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# Fermion nonminimal gravitational coupling and the "solar neutrino problem"*

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In this work a universal magnetic-dipole interaction between massive fermions is considered, with the coupling mediated by the local spacetime curvature. The strong principle of equivalence is not valid for fermions because of their intrinsic spin. Hence, the associated principle of "minimal gravitational coupling" for the Dirac equation coupled to electromagnetic fields in the presence of gravity is an assumption which is unsupported by either theory or experiment. We show that relaxing the arbitrary minimal-coupling constraint leads to a simple kind of nonminimal gravitational coupling (NMGC) which can generate curvature-dependent magnetic-moment effects, in background gravitational fields, for fermions coupled electromagnetically. Application of this model to the case of solar neutrinos yields a simple explanation of the low terrestrial neutrino flux in terms of sufficient neutrino energy loss (via multiple neutrino-electron magnetic elastic scattering in the solar plasma) to account for the very low detection rate on earth. Terrestrial tests of this type of NMGC effect for neutrinos would appear in high-energy neutrino-nucleon scattering, in terms of anomalous (charge-dependent) neutrino-deuteron interactions, which could not be explained by charge-independent neutral-current models alone.

## I. INTRODUCTION

In this work a universal magnetic-dipole interaction between massive fermions is considered, with the coupling mediated by the local spacetime curvature. There are two motivations for considering such an interaction. It is well known that massive spinning particles do not move along geodesics since their spins couple directly to the local spacetime curvature. The precise form of the gravitational coupling is unknown since experiments have not yet achieved the necessary sensitivity. Theory invokes minimal coupling only because of its validity for particles without intrinsic spin. Nonminimal coupling is considered here as a logical possibility which should be ruled upon by experiment. A second motivation is the possibility of augmenting the standard four-fermion interactions in the light of the continuing solar neutrino puzzle. Clark and Pedigo[1] have examined an energy-loss mechanism for solar neutrinos via multiple $\nu_e - e$ scattering with a finite neutrino magnetic moment. Their purely electromagnetic interaction requires an unphysically large neutrino magnetic form factor to provide scattering to account for Davis's low capture rate. With the model considered here, the necessarily large magnetic form factor is automatically accounted for by local curvature effects within the solar plasma, and suggests that the Clark-Pedigo model is viable within the nonminimal-gravitational-coupling (NMGC) framework.

In Sec. II, the associated Dirac-Einstein-Maxwell field equations are derived from an action principle. The magnetic-dipole interaction with nonminimal gravitational coupling is written as the simplest term which mixes all three fields and satisfies gauge invariance of the first and second kinds. It is our expectation that currently existing scattering data for charged lepton-hadron processes will not significantly constrain this particular NMGC term, because any electromagnetic or hadronic interaction will always mask the much smaller NMGC effect. Hence, effects of NMGC can be observed only when electromagnetic and strong interactions are absent, that is, for neutral leptons (i.e., neutrinos).

In Sec. III, NMGC is applied to the solar neutrino problem. The gravitational coupling, together with electron mass density (spread over a region determined by average charge neutrality), will act to multiply the size of the neutrino magnetic form factor and make it large enough to allow direct use of Clark and Pedigo's calculation. The NMGC constant is given an upper limit from the terrestrial scattering data of Cowan and Reines.[2]

Since the NMGC coupling is assumed to be universal, terrestrial tests are considered in Sec. IV. High-energy $\nu_e - p$ scattering is discussed wherein the neutrino energy is large enough to sample the proton density (over its wave packet) via NMGC. Neutral-current effects are thought to also occur within this scattering domain; hence to distinguish between an NMGC and neutral-current process, it is suggested that neutrino-deuteron processes might yield an anomalous charge dependence (considered as due to NMGC effects associated with this model), which neutral-current theories alone could not account for.

