Nuclear bonding by quark exchange

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The nuclear force is attributed to the exchange of quarks, analogous to the exchange of electrons in the hydrogen molecule.

I. INTRODUCTION

The force which binds nucleons together has been determined empirically from scattering experiments. Many people have attempted to account for this force in terms of meson exchange. The trouble with this approach is that a number of mesons are required, plus direct-channel effects. The phenomenon cries out for a simpler, more fundamental interpretation.

Nucleons are believed to be made of quarks. In the following we will show that the exchange of quarks, analogous to the exchange of electrons in the hydrogen molecule, leads to a nuclear force with just the right characteristics. Our model is based on two assumptions:

1. A nucleon is bound state of a quark and a diquark, which is two quarks weakly bound in an S state. This hypothesis is consistent with the observed baryon spectrum.[1]
2. The quark and diquark potentials are

$$V_{qq}=\frac{g^2}{r}(1-r^2/r_0^2)-\frac{g^2}{2m^2c^2r}(\vec{p}_1\cdot\vec{p}_2+\vec{p}_1\cdot\hat{r}\vec{p}_2\cdot\hat{r})-\frac{g^2}{m^2c^2}\pi\delta(\vec{r})(\hbar^2+\tfrac{8}{3}\vec{S}_1\cdot\vec{S}_2)$$
$$-\frac{g^2}{m^2c^2r^3}(3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{S}_2)-\frac{3g^2}{4m^2c^2r^3}(\vec{S}_1+\vec{S}_2)\cdot\vec{r}\times(\vec{p}_2-\vec{p}_1), \tag{1}$$

$$V_{dd}=\frac{g^2}{R}(1-R^2/r_0^2)-\frac{g^2}{8m^2c^2R}(\vec{p}_a\cdot\vec{p}_b+\vec{p}_a\cdot\hat{R}\vec{p}_b\cdot\hat{R})-\frac{g^2}{4m^2c^2}\pi\delta(\vec{R})(4\hbar^2+\tfrac{8}{3}\vec{S}_a\cdot\vec{S}_b)$$
$$-\frac{g^2}{4m^2c^2R^3}(3\vec{S}_a\cdot\hat{R}\vec{S}_b\cdot\hat{R}-\vec{S}_a\cdot\vec{S}_b)-\frac{3g^2}{16m^2c^2R^3}(\vec{S}_a+\vec{S}_b)\cdot\vec{R}\times(\vec{p}_b-\vec{p}_a), \tag{2}$$

$$V_{qd}=-\frac{g^2}{r_{1a}}(1-r_{1a}^2/r_0^2)+\frac{g^2}{4m^2c^2r_{1a}}(\vec{p}_1\cdot\vec{p}_a+\vec{p}_1\cdot\hat{r}_{1a}\vec{p}_a\hat{r}_{1a})+\frac{g^2}{2m^2c^2}\pi\delta(\vec{r}_{1a})(2\hbar^2+\tfrac{8}{3}\vec{S}_1\cdot\vec{S}_a)$$
$$+\frac{g^2}{2m^2c^2r_{1a}^3}(3\vec{S}_1\cdot\hat{r}_{1a}\vec{S}_a\cdot\hat{r}_{1a}-\vec{S}_1\cdot\vec{S}_a)+\frac{3g^2}{8m^2c^2r_{1a}^3}(\vec{S}_1+\vec{S}_a)\cdot\vec{r}_{1a}\times(\vec{p}_1-\vec{p}_a), \tag{3}$$

$$V_{q\bar{q}}=-V_{qq}. \tag{4}$$

Our $q\bar{q}$ potential is identical to that used to compute the meson mass spectrum,[2] where the following values were obtained for the quark mass and the gluon coupling constants:

$$m=340\ \text{MeV}/c^2,\quad g^2/\hbar c=0.42,\quad r_0=0.27\ \text{F}. \tag{5}$$

At short distances the potential is Coulombic, while at large distances there is a constant attractive force serving to keep the quarks permanently bound. The relativistic terms (those proportional to c^{-2})—the Breit term, the Darwin term, the spin-spin term, the tensor term, and the LS term—are characteristic of spin-$\frac{1}{2}$ particles.[3] The relativistic terms in the diquark potentials were obtained by summing the interactions of its constituent quarks.

