hat{T}_\mu{}^\mu\rangle=im^2\lim_{x'\to x}\operatorname{tr}G_\infty(x,x')\,.$$

We can now imagine pursuing the same path that led to (6). There is really no need to introduce $\mathcal{L}^{(1)}$ for this purpose. All that is needed is the proper-time expansion of $\operatorname{tr}G_\infty(x,x)$, and this is known from the work of DeWitt.[3] It is clear that the answer will be

$$\lim_{m\to 0}\langle \hat{T}_\mu{}^\mu\rangle_{\text{fin}}=\frac{1}{16\pi^2}\operatorname{tr}a_2\,, \qquad (9)$$

where a_2 is given on p. 158 of Ref. 3.

The spin-1 case can also be treated in a similar manner although one must be prepared to start from a massive theory which makes sense only in the massless limit, if one is to obtain the correct massless results. This is because the ghost contribution comes in with different strengths in the massive and strictly massless cases. Thus we choose the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\tfrac{1}{2}(A^\mu{}_{\Vert\mu})^2+\tfrac{1}{2}m^2A_\mu A^\mu-\overline{c}_{\Vert\mu}c^{\Vert\mu}+m^2\overline{c}c$$

with $F_{\mu\nu}=A_{\mu\Vert\nu}-A_{\nu\Vert\mu}$ and where c is the *complex* ghost field. The stress-energy tensor is then given by

$$T_{\mu\nu}=-F_{\mu\alpha}F_\nu{}^\alpha+\tfrac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}+m^2A_\mu A_\nu-\tfrac{1}{2}m^2g_{\mu\nu}A_\rho A^\rho-\tfrac{1}{2}g_{\mu\nu}(A^\rho{}_{\Vert\rho})^2-g_{\mu\nu}A^\rho A^\sigma{}_{\Vert\sigma\rho}+A_\mu A^\rho{}_{\Vert\rho\nu}$$
$$+A_\nu A^\rho{}_{\Vert\rho\mu}-\overline{c}_{\Vert\mu}c_{\Vert\nu}-\overline{c}_{\Vert\nu}c_{\Vert\mu}+g_{\mu\nu}(\overline{c}_{\Vert\rho}c^{\Vert\rho}-m^2\overline{c}c)\,.$$

The vacuum average of the formal trace of $\hat{T}_{\mu\nu}$ is found to be

$$\langle \hat{T}_\mu{}^\mu\rangle=-im^2\lim_{x'\to x}[G_\mu{}^{\mu'}(x,x')-2G_0(x,x')]\,, \qquad (10)$$

where the Green's functions G_0 and $G_\mu{}^{\mu'}$ satisfy the equations

$$G^\nu{}_{\nu'\Vert\mu}{}^\mu-R^\nu{}_\mu G^\mu{}_{\nu'}+m^2G^\nu{}_{\nu'}=g^\nu_{\nu'}\delta(x,x') \qquad (11)$$

and

$$G_{0\Vert\mu}{}^\mu+m^2G_0=\delta(x,x')\,.$$

Note that the ghost Green's function, G_0, satisfies the minimal Klein-Gordon equation and not the conformal one.

We now need the proper-time expansions of $G_0(x,x)$ and $G^\mu{}_\mu(x,x)$, in particular the coefficients of $(i\tau)^2$. If these are denoted by a_2^0 and a_2^1, for G_0 and $G_\mu{}^\nu$, respectively, the vector anomaly can be written

$$\lim_{m^2\to 0}\langle \hat{T}_\mu{}^\mu\rangle_{\text{fin}}=-\frac{1}{16\pi^2}(\operatorname{tr}a_2^1-2a_2^0)\,. \qquad (12)$$

The numerical results are given and discussed in the next section.

III. RESULTS AND DISCUSSION

In nearly the notation of Christensen and Fulling[4] the anomaly is written in the general form suggested by Deser, Duff, and Isham[5]

$$-\langle \hat{T}_\mu{}^\mu\rangle_{\text{fin}}=(2880\pi^2)^{-1}[k_1C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}+k_2(R_{\mu\nu}R^{\mu\nu}-\tfrac{1}{3}R^2)-k_3\Box R+k_4R^2]$$
$$\equiv(2880\pi^2)^{-1}k_\alpha I^\alpha\,.$$

(Our conventions are a negative signature for $g_{\mu\nu}$ and Schouten's[6] definitions for $R_{\mu\nu\rho\sigma}$, etc.)