## II. A SIMPLE FORM OF NONMINIMAL GRAVITATIONAL COUPLING IN THE DIRAC-EINSTEIN-MAXWELL SYSTEM

If we eliminate the arbitrary assumption of minimal gravitational coupling in the Dirac-Einstein-Maxwell equations, consistent with the fact that the strong principle of equivalence[3] (SPE) is not valid for fermions, the simplest gauge-invariant[4] nonminimal gravitational interaction which mixes all three fields (and has dimensions of an energy density) is a magnetic-moment interaction which is curvature dependent in the form

$$\mathcal{L}_{\rm NMGC} = \eta R(\tfrac{1}{2}\mu_0)\overline{\Psi}\sigma_{\mu\nu}\Psi F^{\mu\nu}\,, \tag{1}$$

where $\eta$ has dimensions of (length)$^2$ and $\mu_0 \equiv e/2m_e$. Units of magnetic moment are used and $\hbar = c = 1$. Inserting (1) into the standard Dirac-Einstein-Maxwell action yields

$$I = \int d^4x(-g)^{1/2}[R + \kappa(\mathcal{L}_{\rm Dirac} + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + J_\mu A^\mu + \mathcal{L}_{\rm NMGC})]\,, \tag{2}$$

where the NMGC term has a strength determined by the value of $\eta\kappa$, and is added directly into the original minimally coupled action principle. Then variation $\delta I = 0$ with respect to $\delta g_{\mu\nu}$, $\delta A_\mu$, and $\delta\Psi$ gives[5] respectively a modified energy-momentum tensor in the Einstein equation

$$\begin{aligned} G_{\mu\nu} &= -\kappa(1+\eta\kappa S)^{-1}\left[t_{\mu\nu({\rm Dirac})} + t_{\mu\nu({\rm Maxwell})} + \eta\left(R\frac{\partial S}{\partial g^{\mu\nu}} + g_{\mu\nu}S_{;\alpha}{}^{\alpha} - S_{;\mu\nu}\right)\right] \\ &= -\kappa T_{\mu\nu}(\eta)\,, \end{aligned} \tag{3}$$

where $S = \frac{1}{2}\mu_0\overline{\Psi}\sigma_{\mu\nu}\Psi F^{\mu\nu}$, and the Einstein gravitational constant is $\kappa = 8\pi G c^{-4}$; a modified current in the Maxwell equation

$$F_{\mu\nu}{}^{;\nu} = J_\mu(\eta) = [e\overline{\Psi}\gamma_\mu\Psi - (\tfrac{1}{2}\eta R\mu_0\overline{\Psi}\sigma_{\mu\nu}\Psi)^{;\nu}]\,, \tag{4}$$

and finally a modified Dirac equation with a curvature-dependent anomalous magnetic moment

$$(-i\gamma^\mu\nabla_\mu + m + e\not{A} + \tfrac{1}{2}\eta R\mu_0\sigma_{\mu\nu}F^{\mu\nu})\Psi = 0\,. \tag{5}$$

Note that the NMGC terms do not vanish in a local geodesic frame, as expected.

## III. APPLICATION TO THE SOLAR NEUTRINO PROBLEM

From Sec. II, we see that the effect of the NMGC term (1) on the action (2) is to give the fermion an anomalous magnetic moment $\mu = \eta R\mu_0$, depending on the scalar curvature $R$.[6] Let us suppose that we choose the value of $\eta$ subject to the constraint that this effect is dwarfed by the presence of the much stronger electromagnetic and strong interactions (which would account for our lack of any experimental evidence of this term for charged leptons and hadrons). Then it still might be observable for neutrinos, since they have zero charge, and only a very weak interaction with matter. To test this hypothesis we will apply it to the case of the solar neutrinos. The possibility of the neutrinos undergoing multiple magnetic elastic scattering on the electrons in the solar plasma has been previously investigated[1] as a possible solution to the "solar neutrino problem." Since there exists some evidence that neutrinos may have a small nonzero mass[7] we will assume in our calculations that the neutrino is described by (5) with $e = 0$ and $m_\nu < 60$ eV. We take the general point of view[8] that parity nonconservation is a property of the weak-interaction matrix element itself [hence if $\Psi$ is a solution to (5) with $e = 0$ and $m_\nu \neq 0$, then only $\frac{1}{2}(I - i\gamma^5)\Psi$ contributes to the weak-interaction matrix element (even if $m_\nu \neq 0$)]. In the sun, the metric is slowly varying over microscopic dimensions so we can calculate effectively in a local freely falling geodesic frame where the first derivatives of $g_{\mu\nu}$ vanish. If we also assume that gravity is treated as an external field, then the equations for solar neutrinos inside the solar electron-proton plasma can be specialized from (5) to the simpler form (6) below under the following assumptions: First, for solar neutrinos $E_\nu < 10$ MeV, so this means that $\lambda_\nu \sim \hbar c/E_\nu > 10^{-12}$ cm; thus the protons are seen by the neutrinos as if they are massive charged pointlike structures surrounded by unbound negative-charged electron distributions; hence the proton dynamics can be neglected at this energy, and only neutrino-electron scattering will be important. Second, the universal NMGC is assumed to be masked, in the electron equation, by the electromagnetic term $-e\not{A}$, hence it is neglected in this approximation. Under these assumptions the local equations for NMGC solar neutrinos in the solar plasma are approximated to first order in $\eta\kappa$ as

