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Can couplings of charged heavy leptons to the W^0 be measured?

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We consider effects of the W^0 on the one-particle inclusive distribution for the reaction $e^+e^- \to L^+L^- \to (l\nu\bar{\nu})(sX)$, where the energy and/or angle of l are observed, and s is any specific state detected along with l as the signature of the L^+L^- pair, and discuss the possibility of deducing W^0 couplings from measurements.

Apparently charged heavy leptons with mass near 2 GeV are produced in pairs by e^-e^+ annihilation.[1] Although little is known about their properties, a number of authors have considered how further observations can determine their decay parameters[2-5] and electromagnetic form factors.[4,5] Some theoretical models predict additional heavy leptons. For example, the E_7 gauge model of weak, electromagnetic, and color interactions predicts the existence of four charged leptons.[6] If there exists more than one type of heavy lepton, future e^-e^+ storage rings may discover a fascinating lepton spectroscopy.

Another interesting task for future storage rings will be measuring W^0 couplings. Many papers have discussed the possibility of deducing these couplings from measurements of photon-W^0 interferences. (For examples of theoretical aspects see Refs. 7, 8 and for examples of experimental aspects see Refs. 9, 10.) It would be especially interesting to measure W^0 couplings of heavy leptons since unified models make specific predictions of these couplings (for example, see Ref. 11).

In order to learn whether this is possible, we examine the reaction $e^+e^- \to L^+L^- \to (l\nu\bar{\nu})(sX)$, where $l = e$ or μ and s is any specific state detected along with l as the signature of the L^+L^- pair. We calculate the distribution of the laboratory angle and energy fraction x of the l, $d\sigma/d\Omega\, dx$. We do not include the effects of kinematic cuts on the state s. These effects should be negligible at high energies with wide-angle detectors.

Our method of calculation was described in Ref. 5. We calculate the differential cross section for $e^+e^- \to L^+L^-$ assuming one photon and one W^0 exchange, with arbitrary initial and final spins. The result is given in Appendix A. We consider the case where one of the heavy leptons decays to $l^\pm$ and two massless fermions, such as $\nu\bar{\nu}$, by a four-fermion Lagrangian. The other lepton can decay to any class of final states. We symbolize this class sX, where s is a particle or set of particles. We fold the decay probabilities for $L^\pm \to l^\pm\nu\bar{\nu}$ and $L^\mp \to sX$ into the cross section in Appendix A, then integrate over the momentum of s and the angles of the heavy-lepton pair. The first integration removes all kinematic details of $L^\mp \to sX$, leaving only its branching ratio B_s. The result is the one-particle inclusive distribution $d\sigma^\pm/d\Omega\, dx$, which is given in Appendix B.

If, as generally assumed, the heavy lepton has pointlike form factors, and if M_{W0}/E and E/M_L are large, our result is

$$\frac{4q^2}{\alpha^2 B_p B_s}\frac{d\sigma^\pm}{d\Omega\, dx} = [(1+\mathcal{P}_-^L\mathcal{P}_+^L)(1+\cos^2\theta) + \mathcal{P}_-^T\mathcal{P}_+^T\sin^2\theta\cos(2\varphi - \varphi_+ - \varphi_-)][(1+2g_V g_V' R)P(x,\rho) \mp 2g_V g_A' R\xi Q(x,\delta)]$$
$$+ (1+\mathcal{P}_-^L\mathcal{P}_+^L)2\cos\theta[2g_A g_A' RP(x,\rho) \mp 2g_A g_V' R\xi Q(x,\delta)]$$
$$+ (\mathcal{P}_-^L + \mathcal{P}_+^L)(1+\cos^2\theta)[2g_A g_V' RP(x,\rho) \mp 2g_A g_A' R\xi Q(x,\delta)]$$
$$+ (\mathcal{P}_-^L + \mathcal{P}_+^L)2\cos\theta[2g_V g_A' RP(x,\rho) \mp (1+2g_V g_V' R)\xi Q(x,\delta)] \qquad (1)$$

with kinematics, form factors, decay parameters, and functions $P(x, \rho)$ and $\xi Q(x, \delta)$ defined in the appendixes.

When E/M_{W^0} is small, the dominant terms are the one-photon terms, i.e., the terms with no R. By measuring these, $P(x, \rho)$ and (if longitudinal beam polarization is available) $\xi Q(x, \delta)$ can be deduced. Their predicted shapes for $V \pm A$ decays are given in Figs. 1 and 2.