We do not write out the values for a_2 and a_2^0 for spin 0 and spin $\frac{1}{2}$ since they are well known and are given, e.g., in Refs. 2 and 3. However, the spin-1 (vector) expressions are not so widely known. We take them from the work of Gilkey[7] and list them here in our conventions for convenience,

$$\mathrm{tr}\, a_0^1 = 4\,,$$

$$\mathrm{tr}\, a_1^1 = R/3\,,$$

$$\mathrm{tr}\, a_2^1 = -\tfrac{1}{9}R^2 + \tfrac{43}{90}R_{\mu\nu}R^{\mu\nu} - \tfrac{11}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \tfrac{1}{30}R_{\|\mu}{}^{\mu}.$$

(In Misner-Thorne-Wheeler conventions the sign of R would be reversed.)

These values are confirmed by the work of Donnelly[8] on Kaehler manifolds. If the results for the three spins are collected we find the sets of values for the anomaly coefficients,

$$\{k_\alpha\} = (1, 1, 1, 0) \text{ for spin 0} \qquad (13)$$

$$= (\tfrac{7}{4}, \tfrac{11}{2}, 3, 0) \text{ for ``neutrino'' (massless Dirac)} \qquad (14)$$

$$= (-13, 62, -18, 0) \text{ for photon.} \qquad (15)$$

We have divided the value (9) by two to give the result for a single "neutrino" (= massless real Dirac spinor).

The values (13) agree with those in Ref. 4 where they were tentatively identified from basically the same term that we used. Christensen and Fulling use the point-separated $\langle \hat{T}_{\mu\nu}\rangle$ derived by Christensen,[9] and so the status of the anomaly as a consequence of a straight subtraction is slightly obscured. Also the hesitancy shown by these authors about drawing conclusions for the massless case from an asymptotic expansion, like that for $\mathcal{L}^{(1)}$, is, in our view, unjustified. Thus $\mathcal{L}^{(1)}_{\mathrm{fin}}$ is defined by (5) and we do not use its asymptotic expansion [which is just the last sum in (4)]. The only difficulty that might arise is an infrared one.

Brown[10] in an extensive calculation has derived the scalar anomaly (13) using a variant of dimensional regularization.

The spin-$\frac{1}{2}$ coefficients agree with those found by Bunch and Davies,[11] who use a completely different method but one which involves feeding-in the value of k_3 from the perturbation-theory calculations of Capper and Duff.[12] The value $k_\lambda = \frac{7}{4}$ was also given in Ref. 4.

For spin 1 our value of k_3 differs from that derived by Capper and Duff[12] and by Brown and Cassidy.[13] We attribute this to their use of dimensional regularization. The vector coefficients derived in Ref. 13 satisfy the relation $3k_3 - k_2 - 2k_1 = 0$ derived by M. J. Duff, again on the basis of dimensional regularization.

If dimensional regularization had been used here it would have been necessary to include the dimension dependence of the vector coefficients $\mathrm{tr}\, a_n^1$. For reference we give the expressions in 2ω dimensions of the first three coefficients. These are

$$\mathrm{tr}\, a_0^1(\omega) = 2\omega\, a_0^0\,,$$

$$\mathrm{tr}\, a_1^1(\omega) = \mathrm{tr}\, a_1^1(1) + 2(\omega - 1)a_1^0\,, \qquad (16)$$

$$\mathrm{tr}\, a_2^1(\omega) = \mathrm{tr}\, a_2^1(2) + 2(\omega - 2)a_2^0\,, \qquad (17)$$

where the a_n^0 are the standard scalar coefficients, for the *minimal* equation,[3] and where $\mathrm{tr}a_2^1(2)$ is given above and $\mathrm{tr}a_1^1(1) = 2R/3$. Clearly the scalar additions will mix in with the effect of the ghost, but we do not intend following this line of developement further now.

IV. SCALE TRANSFORMATIONS. GENERAL DERIVATION OF THE ANOMALY

Perhaps this is an appropriate moment to make our attitude towards scale transformations clear and, at the same time, begin to place the discussion on a more general footing. To do this we avail ourselves of the general expression for the one-loop effective Lagrangian $\mathcal{L}^{(1)}$ derived in Ref. 1 in terms of the ζ-function $\zeta(\nu, m^2)$, viz.,

$$\mathcal{L}^{(1)} = -\frac{i}{2}\left[\left(\frac{1}{\nu - 1} + \ln L^{-2}\right)\zeta(0, m^2) + \zeta'(0, m^2)\right], \qquad (18)$$

where we have included the contribution from the arbitrary scale length L.

Under the constant rescaling $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$, the quantum-mechanical propagator K transforms as $K \to \lambda^{-4}K$ and the proper time as $\tau \to \lambda^2\tau$ so that for $m^2 = 0$ the ζ function rescales by

$$\zeta(\nu, 0) \to \lambda^{2(\nu-2)}\zeta(\nu, 0)\,. \qquad (19)$$

(This also follows from the transformation of the massless Green's function $G \to \lambda^{-2}G$.)