$$(-i\not{\partial} + m_{(\nu)} + \tfrac{1}{2}\eta R\mu_0\sigma_{\mu\nu}F^{\mu\nu})\Psi_{(\nu)} = 0\,, \tag{6a}$$

$$(-i\not{\partial} + m_e - e\not{A})\Psi_e = 0\,, \tag{6b}$$

$$F_{\mu\nu}{}^{,\nu} = -e\Psi_e\gamma_\mu\Psi_e - \tfrac{1}{2}(\eta R\mu_0\overline{\Psi}_{(\nu)}\sigma_{\mu\nu}\Psi_{(\nu)})^{,\nu}\,, \tag{6c}$$

$$\begin{aligned} R &\simeq \kappa(T_e + T_p) \\ &\simeq \kappa c^2(S_e + S_p)\,, \end{aligned} \tag{6d}$$

where $\mu_0 \equiv e^2/2m_e$ and $S_e, S_p$ are the electron and proton mass densities. From (6a) we see that for the solar neutrinos specifically, the scalar curvature in (6d) generates an anomalous magnetic moment term so that the neutrino equation of motion is

$$[-i\not\partial + m_{(\nu)} + \tfrac{1}{2}\eta\kappa c^2 \mu_0 (S_e + S_p)\sigma_{\mu\nu}F^{\mu\nu}]\Psi_{(\nu)} = 0. \qquad (7)$$

However, in the neutral solar plasma, since $\lambda_\nu > 10^{-12}$ cm, $S_p$ is seen by the neutrino to be a superposition of pointlike proton mass densities surrounded by spread out $S_e(x)$ electron-wave mass densities. Since the average density of the solar plasma $\sim 1$ g/cm$^3$, this implies that there are $10^{25}$ protons/cm$^3$; hence there is one proton every $10^{-8}$ cm along the neutrino path. The average neutrality of the plasma then implies that there is also one electron wave spread out over the $10^{-8}$ cm in between each pointlike proton, as seen by the solar neutrinos moving through the plasma. Thus, in (7) only the spread out structure of the electron clouds will be sensed most sensitively by neutrinos with $\lambda_\nu > 10^{-12}$ cm (while the more massive pointlike protons will exert a much smaller effect). For elastic scattering, most of the recoil from neutrino scattering will be taken up by the electrons,[9] and for this case (7) can be written in the more revealing form

$$[-i\not\partial + m_{(\nu)} + \tfrac{1}{2}\mu_{(\nu)}(x)\sigma_{\mu\nu}F^{\mu\nu}]\Psi_{(\nu)} = 0\,, \qquad (8)$$

where $\mu_{(\nu)}(x)$ is the effective NMGC-induced neutrino magnetic moment inside the solar plasma which participates in neutrino-electron magnetic elastic scattering

$$\mu_{(\nu)}(x) \simeq [\eta\kappa c^2 S_e(x)]\mu_0. \qquad (9)$$

Now $|S_e| \simeq [10^{-28}/(10^{-8})^3] \simeq 10^{-4}$ g/cm$^3$ in the solar plasma, so that the NMGC-induced $\mu_{(\nu)}(x)$ is spread out over $10^{-8}$ cm with an average density of $10^{-4}$ g/cm$^3$. Hence we can write $\mu_{(\nu)}(x)$ in the form

