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Charged-particle multiplicity distributions in 100-GeV/c pd, π^+d, p-nucleon, and π-nucleon interactions

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In an experiment in the Fermilab 30-inch deuterium bubble chamber, we have determined the pd and π^+d charged-particle multiplicity distributions. We use corrected odd-prong distributions plus a no-cascade model to extract the pn and π^+n multiplicity distributions, and indicate how a cascade model could alter the distributions. We examine the relations between pn and pp, and π^+n and π^-p, multiplicity distributions. For each pair, a relation is found involving quantities that give the probability, at each multiplicity, that a struck proton remains a proton. These quantities are evaluated and compared with expectations.

I. INTRODUCTION

Experiments in deuterium bubble chambers yield directly hadron-deuteron charged-particle multiplicity distributions. Such distributions provide useful information for studies of hadron-nucleus interactions. In addition, if some assumptions are made about hadron-deuteron interactions, then hadron-neutron multiplicity distributions can be extracted from the deuterium experiments. These hadron-neutron distributions may provide valuable new information on the nature of hadron-hadron interactions. Alternatively, if the hadron-neutron distributions, or some of their properties, are known from some source other than deuterium experiments, then a comparison can test the assumptions made about hadron-deuteron interactions.

In this paper, we report on multiplicity distributions from a deuterium bubble-chamber experiment with a tagged 100-GeV/c incident beam that consisted mainly of protons and positive pions. The present distributions supersede those in an earlier brief publication[1]; there is a 25% increase in statistics, and some possible experimental biases have been checked in detail. We extract hadron-neutron multiplicity distributions from our odd-prong deuterium distributions, and discuss the assumptions involved in the extraction. We then look at the relations between π^+n and π^-p distributions and between pn and pp distributions. We examine how well the hadron-neutron distributions can be predicted from knowledge of hadron-proton interactions, and we see what new information our distributions can yield.

Multiplicity distributions have been reported from other high-energy ($\gtrsim$20 GeV/c) deuterium experiments[2-6] at incident momenta of 21 GeV/c (π^-d), 195 GeV/c (pd and π^+d), 200 GeV/c (pd), 205 GeV/c (π^-d), and 300 GeV/c (pd). These experiments all report "effective" hadron-neutron distributions obtained from odd-prong events, or odd-prong plus backward-spectator events, assuming a spectator model. However, because considerable rescattering occurs (in ~15% of events), an assumption about the rescattering mechanism has to be made in order to arrive at hadron–free-neutron distributions. Assumptions made have included a no-cascade model,[4] an intermediate cascade model,[3] and an extreme cascade model.[6] The two π^-d experiments[2,5] make assumptions about the relations between π^-n and π^+p distributions (more than just charge symmetry is required), and then use their "effective" π^-n distributions plus π^+p data to argue against an extreme cascade model. In the present paper we will assume a no-cascade model, but will indicate how some cascading could affect the results. It turns out that, at the present level of statistics, the physics results are relatively insensitive to the no-cascade assumption. Our study of rescattering, published elsewhere,[7] argued against an extreme cascade model but could not rule out some cascading. The relations between hadron-neutron and hadron-proton multiplicity distributions, which we study in detail, are mentioned only in Ref. 3 and in our own earlier publication.[1]

Throughout this paper, "multiplicity" refers to charged-particle multiplicity unless explicitly stated otherwise.

II. MULTIPLICITY DISTRIBUTIONS

The data come from the analysis of 26 000 pictures of the Fermilab 30-in. deuterium-filled bubble chamber exposed to a secondary beam of 100-GeV/c positive particles produced at 6 mr by the 300-GeV/c primary extracted proton beam. The beam composition was approximately 57% proton, 39% π^+, 2% μ^+, and 2% K^+, as determined by a Čerenkov counter in the beam. This counter, along with three sets of proportional wire chambers, formed a tagging system[8] which allowed the determination of the mass of each beam track and its location in the bubble chamber.

The film was scanned twice for all interactions with three or more outgoing prongs produced by beamlike tracks in a fiducial volume 47 cm long. The resulting scan efficiency was (99±1)%, independent of multiplicity. The number of prongs was counted on each event on each scan; the prong count included any short visible stubs. Odd-prong events, in which there is presumably an unobservably short proton or deuteron track, constituted 29% of the events. All events with prong-count discrepancies between the two scans were examined by a physicist or a third scanner, who made the final prong-count decision. About 2.7% of the events were assigned to an uncountable category; most such events had a secondary interaction close to the primary vertex. Since it is important to separate odd-prong events from even-prong events, the uncountable events included some events that had an uncertainty of only one in the prong count.

The multiplicity distributions of the uncountable events were estimated as follows: Each such event was given a minimum and maximum primary-vertex prong count at the scan table. Then each event was allowed to contribute to every multiplicity between its minimum and its maximum according to the multiplicity distribution of the countable events, with a total contribution of one per event. The resulting mean assigned prong count of the uncountable events was 9.1. For comparison, an alternative method was tried, in which the uncountable events were randomly assigned to be odd-prong or even-prong events in the ratio of 29% to 71%; then each odd-prong (even-prong) event was assigned to a random odd-prong (even-prong) count between its minimum and its maximum. It was found that adoption of this alternative method would have made only very small changes to the final results; for example, the final mean multiplicities (countable plus uncountable events) would have increased by 0.01 (odd prongs) or 0.005 (even prongs).

