Nonleptonic meson decays in the bag model

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Nonleptonic meson decays are discussed in the current-current picture making use of the MIT bag model. It is shown that upon inclusion of an enhancement factor due to asymptotic freedom the decay amplitude for $K_S^0 \to \pi^0\pi^0$ is also of the correct order of magnitude. Furthermore, we have computed the enhancement-independent ratio $a(K_S^0 \to \pi^0\pi^0)/A(A_0^0)$ and have found it to be equal to 833 MeV, which is to be compared with the experimental ratio of 1142 MeV. We conclude by presenting a prediction for the decay width for $D \to K^-\pi^+$, where D^0 is the lowest charmed meson.

I. INTRODUCTION

Considerable interest has recently been devoted to the MIT bag model.[1-8] One of the main virtues of this model lies in providing a consistent relativistic framework in which the quark wave functions may by explicitly calculated at least in the cavity approximation. In particular, the model has been successfully applied to the calculation of the low-lying hadron mass spectrum, static parameters of light hadrons, and various types of hadron decays.

In Refs. 6 and 8, the bag model was first applied to the discussion of nonleptonic baryon decays. In this paper we complete the discussion of those references and investigate the application of the bag model to nonleptonic meson decays.

We start by writing down the matrix elements for the S-wave nonleptonic meson decays. These are

$$(2k_0)^{1/2}(2\pi)^{3/2}\langle p_\alpha \pi_\beta |\mathcal{H}_w^{pv}(0)|p_\gamma\rangle \equiv i\left(\frac{1}{2\pi}\right)^3 \frac{1}{(4p_{0\alpha}p_{0\gamma})^{1/2}} Q^\beta_{\alpha\gamma}\,. \quad (1)$$

Furthermore, the commutator contribution to Eq. (1) is given by[9]

$$-\frac{\sqrt{2}}{f_\pi} i\langle p_\alpha |[Q_5^\beta, \mathcal{H}_w^{pv}(0)]|p_\gamma\rangle = -\frac{\sqrt{2}}{f_\pi} i\langle p_\alpha |[Q^\beta, \mathcal{H}_w^{pc}(0)]|p_\gamma\rangle \equiv i\left(\frac{1}{2\pi}\right)^3 \frac{1}{(4p_{0\alpha}p_{0\gamma})^{1/2}} (Q_c)^\beta_{\alpha\gamma}\,.$$

In the above, $\mathcal{H}_w^{pv}(0)$ and $\mathcal{H}_w^{pv}(0)$ denote the parity-violating and parity-conserving parts of the nonleptonic current-current weak Hamiltonian, which may be expressed in terms of quarks in the standard fashion.

We now proceed to calculate the matrix elements in Eqs. (1) and (2). For this purpose, we first note that all of these decay amplitudes may be expressed in terms of $\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle$ and $\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$ in the standard fashion,[6,8,9] and then proceed to calculate these matrix elements in the bag model in precisely the same way as for the baryons. However, before doing so, we first wish to stress the strong similarity in the calculation of $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$ and $\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$. In fact, in computing both of these matrix elements, we are dealing with sums of contributions of the type $\overline{q}_1\Gamma_\mu q_2\overline{q}_3\Gamma^\mu q_4$ for baryons and $\overline{q}_1\Gamma_\mu q_2\overline{q}\,\Gamma^\mu q_3$ for mesons, where the indices 2 and 4 denote the quarks in the initial state while 1 and 3 denote the quarks in the final state. On the other hand, in computing $\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle$ we are dealing with sums of contributions of the type $\overline{q}_3\Gamma_\mu q_1\overline{q}_4\Gamma^\mu q_2$ with the same convention for the indices as above. In other words, $\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle$ corresponds to a sum of annihilation diagrams while that is not the case for $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$ and $\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$. Since in bag-model calculations one makes use of quark wave functions in a fixed-sphere approximation, one could expect that annihilation diagrams might not be correctly described in this framework. For example, if one naively computes the matrix element $\langle\Omega|A_\mu|K\rangle$ in the bag model, one obtains zero as a result. On the other hand, if one calculates in a somewhat more sophisticated way, as is done in Ref. 5, one arrives at a result which is in reasonable agreement with experiment only if color is neglected altogether for the quarks. If, however, color is included, one arrives at a result which is three times larger and, hence, in contradiction with experiment.