As E increases, R increases, allowing the possibility of measuring W^0 couplings. With unpolarized beams, our result for the energy distribution is

$$\frac{d\sigma^\pm}{dx} = \frac{4\pi\alpha^2 B_p B_s}{3q^2}[(1+2g_V g_V' R)P(x,\rho) \mp 2g_V g_A' R\xi Q(x,\delta)], \quad (2)$$

and for the forward-backward asymmetry is

$$A^\pm(x) = \frac{N_F^\pm - N_B^\pm}{N_F^\pm + N_B^\pm} = \frac{3}{4}\left[\frac{2g_A g_A' R P(x,\rho) \mp 2g_A g_V' R \xi Q(x,\delta)}{(1+2g_V g_V' R)P(x,\rho) \mp 2g_V g_A' R\xi Q(x,\delta)}\right]. \quad (3)$$

The $g_V g_V'$ and $g_A g_A'$ terms in these results could be masked by higher-order QED contributions (see Appendix A). This complication might be avoided by measuring the charge asymmetry:

$$\frac{d\sigma^-}{d\cos\theta dx} - \frac{d\sigma^+}{d\cos\theta dx} = \frac{\pi\alpha^2 B_p B_s}{q^2}\xi Q(x,\delta)[(1+\cos^2\theta)2g_V g_A' R + 2\cos\theta 2g_A g_V' R] \quad (4)$$

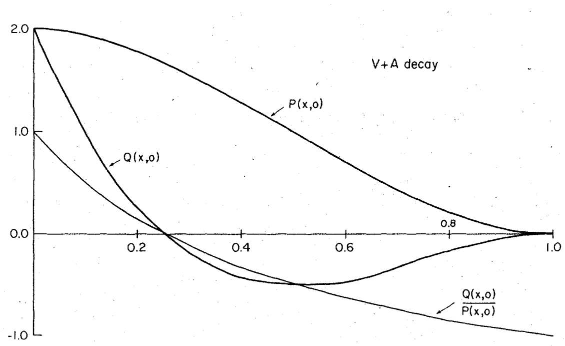

FIG. 1. $P(x, \rho)$, $\xi Q(x,\delta)$, and their ratio versus x for $V+A$ decays ($\rho=\delta=0$, $\xi=1$).

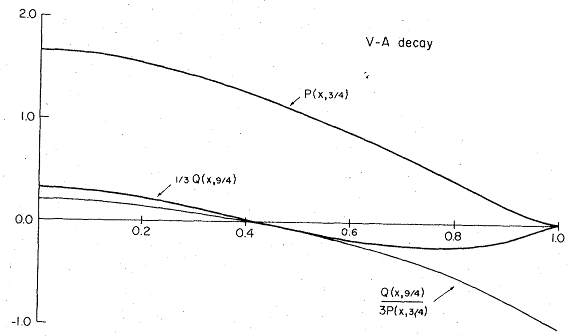

FIG. 2. $P(x, \rho)$, $\xi Q(x,\delta)$, and their ratio versus x for $V-A$ decays ($\rho=\frac{3}{4}$, $\delta=\frac{9}{4}$, and $\xi=\frac{1}{3}$).

although, if the decay is $V-A$, the factor $\xi Q(x, \delta)$ is small.

To consider the feasibility of measuring the W^0 couplings at PEP and PETRA energies, suppose the W^0 coupling is $V \pm A$ with $g_V = g_V' = \pm g_A = \pm g_A' = M_{W^0}(G/\sqrt{2})^{1/2}$. If $M_{W^0} = 70$ GeV, then at $q^2 = (30\ \text{GeV})^2$ the W^0 term will have the magnitude of $|2g_V g_A' R| = 0.20$, i.e., 20% of the one-photon contribution. If the factor $B_p B_s$ is around 0.1, then an experimental run at PEP lasting two months[9] should yield around 1200 events of the type

$$e^+e^- \to \tau^+\tau^- \to e^\pm \mu^\mp + \text{neutrals}.$$

This could permit the rate to be measured to within 3%.

An experiment at PEP or PETRA should be able to establish whether the W^0 couples to the L, and if it does, to measure approximately g_V' and g_A'. (Some information on g_V and g_A can be expected[8] from experiments on $e^+e^- \to e^+e^-$ or $e^-e^- \to e^-e^-$.) This would allow one to test the assumption of "e/L universality" that g_V' and g_A' are equal to g_V and g_A, respectively. Extracting more detailed information would require more ambitious experiments than those planned at PEP and PETRA.

The next generation of storage rings after PEP and PETRA may have sufficiently high energy to reach the W^0 pole.[10] There, the R^2 terms we dropped will dominate. At these energies, there may be too many heavy states decaying to know with what the W^0 is coupling.

Our results indicate that the W^0 coupling to charged heavy leptons can be measured in e^+e^- annihilation in future storage rings if long experimental runs and calculations of the QED background are available.