The raw multiplicity distributions, after assignment of uncountable events, are given in Tables I and II. Fractions resulting from the uncountable events have been rounded off. For the pd and π^+d numbers in Table II, both odd- and even-prong events are included, with one added to each odd-prong count to account for the presumed unobserved slow proton or deuteron.

In deuterium experiments it is customary to assume implicitly that the probability of seeing a very short proton track in the bubble chamber is independent of multiplicity. That is, the odd-prong multiplicity distribution is assumed to be the same as that for events in which the spectator proton has a range less than a specific value (for example, ~2 mm). We have examined this point in detail, using a sample of measured events in which all slow (momentum $P \lesssim 1500$ MeV/c) tracks were measured on all odd-prong events and on even-prong events with a stopping dark track. Thus, we can compare the multiplicity distributions of odd-prong events and of even-prong events with a slow ($P < 140$ MeV/c) proton. We have made small corrections for deuteron final-state events and for the very small fraction (~0.3%) of proton target events that after Fermi motion smearing will have a slow ($P < 140$ MeV/c) proton. We assume that, after making these corrections, we are dealing with interactions on only the neutron in the deuteron, and hence that the multiplicity distribution is independent of the spectator-proton momentum, at least for spectator momenta < 140 MeV/c. We find that the data are consistent with the detectability of

TABLE I. Odd-prong multiplicities. "Raw events" includes uncountable events, assigned as explained in text. "Corrected events" has been corrected for proton visibility factor, missed Dalitz pairs, and missed close vees, γ conversions, and secondary interactions.

	pd		π^+d	
N	Raw events	Corrected events	Raw events	Corrected events
3	410	469 ± 24	174	201 ± 16
5	492	550 ± 26	215	242 ± 17
7	420	458 ± 24	197	216 ± 16
9	289	307 ± 19	121	127 ± 13
11	168	173 ± 15	87	91 ± 11
13	82	81 ± 11	46	45 ± 7
15	33	30 ± 7	22	22 ± 5
17	10	8 ± 4	3	2 ± 2
19	4	4 ± 2	1	1 ± 1
21	2	2 ± 1	1	1 ± 1
23	1	1 ± 1	0	0
Total	1911	2083	867	948

TABLE II. Deuteron multiplicities, even- and odd-prong events. The prong count for odd-prong events is increased by one; deuteron final-state events have not been removed. "Raw events" includes uncountable events, assigned as explained in text. "Corrected events" has been corrected for missed Dalitz pairs and missed close vees, γ conversions, and secondary interactions.

	pd			π^+d		
N	Raw events	Corrected events	Cross section (mb)	Raw events	Corrected events	Cross section (mb)
4	1549	1576±41	12.81±0.58	660	676±27	7.57±0.48
6	1719	1732±43	14.08±0.63	735	742±28	8.31±0.51
8	1442	1439±40	11.70±0.54	672	672±28	7.53±0.48
10	959	943±33	7.67±0.39	412	402±22	4.50±0.33
12	554	538±25	4.37±0.26	268	262±18	2.93±0.25
14	262	248±18	2.02±0.16	129	121±13	1.36±0.16
16	134	126±13	1.02±0.11	52	48±8	0.54±0.09
18	35	29±7	0.24±0.06	16	14±5	0.16±0.06
20	10	9±3	0.07±0.02	6	6±3	0.07±0.03
22	2	2±1	0.02±0.01	2	2±1	0.02±0.01
24	1	1±1	0.01±0.01	1	1±1	0.01±0.01
Total	6667	6643	54.00±2.00	2953	2946	33.00±1.60

protons being independent of multiplicity for $P = 120-140$ MeV/c, but not for $P < 120$ MeV/c. A good fit is obtained to the hypothesis that below 120 MeV/c the detectability varies linearly with multiplicity. Specifically, the data fitted the relation $M_{N,120} = k(1-bN)M_{N-1}$, where $M_{N,120}$ is the number of N-prong events (N even) with a detected proton of momentum less than 120 MeV/c, M_{N-1} is the number of $(N-1)$-prong events, and k and b are independent of N, with nonzero b implying multiplicity-dependent detectability. The result is a correction factor of $1.144\,[1-(0.006\pm0.001)N]$ to our odd-prong multiplicity distributions. The arbitrary factor of 1.144 is such that 21- and 22-prong numbers are unchanged. Thus, at each odd-prong multiplicity less than 21, the number of events is increased at the expense of the next higher even-prong event numbers, the increase being proportionately largest at $N = 3$.

In addition to the above proton visibility correction, we have made corrections to the raw data for unobserved Dalitz pairs and for missed vees, γ conversions, and secondary interactions close to the primary vertex. The small changes in the mean multiplicities $\langle N\rangle$ and in the dispersions D (both for $N \geq 3$) produced by each of these corrections are given in Table III. The corrected multiplicity distributions are given in Tables I and II, with errors that include statistical errors and errors in the corrections applied. Numbers of

TABLE III. Changes in multiplicity means $\langle N\rangle$ and dispersions D produced by corrections. The quantity entered in the table is corrected value minus uncorrected value.

		pd		π^+d	
Correction		Odd-prong events	Odd- and even-prong events	Odd-prong events	Odd- and even-prong events
Slow-proton visibility	$\langle N\rangle$	−0.07	···	−0.07	···
	D	−0.03	···	−0.02	···
Missed Dalitz pairs	$\langle N\rangle$	−0.05	−0.05	−0.06	−0.06
	D	−0.02	−0.02	−0.03	−0.02
Close vees, γ	$\langle N\rangle$	−0.01	−0.01	−0.02	−0.01
	D	−0.01	−0.01	0.00	−0.01
Close secondaries	$\langle N\rangle$	−0.02	−0.01	−0.01	−0.01
	D	−0.02	−0.01	−0.01	0.00

events are converted to cross sections using the sensitivities of 8.13 ± 0.30 μb per event for pd and 11.20 ± 0.54 μb per event for π^+d, which follow from our measured cross sections for $N\geq3$ of 54.0 ± 2.0 mb (pd) and 33.0 ± 1.6 mb (π^+d).

