Electrogravitational conversion cross sections in static electromagnetic fields

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We use Feynman perturbation techniques to analyze a classical process: the conversion of gravitational waves into electromagnetic waves (and vice versa) under the "catalytic" action of a static electromagnetic background field. Closed-form differential cross sections are presented for conversion in the Coulomb field of a point charge, electric and magnetic dipole fields, and uniform electrostatic and magnetostatic fields. Using the model calculation of conversion in a Coulomb field, we discuss the problems we must face when calculating non-gauge-invariant quantities, as is frequently done in literature. The cross sections are extremely small, but may lead to observable effects if allowed to act on astrophysical distance and time scales. The calculations also provide additional insight into the physics of electromagnetic detectors of gravitational waves.

I. INTRODUCTION

The principle of equivalence in general relativity theory dictates that gravity couple to the energy-momentum tensor of all fields. Past research in general relativity theory has emphasized the coupling of gravity to heavy objects (stars, galaxies, the universe, Weber bars, and the like), since the cumulative effect of a large mass tends to offset the extreme smallness of the gravitational coupling constant. Less effort has been spent on investigating the interaction of gravitons with other elementary particles, such as the photon.[1] In certain extreme astrophysical situations (pulsars, quasars, collapsing stars, active galactic nuclei, and early universe), however, intense gravitational and/or electromagnetic fields exist, and we may not discard *a priori* the interaction between gravitons and photons.

In particular, recent work indicates that in any spacetime permeated by an electromagnetic background, a nontrivial coupling exists between electromagnetic and gravitational perturbations. Whereas the total energy in these perturbations is conserved, photon and graviton numbers individually are not. This implies the existence of conversion cross sections, expressing the fact that a static electromagnetic field may serve as a catalyst for converting electromagnetic waves into gravitational waves and vice versa.

It was hoped that the electromagnetic-gravitational resonance near a Reissner-Nordström (charged, nonrotating) black hole would have observationally detectable consequences. Insight into the details of electromagnetic-gravitational resonance has been provided by Gerlach,[2] who originally found the coupled electromagnetic-gravitational perturbation equations in the WKB limit. The Newman-Penrose formalism was used by Chitre *et al.*[3] to separate the wave equations for mixed gravitational and electromagnetic perturbations. However, numerical studies[4,5] have shown that the electrogravitational interconversion can become efficient only when the charge-to-mass ratio Q/M of the black hole is near unity (in geometrized units, i.e., $G=c=1$). Black holes with such an extreme Q/M ratio are unlikely to exist. Nevertheless, the problem of coupled electromagnetic and gravitational perturbations in the vicinity of a Reissner-Nordström black hole remains interesting in principle, and Matzner[6] has recently calculated the conversion cross sections in the long-wavelength limit for quadrupole waves.

We shall not address ourselves to the exact strong-field problem, which requires the use of the full mathematical apparatus of general relativity theory. Rather, we study the simplified case of Minkowski spacetime permeated by various static electromagnetic backgrounds. This simplification makes the problem mathematically tractable, without excluding the essential physical features. Although the conversion efficiencies are extremely small, they may lead to observable effects if allowed to act on sufficiently long distance and time scales.

Conversion scattering may also play a role in the laboratory generation[7] and detection[8,9] of high-frequency gravitational waves ("Hertz-type" experiment). In the laboratory we may compensate for the smallness of the effects by exploiting the resonance and coherence of the electromagnetic wave and the gravitational wave. It appears that the most promising approach would be to use an electromagnetic resonator to generate coherently a gravitational wave with a precise frequency and phase. This gravitational wave would subsequently be detected by a second electromagnetic resonator with a set of natural frequencies which are matched to the gravitational wave. Resonant reception occurs when the frequency of the gravitational wave is equal to the difference of two electromagnetic natural frequencies.

It has also been suggested that electromagnetic resonators be used to detect high-frequency gravitational noise, which has been predicted by Grishchuk.[10] The lack of coherence, however, will make the latter experiment even more difficult than a pure laboratory Hertz-type experiment.

Electrogravitational conversion was known to Whittaker[11] as early as 1947. Gertsenshteĭn,[12] however, was the first to actually calculate a conversion efficiency. In 1961 he used Einstein's linearized theory to consider the resonance of an electromagnetic wave and a gravitational wave in a strong uniform magnetostatic field. Weber and Hinds[13] investigated similar conversion processes by employing the Hamiltonian formulation of general relativity theory. The problem of the electromagnetic response of a capacitor to an incident gravitational wave has been investigated by Lupanov.[14] We take special note of a series of papers by an Italian research group,[15-19] in which various conversion mechanisms are studied. Both a Lagrangian-based quantum theory of gravity and classical general relativity theory are used. Their conclusions include possible astrophysical consequences and suggestions for gravitational-wave experiments. Papini and Valluri[20] used a Lagrangian-based quantum theory of gravity to study the role of conversion scattering in pulsars. Ginzburg and Tsytovich[21] recently calculated conversion cross sections by using the formal analogy between conversion scattering and dielectric wave-induced transition radiation.

This paper is the second in a series advocating the reassessment of the quantum approach in the investigation of the role of gravitation in astrophysics.[22] Using Feynman perturbation techniques we have derived conversion cross sections in closed form and have analyzed in detail their dependence on the polarization of the incident wave. Many of our results have been obtained before by the use of some other method. The reader is invited to compare the ease with which results can be obtained by the Feynman perturbation technique as opposed to the calculations hitherto used.

The paper is in eight sections. Section II summarizes the relevant Feynman rules. Section III treats interactions with a nonspinning test charge. In Secs. IV and V we calculate conversion cross sections in magnetic and electric dipole fields. Sections VI and VII are devoted to conversion in uniform magnetostatic and electrostatic fields. Finally, in Sec. VIII we discuss our results in the light of previous investigations and make remarks about some inaccuracies in the literature.

In the following we shall use natural units ($G = \hbar = c = 1$) and a metric $g_{\alpha\beta}$ with signature $+2$. Semicolons denote covariant derivatives and commas denote partial derivatives. Greek indices take values from 0 to 3, Latin indices from 1 to 3. We shall also use the abbreviation:

$$\underline{a}\cdot\underline{b} = \vec{a}\cdot\vec{b} - a_0 b_0\,, \tag{1.1}$$

where $\vec{a}$ ($\vec{b}$) is the spatial part of $\underline{a}$ ($\underline{b}$).

II. THE FEYNMAN RULES

We review here the Feynman rules which will be relevant for our purposes. The Lagrangian density describing the interaction of a charged massive scalar field (e.g., a pion) and a photon field in a Minkowski background is

$$\mathcal{L} = -\{\eta^{\mu\nu}[(\partial_\mu + ieA_\mu)\psi^*][(\partial_\nu - ieA_\nu)\psi] + M^2\psi^*\psi + \tfrac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\}. \tag{2.1}$$

Here ψ is the scalar field, M is the scalar field's mass and e is its charge in Lorentz-Heaviside (rationalized) units (for an electronic charge $e^2/4\pi = \frac{1}{137}$). A_μ is the Maxwell 4-potential, and $F_{\mu\nu}$ is the electromagnetic field tensor computed from A_μ by

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \tag{2.2}$$

Through minimal substitution we obtain from (2.1) the corresponding manifestly covariant Lagrangian density in a curved background[32]:

$$\mathcal{L} = -\sqrt{-g}\{g^{\mu\nu}[(\partial_\mu + ieA_\mu)\psi^*][(\partial_\nu - ieA_\nu)\psi] + M^2\psi^*\psi + \tfrac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\}. \tag{2.3}$$

An infinitesimal variation of ψ^* in the action $\mathcal{S} = \int \mathcal{L}d^4x$ yields the following field equation for ψ:

$$\frac{1}{\sqrt{-g}}[(\partial_\mu - ieA_\mu)\sqrt{-g}\,g^{\mu\nu}(\partial_\nu - ieA_\nu)]\psi - M^2\psi = 0. \tag{2.4}$$

Similarly, varying the action $\mathcal{S}$ with respect to A_μ provides a set of Maxwell equations

$$F^{\mu\nu}{}_{;\nu} = ej^\mu\,, \tag{2.5}$$

where the conserved current j^μ is defined as

$$j^\mu = ig^{\mu\nu}[\psi(\partial_\nu + ieA_\nu)\psi^* - \psi^*(\partial_\nu - ieA_\nu)\psi]. \tag{2.6}$$

The other Maxwell equations

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0 \tag{2.7}$$

follow from (2.2).