APPENDIX A: THE CROSS SECTION FOR $e^+e^-\to L^+L^-$

We denote the momenta of the beams by $k_\pm$ and the momenta of the $L^\pm$ by $k'_\pm$. In the laboratory frame they can be written as

$$k_-^\mu = E(1,0,0,1)\,,$$
$$k'^\mu_- = E(1, \beta\sin\theta'\cos\varphi', \beta\sin\theta'\sin\varphi', \beta\cos\theta') \tag{A1}$$

with $(k_+ + k_-)^2 = q^2 = 4E^2$ and $\beta = (1-1/\gamma^2)^{1/2}$, where $\gamma = E/M_L$ and M_L is the mass of L. The beam polarizations can be written

$$\vec{\mathcal{P}}_\pm = (\mathcal{P}^T_\pm\cos\varphi_\pm, \mathcal{P}^T_\pm\sin\varphi_\pm, \mathcal{P}^L_\pm) \tag{A2}$$

(with beam helicities $h_\pm = \mp\mathcal{P}^L_\pm$). The spin vectors of the $L^\pm$ have components

$$S'^\mu_\pm = (0, S^x_\pm, S^y_\pm, S^z_\pm) \tag{A3}$$

in their respective rest frames. More details are given in Ref. 5.

Using these definitions, we assumed the reduced amplitude for $e^+e^-\to L^+L^-$ is given by

$$\mathfrak{M} = \frac{ie^2}{q^2}\langle L^-L^+|J^{\rm em}_\alpha|0\rangle\,\bar{v}(k_+)\gamma^\alpha u(k_-) + \frac{i}{q^2-M_{W^0}{}^2}\langle L^-L^+|J^W_\alpha|0\rangle \times \bar{v}(k_+)\gamma^\alpha(g_V+g_A\gamma_5)u(k_-)\,, \tag{A4}$$

where M_{W^0} is the mass of the W^0, and g_V and g_A are the coupling of the W^0 to the electron. The matrix elements of the heavy lepton (or any spin-$\frac{1}{2}$ particle) define the form factors as follows:

$$\langle L^-L^+|J^{\rm em}_\alpha|0\rangle \equiv \bar{u}(k'_-S'_-)[(k'_+-k'_-)_\alpha F+\gamma_\alpha G]v(k'_+,S'_+)$$
$$\equiv \bar{u}(k'_-,S'_-)\left[(k'_+-k'_-)_\alpha\frac{M_L(G_E-G_M)}{2(E^2-M_L{}^2)}+\gamma_\alpha G_M\right]v(k'_+,S'_+)\,, \tag{A5}$$

$$\langle L^-L^+|J^W_\alpha|0\rangle \equiv \bar{u}(k'_-,S'_-)\{(k'_+-k'_-)_\alpha F_V+\gamma_\alpha G_V+(k'_++k'_-)H_V$$
$$+[(k'_+-k'_-)_\alpha F_A+\gamma_\alpha G_A+(k'_++k'_-)_\alpha H_A]\gamma_5\}v(k'_+,S'_+)$$
$$\equiv \bar{u}(k'_-,S'_-)\left\{(k'_+-k'_-)_\alpha\frac{M_L(G_z-G_V)}{2(E^2-M_L{}^2)}+\gamma_\alpha G_V+(k'_++k'_-)_\alpha H_V\right.$$
$$\left.+[(k'_+-k'_-)_\alpha\frac{iG_D}{M_L}+\gamma_\alpha G_A+(k'_++k'_-)_\alpha H_A]\gamma_5\right\}v(k'_+,S'_+) \tag{A6}$$

The W^0 form factors are the most general for vector and axial-vector currents J^W_α. The nonconserved current form factors H_V and H_A do not appear in the cross section for $e^-e^+\to L^-L^+$. The vector form factors G_Z and G_V are the W^0 analogs of G_E and G_M, and the axial-vector G_D the analog of the electric-dipole-moment form factor.

The form factors are functions of q^2. Since J^W is Hermitian, for spacelike q^2, G_V, G_A, and F_V will be real and F_A will be imaginary, implying time-reversal violation unless it vanishes. For q^2 positive, Hermiticity will not fix the phases of the form factors; however, field theory implies that there are contributions for F_V and F_A which are hard,[12] i.e., approximately constant up to $q^2 = M_{W^0}{}^2$. These will remain real and imaginary, respectively. Although they are expected to be small, their contributions to the cross section will increase with q^2 faster than the usual weak contribution, suggesting a slight possibility of observing them. Also there should be soft contributions to F_V and F_A (of order $M_L{}^2/q^2$) with both real and imaginary parts.