III. DEUTERON MULTIPLICITIES AND MOMENTS

We have previously[7] estimated the cross sections and multiplicity distributions of deuteron final-state (or, coherent deuteron) events, that is, events of the type $pd\to dX$ and $\pi^+d\to dX$. We also estimated the probability that a deuteron breakup event with meson production has charge multiplicity $N=2$. We have used these estimates plus our present data to obtain multiplicity distributions for deuteron breakup events with meson production. The low-order moments of these distributions are given in Table IV.

In experiments with heavy nuclei,[9] the mean charged-particle multiplicities reported generally exclude slow particles. We have taken slow to mean velocity $\beta<0.7$, and have made use of our sample of measured events to arrive at shower ($\beta>0.7$) particle multiplicity distributions. The resulting low-order moments for shower particles are given in Table IV. The quantity R, which gives the ratio of the mean shower-particle multiplicity in deuterium to that in hydrogen, is then 1.046 ± 0.022 for incident protons and 1.036 ± 0.025 for incident π^+. (We have taken pp and π^+p mean shower-particle multiplicities to be 6.04 ± 0.05 and 6.31 ± 0.09, respectively.)[10–12] These values of R agree with the values of 1.048 (incident protons) and 1.035 (incident π^+) predicted by the formula $R=0.5+0.5\,\bar{\nu}$, which fits heavier-nuclei data[13,14] (here $\bar{\nu}$ is the average nucleus thickness in units of the mean free path of the incident particle).

IV. RELATION BETWEEN ODD-PRONG AND FREE-NEUTRON MULTIPLICITY DISTRIBUTIONS

In a spectator model of high-energy interactions with deuterons, odd-prong events arise primarily from interactions with the neutron in the deuteron, with the proton in the deuteron remaining as a spectator. Then the odd-prong multiplicity distributions are related to free-neutron multiplicity distributions. Three effects that can contribute to differences between these two distributions are coherent deuteron interactions (i.e., $hd\to dX$), symmetry requirements on the final-state wave function,[15] and rescattering.

In the Appendix, we derive expressions that relate odd-prong and free-neutron multiplicity distributions, taking into account coherent deuteron interactions and wave-function symmetry requirements. The derivation assumes that the impulse approximation is valid. If we assume further that rescattering (see below) and screening are multiplicity-independent, then we can use Eq. (A13) and our data to obtain hadron-neutron topological cross sections. For the pn inelastic cross section, we use the measured pn total cross section,[16] and assume that the ratios of elastic to total cross sections[17,18] for pp and for pn are equal. For the π^+n cross section, charge symmetry allows us to use measured π^-p cross sections.[17,18] We take, for the quantities f and f_d in Eq. (A13), values of 0.64 ± 0.04 and 0.54 ± 0.06, respectively, corresponding to a visibility cutoff of ~2 mm. For $\sigma(hd\to dX)$ in Eq. (A13) we use our earlier estimates.[7] The resulting topological cross sections are given in Table V. The $N=1$ estimates are explained in Secs. V and VI below [see Eqs. (16) and (7)].

The $hd\to dX$ reactions contribute primarily to $N\leq6$ multiplicities. Therefore, the result of using Eq. (A13) rather than simply assuming that

TABLE IV. Deuteron breakup events with meson production: low-order moments for all charged particles and for shower (velocity $\beta>0.7$) particles. An estimated contribution from $N=1$ and 2 events is included.

		pd	π^+d
All charged particles	$\langle N\rangle$	7.05 ± 0.11	7.29 ± 0.12
	D	3.43 ± 0.05	3.42 ± 0.07
	f_2	4.70 ± 0.42	4.41 ± 0.53
Shower particles ($\beta>0.7$)	$\langle N\rangle$	6.32 ± 0.12	6.54 ± 0.13
	D	3.53 ± 0.05	3.50 ± 0.07
	f_2	6.15 ± 0.44	5.71 ± 0.55

TABLE V. Inelastic hadron-neutron topological cross sections.

N	$\sigma(pn)$ (mb)	$\sigma(\pi^+n)$ (mb)
1	2.81 ± 0.50 [a]	1.17 ± 0.25 [b]
3	6.17 ± 0.37	3.88 ± 0.38
5	7.73 ± 0.37	5.00 ± 0.36
7	6.53 ± 0.34	4.55 ± 0.34
9	4.38 ± 0.27	2.69 ± 0.28
11	2.47 ± 0.21	1.93 ± 0.23
13	1.15 ± 0.16	0.95 ± 0.15
15	0.43 ± 0.10	0.47 ± 0.11
17	0.11 ± 0.06	0.04 ± 0.04
19	0.06 ± 0.03	0.02 ± 0.02
21	0.03 ± 0.01	0.02 ± 0.02
23	0.01 ± 0.01	0.00
Total [c]	31.88 ± 0.44	20.72 ± 0.15

[a] Estimated as $(0.088\pm0.015)\ \sigma_{\text{inel}}(pn)$.
[b] Estimated as $(0.056\pm0.012)\ \sigma_{\text{inel}}(\pi^+n)$.
[c] From Refs. 16–18; see text.