Following Feynman[23] and Gupta[24] we define the gravitational field as the deviation from Minkowski space-time:

$$\sqrt{-g}\,g^{\alpha\beta} \equiv \mathfrak{g}^{\alpha\beta} \equiv \eta^{\alpha\beta} - 2\lambda\bar{h}^{\alpha\beta}, \tag{2.8}$$

where the gravitational coupling constant $\lambda=\sqrt{8\pi}$. The indices of the trace-reversed metric perturbation $\bar{h}^{\alpha\beta}$ are lowered with the Minkowski metric $\eta_{\alpha\beta}=\text{diag}(-1,1,1,1)$. The contravariant components of the metric and the determinant factor are now expressed as infinite series in λ,

$$g^{\alpha\beta}=\eta^{\alpha\beta}-2\lambda(\bar{h}^{\alpha\beta}-\tfrac{1}{2}\eta^{\alpha\beta}\bar{h})+O(\lambda^2), \tag{2.9a}$$

$$\sqrt{-g}=(-\det\|g_{\alpha\beta}\|)^{1/2}=(-\det\|g^{\alpha\beta}\|)^{1/2}$$
$$=1-\lambda\bar{h}+O(\lambda^2), \tag{2.9b}$$

where the trace of the metric perturbation is denoted by $\bar{h}=\bar{h}_\alpha{}^\alpha$. Substituting (2.8) and (2.9) into the Lagrangian density (2.3) we find that we may break up the Lagrangian density into three pieces:

$$\mathcal{L}=\mathcal{L}_S+\mathcal{L}_{\text{em}}+\mathcal{L}_I,$$

$$\mathcal{L}_S=-(\eta^{\mu\nu}\psi^*_{,\mu}\psi_{,\nu}+M^2\psi^*\psi), \tag{2.10a}$$

$$\mathcal{L}_{\text{em}}=-\tfrac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}, \tag{2.10b}$$

$$\begin{aligned}\mathcal{L}_I=&-ie\eta^{\mu\nu}(A_{(\mu}\psi_{,\nu)}\psi^*-A_{(\mu}\psi^*_{,\nu)}\psi)\\&+2\lambda\bar{h}^{\mu\nu}(\psi^*_{,(\mu}\psi_{,\nu)}+\tfrac{1}{2}\eta_{\mu\nu}M^2\psi^*\psi)\\&+2ie\lambda\bar{h}^{\mu\nu}(A_{(\mu}\psi_{,\nu)}\psi^*-A_{(\mu}\psi^*_{,\nu)}\psi)\\&+\lambda(\bar{h}^{\mu\alpha}\eta^{\nu\beta}-\tfrac{1}{4}\bar{h}\eta^{\mu\alpha}\eta^{\nu\beta})F_{\mu\nu}F_{\alpha\beta}+O(e^2,\lambda^2),\end{aligned} \tag{2.10c}$$

where we have used the notation $A_{(\mu}B_{\nu)}=\frac{1}{2}(A_\mu B_\nu+A_\nu B_\mu)$. $\mathcal{L}_S$ and $\mathcal{L}_{\text{em}}$ describe how the free massive scalar and photon fields propagate in Minkowski space. From (2.10a) and (2.10b), we can deduce the propagators for the massive scalar field and the photon field in momentum space

$$D^S(p)=\frac{1}{p^2+M^2-i\epsilon}, \tag{2.11a}$$

$$D^{\text{em}}_{\mu\nu}(k)=\frac{\eta_{\mu\nu}}{k^2-i\epsilon}, \tag{2.11b}$$

where ϵ is a small real positive number.

The Lagrangian density $\mathcal{L}_I$ describes the mutual interaction of the scalar, photon, and graviton fields and yields the Feynman vertex functions (see Fig. 1):

(a) the scalar-particle–scalar-particle–photon vertex

$$T_\mu({}^1p,{}^2p,{}^\gamma k)=e({}^1p+{}^2p)_\mu, \tag{2.12a}$$

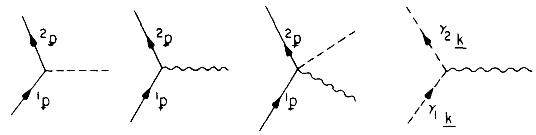
(b) the scalar-particle–scalar-particle–graviton vertex

$$T_{\mu\nu}({}^1p,{}^2p,{}^g k)=2\lambda({}^1p_{(\mu}{}^2p_{\nu)}+\tfrac{1}{2}M^2\eta_{\mu\nu}), \tag{2.12b}$$

(c) the scalar-particle–scalar-particle–graviton–photon vertex

$$T_{\mu\nu,\alpha}({}^1p,{}^2p,{}^g k,{}^\gamma k)=-2e\lambda({}^1p+{}^2p)_{(\mu}\eta_{\alpha\nu)}, \tag{2.12c}$$

(d) the graviton-photon-photon vertex



(a) (b) (c) (d)

FIG. 1. The Feynman vertices. The solid lines represent scalar quanta, the dashed lines represent photons, the wavy lines represent gravitons.

$$\begin{aligned}T_{\mu\nu,\alpha,\beta}&({}^g k,{}^{\gamma1}k,{}^{\gamma2}k)\\&=2\lambda[{}^{\gamma1}k_{(\mu}{}^{\gamma2}k_{\nu)}\eta_{\alpha\beta}+\eta_{\alpha(\mu}\eta_{\nu)\beta}{}^{\gamma1}\underline{k}\circ{}^{\gamma2}\underline{k}-{}^{\gamma1}k_{(\mu}\eta_{\nu)\beta}{}^{\gamma2}k_\alpha\\&\quad-{}^{\gamma2}k_{(\mu}\eta_{\nu)\alpha}{}^{\gamma1}k_\beta-\tfrac{1}{2}\eta_{\mu\nu}({}^{\gamma1}\underline{k}\circ{}^{\gamma2}\underline{k}\eta_{\alpha\beta}-{}^{\gamma1}k_\beta{}^{\gamma2}k_\alpha)].\end{aligned} \tag{2.12d}$$

The transition matrix elements above have been normalized by the definition

$$S_{fi}=\delta_{fi}+i(2\pi)^4\delta^4(\textstyle\sum\underline{p}_i-\sum\underline{p}_f)T_{fi}, \tag{2.13}$$

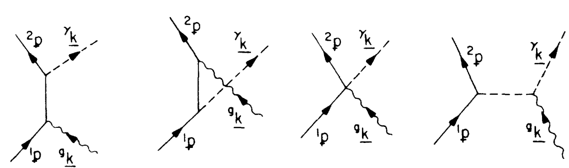
where S_{fi} is the scattering matrix element $\langle f|S|i\rangle$.

III. EXCHANGE COMPTON SCATTERING

We are now in a position to work out the cross sections for the conversion of gravitational waves into electromagnetic waves in the electrostatic field of a charged scalar particle.

Let the initial and final 4-momenta of the scalar particle be ${}^1\underline{p}=({}^1E,{}^1\vec{p})$ and ${}^2\underline{p}=({}^2E,{}^2\vec{p})$, and those of the incident graviton and scattered photon ${}^g\underline{k}=({}^g\omega,{}^g\vec{k})$ and ${}^\gamma\underline{k}=({}^\gamma\omega,{}^\gamma\vec{k})$, respectively. The polarizations of the graviton and photon are denoted by $\bar{e}^{\mu\nu}$ and ϵ^μ. The lowest-order diagrams for exchange Compton scattering are shown in Fig. 2. Figures 2(a) and 2(b) are the pion-pole terms, and Fig. 2(c) is the seagull term familiar from meson theory. The t-pole term, exhibited in Fig. 2(d) is a unique feature of gravitation. It arises from the fact that the gravitational wave interacts not only with the mass of the particle, but also with the energy associated with its long-range electrostatic field.