Assuming the matrix elements given above, the cross section for $e^+e^-\to L^+L^-$ in the laboratory frame is

$$
\begin{aligned}
\frac{16q^2}{\alpha^2\beta}\frac{d\sigma}{d\Omega_L} = {} & \mathcal{L}_1\{[(1+\cos^2\theta')U+\sin^2\theta' T_c](1+S_-^z S_+^z)+2\cos\theta' L(S_-^z+S_+^z) \\
& \quad +[\sin^2\theta' U+(1+\cos^2\theta')T_c](S_-^xS_+^x - S_-^yS_+^y) - 2\cos\theta' T_s(S_-^xS_+^y+S_-^yS_+^x)\} \\
& +\mathcal{L}_2\gamma^{-2}\sin^2\theta'(U-T_c)(1-S_-^zS_+^z+S_-^xS_+^x+S_-^yS_+^y) \\
& -\mathcal{L}_3\gamma^{-1}2\sin\theta'[L(S_-^x+S_+^x)+\cos\theta'(U-T_c)(S_-^xS_+^z+S_-^zS_+^x)+T_s(S_-^yS_+^z+S_-^zS_+^y)] \\
& +\mathcal{L}_4\beta\{2\cos\theta' L(1+S_-^zS_+^z)+[(1+\cos\theta')U+\sin^2\theta' T_c](S_-^z+S_+^z)\} \\
& +\mathcal{L}_5[(1+\cos^2\theta')L(1+S_-^zS_+^z)+2\cos\theta' U(S_-^z+S_+^z)+\sin^2\theta' L(S_-^xS_+^x-S_-^yS_+^y)] \\
& +\mathcal{L}_6\beta[2\cos\theta' U(1+S_-^zS_+^z)+(1+\cos^2\theta')L(S_-^z+S_+^z)+2\cos\theta' T_c(S_-^xS_+^x-S_-^yS_+^y) \\
& \quad -(1+\cos^2\theta')T_s(S_-^xS_+^y+S_-^yS_+^x)] \\
& -\mathcal{L}_7\gamma^{-1}\beta\sin\theta'[\cos\theta'(U-T_c)(S_-^x+S_+^x)+T_s(S_-^y+S_+^y)+L(S_-^xS_+^z+S_-^zS_+^x)] \\
& -\mathcal{L}_8\gamma^{-1}\sin\theta'[U(S_-^x+S_+^x)+\cos\theta' L(S_-^xS_+^z+S_-^zS_+^x)] \\
& -\mathcal{L}_9\gamma^{-1}\sin\theta'[T_c(S_-^x+S_+^x)-\cos\theta' T_s(S_-^y+S_+^y)] \\
& -\mathcal{L}_{10}\gamma^{-1}\beta\sin\theta'[\cos\theta' L(S_-^x+S_+^x)+(U-T_c)(S_-^xS_+^z+S_-^zS_+^x)+\cos\theta' T_s(S_-^yS_+^z+S_-^zS_+^y)] \\
& +\mathcal{L}_{11}\gamma^{-2}\sin^2\theta' L(1-S_-^zS_+^z+S_-^xS_+^x+S_-^yS_+^y) \\
& -\mathcal{L}_{12}2\gamma\beta\sin\theta'[L(S_-^y-S_+^y)-T_s(S_-^xS_+^z-S_-^zS_+^x)+\cos\theta'(U-T_c)(S_-^yS_+^z-S_-^zS_+^y)] \\
& -\mathcal{L}_{13}2\gamma\beta\sin\theta'[\cos\theta' T_s(S_-^x-S_+^x)+(U+T_c)(S_-^y-S_+^y)+\cos\theta' L(S_-^yS_+^z-S_-^zS_+^y)] \\
& -\mathcal{L}_{14}2\beta\sin^2\theta'(U-T_c)(S_-^xS_+^y-S_-^yS_+^x) \\
& -\mathcal{L}_{15}2\beta\sin^2\theta'[T_s(S_-^z-S_+^z)+L(S_-^xS_+^y-S_-^yS_+^x)] \\
& -\mathcal{L}_{16}2\gamma^{-1}\sin\theta'[\cos\theta'(U-T_c)(S_-^y+S_+^y)-T_s(S_-^x+S_+^x)+L(S_-^yS_+^z+S_-^zS_+^y)] \\
& -\mathcal{L}_{17}\beta\{[\sin^2\theta' U+(1+\cos^2\theta')T_c](S_-^xS_+^y+S_-^yS_+^x)+2\cos\theta' T_s(S_-^xS_+^x-S_-^yS_+^y)\} \\
& -\mathcal{L}_{18}\{T_s[\sin^2\theta'(1+S_-^zS_+^z)+(1+\cos^2\theta')(S_-^xS_+^x-S_-^yS_+^y)]+2\cos\theta' T_s(S_-^xS_+^y+S_-^yS_+^x)\} \\
& -\mathcal{L}_{19}\beta\sin^2\theta'[T_s(S_-^z+S_+^z)+L(S_-^xS_+^y+S_-^yS_+^x)] \\
& +\mathcal{L}_{20}\gamma^{-1}\beta\sin\theta'[L(S_-^y+S_+^y)+\cos\theta'(U-T_c)(S_-^yS_+^z+S_-^zS_+^y)-T_s(S_-^xS_+^z+S_-^zS_+^x)] \\
& -\mathcal{L}_{21}\gamma^{-1}\sin\theta'[\cos\theta' L(S_-^y+S_+^y)+U(S_-^yS_+^z+S_-^zS_+^y)] \\
& -\mathcal{L}_{22}\gamma^{-1}\sin\theta'[\cos\theta' T_s(S_-^xS_+^z+S_-^zS_+^x)+T_c(S_-^yS_+^z+S_-^zS_+^y)] \\
& +\mathcal{L}_{23}\gamma^{-1}\beta\sin\theta'[-\cos\theta' T_s(S_-^x+S_+^x)+(U-T_c)(S_-^y+S_+^y)+\cos\theta' L(S_-^yS_+^z+S_-^zS_+^y)] \\
& +\mathcal{L}_{24}\gamma^{-2}\sin^2\theta' T_s(1-S_-^zS_+^z+S_-^xS_+^x+S_-^yS_+^y) \\
& +\mathcal{L}_{25}2\gamma\beta\sin\theta'[\cos\theta'(U-T_c)(S_-^x-S_+^x)+T_s(S_-^y-S_+^y)+L(S_-^xS_+^z-S_-^zS_+^x)] \\
& +\mathcal{L}_{26}2\gamma\beta\sin\theta'[\cos\theta' L(S_-^x-S_+^x)+(U+T_c)(S_-^xS_+^z-S_-^zS_+^x)-\cos\theta' T_s(S_-^yS_+^z-S_-^zS_+^y)] \\
& +\mathcal{L}_{27}2\beta\sin^2\theta'(U-T_c)(S_-^z-S_+^z) \\
& +\mathcal{L}_{28}2\beta\sin^2\theta'[L(S_-^z-S_+^z)-T_s(S_-^xS_+^y-S_-^yS_+^x)]. \qquad \text{(A7)}
\end{aligned}
$$