TABLE VI. Low-order moments of the hadron-neutron distributions with no-cascade and extreme cascade models. See text for definitions.

	pn		π^+n	
	No cascade	Extreme cascade	No cascade	Extreme cascade
$\langle N\rangle$	6.20 ± 0.11	6.37 ± 0.12	6.57 ± 0.13	6.70 ± 0.14
D	3.38 ± 0.07	3.47 ± 0.08	3.38 ± 0.09	3.42 ± 0.09
f_2	5.25 ± 0.51	5.66 ± 0.55	4.89 ± 0.59	5.03 ± 0.60

odd-prong cross sections are proportional to free-neutron cross sections is to reduce slightly the $N \leqslant 5$ contributions to the free-neutron cross sections. Thus the means of the distributions in Table V exceed the means of the corrected odd-prong distributions by 0.07 and 0.09 for incident proton and π^+, respectively, both for $N \geqslant 3$. It is perhaps amusing that in this experiment the corrections to the mean free-neutron multiplicities produced by the proton visibility factor and by the $\sigma(hd \to dX)$ term in Eq. (A13) almost exactly cancel.

The assumption that rescattering is multiplicity-independent may be called a no-cascade assumption. Both heavy-nuclei experiments[9,19] and deuteron rescattering studies[2,5,7] suggest that this assumption, which we will adopt, is quite reasonable. For example, the observation that the fraction of πd events in which rescattering occurs is constant, within errors, over the incident momentum interval 20–200 GeV/c, while the mean πp multiplicity increases from 4.6 to 8.0, supports a no-cascade model. However, we will indicate how our results would be affected if a cascade model was correct. In an extreme cascade model, the probability of a rescatter on the second nucleon in the deuteron is equal to $\beta N_1'$, where N_1' is the multiplicity (charged plus neutral) of the interaction on the first nucleon, and β is given by $F_{rs}/\langle N_1'\rangle$, which is 0.020 for pd and 0.014 for π^+d. Here F_{rs} is the fraction of events with rescattering, and we assume that such events will be even-prong events. Then correction factors of $(1+0.023N)$ for pd, and $(1+0.017N)$ for π^+d, are required to arrive at the free-neutron multiplicity distributions. The resulting multiplicity distribution moments are compared with the no-cascade moments in Table VI. A less extreme cascade model could have the probability of a rescatter on the second nucleon equal to $\alpha + \beta' N_1'$, with $\alpha > 0$ and $0 < \beta' < F_{rs}/\langle N_1'\rangle$, leading to a distribution intermediate between the no-cascade and extreme cascade distributions (taking β' to be zero corresponds to our no-cascade assumption). Thus we see that the mean multiplicity $\langle N\rangle$ could be increased by up to 0.17 (pn) or 0.13 (π^+n) units if a cascade model was correct. These increases are of the same magni-

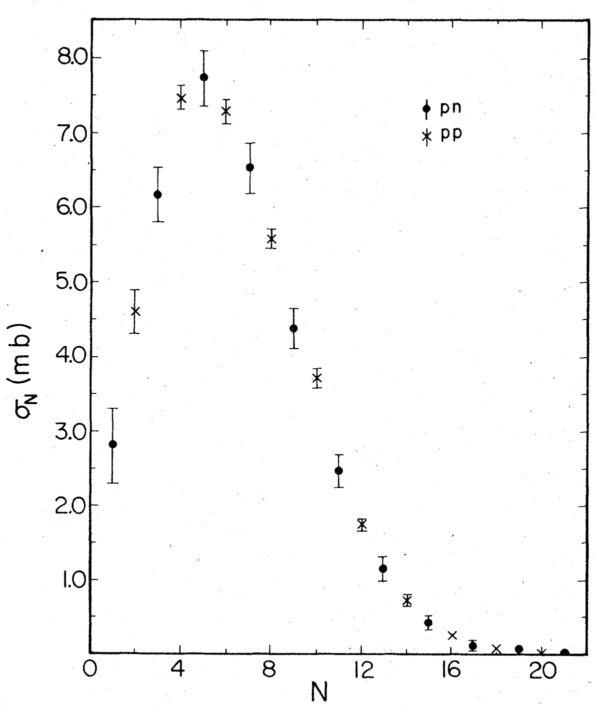

FIG. 1. The pn inelastic topological cross sections σ_N versus charged-particle multiplicity N. Also shown are the 100-GeV/c pp topological cross sections, from Refs. 10–12 (we take a simple average of the three experiments, after normalizing each to the total inelastic cross section obtained from Refs. 17 and 18).

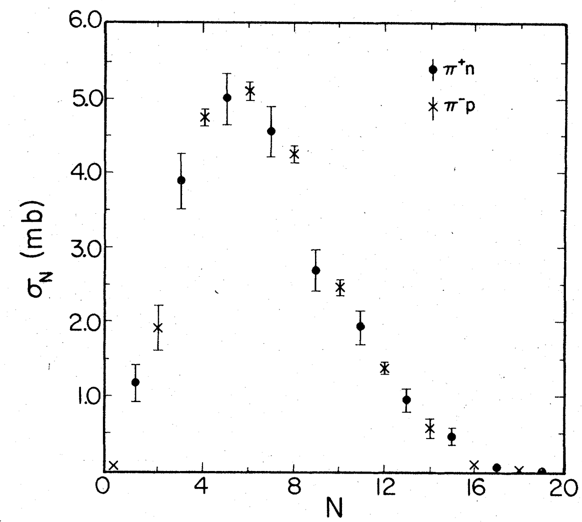

FIG. 2. The π^+n inelastic topological cross sections σ_N versus charged-particle multiplicity N. Also shown are the 100-GeV/c π^-p topological cross sections, from Ref. 20 (normalized to the total inelastic cross section obtained from Refs. 17 and 18).