A straightforward application of the Feynman rules summarized in Sec. II yields for the individual contributions of the separate graphs



(a) (b) (c) (d)

FIG. 2. Feynman graphs for exchange Compton scattering (graviton → photon) by a charged spin-0 meson.

$$T_a=2\lambda e({}^1p_\mu \bar{e}^{\mu\nu}\,{}^1p_\nu+\tfrac{1}{2}M^2\bar{e})^2\underline{p}\cdot\underline{\epsilon}^*({}^1\underline{p}\cdot{}^g\underline{k})^{-1}, \tag{3.1a}$$

$$T_b=-2\lambda e({}^2p_\mu \bar{e}^{\mu\nu}\,{}^2p_\nu+\tfrac{1}{2}M^2\bar{e})^1\underline{p}\circ\underline{\epsilon}^*({}^1\underline{p}\circ{}^\gamma\underline{k})^{-1}, \tag{3.1b}$$

$$T_c=-2\lambda e\,\bar{e}^{\mu\nu}({}^1p+{}^2p)_{(\mu}\epsilon^*_{\nu)}, \tag{3.1c}$$

$$\begin{aligned}T_d=-\lambda e\{&{}^\gamma k_\mu \bar{e}^{\mu\nu}\,{}^\gamma k_\nu({}^1\underline{p}+{}^2\underline{p})\circ\underline{\epsilon}^*-\bar{e}^{\mu\nu}({}^1p+{}^2p)_{(\mu}\epsilon^*_{\nu)}{}^g\underline{k}\circ{}^\gamma\underline{k}\\&-\bar{e}^{\mu\nu}\,{}^\gamma k_{(\mu}\epsilon^*_{\nu)}{}^\gamma\underline{k}\circ({}^1\underline{p}+{}^2\underline{p})+\bar{e}^{\mu\nu}\,{}^\gamma k_{(\mu}({}^1p+{}^2p)_{\nu)}{}^g\underline{k}\circ\underline{\epsilon}^*\\&+\tfrac{1}{2}\bar{e}[{}^g\underline{k}\cdot{}^\gamma\underline{k}({}^1\underline{p}+{}^2\underline{p})\circ\underline{\epsilon}^*-{}^g\underline{k}\circ\underline{\epsilon}^*\,{}^\gamma\underline{k}\circ({}^1\underline{p}+{}^2\underline{p})]\}({}^g\underline{k}\circ{}^\gamma\underline{k})^{-1}.\end{aligned} \tag{3.1d}$$

Here

$$\bar{e}=\bar{e}_\mu{}^\mu. \tag{3.2}$$

To obtain the above, we have used

$${}^1\underline{p}\circ{}^1\underline{p}={}^2\underline{p}\cdot{}^2\underline{p}=-M^2, \tag{3.3a}$$

$${}^g\underline{k}\circ{}^g\underline{k}={}^\gamma\underline{k}\circ{}^\gamma\underline{k}=0, \tag{3.3b}$$

$${}^g k_\mu \bar{e}^{\mu\nu}={}^g k_\nu \bar{e}^{\mu\nu}={}^\gamma k_\mu \epsilon^\mu=0. \tag{3.3c}$$

To investigate the gauge invariance of the scattering amplitude, we consider the tranformations

$$\bar{e}^{\mu\nu}\to\bar{e}^{\mu\nu}+{}^g k^{(\mu}\chi^{\nu)}, \tag{3.4a}$$

$$\epsilon^\mu\to\epsilon^\mu+f\,{}^\gamma k^\mu, \tag{3.4b}$$

where ${}^g k_\mu\chi^\mu=0$, and f and χ^μ are otherwise arbitrary functions.

It can readily be shown that the individual terms are not gauge invariant though their sum is. Indeed, the fact that the sum turns out to be invariant under gauge transformations is a strong test which assures us that no algebraic errors have entered into the calculation.

In our expressions for the cross section we shall use the laboratory frame, in which

$$\begin{aligned}&{}^1\vec{p}=0,\quad {}^1E=M,\\&{}^g\vec{k}={}^2\vec{p}+{}^\gamma\vec{k},\\&{}^g\omega+M={}^\gamma\omega+{}^2E.\end{aligned} \tag{3.5}$$

We remove the gauge freedom for the electromagnetic field by choosing the photon polarization ϵ^μ to be purely spacelike ($\epsilon^0=0$). The gravitational gauge freedom is specified by choosing the transverse-traceless (TT) gauge ($\bar{e}^{\mu 0}=\bar{e}^{0\mu}=0$; $\bar{e}=0$).

We then see that the contributions of the diagrams (a) and (b) vanish and the remaining terms take a much simpler form:

$$T_c=2\lambda e\,\bar{e}^{ij}\,{}^\gamma k_{(i}\epsilon^*_{j)}, \tag{3.6a}$$

$$T_d=2\lambda e\,\frac{{}^\gamma\omega}{{}^g\omega-{}^\gamma\omega}\,\bar{e}^{ij}\,{}^\gamma k_{(i}\epsilon^*_{j)}, \tag{3.6b}$$

where we have used

$${}^2\vec{p}\circ\vec{\epsilon}={}^g\vec{k}\circ\vec{\epsilon}, \tag{3.7a}$$

$$\bar{e}^{ij2}p_{(i}\epsilon_{j)}=-\bar{e}^{ij}\,{}^\gamma k_{(i}\epsilon_{j)}, \tag{3.7b}$$

$$\bar{e}^{ij}\,{}^\gamma k_{(i}{}^2p_{j)}=-\bar{e}^{ij}\,{}^\gamma k_{(i}{}^\gamma k_{j)}, \tag{3.7c}$$

$${}^g\underline{k}\circ{}^\gamma\underline{k}=M({}^\gamma\omega-{}^g\omega), \tag{3.7d}$$

$${}^\gamma\underline{k}\circ({}^1\underline{p}+{}^2\underline{p})=-M({}^\gamma\omega+{}^g\omega). \tag{3.7e}$$

The above relations follow from conservation of energy-momentum and the transverse nature of the photon and graviton.

The frequency of the outgoing photon is related to the frequency of the incident graviton through the Compton relation

$${}^\gamma\omega=\frac{{}^g\omega}{1+2({}^g\omega/M)\sin^2(\frac{1}{2}\theta)}, \tag{3.8}$$

where θ is the angle between ${}^g\vec{k}$ and ${}^\gamma\vec{k}$.

The differential cross section for converting a graviton with frequency ${}^g\omega$ and polarization $\bar{e}^{\mu\nu}$ into a photon with frequency ${}^\gamma\omega$ and polarization ϵ^μ is

$$d\sigma=\frac{2\pi}{2M2^g\omega 2^2E2^\gamma\omega}\,|T_c+T_d|^2D, \tag{3.9}$$

where D denotes the density of final states

$$D=\frac{1}{(2\pi)^3}\,\frac{{}^2E^\gamma\omega^3}{M^g\omega}\,d\Omega. \tag{3.10}$$

Substitution of (3.6) and (3.10) in (3.9) and use of (3.8) yields

$$\frac{d\sigma}{d\Omega}=\frac{e^2}{8\pi\sin^4(\frac{1}{2}\theta)}\left[\frac{1}{1+(2^g\omega/M)\sin^2(\frac{1}{2}\theta)}\right]^2|\bar{e}^{ij}\,{}^\gamma\hat{k}_{(i}\epsilon^*_{j)}|^2, \tag{3.11}$$

where ${}^\gamma\hat{k}$ is a unit vector in the direction of the outgoing photon.

In the nonrelativistic (NR) region, i.e., ${}^g\omega \ll M$, there is negligible recoil of the scatterer, and (3.11) reduces to

$$\frac{d\sigma}{d\Omega}=\frac{e^2}{8\pi\sin^4(\frac{1}{2}\theta)}\,|\bar{e}^{ij}\,{}^\gamma\hat{k}_{(i}\epsilon^*_{j)}|^2 \tag{3.12}$$

(NR limit,[25] in laboratory frame).

It is easily seen that the cross section (3.12) is solely due to the contribution of the t-pole diagram. We therefore conclude that although the t-pole term is not invariant by itself with respect to gravitational gauge transformations, it yields the correct nonrelativistic scattering amplitude in the laboratory frame, *but only if one chooses the TT gauge for the graviton*. One is free to choose the photon gauge, as the t-pole term is invariant with respect to photon gauge transformations.

When ${}^g\omega \ll M$, the source of the electromagnetic background field is not appreciably affected by the incident graviton. This justifies the use of the external-field approximation[26] in the nonrelativistic limit. In this approximation the differential conversion cross section is given by

$$d\sigma = \frac{2\pi}{2{}^g\omega 2{}^\gamma\omega}\,|T|^2 D, \tag{3.13}$$

with

$${}^g\omega = {}^\gamma\omega \quad \text{and} \quad D = \frac{{}^\gamma\omega^2}{(2\pi)^3}\,d\Omega. \tag{3.14}$$

The transition amplitude to be used in (3.13) is given by (2.12d), where one of the photon polarizations that must be contracted with it stands for the 3-dimensional Fourier transform of the Coulomb potential

$$A_\mu = \frac{e}{4\pi r}\,\eta_{\mu 0}, \tag{3.15}$$

i.e.,

$$\sigma_\mu \equiv \mathfrak{F}\{A_\mu\} = \frac{e}{q^2}\,\eta_{\mu 0}. \tag{3.16}$$

Here q is pure spacelike (no recoil of the scatterer). It is readily checked that the external-field approximation leads to the nonrelativistic cross section (3.12).