The form factors appear in the coefficients $\mathcal{L}_j$ which are defined as follows, where the P, C, and initial (C^i) and final (C^f) state charge conjugation of each term is indicated on the right:

$$\begin{aligned}
&\mathcal{L}_1 = |G_M|^2 + 2\,\mathrm{Re}(g_V G_M^* G_V)R, && \mathcal{L}_2 = |G_E|^2 + 2\,\mathrm{Re}(g_V G_E^* G_Z)R && (++++),\\
&\mathcal{L}_3 = \mathrm{Re}(G_M^* G_E) + \mathrm{Re}(g_V G_M^* G_Z + g_V G_E^* G_V)R, && \mathcal{L}_{16} = \mathrm{Im}(G_M^* G_E) + \mathrm{Im}(g_V G_M^* G_Z - g_V G_E^* G_V)R && (++++),\\
&\mathcal{L}_4 = 2\,\mathrm{Re}(g_V G_M^* G_A)R, && \mathcal{L}_{17} = 2\,\mathrm{Im}(g_V G_M^* G_A)R && (--+-),\\
&\mathcal{L}_5 = 2\,\mathrm{Re}(g_A G_M^* G_V)R, && \mathcal{L}_{18} = 2\,\mathrm{Im}(g_A G_M^* G_V)R && (---+),\\
&\mathcal{L}_6 = 2\,\mathrm{Re}(g_A G_M^* G_A)R, && \mathcal{L}_{19} = 2\,\mathrm{Im}(g_A G_M^* G_A)R && (++--),\\
&\mathcal{L}_7 = 2\,\mathrm{Re}(g_V G_E^* G_A)R, && \mathcal{L}_{20} = 2\,\mathrm{Im}(g_V G_E^* G_A)R && (--+-),\\
&\mathcal{L}_8 = 2\,\mathrm{Re}(g_A G_M^* G_Z + g_A G_E^* G_V)R, && \mathcal{L}_{21} = 2\,\mathrm{Im}(g_A G_M^* G_Z - g_A G_E^* G_V)R && (---+),\\
&\mathcal{L}_9 = 2\,\mathrm{Re}(g_A G_M^* G_Z - g_A G_E^* G_V)R, && \mathcal{L}_{22} = 2\,\mathrm{Im}(g_A G_M^* G_Z + g_A G_E^* G_V)R && (---+),\\
&\mathcal{L}_{10} = 2\,\mathrm{Re}(g_A G_E^* G_A)R, && \mathcal{L}_{23} = 2\,\mathrm{Im}(g_A G_E^* G_A)R && (++--),\\
&\mathcal{L}_{11} = 2\,\mathrm{Re}(g_A G_E^* G_Z)R, && \mathcal{L}_{24} = 2\,\mathrm{Im}(g_A G_E^* G_Z)R && (---+),\\
&\mathcal{L}_{12} = 2\,\mathrm{Re}(g_V G_M^* G_D)R, && \mathcal{L}_{25} = 2\,\mathrm{Im}(g_V G_M^* G_D)R && (-+++),\\
&\mathcal{L}_{13} = 2\,\mathrm{Re}(g_A G_M^* G_D)R, && \mathcal{L}_{26} = 2\,\mathrm{Im}(g_A G_M^* G_D)R && (+--+),\\
&\mathcal{L}_{14} = 2\,\mathrm{Re}(g_V G_E^* G_D)R, && \mathcal{L}_{27} = 2\,\mathrm{Im}(g_V G_E^* G_D)R && (-+++),\\
&\mathcal{L}_{15} = 2\,\mathrm{Re}(g_A G_E^* G_D)R, && \mathcal{L}_{28} = 2\,\mathrm{Im}(g_A G_E^* G_D)R && (+--+).
\end{aligned} \tag{A8}$$