TABLE VII. Low-order moments of 100-GeV/c hadron-nucleon multiplicity distributions. We take a simple average of the values from the three pp experiments (Refs. 10–12), and from the two π^+p experiments (Refs. 11, 12). The π^-p values are from Ref. 20.

	pn	pp	π^+n	π^+p	π^-p
$\langle N\rangle$	6.20 ± 0.11	6.39 ± 0.05	6.57 ± 0.13	6.71 ± 0.09	6.80 ± 0.08
D	3.38 ± 0.07	3.22 ± 0.03	3.38 ± 0.09	3.24 ± 0.05	3.15 ± 0.04
f_2	5.25 ± 0.51	3.97 ± 0.18	4.89 ± 0.59	3.84 ± 0.36	3.14 ± 0.30

tude as the corresponding statistical errors.

For the remainder of this paper, we will assume that the distributions in Table V are the multiplicity distributions for hadrons on free neutrons. The distributions are displayed in Fig. 1 and Fig. 2 for pn and π^+n, respectively, together with 100-GeV/c pp and π^-p distributions.[10-12,20] The low-order moments are compared with those for 100-GeV/c pp and πp distributions in Table VII. Relations between target-proton and target-neutron multiplicity distributions are examined in the following two sections.

V. RELATION BETWEEN πp AND πn MULTIPLICITY DISTRIBUTIONS

In an earlier paper[1] we gave a relation between π^+n and π^-p multiplicity distributions based on charge symmetry. Here we pursue that topic in greater detail. We first derive the relation between the distributions, stating explicitly the assumptions being made. Next we employ this relation to estimate the one-prong π^+n cross section, and then to calculate some properties of π^-p interactions using our π^+n distributions and published π^-p distributions.

If only one nucleon plus pions result from a pion-nucleon interaction, we can write (to avoid confusion, N is consistently even in the following)

$$\sigma(\pi^+n\to N+1)=\sigma(\pi^+n\to p+N)+\sigma(\pi^+n\to n+N+1)\,, \tag{1}$$

where N, $N+1$ refer to numbers of charged particles excluding those explicitly stated. Then, by applying charge symmetry, we have

$$\sigma(\pi^+n\to N+1)=\sigma(\pi^-p\to n+N)+\sigma(\pi^-p\to p+N+1)\,, \tag{2}$$

which can be rewritten

$$\sigma(\pi^+n\to N+1)=(1-Y_N)\,\sigma(\pi^-p\to N)+Y_{N+2}\,\sigma(\pi^-p\to N+2)\,, \tag{3}$$

where

$$Y_N=\frac{\sigma(\pi^-p\to p+N-1)}{\sigma(\pi^-p\to N)}\,. \tag{4}$$

It is straightforward to extend these equations to include kaon pairs, hyperon-kaon pairs, and baryon-antibaryon pairs. We find that Eq. (3) still holds if we modify slightly the physical interpretation of the Y_N and if we make a few reasonable assumptions. Thus we modify Eq. (4) as follows:

$$Y_N=\frac{\sigma(\pi^-p\to p+N-1)+\sigma(\pi^-p\to Y^0K^++N-1)+\sigma(\pi^-p\to Y^\pm K^++N-2)}{\sigma(\pi^-p\to N)}\,, \tag{4$'$}$$

which can be rewritten

$$Y_N=\langle p\rangle_N+\langle YK^+\rangle_N\,. \tag{4$''$}$$

Here Y^0 and $Y^\pm$ refer to hyperons and $\langle p\rangle_N$ and $\langle YK^+\rangle_N$ are, respectively, the average numbers per N-prong π^-p interaction of (nonproduced) protons and hyperon–positive-kaon pairs. The assumptions made, besides that of charge symmetry, are the following: (a) in association with a specified nucleon plus N charged pions, the probability of a K^+K^- ($p\bar{p}$) pair occurring is equal to that for a $K^0\bar{K}^0$ ($n\bar{n}$) pair; (b) in association with a specified nucleon plus N charged pions, the probability of a $Y\bar{p}K^+$ ($\bar{Y}pK^-$) trio occurring is equal to that for a $Y\bar{n}K^0$ ($\bar{Y}n\bar{K}^0$) trio; (c) the probability of producing three or more strange particles is negligible; and (d) the probability of producing two or more antibaryons is negligible.

Equation (4$'$) shows that $0\leq Y_N\leq1$ and that $Y_0=0$. Equation (3) then shows that we should expect the π^+n topological cross sections to be interleaved between those for π^-p. Further, if values of Y_N were available from π^-p data, we could predict the π^+n multiplicity distribution and so check our method of extracting this distribution from a deuterium experiment. However, values of Y_N are not available, because in bubble-chamber experiments protons are not distinguishable from pions at momenta above 1.4 GeV/c. Therefore, we use our π^+n distributions together with π^-p distributions to determine the values of Y_N.