From now on we shall restrict our attention to this more realistic case of nonrelativistic scattering (unless otherwise stated). The relativistic (R) cross sections can be obtained from the nonrelativistic cross sections by multiplication by the appropriate factor

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm R} = \left(\frac{{}^\gamma\omega}{{}^g\omega}\right)^2 \left(\frac{d\sigma}{d\Omega}\right)_{\rm NR} = \left[\frac{1}{1+(2{}^g\omega/M)\sin^2(\frac{1}{2}\theta)}\right]^2 \left(\frac{d\sigma}{d\Omega}\right)_{\rm NR}. \tag{3.17}$$

We choose now for the basis states of the incident graviton and the outgoing photon the circular polarizations

$$\bar{\epsilon}_R = \tfrac{1}{2}[\hat{e}_x\hat{e}_x - \hat{e}_y\hat{e}_y + i(\hat{e}_x\hat{e}_y + \hat{e}_y\hat{e}_x)],$$
$$\bar{\epsilon}_L = \tfrac{1}{2}[\hat{e}_x\hat{e}_x - \hat{e}_y\hat{e}_y - i(\hat{e}_x\hat{e}_y + \hat{e}_y\hat{e}_x)], \tag{3.18a}$$

$$\vec{\epsilon}_R = \frac{1}{\sqrt{2}}\,(\hat{e}_\theta + i\hat{e}_\phi),$$
$$\vec{\epsilon}_L = \frac{1}{\sqrt{2}}\,(\hat{e}_\theta - i\hat{e}_\phi), \tag{3.18b}$$

where $\hat{e}_x$, $\hat{e}_y$, $\hat{e}_\theta$, and $\hat{e}_\phi$ are the unit vectors in the x, y, θ, and ϕ directions. Substituting (3.18) into (3.12), we find

$$\left(\frac{d\sigma}{d\Omega}\right)_{RL} = \left(\frac{d\sigma}{d\Omega}\right)_{LR} = \frac{e^2}{16\pi}\cot^2(\tfrac{1}{2}\theta)(1-\cos\theta)^2, \tag{3.19a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{RR} = \left(\frac{d\sigma}{d\Omega}\right)_{LL} = \frac{e^2}{16\pi}\cot^2(\tfrac{1}{2}\theta)(1+\cos\theta)^2, \tag{3.19b}$$

where the first (second) subscript denotes the graviton (photon) polarization. The cross section for converting circularly polarized gravitons into photons (of any polarization) is then

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \left(\frac{d\sigma}{d\Omega}\right)_L = \frac{e^2}{8\pi}\cot^2(\tfrac{1}{2}\theta)(1+\cos^2\theta). \tag{3.20}$$

For any angle $\theta \neq 0$ the outgoing electromagnetic radiation is not circularly polarized anymore, but elliptically polarized. In the forward direction however the outgoing photon has the same helicity as the incident graviton. We also note that there is no backscatter,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta=\pi} = 0. \tag{3.21}$$

It is worthwhile to compare these conversion cross sections with the Compton-scattering cross sections for photons and gravitons. The photon Compton-scattering cross section (in the nonrelativistic limit) is the familiar Thomson cross section

$$\frac{d\sigma}{d\Omega} = \tfrac{1}{2}\left(\frac{e^2}{4\pi M}\right)^2(1+\cos^2\theta). \tag{3.22}$$

Unlike Thomson scattering the conversion cross sections (3.19b) and (3.20) exhibit a Rutherford peak in the forward direction. This feature is entirely due to the t-pole term and is also present in the cross section for graviton scattering[22]

$$\frac{d\sigma}{d\Omega} = \frac{M^2}{\sin^4\frac{1}{2}\theta)}[\cos^8(\tfrac{1}{2}\theta) + \sin^8(\tfrac{1}{2}\theta)], \tag{3.23}$$

where M is the mass of the scatterer.

We turn now to linear polarizations. We choose for the graviton basis states

$$\bar{\epsilon}_+ = \frac{1}{\sqrt{2}}\,(\hat{e}_x\hat{e}_x - \hat{e}_y\hat{e}_y), \tag{3.24a}$$

$$\bar{\epsilon}_\times = \frac{1}{\sqrt{2}}\,(\hat{e}_x\hat{e}_y + \hat{e}_y\hat{e}_x). \tag{3.24b}$$

Substituting (3.24) into (3.12) and summing over the polarizations of the outgoing photon we find

$$\left(\frac{d\sigma}{d\Omega}\right)_+ = \frac{e^2}{4\pi}\cot^2(\tfrac{1}{2}\theta)(\sin^2 2\phi + \cos^2\theta\cos^2 2\phi), \tag{3.25a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\times} = \frac{e^2}{4\pi}\cot^2(\tfrac{1}{2}\theta)(\cos^2 2\phi + \cos^2\theta \sin^2 2\phi). \tag{3.25b}$$

From (3.19) it follows that the outgoing photon is also linearly polarized. For unpolarized incident waves we must average over ϕ, and we recover (3.20).

The differential cross sections can also be expressed in terms of the frequency ${}^{\gamma}\omega$ of the outgoing photon instead of in terms of the scattering angle θ. Using (3.8), we find, after integration over ϕ

$$\frac{d\sigma}{d^{\gamma}\omega} = \frac{e^2}{2^{g}\omega}\left(\frac{M}{^{g}\omega}\right)\left[\frac{^{g}\omega}{M}\left(\frac{^{g}\omega}{^{\gamma}\omega}-1\right)^{-1} - \frac{1}{2}\right] \times\left\{1+\left[1-\frac{M}{^{g}\omega}\left(\frac{^{g}\omega}{^{\gamma}\omega}-1\right)\right]^2\right\}, \tag{3.26}$$

where the range of ${}^{\gamma}\omega$ is

$$\frac{^{g}\omega}{1+2^{g}\omega/M} \le {}^{\gamma}\omega \le {}^{g}\omega. \tag{3.27}$$

The total conversion cross section obtained by integrating (3.20) diverges because of the long-range character of the Coulomb field. This divergence may be avoided by Debye shielding if the scattering takes place in a plasma. In the nonrelativistic limit the interaction of the gravitational wave with the fixed charge is now assumed to take place through a screened Coulomb potential

$$A_\mu = e\,\frac{\exp(-r/\lambda_D)}{4\pi r}\,\eta_{\mu 0}. \tag{3.28}$$

Here the Debye screening length λ_D is given in terms of the electron thermal velocity

$$v_{T_e} = \left(\frac{2kT_e}{M_e}\right)^{1/2}, \tag{3.29}$$

and the plasma frequency

$$\omega_{pe} = \left(\frac{Ne^2}{M_e}\right)^{1/2} \tag{3.30}$$

as

$$\lambda_D = \frac{v_{T_e}}{\sqrt{2}\,\omega_{pe}}. \tag{3.31}$$

The screened Coulomb potential has a spatial Fourier transform

$$\sigma_\mu = \frac{e}{\vec{q}^{\,2}+q_{sc}^{\,2}}\,\eta_{\mu 0}, \tag{3.32}$$

where $\vec{q}$ is the momentum of the spacelike photon mediating the Coulomb interaction and $q_{sc} = 1/\lambda_D$.

Using (3.32) and (2.12d) we find that when shielding occurs, (3.20) must be replaced by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpolarized}} = \left(\frac{d\sigma}{d\Omega}\right)_R = \left(\frac{d\sigma}{d\Omega}\right)_L = \frac{e^2}{32\pi}\,\frac{\sin^2\theta(1+\cos^2\theta)}{[\sin^2(\frac{1}{2}\theta)+(q_{sc}/2\omega)^2]^2}. \tag{3.33}$$

For linearly polarized incident gravitational waves the $\frac{1}{2}(1+\cos^2\theta)$ must be replaced by $(\sin^2 2\phi + \cos^2\theta\cos^2 2\phi)$ or $(\cos^2 2\phi + \cos^2\theta \sin^2 2\phi)$ for the + and × polarizations, respectively.

The total cross section now becomes finite and is given by

$$\sigma = \int_0^{\pi}\int_0^{2\pi}\frac{d\sigma}{d\Omega}\sin\theta\, d\theta\, d\phi = 2e^2[\ln(2\omega\lambda_D) - \tfrac{4}{3}]. \tag{3.34}$$

Note that this result is valid only for tenuous plasmas, i.e., for $\omega\lambda_D \gg 1$. For dense plasmas the electromagnetic index of refraction

$$n(\omega) = (1-\omega_{pe}^{\,2}/\omega^2)^{1/2} \tag{3.35}$$

will not allow the electromagnetic wave to travel with the same phase velocity as the gravitational wave, and therefore the conversion cross section will be reduced to a value considerably less than (3.34).[21]

If the scattering does not take place in a plasma but the incident gravitational wave front has a width D, the Rutherford forward scattering peak is again suppressed and (3.34) applies with the Debye screening length λ_D being replaced by the width D.