The W^0 propagator is contained in the factor

$$R \equiv \frac{q^2}{4\pi\alpha(q^2 - M_{W^0}{}^2)}, \tag{A9}$$

and the beam polarization is contained in the factors

$$\begin{aligned}
&U \equiv 1 + \mathcal{P}_-^L \mathcal{P}_+^L, \quad L \equiv \mathcal{P}_-^L + \mathcal{P}_+^L,\\
&T_c \equiv \mathcal{P}_-^T \mathcal{P}_+^T \cos(2\varphi' - \varphi_+ - \varphi_-),\\
&T_s \equiv \mathcal{P}_-^T \mathcal{P}_+^T \sin(2\varphi' - \varphi_+ - \varphi_-).
\end{aligned} \tag{A10}$$

Landvai *et al.* have also calculated this cross section.[13] They used general form factors and retained the $|W^0|^2$ terms, but dropped lepton mass terms.

In the above cross section, each of the 28 terms $\mathcal{L}_j$ contains a term with the variable R resulting from γ-W^0 interference. Some of the $\mathcal{L}_j$'s contain a $|\gamma|^2$ term. These terms were given in Ref. 5. The $|W^0|^2$ terms are not included here. They are proportional to R^2 with additional weak couplings and will be small when $M_{W^0}{}^2/q^2$ is small. The first 15 terms, $j=1$–15, consist of the real parts of products of coupling constants. Of these, terms 12–15 involve G_D, which is expected to be small. The rest of the terms 16–28 consist of the imaginary parts of the corresponding products and are expected to be very small also. If L has no derivative coupling and does not interact strongly (pointlike coupling), we get the following: $G_E = G_M = 1$, $G_V = G_Z \equiv g'_V$, $G_A \equiv g'_A$, and $F_V = F_A = 0$, so that $\mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3$, $\mathcal{L}_4 = \mathcal{L}_7$, $\mathcal{L}_5 = \frac{1}{2}\mathcal{L}_8 = \mathcal{L}_{11}$, $\mathcal{L}_6 = \mathcal{L}_{10}$, and the rest vanish.

The terms in the cross section which are even under parity (P) and charge conjugation (C) can have two-photon contributions of order $(\alpha^2)^2$ which could dominate the weak-interference terms of order $\alpha G q^2$. To extract the W^0 couplings from measurements involving such terms, these radiative corrections must be calculated. This problem can be avoided by choosing measurements involving terms odd under P and/or C.

APPENDIX B: THE ONE-PARTICLE INCLUSIVE DISTRIBUTION

Since only the decay products of the heavy lepton can be observed, we fold into the cross section in the Appendix A the normalized probabilities for the decays. For the case where the final lepton is an l^+, we assume the decay $L^+ \to l^+ \nu\bar{\nu}$ is described by a local four-fermion Lagrangian. This yields the following differential decay rate:

$$\frac{p^0}{\Gamma}\frac{d^3\Gamma}{d^3\vec{p}}(L^+ \to l^+\nu\bar{\nu}) = \frac{12}{M_L{}^2\pi}\left[yG(y,\rho) - \xi\frac{2}{M_L}(p\cdot S'_+)G(y,\delta)\right], \tag{B1}$$

where Γ is the decay width, ρ, δ, ξ are the decay parameters which depend on the S, P, T, V, A nature of the Lagrangian,[14] and p is the momentum of l, which we assumed to be massless. Further,