Therefore

$$\zeta'(\nu, 0) \to \lambda^{2(\nu-2)}[\zeta'(\nu, 0) + \ln\lambda^2\zeta(\nu, 0)]\,,$$

whence from (18) we find the rescaling law for $\mathcal{L}^{(1)}(m^2 = 0)$

$$\mathcal{L}^{(1)} \to \lambda^{-4}\mathcal{L}^{(1)}\,. \qquad (20)$$

Note that we have taken L to rescale, $L \to \lambda L$. Equation (20) implies the invariance of the unrenormalized action $W^{(1)}$ under rescalings. Renormalization breaks this invariance since the term subtracted from $\mathcal{L}^{(1)}$, $\ln(m^2L^2)$, does not rescale as (20).

This can also be seen directly from (18). The renormalization of $\mathcal{L}^{(1)}$ consists of dropping the entire $\zeta(0, m^2)$ term, and some finite terms from $\zeta'(0, m^2)$ detailed in Ref. 1. It is the first operation that destroys the scale invariance (20) of the massless limit. Note that the scale-breaking term $\ln L^{-2}$ is closely associated with the pole, which suggests, but does not prove, that it is correct to remove the finite terms proportional to a_0, a_1, and a_2.

So long as we restrict ourselves to constant rescalings the situation is as we have described it. However, in order to cover spatially varying, or local, scale transformations, $g_{\mu\nu}(x) \to \lambda^2(x) g_{\mu\nu}(x)$, the ζ function has to be modified, and we would like now to make some observations of a technical nature about this.

The ζ function, as defined in Ref. 1, is the νth space-time matrix power of G_∞,

$$\zeta(\nu, m^2) = (G_\infty)^\nu \,, \tag{21}$$

and the effective one-loop massless action, *without* the introduction of a constant scale length, is [see Eq. (10), Ref. 1]

$$W^{(1)}(1) = \lim_{\nu \to 1} W^{(\nu)}(1) \,,$$

where

$$W^{(\nu)}(1) = -\frac{i}{2}\frac{1}{\nu - 1}\int \operatorname{diag}\zeta(\nu - 1, 0)(-g)^{1/2}\, d^4x \,. \tag{22}$$

Up to a factor, $W^{(\nu)}$ can be thought of graphically as a closed loop of $(\nu - 1)$ Green's functions with $(\nu - 1)$ space-time integrations.

Ask now what is the effect of a local rescaling. The massless Green's function transforms as

$$G_\infty(x, x') \to \lambda^{-1}(x) G_\infty(x, x') \lambda^{-1}(x') \,,$$

while the invariant space-time volume element changes as

$$(-g)^{1/2}\, d^4x \to \lambda^4(x)(-g)^{1/2}\, d^4x \,.$$

Thus the rescaling amounts to the introduction of a factor $\lambda^2(x)$ at each of the $(\nu - 1)$ integrations in (22), and so the action is not invariant (as expected since the scale length has been omitted).

If $\lambda(x)$ were uniform then an invariant action is obtained by introducing the constant scale length L as a simple multiplier,

$$W^{(\nu)}(L) = L^{-2(\nu-1)} W^{(\nu)}(1) \,. \tag{23}$$

We leave the proof to the reader. It has been given, for the limit $\nu \to 1$, in the preceding section.

For $\lambda(x)$ nonuniform, (23) is *not* the correct way to introduce the scale length. Rather it is necessary to insert at each integration in (21), or (22), an arbitrary weight $\rho(x)$ which transforms as $\rho(x) \to \lambda^{-2}(x)\rho(x)$ in order to cancel the effect of the local transformations of G_∞ and $(-g)^{1/2} d^4x$. We denote the corresponding action $W^{(\nu)}[\rho]$.

It does not seem possible to give a simple expansion for $W^{(\nu)}[\rho]$ in terms of the a_n as it is for $W^{(\nu)}[1] = W^{(\nu)}(1)$ [see Eqs. (12) and (17) in Ref. 1]. However, it is straightforward to derive the functional derivatives of $W^{(\nu)}[\rho]$ with respect to ρ. For example, , we have

$$\left.\frac{\delta W^{(\nu)}[\rho]}{\delta \rho(x)}\right|_{\rho=1} = -\frac{i}{2}\operatorname{diag}_x \zeta(\nu - 1, 0)[-g(x)]^{1/2} \,, \tag{24}$$

so that in the case when ρ is nearly unity, i.e., $\ln\rho$ small, an expansion which takes the place of (4) can be derived without difficulty. Thus the first two terms in the Taylor-series expansion of $W^{(\nu)}[\rho]$ in powers of $\ln\rho$ are

$$W^{(\nu)}[\rho] \approx W^{(\nu)}[1] + \int \left.\frac{\delta W^{(\nu)}(\rho)}{\delta \rho(x)}\right|_{\rho=1} \ln\rho(x) d^4x \,, \tag{25}$$

so that the Lagrangian density $\mathcal{L}^{(\nu)}(x)$ is, if (24) is taken into account,

$$\mathcal{L}^{(\nu)}(x) \approx -\frac{i}{2}\left[\frac{1}{\nu - 1} + \ln\rho(x)\right]\operatorname{diag}_x \zeta(\nu - 1, m^2) \,. \tag{26}$$