The gluon coupling constants which we postulate are *not* those predicted by the color theory of quark binding. In that theory the couplings are proportional to

$$g_{12}^2\propto\tfrac{1}{2}(C^2-C_1^2-C_2^2), \tag{6}$$

where C^2 is the Casimir operator of color SU(3). Only color-singlet states of red, green, and blue quarks are realized in nature. Since the quark is a color triplet, the diquark must be a color $\bar{3}$. Within a nucleon, quarks attract:

$$g_{qd}^2=2g_{qq}^2=g_{q\bar{q}}^2\equiv-g^2. \tag{7}$$

However, the quarks in different nucleons have

no color interaction:

$$3 \otimes 3 = \bar{3} \oplus 6, \tag{8}$$

$$g_{qq}{}^2 \propto 3(-\tfrac{2}{3}) + 6(\tfrac{1}{3}) = 0. \tag{9}$$

Each point within a nucleon is, on the average, neutral in color. There will still be repulsion at short distances due to the kinetic energy of the quarks, which rises as the nucleons come closer together. (This result was previously obtained by a much less transparent argument.[4])

It should be clear that there can be no color attraction between nucleons so long as color symmetry is preserved. Let us break the symmetry by making the red and green quarks more attracted to each other than to the blue quark. Within a nucleon, then, the diquark will always be yellow (a mixture of red and green) and the lone quark will always be blue. This means that, on the average, nucleons are yellow on the inside and blue on the outside—just as atoms are positive on the inside and negative on the outside. Bonding is possible because blue and yellow attract. There can be no bonding in the color-symmetric theory because nucleons would be white (neutral) throughout.

II. CALCULATIONS AND RESULTS

A system of two nucleons is like a hydrogen molecule, with the quarks as the electrons and the diquarks as the nuclei. Assuming, for simplicity, that the diquarks are stationary ($p_a = p_b = 0$), let us make a variational calculation of the bonding energy using a Heitler-London wave function for the quarks,

$$\psi(r_{1a})\psi(r_{2b}) \pm \psi(r_{1b})\psi(r_{2a}), \tag{10}$$

where

$$\psi(r) = (\alpha^3/\pi)^{1/2} e^{-\alpha r} \tag{11}$$

is a hydrogenic wave function, and α is a variational parameter. In analogy with atomic physics it is convenient to work in hadronic units, where

$$\hbar = m = g = 1, \quad 1/c = 0.42. \tag{12}$$

Energy and length are now expressed in terms of

$$\text{hadronic Hartree} = mg^4/\hbar^2 = 60 \text{ MeV}, \tag{13}$$

$$\text{Bohr radius} = \hbar^2/mg^2 = 1.37 \text{ F}. \tag{14}$$

Using the wave function (10), the potentials (1)–(3), and the quark-kinetic-energy terms,

$$\text{KE} = \frac{p_1^2}{2m} - \frac{p_1^4}{8m^2c^2} + \frac{p_2^2}{2m} - \frac{p_2^4}{8m^2c^2}, \tag{15}$$

we obtain the nuclear potential (in hadronic units)