Finally, note that the formulas derived above for a point charge are also valid for a charge distribution confined to the coherence volume

$$V_c \ll \left(\frac{2\pi}{\omega}\right)^3. \tag{3.36}$$

All of the previous formulas applied to gravitational-electromagnetic wave conversion. The inverse process is also possible and is described by the Feynman diagrams in Fig. 3. Straightforward calculations similar to the ones above lead to the differential cross section for converting an electromagnetic wave with frequency ${}^{\gamma}\omega$, polarization $\vec{\epsilon}$ and propagation direction ${}^{\gamma}\hat{k}$ into a gravitational wave with frequency ${}^{g}\omega$ and polarization $\overline{\overline{e}}$:

$$\frac{d\sigma}{d\Omega} = \frac{e^2}{8\pi\sin^4(\frac{1}{2}\theta)}\left[\frac{1}{1+(2^{\gamma}\omega/M)\sin^2(\frac{1}{2}\theta)}\right]^2 \times|\overline{e}^{ij*}\,{}^{\gamma}\hat{k}_{(i}\epsilon_{j)}|^2 \tag{3.37}$$

(valid for all ${}^{\gamma}\omega/M$).

The cross sections for circularly polarized or unpolarized incident electromagnetic waves are the same as those for the corresponding inverse

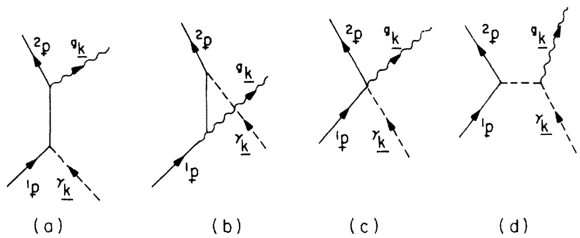


FIG. 3. Feynman graphs for exchange Compton scattering (photon → graviton) by a charged spin-0 meson.

conversion process. Owing to the different spin nature of the incident quanta, the cross sections for linearly polarized incident electromagnetic waves, however, show a different ϕ dependence than the corresponding inverse-process results. Specifically, for electromagnetic wave polarizations along the x and y axes we find (in the nonrelativistic approximation)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\hat{e}_x} = \frac{e^2}{4\pi}\cot^2(\tfrac{1}{2}\theta)(\sin^2\phi + \cos^2\theta\cos^2\phi), \tag{3.38a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\hat{e}_y} = \frac{e^2}{4\pi}\cot^2(\tfrac{1}{2}\theta)(\cos^2\phi + \cos^2\theta\sin^2\phi). \tag{3.38b}$$

IV. CONVERSION IN A MAGNETIC DIPOLE FIELD

We turn now to electrogravitational conversion with a magnetic dipole field acting as a catalyst and utilize the external-field approximation. The magnetic dipole field[27]

$$\vec{B} = \frac{3\vec{r}(\vec{m}\cdot\vec{r}) - \vec{m}(\vec{r}\cdot\vec{r})}{4\pi r^5} + \tfrac{2}{3}\vec{m}\delta^3(\vec{r}) \tag{4.1}$$

is obtained by applying (2.2) to the Maxwell 4-potential

$$A_0 = 0, \tag{4.2a}$$

$$A_j = \frac{(\vec{m}\times\vec{r})_j}{4\pi r^3}. \tag{4.2b}$$

Here $\vec{m}$ is the dipole moment. The Fourier transforms of the A_μ are given by

$$\sigma_0 = 0, \tag{4.3a}$$

$$\sigma_j = -i\,\frac{(\vec{m}\times\vec{q})_j}{q^2+\mu^2} \tag{4.3b}$$

Here $\vec{q}$ is the pure spacelike momentum transfer and μ is an arbitrarily small positive number, which can be thought of as originating from introducing a factor $\exp(-\mu r)$ in (4.2). Its meaning will be made clear below. We choose a pure spacelike photon polarization and the TT gauge for the graviton and use Eqs. (2.12d), (3.13), (3.14), and (4.3). The result for scattering a gravitational wave with polarization $\bar{\epsilon}$, frequency ω, and propagation direction ${}^g\hat{k}$ into an electromagnetic wave with polarization $\vec{\epsilon}$, frequency ω, and propagation direction ${}^r\hat{k}$ is

$$\frac{d\sigma}{d\Omega} = \frac{\omega^2 m^2}{8\pi[\sin^4(\frac{1}{2}\theta)+\eta]}\left|\bar{e}^{ij}\{{}^r\hat{k}_{(i}{}^r\hat{k}_{j)}\vec{\epsilon}^{*}\cdot[\hat{m}\times({}^r\hat{k}-{}^g\hat{k})] + \epsilon^{*}_{(i}[\hat{m}\times({}^r\hat{k}-{}^g\hat{k})]_{j)}(1-{}^r\hat{k}\cdot{}^g\hat{k}) + {}^r\hat{k}_{(i}\epsilon^{*}_{j)}{}^r\hat{k}\cdot(\hat{m}\times{}^g\hat{k}) + {}^r\hat{k}_{(i}[\hat{m}\times({}^r\hat{k}-{}^g\hat{k})]_{j)}{}^g\hat{k}\cdot\epsilon^{*}\}\right|^2. \tag{4.4}$$

Here $\hat{m}$ is a unit vector along the direction of the dipole moment and η is a small positive number, which is a function of μ.

We first consider circular polarizations and find from (4.4) after some algebraic manipulations

$$\left(\frac{d\sigma}{d\Omega}\right)_{RR} = \left(\frac{d\sigma}{d\Omega}\right)_{LL} = \frac{\omega^2 m^2\sin^4\theta}{64\pi[\sin^4(\frac{1}{2}\theta)+\eta]}\{[\cos\alpha\sin\theta + \sin\alpha(1-\cos\theta)\cos\phi]^2 + (2\sin\alpha\sin\phi)^2\}, \tag{4.5a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{RL} = \left(\frac{d\sigma}{d\Omega}\right)_{LR} = \frac{\omega^2 m^2(1-\cos\theta)^4}{64\pi[\sin^4(\frac{1}{2}\theta)+\eta]}\{[\cos\alpha\sin\theta + \sin\alpha(1-\cos\theta)\cos\phi]^2 + (2\sin\alpha\sin\phi)^2\}, \tag{4.5b}$$

where the first (second) subscript denotes the graviton (photon) polarization. The angles θ, ϕ, and α are defined in Fig. 4.

Summing over final polarizations, we obtain the conversion sections for circularly polarized (or unpolarized) waves

$$\left(\frac{d\sigma}{d\Omega}\right)_{R} = \left(\frac{d\sigma}{d\Omega}\right)_{L} = \frac{\omega^2 m^2}{8\pi}\,\frac{\sin^4(\frac{1}{2}\theta)(1+\cos^2\theta)}{\sin^4(\frac{1}{2}\theta)+\eta}\{[\cos\alpha\sin\theta + \sin\alpha(1-\cos\theta)\cos\phi]^2 + (2\sin\alpha\sin\phi)^2\}. \tag{4.6}$$

Note that for any direction but the forward and backward, the outgoing wave will be elliptically polarized. Unlike the scattering by a charge, the conversion cross section does not vanish in the backward direction

$$(d\sigma/d\Omega)_{\theta=\pi} = \pi^{-1}\omega^2 m^2\sin^2\alpha. \tag{4.7}$$

The meaning of the arbitrarily small positive number η is now obvious. For any direction but the forward direction ($\theta\neq 0$), η may be omitted. However, for $\theta = 0$, the presence of η ensures that the cross sections converge to a unique limit (namely zero). If we had not introduced the factor $\exp(-\mu r)$, the cross sections in the forward direction would be nonzero and dependent on ϕ.

For linearly polarized incident gravitational waves one finds from (4.4), after summing over final polarizations,

$$\left(\frac{d\sigma}{d\Omega}\right)_+ = \frac{\omega^2 m^2}{4\pi}\,\frac{\sin^4(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)+\eta}\{[(\cos\alpha\sin\theta+\sin\alpha(1-\cos\theta)\cos\phi)\sin 2\phi-2\sin\alpha\sin\phi\cos 2\phi]^2$$
$$+\cos^2\theta[(\cos\alpha\sin\theta+\sin\alpha(1-\cos\theta)\cos\phi)\cos 2\phi+2\sin\alpha\sin\phi\sin 2\phi]^2\}, \qquad (4.8a)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_\times = \frac{\omega^2 m^2}{4\pi}\,\frac{\sin^4(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)+\eta}\{[(\cos\alpha\sin\theta+\sin\alpha(1-\cos\theta)\cos\phi)\cos 2\phi+2\sin\alpha\sin\phi\sin 2\phi]^2$$
$$+\cos^2\theta[(\cos\alpha\sin\theta+\sin\alpha(1-\cos\theta)\cos\phi)\sin 2\phi-2\sin\alpha\sin\phi\cos 2\phi]^2\}. \qquad (4.8b)$$

As can be seen from (4.5), the outgoing electromagnetic wave will still be linearly polarized.