The encouraging results of the calculation for $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$ suggest that the calculation for $\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$ might also be in reasonable agreement with experiment. We shall see that this is in fact the case. We then also consider the matrix element $\langle\overline{K}^0|\mathcal{H}_w^{pc}(0)|D^0\rangle$ (where D^0 denotes the lowest charmed meson corresponding to $c\overline{u}$ in quark language) and present a prediction for the decay width for $D^0 \to K^-\pi^+$.

We next proceed to calculate the amplitude $\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$. For this purpose, we note that all the terms V_0V_0, V_kV_k, A_0A_0, and A_kA_k of the parity-conserving weak Hamiltonian yield a nonzero contribution. The only difference with respect to the baryon calculation is that in the latter only terms of the type $\bar{\psi}_4\Gamma_\mu\psi_3\bar{\psi}_1\Gamma^\mu\psi_2$ appear, while in the meson computation we have contributions of the type $\bar{\varphi}_4\Gamma_\mu\varphi_3\bar{\psi}_1\Gamma^\mu\psi_2$, where ψ and φ are the quark and antiquark wave functions, respectively. In this way, we thus arrive at the following expressions:

$$\frac{1}{\sqrt{2}}\frac{1}{f_\pi}\left\langle \pi^0\left|\int_{\text{bag}}\mathcal{H}_w^{pc}d^3x\right|K^0\right\rangle = \frac{G}{\sqrt{2}}\sin\theta\cos\theta\frac{1}{f_\pi\sqrt{2}}\frac{2}{\sqrt{2}}I_M\,, \quad (3)$$

where

$$I_M=\left(\frac{N_1N_2N_3N_4}{4\pi}\right)\int_0^R r^2dr(\alpha_1\alpha_2\alpha_3\alpha_4+\beta_1\beta_2\beta_3\beta_4 -\alpha_1\beta_2\beta_3\alpha_4-\beta_1\alpha_2\alpha_3\beta_4 +\tfrac{1}{3}\alpha_1\alpha_2\beta_1\beta_2+\tfrac{1}{3}\beta_1\beta_2\alpha_3\alpha_4 +\tfrac{1}{3}\alpha_1\beta_2\alpha_3\beta_4+\tfrac{1}{3}\beta_1\alpha_2\beta_3\alpha_4)\,, \quad (4)$$

with α, β, and N as defined in Ref. 8.

Furthermore, in the above equations, the indices 1 and 3 correspond to the quark-antiquark pair in the pion state, while 2 and 4 correspond to the quark-antiquark pair in the kaon state. In addition, R_1 and R_2 denote the radii for the π and K states, respectively. We have numerically calculated the integral in Eq. (4) for the two sets of parameters corresponding to the fit in Ref. 3.

In the case in which the nonstrange quark has zero mass and the strange quark has a mass of 279 MeV (which corresponds to the first set of parameters in Ref. 3), we then arrive at

$$\frac{1}{\sqrt{2}}\frac{1}{f\pi}\left\langle \pi^0\left|\int_{\text{bag}}\mathcal{H}_w^{pc}d^3x\right|K^0\right\rangle = 1.87\times10^{-7} \quad (5)$$

if the same enhancement factor as used for the baryons in Ref. 8 is employed, namely an enhancement factor of 3.