$$p^\mu = p^0(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$

in the laboratory frame, and

$$y = \frac{2}{M_L{}^2} p \cdot k'_+ ,$$

$$G(y,\rho) = 1 - y + \tfrac{2}{3}\rho(\tfrac{4}{3}y - 1). \tag{B2}$$

If an l^- is observed instead of an l^+ then the differential decay rate for $L^- \to l^- \nu\bar{\nu}$ is given by $\xi \to -\xi$ and $S'_+ \to S'_-$. If the decay $L \to sX$ is of the form $L \to l\nu\bar{\nu}$ also, its decay rate would be given by the above expression as well. We fold the decay probabilities for $L \to l\nu\bar{\nu}$ and $L \to sX$ into the cross section in Appendix A, then integrate over the momentum of the state s and the angles θ', and ϕ' of the heavy leptons. The first integration removes all details of the $L \to sX$ decays, leaving only its branching ratio B_s in $d\sigma/d\Omega dx$.

The result for $d\sigma^{\pm}/d\Omega dx$, in the laboratory frame, is the following, where the $\pm$ refers to the sign of the heavy lepton decaying to $l^{\pm}\nu\bar{\nu}$, with branching ratio B_l, and $x = p^0/(p^0)_{\max}$:

$$\begin{aligned}
\frac{12q^2}{\alpha^2\beta B_l B_s}\frac{d\sigma^{\pm}}{d\Omega dx} = {}& (1+\mathcal{P}^L_-\mathcal{P}^L_+)\{2(2\mathcal{L}_1+\mathcal{L}_2\gamma^{-2})A(x,\rho)+6\cos\theta\,\mathcal{L}_6[\beta^2C(x,\rho)-D(x,\rho)]\\
&\quad +\xi 4(\mathcal{L}_{27}\mp\mathcal{L}_4)D(x,\delta)\pm\xi 3\cos\theta[2\mathcal{L}_5A(x,\delta)-(2\mathcal{L}_5-\mathcal{L}_8\gamma^{-2})C(x,\delta)]\}\\
&+3[(1+\mathcal{P}^L_-\mathcal{P}^L_+)(\cos^2\theta-\tfrac{1}{3})+\mathcal{P}^T_-\mathcal{P}^T_+\sin^2\theta\cos(2\varphi-\varphi_+-\varphi_-)]\\
&\times[(\mathcal{L}_1-\mathcal{L}_2\gamma^{-2})B(x,\rho)+\xi(2\mathcal{L}_{27}\pm\mathcal{L}_4)E(x,\delta)-\xi(2\mathcal{L}_{25}\pm\mathcal{L}_7\gamma^{-2})F(x,\delta)]\\
&-3\mathcal{P}^T_-\mathcal{P}^T_+\sin^2\theta(2\varphi-\varphi_+-\varphi_-)[(\mathcal{L}_{18}-\mathcal{L}_{24}\gamma^{-2})B(x,\rho)-\xi(2\mathcal{L}_{15}\mp\mathcal{L}_{19})E(x,\delta)+\xi(2\mathcal{L}_{13}\mp\mathcal{L}_{23}\gamma^{-2})F(x,\delta)]\\
&+2(\mathcal{P}^L_-+\mathcal{P}^L_+)\{(2\mathcal{L}_5+\mathcal{L}_{11}\gamma^{-2})A(x,\rho)+3\cos\theta\,\mathcal{L}_4[\beta^2C(x,\rho)-D(x,\rho)]\\
&\quad +\xi 2(\mathcal{L}_{28}\mp\mathcal{L}_6)D(x,\delta)\pm\xi 3\cos\theta[\mathcal{L}_1A(x,\delta)-(\mathcal{L}_1-\mathcal{L}_3\gamma^{-2})C(x,\delta)]\}\\
&+3(\mathcal{P}^L_-+\mathcal{P}^L_+)(\cos^2\theta-\tfrac{1}{3})[(\mathcal{L}_5-\mathcal{L}_{11}\gamma^{-2})B(x,\rho)+\xi(2\mathcal{L}_{28}\pm\mathcal{L}_6)E(x,\delta)-\xi(2\mathcal{L}_{26}\pm\mathcal{L}_{10}\gamma^{-2})F(x,\delta)]\ .
\end{aligned}\tag{B3}$$