An obvious feature of these cross sections is the absence of a Rutherford peak in the forward direction, a manifestation of the fact that the dipole field falls off faster than r^{-1}. This yields a finite total cross section,

$$\sigma = \tfrac{9}{5}\omega^2 m^2(1-\tfrac{7}{9}\cos^2\alpha). \qquad (4.9)$$

Electromagnetic-gravitional conversion is also described by (4.4) with the following substitutions:

$$\overline{\epsilon} \to \overline{\epsilon}^{\,*}, \qquad (4.10a)$$

$$\vec{\epsilon}^{\,*} \to \vec{\epsilon}. \qquad (4.10b)$$

For circularly polarized or unpolarized incident electromagnetic waves, the conversion cross sections are the same as the corresponding gravitational-electromagnetic cross sections. For linearly polarized incident electromagnetic waves, the cross sections exhibit a different ϕ dependence when contrasted with (4.8)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\hat{e}_x} = \frac{\omega^2 m^2}{4\pi}\,\frac{\sin^4(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)+\eta}(\sin^2\phi+\cos^2\theta\cos^2\phi)\{[\cos\alpha\,\sin\theta+\sin\alpha\,(1-\cos\theta)\cos\phi]^2+(2\sin\alpha\sin\phi)^2\}, \qquad (4.11a)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\hat{e}_y} = \frac{\omega^2 m^2}{4\pi}\,\frac{\sin^4(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)+\eta}(\cos^2\phi+\cos^2\theta\sin^2\phi)\{[\cos\alpha\,\sin\theta+\sin\alpha\,(1-\cos\theta)\cos\phi]^2+(2\sin\alpha\,\sin\phi)^2\}. \qquad (4.11b)$$

V. CONVERSION IN AN ELECTRIC DIPOLE FIELD

The electric dipole field[27]

$$\vec{E} = \frac{3\vec{r}(\vec{p}\cdot\vec{r})-\vec{p}(\vec{r}\cdot\vec{r})}{4\pi r^5} - \tfrac{1}{3}\vec{p}\delta^3(\vec{r}) \qquad (5.1)$$

may be obtained from the Maxwell 4-potential

$$A_0 = -\frac{\vec{p}\cdot\vec{r}}{4\pi r^3}, \qquad (5.2a)$$

$$A_j = 0. \qquad (5.2b)$$

The Fourier transform of A_μ is

$$\sigma_0 = i\frac{\vec{p}\cdot\vec{q}}{q^2+\mu^2}, \qquad (5.3a)$$

$$\sigma_j = 0. \qquad (5.3b)$$

Here $\vec{p}$ is the electric dipole moment. Again we choose a pure spacelike polarization for the photon and the TT gauge for the graviton. Using (5.3), (2.12d), (3.13), and (3.14), we find the differential cross section for converting a gravitational wave with polarization $\overline{\epsilon}$, frequency ω, and propagation direction ${}^g\hat{k}$ into an electromagnetic wave with polarization $\vec{\epsilon}$, frequency ω, and propagation direction ${}^\gamma\hat{k}$,

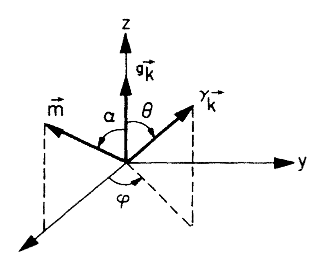


FIG. 4. The spatial orientation of the magnetic moment $\vec{m}$ and the direction ${}^\gamma\vec{k}$ of the outgoing photon relative to the direction ${}^g\vec{k}$ of the incident graviton.

$$\frac{d\sigma}{d\Omega}=\frac{\omega^2p^2}{8\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,|\bar{e}^{ij}\,{}^r\hat{k}_{(i}\epsilon^*_{j)}|^2\,. \tag{5.4}$$

Using the notations of the previous sections, we find that the cross sections for various polarizations are given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{RR}=\left(\frac{d\sigma}{d\Omega}\right)_{LL}=\frac{\omega^2p^2\sin^2\theta(1+\cos\theta)^2}{64\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,, \tag{5.5a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{RL}=\left(\frac{d\sigma}{d\Omega}\right)_{LR}=\frac{\omega^2p^2\sin^2\theta(1-\cos\theta)^2}{64\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2, \tag{5.5b}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{R}=\left(\frac{d\sigma}{d\Omega}\right)_{L}=\frac{\omega^2p^2\sin^2\theta(1+\cos^2\theta)}{32\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,, \tag{5.6}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{+}=\frac{\omega^2p^2\sin^2\theta(\sin^22\phi+\cos^2\theta\cos^22\phi)}{16\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,, \tag{5.7a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\times}=\frac{\omega^2p^2\sin^2\theta(\cos^22\phi+\cos^2\theta\sin^22\phi)}{16\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,. \tag{5.7b}$$

As before, electromagnetic-gravitational conversion is described by (5.4) modulo the substitutions

$$\bar{e}^{ij}\to\bar{e}^{ij*}\,, \tag{5.8a}$$

$$\epsilon^*_j\to\epsilon_j\,. \tag{5.8b}$$

Formulas (5.5) and (5.6) remain the same but (5.7) must be replaced by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\hat{e}_x}=\frac{\omega^2p^2\sin^2\theta(\sin^2\phi+\cos^2\theta\cos^2\phi)}{16\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,, \tag{5.9a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\hat{e}_y}=\frac{\omega^2p^2\sin^2\theta(\cos^2\phi+\cos^2\theta\sin^2\phi)}{16\pi[\sin^4(\frac{1}{2}\theta)+\eta]}[\sin\alpha\,\sin\theta\cos\phi-\cos\alpha(1-\cos\theta)]^2\,. \tag{5.9b}$$

Unlike scattering off a magnetic dipole, the conversion cross sections vanish in the backward direction. The total cross section is finite and is given by

$$\sigma=\tfrac{7}{15}\,\omega^2p^2(1-\tfrac{1}{7}\cos^2\alpha)\,. \tag{5.10}$$

VI. CONVERSION IN A UNIFORM MAGNETOSTATIC FIELD

We first study the inverse Gertsenshteĭn process,[12] i.e., gravitational-electromagnetic conversion in a homogeneous magnetostatic background. Consider a plane gravitational wave

$$\bar{h}^{\mu\alpha}=\bar{e}^{\mu\alpha}e^{i^g\mathbf{k}\cdot\mathbf{x}}\,, \tag{6.1}$$

propagating along the z axis and incident on a uniform magnetostatic field $\vec{B}$ (see Fig. 5). This magnetic background is confined to the region between the planes $z=-l/2$ and $z=l/2$ and makes an angle α with ${}^g\vec{k}$:

$$\vec{B}=B\,\mathrm{rect}\left(\frac{z}{l}\right)(\sin\alpha\,\hat{e}_x+\cos\alpha\,\hat{e}_z)\,, \tag{6.2}$$

where the *rectangle* function is defined by

$$\mathrm{rect}(x)=\begin{cases}1 & |x|\le\frac{1}{2}\\ 0 & \text{otherwise}\,.\end{cases} \tag{6.3}$$

In the TT gauge the conversion process is described by the 2-photon–graviton interaction func-

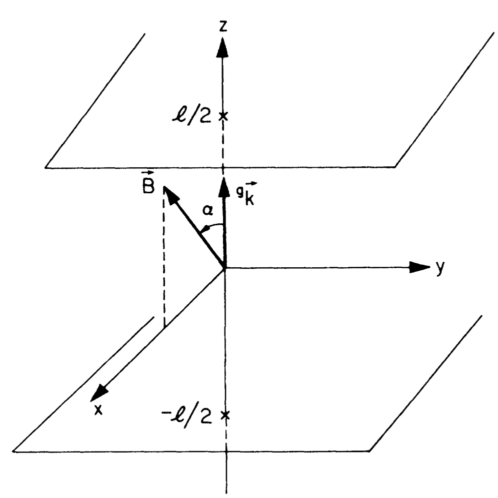


FIG. 5. The spatial orientation of the uniform magnetic background $\vec{B}$ relative to the direction ${}^g\vec{k}$ of the incident graviton.