The following expression for Y_N follows from Eq. (3):

$$Y_N = \frac{\sigma(\pi^- p \to \geq N) - \sigma(\pi^+ n \to \geq N+1)}{\sigma(\pi^- p \to N)}, \tag{5}$$

where $\sigma(\pi^- p \to \geq N)$ is the cross section for producing N or more charged particles in an inelastic π^-p interaction, and similarly for $\sigma(\pi^+ n \to \geq N+1)$. The mean value of Y_N is given by

$$\langle Y_N \rangle = \sum_{N=0}^{\infty} \sigma(\pi^- p \to N)\, Y_N / \sigma_{\text{inel}}(\pi^- p)$$
$$= 0.5(\langle N \rangle_{\pi^- p} - \langle N \rangle_{\pi^+ n}) + 0.5, \tag{6}$$

where $\sigma_{\text{inel}}(\pi^- p)$ is the π^-p inelastic cross section.

To apply Eqs. (5) and (6) to our data, we must assume a value for the inelastic one-prong cross section, $\sigma(\pi^+ n \to 1)$, or equivalently via Eq. (3) a value for Y_2. From published cross sections[21] for two-prong π^-p inelastic interactions at 150 GeV/c we find Y_2 (150 GeV/c) = 0.61±0.03 (we note that these cross sections do include assumptions about protons with momenta above 1.4 GeV/c). Further, the values for Y_N, $N \geq 4$, derived below are ~0.5–0.6. Therefore we make the assumption

$$\sigma(\pi^+ n \to 1) = (0.6 \pm 0.1)\, \sigma(\pi^- p \to 2). \tag{7}$$

Equation (7) corresponds to $Y_2 = 0.6 \pm 0.1$, to the extent that $\sigma(\pi^- p \to 0)$ is negligible.

The resulting values for Y_N and $\langle Y_N \rangle$ from our π^+n multiplicity distribution plus the 100-GeV/c π^-p distributions,[20] are given in Table VIII. We see that the Y_N values are consistent with being independent of N or with a slow decrease with increasing N. The latter is expected in a simple two-component model of high-energy interactions, where the diffraction component has $Y_N > 0.5$ and the nondiffraction component has $Y_N = 0.5$.

Some remarks should be made on the relation studied above between the π^+n and π^-p multiplicity distributions. Firstly, Eq. (3), with no restriction on the Y_N values, is valid without invoking charge symmetry; the Y_N can be considered to be defined by Eq. (5). The result of invoking charge symmetry, plus the additional assumptions mentioned, is to give a physical significance to Y_N, and then to place limits on allowed Y_N values. Secondly, Eq. (3) enables us to express any property of the π^+n distribution in terms of the Y_N and the π^-p distribution; Eq. (6) is an example. Another example is the following expression for the dispersion D:

TABLE VIII. Y_N values calculated from 100-GeV/c π^+n and π^-p multiplicity distributions.

N	Y_N
2	0.60±0.10[a]
4	0.65±0.09
6	0.65±0.08
8	0.66±0.09
10	0.50±0.12
12	0.49±0.14
14	0.40±0.21
16	0.85±0.41
$\langle Y_N \rangle = 0.61 \pm 0.08$	

[a] Assumed value (see text).

$$D_{\pi^+ n}^2 = D_{\pi^- p}^2 + 4[\langle N \rangle_{\pi^- p} \langle Y_N \rangle - \langle N Y_N \rangle_{\pi^- p} + \langle Y_N \rangle (1 - \langle Y_N \rangle)]. \tag{8}$$

Equation (8) shows that if Y_N is either a constant or a decreasing function of N, then $D_{\pi^+ n} > D_{\pi^- p}$, in agreement with the data. Thus, any similarities or differences between the two distributions can be thought of as resulting from the Y_N values, which tell how often a struck proton yields a proton or a YK^+ pair rather than a neutron or a YK^0 pair. Similarities need not be thought of as supporting ideas of universal properties of multiplicity distributions, and differences need not be thought of as undermining such ideas.

VI. RELATION BETWEEN pp AND pn MULTIPLICITY DISTRIBUTIONS

A relation between pp and pn multiplicity distributions similar to the above one for πp and πn cannot be derived from charge symmetry. However, relations analogous to Eqs. (3) and (4) do follow if one simple assumption is made. The assumption is that the cross section for two ordered initial-state nucleons to yield two ordered final-state nucleons plus N additional charged particles depends only on how many nucleons (0, 1, or 2) have flipped charge state and on N. The following relations are thus assumed:

$$\sigma(pn \to pn + N) = \sigma(pp \to pp + N), \tag{9}$$

$$\sigma(pn \to np + N) = \sigma(pp \to nn + N), \tag{10}$$

$$\sigma(pn \to nn + N) = \sigma(pp \to np + N), \tag{11}$$

$$\sigma(pn \to pp + N) = \sigma(pp \to pn + N), \tag{12}$$

where the first written nucleon in a pair is the one with the larger laboratory momentum. Equation (12) also follows from Eq. (11) plus charge symmetry.

Equations (9)–(12), and hence the original as-

TABLE IX. X_N values, calculated from 100-GeV/c pn and pp multiplicity distributions.

N	X_N
2	0.60 ± 0.10 [a]
4	0.57 ± 0.07
6	0.60 ± 0.06
8	0.64 ± 0.07
10	0.62 ± 0.08
12	0.58 ± 0.11
14	0.57 ± 0.17
16	0.45 ± 0.29
$\langle X_N \rangle = 0.60 \pm 0.06$	

[a] Assumed value (see text).

sumption, follow from vertex independence and charge symmetry, plus assumptions about K^+K^- and $K^0\bar{K}^0$, and $p\bar{p}$ and $n\bar{n}$, pairs similar to those needed in the pion-nucleon case.