The functions A, B, and C are defined in Ref. 5. The definition of D is

$$D(x,\rho)=\begin{cases}\dfrac{4\beta^2}{3(1+\beta)}\left(\dfrac{x}{x_0}\right)^2\left[\dfrac{4}{1+\beta}\left(\dfrac{x}{x_0}\right)r(\rho)+3s(\rho)\right], & 0\le x<x_0\\[2ex] \dfrac{(1-x)(1+\beta)}{3\beta}\{[2-x(1+x)(1+3\beta)]r(\rho)+3[1-x(1+2\beta)]s(\rho)\}, & x_0\le x\le 1\ ,\end{cases}\tag{B4}$$

$$\int_0^1 dx\,D(x,\rho)=0\ ,$$

where $x_0=(1-\beta)/(1+\beta)$, $r(\rho)=8\rho/3-3$, and $s(\rho)=3-2\rho$. For $V-A$, $r(\rho)=-1$, $s(\rho)=\frac{3}{2}$, $r(\delta)=3$, and $s(\delta)=-\frac{3}{2}$, and for $V+A$, $r(\rho)=-3$, $s(\rho)=3$, $r(\delta)=-3$, and $s(\delta)=3$. D is plotted in Fig. 3 in the case $\gamma=4$ with the $V\pm A$ values. The functions E and F are defined by

$$\begin{aligned}E(x,\rho)&=A(x,\rho)-B(x,\rho)-\frac{3}{2\gamma^2}C(x,\rho)-D(x,\rho)\ ,\\ F(x,\rho)&=A(x,\rho)-B(x,\rho)-\tfrac{3}{2}C(x,\rho)\ .\end{aligned}\tag{B5}$$

In the limit $\gamma=E/M_L\to\infty$ ($\beta\to 1$ and $x_0\to 0$), the functions A, B, C, D, and their combinations appearing in $d\sigma/d\Omega dx$ having the following limits:

$$\begin{aligned}A(x,\rho),B(x,\rho)\to P(x,\rho)&\equiv 2(1-3x^2+2x^3)\\ &\quad-\tfrac{4}{9}\rho(1-9x^2+8x^3)\ ,\\ D(x,\rho),-E(x,\rho)\to Q(x,\rho)&\equiv 2(1-6x+9x^2-4x^3)\\ &\quad-\tfrac{4}{9}\rho(1-12x+27x^2-16x^3),\\ C(x,\rho),-\tfrac{2}{3}F(x,\rho)&\to P(x,\rho)+Q(x,\rho)\ .\end{aligned}\tag{B6}$$

The functions $P(x,\rho)$, $\xi Q(x,\delta)$, and their ratio are plotted in Figs. 1, 2 for $V\pm A$.

The result for the one-particle inclusive distribution contains 20 of the $\mathcal{L}_i$'s. In order to derive a measurable result involving the missing 8, one

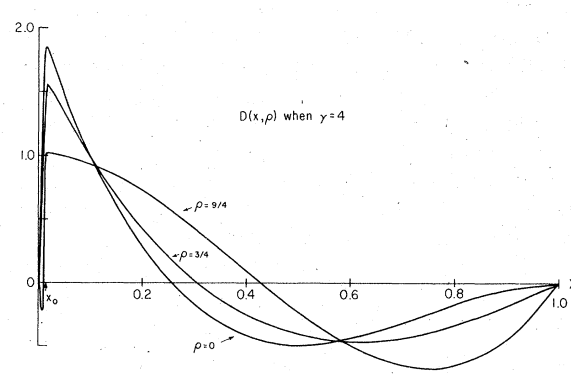

FIG. 3. $D(x,\rho(\text{or }\delta))$ versus x when $\gamma=4$.

would have to restrict the integration over the momentum of state s, or else consider a multiparticle inclusive distribution.

A number of obstacles stand in the way of extracting form factors from measurements of $d\sigma/d\Omega dx$. (1) Many of the form factors are expected to be small for reasons discussed in Appendix A. This is why we concentrate on the pointlike case in the text. (2) Many of the terms vanish unless the initial beams have transverse or longitudinal polarization. It may prove impossible to polarize, or to control the polarization of the beams, in which case at most only the terms $j = 1, 2, 4, 5, 6, 7, 8, 25$, and 27 will be observable. (3) There will be significant higher-order QED contributions to the terms in $d\sigma/d\Omega dx$ which result from terms in $d\sigma/d\Omega_L$ which are even under P and C. These contributions would have to be computed and subtracted from measurements of these terms. Thirteen of the terms in the general $d\sigma/d\Omega dx$ come from terms in $d\sigma/d\Omega_L$ which are odd under P and/or C. Measurements of these would be unambiguous evidence for weak effects. (4) The event rate is expected to be very small, implying that at most, only integrals of $d\sigma/d\Omega dx$ will be observable.

We have examined only the one-particle distribution for $e^+e^- \to lX$ here. There are a number of reasons for looking at the two-particle distribution for $e^+e^- \to l^+l^-X$. First, the two-particle distributions will furnish additional information on couplings and decay constants. Second, we have assumed that there are no experimental cuts on the final lepton whose momentum we integrated over. If this is not the case, the one-particle distribution will only be calculable if the two-particle distribution is known. It is conceptually simple to calculate the two-particle distribution using the cross section for $e^+e^- \to L^+L^-$ in Appendix A and a decay rate. In the case of the local four-fermion Lagrangian, the result is horrendous.

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