tional [cf., Eq. (2.10c)]

$$\mathcal{S}_I = \lambda \int h^{ij}\eta^{\nu\beta}F_{i\nu}F_{j\beta}d^4x\,, \qquad (6.4)$$

where $F_{i\nu}$ and $F_{j\beta}$ stand both for the outgoing electromagnetic wave

$$F_{\mu\nu} = -i({}^{\gamma}k_\mu \epsilon_\nu^* - {}^{\gamma}k_\nu \epsilon_\mu^*)e^{-i{}^{\gamma}\underline{k}\cdot\underline{x}}\,, \qquad (6.5)$$

and the magnetic background

$$\begin{aligned} F_{0\nu} &= -F_{\nu 0} = 0\,,\\ F_{13} &= -F_{31} = 0,\\ F_{12} &= -F_{21}\\ &= (2\pi)^3 Bl\cos\alpha\\ &\quad\times\int \operatorname{sinc}\left(\frac{q_3 l}{2\pi}\right)\delta(q_1)\delta(q_2)\delta(q_0)\, e^{i\underline{q}\cdot\underline{x}}\frac{d^4q}{(2\pi)^4}\,, \qquad (6.6)\\ F_{23} &= -F_{32}\\ &= (2\pi)^3 Bl\sin\alpha\\ &\quad\times\int \operatorname{sinc}\left(\frac{q_3 l}{2\pi}\right)\delta(q_1)\delta(q_2)\delta(q_0)\, e^{i\underline{q}\cdot\underline{x}}\frac{d^4q}{(2\pi)^4}\,. \end{aligned}$$

The sinc function is defined by

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = j_0(\pi x)\,, \qquad (6.7)$$

where j_0 is the zero-order spherical Bessel function. For the electromagnetic wave we choose the pure spacelike gauge ($\epsilon_0 = 0$). From (6.4) we deduce the transition matrix element

$$\begin{aligned} T_{fi} &= i2(2\pi)^2\lambda[({}^{\gamma}\hat{k}\times\hat{B})\cdot\overline{\epsilon}\cdot\vec{\epsilon}^*]\\ &\quad\times Bl\operatorname{sinc}\left[\frac{({}^{\gamma}k_3 - {}^{g}k_3)l}{2\pi}\right]{}^{\gamma}k_3\\ &\quad\times\delta({}^{\gamma}k_1 - {}^{g}k_1)\delta({}^{\gamma}k_2 - {}^{g}k_2)\,. \qquad (6.8) \end{aligned}$$

The presence of δ functions in (6.8) means, among other things, that the electromagnetic wave is constrained to travel the $\pm z$ directions.

To obtain the transition probability per second we must square (6.8) and substitute into the "golden rule"

$$\frac{\text{transition probability}}{\text{sec}} = \frac{2\pi}{2{}^{\gamma}k^0 2{}^{g}k^0}|T_{fi}|^2 D\delta({}^{\gamma}k^0 - {}^{g}k^0)\,, \qquad (6.9)$$

where D is the density of final states

$$D = \frac{d^3\,{}^{\gamma}k}{(2\pi)^3}\,. \qquad (6.10)$$

This transition probability exhibits quadratic dependence on $\delta({}^{\gamma}k_1 - {}^{g}k_1)$ and $\delta({}^{\gamma}k_2 - {}^{g}k_2)$. Following the usual procedure,[28] we put

$$|\delta({}^{\gamma}k_1 - {}^{g}k_1)|^2 = \frac{L}{2\pi}\delta({}^{\gamma}k_1 - {}^{g}k_1)\,, \qquad (6.11)$$

$$|\delta({}^{\gamma}k_2 - {}^{g}k_2)|^2 = \frac{L}{2\pi}\delta({}^{\gamma}k_2 - {}^{g}k_2)\,, \qquad (6.12)$$

with L an arbitrarily large but finite length. Ill-defined mathematical expressions containing squares of δ functions can be avoided if one uses wave packets to represent the ingoing and outgoing waves. The infinities in $|T_{fi}|^2$ arise as a consequence of the infinite extent of the interaction region (infinitely wide wavefronts propagating in a magnetic background, which itself is infinitely extended in the transverse directions).

Therefore we calculate the transition rate per unit area

$$\begin{aligned} \Gamma &\equiv \left(\frac{\text{transition probability}}{\text{cm}^2\,\text{sec}}\right)\\ &= \frac{2\pi}{2{}^{\gamma}k^0 2{}^{g}k^0}\frac{|T_{fi}|^2}{L^2}D\delta({}^{\gamma}k^0 - {}^{g}k^0)\,, \qquad (6.13) \end{aligned}$$

and we obtain, after using (6.8), (6.10), (6.11), and (6.12)

$$\Gamma_{\text{forward}} = 8\pi B^2 l^2 \sum_{\vec{\epsilon}} |({}^{\gamma}\hat{k}\times\hat{B})\cdot\overline{\epsilon}\cdot\vec{\epsilon}^*|^2\,, \qquad (6.14\text{a})$$

$$\Gamma_{\text{backward}} = 8\pi B^2 l^2 \operatorname{sinc}^2\left(\frac{\omega l}{\pi}\right)\sum_{\vec{\epsilon}} |({}^{\gamma}\hat{k}\times\hat{B})\cdot\overline{\epsilon}\cdot\vec{\epsilon}^*|^2\,. \qquad (6.14\text{b})$$

Here $\sum_{\vec{\epsilon}}$ denotes summation over the final photon polarizations and $\omega \equiv {}^{g}k^0 = {}^{\gamma}k^0$. Evaluating

$$P = |({}^{\gamma}\hat{k}\times\hat{B})\cdot\overline{\epsilon}\cdot\vec{\epsilon}^*|^2$$

for different choices of initial and final polarizations we arrive at

$$P_{+\to\hat{e}_x} = P_{\times\to\hat{e}_y} = 0\,, \qquad (6.15\text{a})$$

$$P_{\times\to\hat{e}_x} = P_{+\to\hat{e}_y} = \tfrac{1}{2}\sin^2\alpha\,, \qquad (6.15\text{b})$$

$$P_{RL} = P_{LR} = 0\,, \qquad (6.15\text{c})$$

$$P_{RR} = P_{LL} = \tfrac{1}{2}\sin^2\alpha\,, \qquad (6.15\text{d})$$

i.e., linearly polarized gravitons generate linearly polarized photons, whereas circularly polarized gravitons generate circularly polarized photons with the same helicity.

Note that these transition probabilities have been computed for an incident number flux = 1 particle/cm^2 sec. It follows that

$$T^{03}_{\text{EMW}} = (\pm)\Gamma T^{03}_{\text{GW}} = \begin{pmatrix} +1 \\ -\operatorname{sinc}^2\dfrac{\omega l}{\pi}\end{pmatrix} 4\pi B^2\sin^2\alpha\; l^2 T^{03}_{\text{GW}}\,, \qquad (6.16)$$

where $T^{03}_{\rm EMW}$ and $T^{03}_{\rm GW}$ are the power flux of the electromagnetic wave and gravitational wave, respectively, and where the upper (lower) sign refers to forward (backward) outgoing electromagnetic radiation. The electromagnetic power flux in the backward direction is smaller by a factor $\mathrm{sinc}^2(\omega l/\pi)$ as compared to the flux in the forward direction and vanishes if the condition $l=n\lambda/2$ ($n=1,2,\ldots$; $\lambda=2\pi/\omega$) is met. The fact that the conversion efficiency Γ is quadratic in l depends critically on the equality of the propagation velocities of the electromagnetic wave and the gravitational wave. If we introduce a medium with a dielectric constant $\neq 1$, we destroy the coherence between the gravitational and electromagnetic perturbations and thereby put a limit on the useful length l. Note also that for propagation along the field lines of $\vec{\mathrm{B}}$ resonant conversion does not occur.

A magnetic field with finite transverse directions $\sim L (L \gg 2\pi/\omega)$ has a conversion cross section of the order

$$\begin{aligned}\sigma &\sim 4\pi B^2 L^2 l^2 \sin^2\alpha \\ &\sim 4\pi B^2 V t \sin^2\alpha ,\end{aligned} \tag{6.17}$$

where V is the volume of the magnetic field region and t is the travel time of the gravitational perturbation through the magnetostatic background. The propagation direction of the outgoing electromagnetic wave is not confined to only ($\pm$) the direction of the incident gravitational wave but can be within a solid angle centered about this direction of incidence.

If the magnetic background is chaotic with an ordered structure on some scale $l_c \gg 2\pi/\omega$ (l_c stands for correlation length), the electromagnetic waves generated in different cells are incoherent. One must therefore add their energies, and one obtains for the conversion efficiency

$$\Gamma \sim 2\pi \langle B^2\rangle l_c t , \tag{6.18}$$

where t is the time of passage of the gravitational wave through the magnetic background. For the Gertsenshtein process (electromagnetic-gravitational conversion) all of the formulas above apply, allowing the substituion $\vec{\epsilon}^{\,*} \to \vec{\epsilon}$, $\bar{\epsilon} \to \bar{\epsilon}^{*}$ in (6.8) and (6.14).