Given Eqs. (9)–(12), it is straightforward to derive the following relations:

$$\sigma(pn \to N+1) = (1 - X_N)\sigma(pp \to N) + X_{N+2}\sigma(pp \to N+2), \quad (13)$$

$$X_N = \frac{\sigma(pp \to pp + N - 2) + \sigma(pp \to np + N - 1)}{\sigma(pp \to N)}. \quad (14)$$

To take account of kaon-hyperon pairs, where the hyperon is assumed to be a Λ or a Σ, the original assumption can be simply extended so that Eqs. (9)–(12) remain true if any final-state p (n) is replaced by a YK^+ (YK^0) pair. For example

$$\sigma(pn \to Y^0K^+n + N) = \sigma(pp \to Y^0K^+p + N), \quad (9')$$

$$\sigma(pn \to pY^+K^0 + N) = \sigma(pp \to pY^-K^+ + N). \quad (9'')$$

Then Eq. (14) must be rewritten, analogously to Eq. (4′), to include YK^+, and we finally have

$$X_N = 0.5[\langle p \rangle_N + \langle YK^+ \rangle_N], \quad (15)$$

where $\langle p \rangle_N$ and $\langle YK^+ \rangle_N$ are, respectively, the average numbers per N-prong pp interaction of (nonproduced) protons and YK^+ pairs. Thus X_N is similar to Y_N in the pion-nucleon case, and in a model with vertex independence we would expect $\langle X_N \rangle = \langle Y_N \rangle$.

Values of X_N are not available from pp experiment. However, an estimate of $\langle X_N \rangle$ can be obtained from pp data as follows. The quantity $d\sigma/dX$ for the reaction $pp \to pX$ at 100 GeV/c has been measured[22] in the x range 1.0 to 0.5 (here x is the Feynman scaling variable). The results show that $d\sigma/dx$ has little or no x dependence in the range 0.9 to 0.5. Also, we know from symmetry that $d\sigma/dx$ must have zero slope at $x = 0$. Therefore we assume that the average $d\sigma/dx$ value in the range 0.5 to 0.0 equals that in the range 0.82 to 0.5, and we integrate over all x to arrive at a value for $\langle p \rangle$ of 1.07 ± 0.03. To obtain $\langle YK^+ \rangle$, we take $\langle YK \rangle \approx 1.5(\langle \Lambda \rangle - \langle \bar{\Lambda} \rangle)$, where Λ includes Σ^0, and take $\langle YK^+ \rangle \approx 0.5 \langle YK \rangle$. Then pp data[23,24] yield $\langle YK^+ \rangle \approx 0.07$. Thus we finally have $\langle X_N \rangle = 0.57 \pm 0.02$, where the error includes an estimated 50% uncertainty in $\langle YK^+ \rangle$.

As in the πn case, we can use our pn multiplicity data along with pp multiplicities[10,12] to solve for X_N, with equations exactly analogous to Eqs. (5) and (6), once we have a value for $\sigma(pn \to 1 \text{ prong})$. We assume that $X_2 = Y_2$, and then Eq. (13) yields

$$\sigma(pn \to 1) = (0.6 \pm 0.1)\sigma(pp \to 2). \quad (16)$$

The resulting values of X_N and $\langle X_N \rangle$ are given in Table IX. We see that $\langle X_N \rangle$ agrees with the estimate from pp data and with $\langle Y_N \rangle$. The X_N values, similarly to Y_N, are consistent with a constant or slowly falling value as N increases, again in agreement with a simple two-component model.

It is possible that the agreement between the $\langle X_N \rangle$ values from the pp data and from the pp and pn multiplicity distributions is fortuitous. This is because assumptions have been made in obtaining $\langle X_N \rangle$ from the pp data, in extracting the pn multiplicity distribution from pd data, and in estimating $\sigma(pn \to 1)$. With regard to the last of these, we note that a change of 0.1 in the value of X_2 would change our $\langle X_N \rangle$ value by 0.04. However, if we set aside this fortuitous possibility, then the agreement between the $\langle X_N \rangle$ values supports the assumptions made in Eqs. (9)–(12) and their extensions. The agreement between the X_N values and the Y_N values also supports these assumptions. Then, analogously to the incident-pion case above, any similarities or differences between the pn and pp multiplicity distributions can be thought of as resulting from how often a struck proton yields a proton or a YK^+ pair.

VII. CONCLUSIONS

The main results and conclusions from this work are as follows.

We have determined charged-particle multiplicity distributions for pd and π^+d interactions. The mean multiplicities for shower particles (i.e., those with velocity $\beta > 0.7$) agree with the predictions of a formula that describes heavier-nuclei mean shower multiplicities.

From corrected odd-prong distributions, plus a no-cascade assumption and one-prong estimates, we have obtained inelastic pn and π^+n charged-particle multiplicity distributions. We have made small corrections for a multiplicity-dependent slow proton visibility and for coherent deuteron events and wave-function symmetry effects. The mean charged-particle multiplicities are 6.20±0.11 (pn) and 6.57±0.13 (π^+n).

We have related π^+n and π^-p charged-particle multiplicity distributions via a set of quantities Y_N (N = charge multiplicity), where Y_N equals the average number of (nonproduced) protons plus hyperon–charged-kaon pairs per N-prong π^-p inelastic interaction. From the π^+n and π^-p multiplicity distributions, we evaluate the Y_N. We find that Y_N has at most a small N dependence, and that $\langle Y_N\rangle = 0.61\pm0.08$.