$$\mu_{(\nu)}(x) \simeq (\eta\kappa c^2\mu_0)\times 10^{-4}F(x)\,, \qquad \int F(x)dx^3 = 1\,, \qquad (10)$$

and where the density structure $F(x)$ acts like a "magnetic form factor." Applying (8) through (10) to the work of Clark and Pedigo,[1] we see that the NMGC effect gives the neutrino a very large spread for its effective magnetic form factor $\sim 10^{-8}$ cm. This is precisely the kind of effect which enables the neutrino-electron multiple magnetic-elastic scattering models to account for sufficient neutrino energy loss to fall below the detection threshold of the Davis experiment.[10] To see this in more detail, we note that the NMGC neutrino equations (8), (9), and (10) can be written as

$$[-i\not\partial + m_\nu + \eta\kappa c^2(\mu_0)S_e\sigma_{\mu\nu}\partial^\nu A^\mu]\Psi_{(\nu)} = 0\,. \qquad (11)$$

To lowest order the magnetic-elastic scattering matrix element for the neutrinos and electrons is of the same form as that used by Clark and Pedigo, except for the fact that our magnetic form factor $F(x)$ is related to the local electron mass density in the solar plasma as

$$S_e(x) \simeq 10^{-4}F(x)\,. \qquad (12)$$

Hence we may directly use their approach to offer a possible solution to the solar neutrino problem. Since they use the results of Cowan and Reines,[2] that experimentally for terrestrial $\nu_e - e$ scattering $\mu_{(\nu)} < 10^{-9}\mu_0$, this means that from (10) we can determine an upper limit on $(\eta\kappa)$ as

$$(\eta\kappa)c^2 10^{-4} < 10^{-9}\,,$$

which yields an upper limit of

$$(\eta\kappa) < 10^{-26}\,. \qquad (13)$$

This is the terrestrial upper limit on the coupling $(\eta\kappa)$ of the NMGC-induced magnetic moment of the neutrino. Since our model predicts a spread out "solar plasma" form factor $\langle r\rangle_\nu \sim 10^{-8}$ cm, then the results of Clark and Pedigo[1] will satisfactorily hold and indicate that choices of $F(x)$ which imply $\langle r\rangle_{(\nu)} \gg 7\times 10^{-10}$ cm are explainable by the NMGC hypothesis.

## IV. TERRESTRIAL TESTS OF THE NMGC HYPOTHESIS FOR NEUTRINOS

To study the possibility of terrestrial tests we recall that we have assumed universality of the NMGC. This means we assume that anomalous-magnetic-moment terms $\mu_i(x) \sim \eta_i\kappa c^2 S(x)\mu_0$ appear in all fermion equations, for various leptons and hadrons. For the case of neutrinos, the largest value that the NMGC effect can have is when the density is (neutrino-nucleon scattering) $|S(x)| \sim 10^{15}$ g/cm$^3$, implying in this extreme that $|\mu(x)| < 10^{-26}\times 10^{15}\times 10^{21}\times\mu_0) \sim 10^{10}\mu_0$ for hadronic interactions. However, since the Bohr magneton [chosen as the unit in which $(\eta\kappa) < 10^{-26}$ was found] is $\mu_0 \sim 10^{-20}$, then $|\mu(x)| < 10^{-10}$, which is comparable to a "weak interaction." However, we assume NMGC would still be masked by the stronger electromagnetic and hadronic forces associated with electrons and nucleons (as we expected it would be earlier). However, for high-energy terrestrial neutrinos, because of the absence of neutrino electric charge, an effect of the new NMGC coupling might actually be seen in terrestrial neutrino-nucleon scattering. To see this we note that for neutrino energies $E_{(\nu)} > 1$ GeV, $\lambda_{(\nu)} < 10^{-14}$ cm and the neutrino will begin to detect the nuclear density (over its wave packet) if it collides with a nucleon. The strength of this neutrino-nucleon magnetic elastic scattering effect for the case of protons is determined from the above estimate as comparable to the weak interaction since $(\eta\kappa) \leq 10^{-26}$ when the neutrino interacts with the proton. Recent data[11] on high-energy neutrino-proton scattering indicate that such an effect occurs. This is currently interpreted as due to neutral-current mechanisms, and if an NMGC effect were also present, its comparable strength to weak interactions would make it difficult to distinguish from neutral-current effects, since the exact symmetry of the neutral current is not yet clearly known.[12] However, since the specific NMGC we chose is sensitive to the charge of the target nucleon, upon which the neutrino scatters, we would expect that if both neutral currents and the NMGC effect were present in neutrino-nucleon scattering then we would see it occur as a difference in the specific nature of neutrino-proton and neutrino-neutron scattering. A practical way of testing this is to study the neutrino-deuteron scattering process. If only neutral currents are present (and $\eta\kappa = 0$) then no charge dependence in the scattering occurs. On the other hand, if $0 < (\eta\kappa) \leq 10^{-26}$, then the NMGC would given an anomalous charge dependence to the process (due to the neutrino magnetic moment interacting differently with the proton and the neutron, respectively), which, if not explainable on the basis of neutral currents, could be terrestrial evidence of NMGC for neutrinos.