VII. CONVERSION IN A UNIFORM ELECTROSTATIC FIELD

Finally, turn to the Lupanov process[14] (and its inverse), i.e., gravitational-electromagnetic conversion (and vice versa) in a homogeneous electrostatic field. Choose the same geometrical configuration as in Fig. 5, with $\vec{\mathrm{B}}$ being replaced by $\vec{\mathrm{E}}$,

$$\vec{\mathrm{E}} = E\,\mathrm{rect}\left(\frac{z}{l}\right)(\sin\alpha\,\hat{e}_x + \cos\alpha\,\hat{e}_z). \tag{7.1}$$

The conversion processes are again described by (6.4) where the electrostatic background is now described by

$$F_{10} = -F_{01} = (2\pi)^3 E l \sin\alpha \times \int \mathrm{sinc}\left(\frac{q_3 l}{2\pi}\right)\delta(q_1)\delta(q_2)\delta(q_0)\, e^{i\underline{q}\cdot\underline{x}}\,\frac{d^4q}{(2\pi)^4}, \tag{7.2}$$

$$F_{30} = -F_{03} = (2\pi)^3 E l \cos\alpha \times \int \mathrm{sinc}\left(\frac{q_3 l}{2\pi}\right)\delta(q_1)\delta(q_2)\delta(q_0)\, e^{i\underline{q}\cdot\underline{x}}\,\frac{d^4q}{(2\pi)^4},$$

all other $F_{\mu\nu}=0$.

The transition amplitude is given by (6.8) with B being replaced by E. The conversion efficiencies and cross sections of Sec. VI are applicable if we substitute B by E.

VIII. CONCLUSIONS AND COMPARISONS WITH PREVIOUS RESULTS

We have computed electrogravitational conversion cross sections using Feynman perturbation methods for various electromagnetic backgrounds of possible astrophysical and/or experimental interest. For reasons of ease and straightforwardness, a quantum approach has been used to calculate a process which is classical in itself (the conversion efficiencies do not depend on $\hbar$).

For the exchange Compton scattering, various authors have obtained conflicting results. Papini and Valluri,[20] and Matzner[6] have obtained finite total cross sections. Our results confirm the findings of Ginzburg and Tsytovich,[21] who exploited the formal analogy with electromagnetic transition radiation and obtained exactly the nonrelativistic limit of our (nonintegrable) differential cross section. The divergence is avoided only after either introducing Debye screening or by limiting the spatial extent of the incident wavefronts. Boughn[20] also arrived at a divergent cross section in the form of a multipole series. The quadrupole term in this series is the most important one but the higher-multipole terms do not fall off fast enough to ensure convergence of the series. For this reason one may not limit oneself to quadrupole waves in computing the total cross section as Matzner does.

It must be stressed that we have calculated a *gauge-invariant* transition matrix element. We have also shown that in the nonrelativistic regime ($\omega \ll M$) one can still obtain the correct transition matrix element by limiting one's attention to the

t-pole diagram, *if one chooses the TT gauge for the gravitational wave*. This is what Ginzburg and Tsytovich, and Boughn have done. If one were to choose a non-TT gauge for the gravitational wave, the t-pole term becomes (in the nonrelativistic limit)

$$T=\frac{2\lambda e}{({}^{\gamma}\vec{k}-{}^{g}\vec{k})^2}\Big[\omega\bar{e}^{ij\gamma}k_{(i}\epsilon^*_{j)}-\omega\bar{e}^{ijg}k_{(i}\epsilon^*_{j)}+\tfrac{1}{2}\omega\bar{e}^{g}\,\vec{k}\cdot\vec{\epsilon}^{\,*} -\tfrac{1}{2}\bar{e}^{i0}\epsilon^*_i({}^{\gamma}\vec{k}-{}^{g}\vec{k})^2-\bar{e}^{\mu 0\gamma}k_{\mu}\,{}^{g}\vec{k}\cdot\vec{\epsilon}^{\,*}\Big]\,. \tag{8.1}$$

For notations see Sec. III. The transition matrix element (8.1) was calculated for a pure spacelike photon gauge. (The t-pole term is independent of the photon gauge.) In the TT gauge only the first term in (8.1) survives. Note that for a non-TT gauge the backscatter is nonzero,

$$T_{({}^{\gamma}\vec{k}=-{}^{g}\vec{k})}=-2\lambda\bar{e}^{j0}\epsilon^*_j\,. \tag{8.2}$$

If we choose to calculate in the TT gauge, however, we find $T_{({}^{\gamma}\vec{k}=-{}^{g}\vec{k})}=0$. This glaringly illustrates the ambiguities we must face if we calculate a transition matrix element which is not gauge invariant. The best we can hope for is that for an appropriate choice of gauge, the effect of the omitted diagrams is negligible. The gauge to choose for this problem is the TT gauge.

Finally note that we have studied exchange Compton scattering only for spinless particles. For spin-$\frac{1}{2}$ fermions the calculations are similar but more complicated owing to the extra spin degrees of freedom. In the nonrelativistic limit, however, the results for scalar particles are valid for spin-$\frac{1}{2}$ fermions as well.

Conversion scattering in the field of dipoles has received attention from Ginzburg and Tsytovich, and Papini and Valluri. Ginzburg and Tsytovich give differential cross sections that are integrated over ϕ. Our differential cross sections for an electric dipole, when integrated over ϕ, agree with the results of Ginzburg and Tsytovich.[30] For magnetic dipoles, however, Ginzburg and Tsytovich find the same results as for electric dipoles, whereas our results are different. This is because they do not use the correct field for a magnetic dipole.[31]

The Gertsenshteĭn and Lupanov resonant processes (and their inverses) have been analyzed rigorously by Boccaletti *et al.*[17] For electromagnetic-gravitational conversion our results are identical with theirs: Only the transverse components of the background field contribute to conversion, the converted wave propagates only in the same or in the opposite direction of the incident wave, the converted wave propagating in the backward direction is weaker than the converted wave propagating forwards and may be absent completely, and the conversion efficiency depends quadratically on the travel time of the perturbation through the background. We also confirm their their numerical correction to Gertsenshteĭn's original results.

There is some disagreement with the results of Boccaletti *et al.* for gravitational-electromagnetic conversion in a homogeneous background. These authors find a backward-travelling electromagnetic wave if the incident gravitational wave propagates *along* the field lines of the background. This erroneous feature (which destroys the electromagnetic-gravitational symmetry) is due to their choice of a gravitational gauge which is not TT. If one chooses to use the TT gauge, the method used by Boccaletti *et al.* reproduces our results.

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[1]Notable exceptions are the "classic" tests of general relativity theory: light bending and quasar radio-wave bending near the sun, gravitational red-shift in the earth's gravitational field, and Shapiro time delay of radar signals. [See any textbook on the general theory of relativity, e.g., C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).] Whereas all these tests probe the geometric-optics limit of electromagnetic-gravitational coupling, the conversion process goes beyond geometric optics.

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[30]There seems to be a typographical error in their equation (19). The term $\frac{1}{2}\sin^2\theta_0(1+\cos^2\theta)$ should be $\frac{1}{2}\sin^2\theta_0(1+\cos\theta)^2$. This does *not* change the total cross section.

[31]Ginzburg and Tsytovich start from the magnetic *scalar* potential $\sigma^0 = -i\,\vec{q}\cdot\vec{m}/q^2$, from which they find the magnetic field $\vec{B} = -\vec{q}\,\vec{q}\cdot\vec{m}/q^2$. This is *not* the Fourier transform of the correct expression (4.1) of the magnetic dipole field, but rather of

$$\vec{B} = \frac{3\hat{r}\,(\vec{m}\cdot\hat{r}) - \vec{m}(\hat{r}\cdot\hat{r})}{4\pi r^5} - \tfrac{1}{3}\vec{m}\,\delta^3(\vec{r}).$$

As the current density does not vanish everywhere, one should really use the magnetic *vector* potential $\vec{\sigma} = -i\,\vec{m}\times\vec{q}/q^2$. From this one finds

$$\vec{B} = \frac{\vec{q}\times(\vec{m}\times\vec{q})}{q^2} = -\vec{q}\,\frac{\vec{q}\cdot\vec{m}}{q^2} + \vec{m},$$

which is the Fourier transform of (4.1). The term $\vec{m}$ (which was neglected by Ginzburg and Tsytovich) is responsible for the different behavior of magnetic and electric dipoles.

[32]Although we will not address the question of renormalization in this paper, it should be pointed out that minimal substitution does not lead to finite higher-order corrections, not is it possible to make these corrections finite by techniques analogous to those employed in quantum electrodynamics. This nonrenormalizability does cast a shadow of uncertainty on results derived from the minimally coupled Lagrangian. However, we feel the close agreement of results obtained in this paper with various classically obtained results is reassuring.