We have related pn and pp multiplicity distributions via a set of quantities X_N, after making an assumption that is suggested by vertex-independence arguments. Here X_N equals one half the average number of (nonproduced) protons plus hyperon–charged-kaon pairs per N-prong pp inelastic interaction. From pn and pp multiplicity distributions, we find values of X_N that show little N dependence, with $\langle X_N\rangle = 0.60\pm0.06$, in agreement with $\langle Y_N\rangle$ and with an estimate of $\langle X_N\rangle$ from $pp\to pX$ and $pp\to\Lambda X$ data.

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APPENDIX

Here we derive approximate expressions that relate odd-prong hadron-deuteron cross sections with hadron-neutron cross sections. We assume that the impulse approximation is valid.

We assume that at small values of the momentum transfer q, where the deuteron form factor $S(q)$ is non-negligible, the hadron-nucleon cross section is spin- and isospin-independent. Then relations derived by Dean[15] within the impulse approximation become

$$\frac{d\sigma}{d\Omega}(hd\to bpp) = \frac{d\sigma}{d\Omega}(hn\to bp)\,, \tag{A1}$$

$$\frac{d\sigma}{d\Omega}(hd\to b'pn, b'd) = \left[\frac{d\sigma}{d\Omega}(hp\to b'p) + \frac{d\sigma}{d\Omega}(hn\to b'n)\right] \times [1+S(q)]\,, \tag{A2}$$

where h is the incident hadron, and the final-state $b'pn$ excludes $b'd$. Equation (A2) is exactly analogous to the scattering differential cross section derived by Franco and Glauber[25] if double scattering terms are neglected.

By analogy with expressions[25] for hd elastic scattering (see also Ref. 26), we obtain (with our approximations)

$$\frac{d\sigma}{d\Omega}(hd\to b'd) = \left[\frac{d\sigma}{d\Omega}(hp\to b'p) + \frac{d\sigma}{d\Omega}(hn\to b'n)\right] \times 2S^2(q/2)\,. \tag{A3}$$

Hence for the deuteron breakup reaction we have

$$\frac{d\sigma}{d\Omega}(hd\to b'pn) = \left[\frac{d\sigma}{d\Omega}(hp\to b'p) + \frac{d\sigma}{d\Omega}(hn\to b'n)\right] \times [1+S(q)-2S^2(q/2)]\,. \tag{A4}$$

We now assume a proton-spectator (p_s) contribution to the reaction as follows:

$$\frac{d\sigma}{d\Omega}(hd\to bp_s p) = \frac{d\sigma}{d\Omega}(hn\to bp)\,, \tag{A5}$$

$$\frac{d\sigma}{d\Omega}(hd\to b'p_s n) = \frac{d\sigma}{d\Omega}(hn\to b'n) \times [1+S(q)-2S^2(q/2)]\,. \tag{A6}$$

Summing over all the states b and b' that contain $N-1$ and N charged particles, respectively (N odd), yields [by our earlier assumptions $(d\sigma/d\Omega)(hn\to bp)$ is zero when $S(q)$ is non-negligible]

$$\frac{d\sigma}{d\Omega}(hd\to p_s+N) = \frac{d\sigma}{d\Omega}(hn\to N) \times [1+S(q)-2S^2(q/2)]\,. \tag{A7}$$

Integrating Eq. (A7) over all angles leads to

$$\sigma(hd\to p_s+N) = \sigma(hn\to N) + (A-B)\,, \tag{A8}$$

$$A = \int d\Omega\, S(q)\, \frac{d\sigma}{d\Omega}(hn\to N)\,, \tag{A9}$$

$$B = \int d\Omega\, 2S^2(q/2)\, \frac{d\sigma}{d\Omega}(hn\to N)\,. \tag{A10}$$

From Eq. (A3) it follows that

$$B = 0.5\sigma(hd\to d+N)\,.$$

We evaluate the quantity A/B in the approximation that $(d\sigma/d\Omega)(hn\to N)$ has a much slower q dependence than $S(q)$, and using for $S(q)$ the sum of three Gaussians.[27] We find $A/B = 0.31$. Hence we have the result

$$\sigma(hd\to p_s+N) = \sigma(hn\to N) - 0.35\sigma(hd\to d+N)\,. \tag{A11}$$

If f is the probability that the proton spectator is invisible ($f\sim0.7$), and f_d is the probability that the final-state deuteron in the reaction $hd\to dX$ is invisible ($f_d\sim0.5$), with both f and f_d assumed to be

N-independent, then we have (we assume no contribution to odd-prong events from neutron spectator events)

$$\sigma(hd\to N)=f\sigma(hd\to p_s+N)+f_d\sigma(hd\to d+N)$$
$$=f\sigma(hn\to N)+(f_d-0.35f)\sigma(hd\to d+N),$$

which can be rewritten as

$$f\sigma(hn\to N)=\sigma(hd\to N)-(f_d-0.35f)\sigma(hd\to d+N). \quad \text{(A12)}$$

Screening and rescattering, which we have neglected above, will destroy the equality in Eq. (A12). However, to the extent that screening and rescattering produce fractional depletions in (p_s+N) and $(d+N)$ final states that are equal and independent of N, then the right-hand side of Eq. (A12) will give the N dependence of $\sigma(hn)$. In that case we have

$$\sigma(hn\to N)=K[\sigma(hd\to N)-(f_d-0.35f)\sigma(hd\to d+N)], \quad \text{(A13)}$$

where K is a normalization factor such that the sum over all N values yields the total hn inelastic cross section.

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