In conclusion, we are suggesting that the solar neutrino problem is an indication of the possibility that very weak SPE-violating NMGC exists in nature, and ultimately terrestrial neutrino-deuteron scattering experiments will determine the validity of this hypothesis, much in the way that the "$\tau - \theta$ puzzle" led physicists to postulate parity-violating weak interactions, which were ultimately tested by $Co^{60}$ decay measurements.

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[1]R. B. Clark and R. D. Pedigo, Phys. Rev. D 8, 2261 (1973).

[2]C. L. Cowan and F. Reines, Phys. Rev. 107, 528 (1957).

[3]Our statement of the SPE is that at each point of space-time the gravitational field variables can be removed from the field equations of matter by a coordinate transformation. This is equivalent to demanding that there be no explicit appearance of curvature terms in the interaction Lagrangian.

[4]By "simplest," we mean that this term is linear in $R$ and $F^{\mu\nu}$, and bilinear in $\Psi$, which is consistent with gauge invariance of both the first and second kinds. It is interesting to note that one might think that a term such as $\eta R \bar{\Psi}\gamma_\mu \Psi A^\mu$ would be simpler, however, this term is not gauge invariant because of $R_{,\mu} \neq 0$ in the general context.

[5]Details on the explicit formalism associated with the generally covariant Dirac equation and variational principles which involve the Dirac-Einstein system are given in D. Leiter, Lett. Nuovo Cimento 12, 633 (1975); D. Leiter and T. Chapman, Am. J. Phys. 44, 858 (1976). See H. C. Ohanian, J. Math. Phys. 14, 1892 (1973), for comments on the relationship of the SPE to general concepts of NMGC.

[6]The scalar curvature $R$ is obtained from Eq. (3) by taking the trace of the Einstein equation over the space-time indices.

[7]V. E. Barnes *et al.*, Phys. Rev. Lett. 38, 1049 (1977).

[8]This is the well-known idea that parity violation in weak interactions may be connected with the inherent chirality invariance of the associated Lagrangian, and not necessarily to the requirement of zero neutrino mass.

[9]This is the same approximation as that used by Clark and Pedigo in Ref. 1.

[10]Good general discussions of this are given in the review article by B. Kuchowicz, Rep. Prog. Phys. 39, 291 (1976), and in the lectures of R. T. Rood, proceedings of the 1976 Summer School "Ettore Majorana," Erice, Italy (unpublished). See also BNL Report No. BNL-21837 (unpublished) for the latest analysis of the data by R. Davis and his group.

[11]See D. Cline *et al.*, Phys. Rev. Lett. 37, 252 (1976); D. Cline *et al.*, *ibid.* 37, 648 (1976); W. Lee *et al.*, *ibid.* 37, 186 (1976).

[12]See Ephraim Fischbach *et al.*, Phys. Rev. D 15, 15 (1977), for good experimental and theoretical analysis of recent neutrino-nucleon scattering experiments, using purely neutral-current models.