It will be noticed that we have reintroduced the mass into Eq. (26). This is because, although we have motivated the analysis using the massless limit, it is in fact independent of this. Of course, only in the massless limit will $W^{(\nu)}[\rho]$ be scale invariant.

The limit of (26) as ν tends to unity is exactly Eq. (4) except that L^2 is replaced by the local scale function $\rho^{-1}(x)$. The same is true of Eq. (18).

We note now that (24) contains essentially the statement of the conformal anomaly. The explanation of this will give us an alternative and more general proof of the anomaly (6) and, moreover, a proof which does not involve taking a massless limit. Rather we can discuss the massless case immediately, as we now do.

We start with the general relation

$$\langle \hat{T}_\mu{}^\mu(x) \rangle = -\lim_{\nu \to 1} (-g)^{-1/2} \left.\frac{\delta W^{(\nu)}[\rho, \lambda]}{\delta \lambda(x)}\right|_{\lambda=1} \,, \tag{27}$$

where $W^{(\nu)}[\rho, \lambda]$ is the action after a local scale change $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$ and we have explicitly displayed the dependence on $\lambda(x)$. In the massless case, the scale invariance says that $W^{(\nu)}[\rho, \lambda]$ is independent of $\lambda(x)$,

$$W^{(\nu)}[\rho, \lambda] = W^{(\nu)}[\rho, 1] = W^{(\nu)}[\rho] \,,$$

so that $\langle \hat{T}_\mu{}^\mu \rangle$ is zero. The action must now be renormalized. One of the terms to be subtracted is the scale-breaking integral in (25). From (24) this is finite in the limit $\nu \to 1$. There will be other subtractions, but these will preserve the scale invariance so we do not need to specify precisely what they are. Let us write the scale-transformed (25) in the shorter form

$$W^{(\nu)}[\rho,\lambda]=W^{(\nu)}[1,\lambda]+B^{(\nu)}[\rho,\lambda], \qquad (28)$$

where $B^{(\nu)}$ stands for the scale-transformed integral, and define the renormalized action as

$$W^{(\nu)}_{\text{fin}}[\rho,\lambda]=W^{(\nu)}[\rho,\lambda]-B^{(\nu)}[\rho,\lambda]-I^{(\nu)}[\rho,\lambda], \qquad (29)$$

where $I^{(\nu)}$ are the scale-invariant subtractions. The finite trace will be given by Eq. (27) with $W^{(\nu)}_{\text{fin}}$ instead of $W^{(\nu)}$. From (29) it is clear that only the scale-breaking term $-B^{(\nu)}$ will contribute to this trace and also, (28) yields the functional-derivative statement

$$\frac{\delta B^{(\nu)}[\rho,\lambda]}{\delta\lambda}=-\frac{\delta W^{(\nu)}[1,\lambda]}{\delta\lambda}.$$

We have met the quantity $W^{(\nu)}[1,\lambda]$ before and the functional derivative is easily evaluated since it amounts only to differentiating with respect to a λ^2 at each $(\nu-1)$ space-time integration. In fact the required derivative is given by (24) since, by definition, $W^{(\nu)}[1,\lambda]=W^{(\nu)}[\lambda^2]$. [This was why we introduced $\rho(x)$ in the first place.] Putting everything together we find

$$\langle \hat{T}_\mu{}^\mu\rangle_{\text{fin}}=i\,\text{diag}\zeta(0,0)$$
$$=-\frac{1}{16\pi^2}a_2$$

from Eq. (16) of Ref. 1.

This derivation of the scalar anomaly is a general one, independent of any massless limit, and since it does not involve the mention of an asymptotic expansion it could replace the discussion of the earlier section with some formal advantages.

V. CONCLUSION

We have tried to emphasize the role of the finite renormalization in producing the conformal anomaly. It seems to us that this has not been brought out sufficiently clearly in other treatments, and we have tried to give both a simple explanation and a general derivation.

It is arguable whether the loss of conformal invariance should be allowed. If it is not then probably some other property of $\langle \hat{T}_{\mu\nu}\rangle$ will have to be given up.

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