This is to be compared with the experimental value of

$$\frac{1}{\sqrt{2}}\frac{1}{f\pi}\left\langle \pi^0\left|\int_{\text{bag}}\mathcal{H}_w^{pc}d^3x\right|K^0\right\rangle = 3.82\times10^{-7}. \quad (6)$$

Furthermore, the ratio $Q(K_S^0\to\pi^0\pi^0)/A(\Lambda_0^0)$ is independent of the enhancement factor and may be computed to be given by

$$\frac{Q(K_S^0\to\pi^0\pi^0)}{A(\Lambda_0^0)} = 833\text{ MeV}\,, \quad (7)$$

which is to be compared with the experimental value of

$$\frac{Q(K_S^0\to\pi^0\pi^0)}{A(\Lambda_0^0)} = 1142\text{ MeV}. \quad (8)$$

On the other hand, for the second set of parameters in Ref. 3 (i.e., for a nonstrange-quark mass of 108 MeV and a strange-quark mass of 353 MeV), we arrive at, say,

$$\frac{1}{\sqrt{2}}\frac{1}{f\pi}\left\langle \pi^0\left|\int_{\text{bag}}\mathcal{H}_w^{pc}d^3x\right|K^0\right\rangle = 97.14\times10^{-7} \quad (9)$$

if an enhancement factor of 3 is used.

It is not astonishing that for this second set of parameters one obtains much too large a value for the matrix element $\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$. This may be easily understood since the kaon radius corresponding to this set is 0.73 GeV^{-1}, which is much smaller than that used for the first set of parameters, i.e., $R=3.26$ GeV^{-1}.

Before proceeding to discuss the charmed mesons we also wish to note that if one computes the matrix element $\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle$ one arrives at a result which is three times larger than that expected from the $\Delta I=\frac{1}{2}$ rule if the color of the quarks is taken into account. On the other hand, if color is neglected altogether, one then arrives precisely at the $\Delta I=\frac{1}{2}$ rule. Furthermore, we also wish to stress that inclusion of color does not change the results of the computation for the matrix elements in Eqs. (3) to (9). The disagreement with experiment of the result of the calculation of $\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle$ with colored quarks should not be taken seriously since we are then dealing with a matrix element corresponding to quark annihilation and, as discussed at the beginning of this section, we do not expect the fixed-sphere bag model to be applicable to such types of processes.

We next proceed to discuss the decay $D^0\to K^-\pi^+$ and start by writing

$$\frac{1}{f\pi}\left\langle \bar{K}^0\left|\int_{\text{bag}}\mathcal{H}_w^{pc}d^3x\right|D^0\right\rangle = \frac{G}{\sqrt{2}}\cos^2\theta\frac{1}{f\pi}\frac{2}{\sqrt{2}}I_M\,, \quad (10)$$

with I_M as defined in Eq. (4) and the set of parameters given in Ref. 4 (with a charmed-quark mass of 1551 MeV).

We thus obtain the prediction

$$\frac{1}{f\pi}\left\langle \bar{K}^0\left|\int_{\text{bag}}\mathcal{H}_w^{pc}d^3x\right|D^0\right\rangle = 17.05\times10^{-7} \quad (11)$$

for an enhancement factor of 3. The widths computed from Eq. (11) may then be written as

$$\Gamma_{D^0 \to K^-\pi^+} = 0.32 \times 10^{12}\,\text{sec}^{-1}. \tag{12}$$

In conclusion, we note that our results indicate that by use of the current-current Hamiltonian reasonable agreement with experiment may be obtained.[8,10]

Note added in proof. It has been argued in Ref. 11 that the enhancement factor in the case of charmed quarks should be somewhat smaller (i.e., about 1.9). With this assumption we then obtain for the decay width $\Gamma_{D^0\to K^-\pi^+}$ the result $(1.9/3)^2 \times 0.32 \times 10^{12}\ \text{sec}^{-1} = 1.5\times 10^{11}\ \text{sec}^{-1}$, in reasonably good agreement with the prediction for this decay width made in Ref. 11 using different techniques.

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