$$\begin{aligned} V_{NN}(R) = {} & -\frac{\alpha^4}{4c^2}(1 \pm X^2)^{-1}[5 \pm (X^2 + 4K + 4K_2)] + \frac{\alpha^3}{c^2}(1 \pm X^2)^{-1}(J'_B + K'_B) + \frac{\alpha^3}{c^2}(1 \pm X^2)^{-1}(2 + 2J_D \pm 4K_D + J'_D \pm K'_D) \\ & + \frac{8}{3}\frac{\alpha^3}{c^2}(1 \pm X^2)^{-1}[\langle \vec{S}_1 \cdot \vec{S}_a \rangle + \langle \vec{S}_1 \cdot \vec{S}_b \rangle (J_D \pm 2K_D) + \langle \vec{S}_1 \cdot \vec{S}_2 \rangle (J'_D \pm K'_D)] + \alpha^2(1 \pm X^2)^{-1}[1 \mp (X^2 + 2K)] \\ & + \alpha(1 \pm X^2)^{-1}(-2 + 2J \pm 4K + J' \pm K') + R^{-1} + (\alpha r_0{}^2)^{-1}(1 \pm X^2)^{-1}(3 + 2J_L \pm 4K_L + J'_L \pm K'_L) - r_0{}^{-2}R \\ & + \frac{\alpha^3}{c^2}(1 \pm X^2)^{-1}[\langle 3\vec{S}_1 \cdot \hat{R}\vec{S}_b \cdot \hat{R} - \vec{S}_1 \cdot \vec{S}_b \rangle (J_T \pm 2K_T) + \langle 3\vec{S}_1 \cdot \hat{R}\vec{S}_2 \cdot \hat{R} - \vec{S}_1 \cdot \vec{S}_2 \rangle (J'_T \pm K'_T)] \\ & - (4c^2R^3)^{-1}\langle 3\vec{S}_a \cdot \hat{R}\vec{S}_b \cdot \hat{R} - \vec{S}_a \cdot \vec{S}_b \rangle, \end{aligned} \tag{16}$$

where X, J, K, etc., are molecular integrals listed in the Appendix.

In order to get (16) into the form

$$V_{NN} = V_c + V_T\langle 3\vec{\sigma}_1 \cdot \hat{R}\vec{\sigma}_2 \cdot \hat{R} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle, \tag{17}$$

involving the Pauli spin matrices of the nucleons, we write

$$\langle \vec{S}_1 \cdot \vec{S}_a \rangle = -\tfrac{1}{2}, \tag{18}$$

$$\langle \vec{S}_1 \cdot \vec{S}_2 \rangle = \tfrac{1}{2}\langle \vec{S}_1 \cdot \vec{S}_b \rangle = \tfrac{1}{36}\langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle, \tag{19}$$

$$\begin{aligned} \langle 3\vec{S}_1 \cdot \hat{R}\vec{S}_2 \cdot \hat{R} - \vec{S}_1 \cdot \vec{S}_2 \rangle &= \tfrac{1}{2}\langle 3\vec{S}_1 \cdot \hat{R}\vec{S}_b \cdot \hat{R} - \vec{S}_1 \cdot \vec{S}_b \rangle \\ &= \tfrac{1}{4}\langle 3\vec{S}_a \cdot \hat{R}\vec{S}_b \cdot \hat{R} - \vec{S}_a \cdot \vec{S}_b \rangle \\ &= \tfrac{1}{36}\langle 3\vec{\sigma}_1 \cdot \hat{R}\vec{\sigma}_2 \cdot \hat{R} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle, \end{aligned} \tag{20}$$

which follow from regarding a quark as $\frac{1}{3}$ of a nucleon.

The exchange symmetry of the nucleons is also possessed by the quarks. If the nucleons are posi-

tive (negative) under spatial exchange, we choose the positive (negative) sign in the wave function (10), and the upper (lower) sign in the nuclear potential (16).

To compute the nuclear potential, the nonrelativistic part of the potential was minimized with respect to the variational parameter α. The results are compared in Fig. 1 with the Reid soft-core potentials.[5] The parameter $r_0 = 0.27$ F gives a potential which is too deep and relativistic, so we have plotted only the results obtained with $r_0 = 2.7$ F and ∞.

The relativistic terms are sometimes larger than the nonrelativistic terms. This is most evident in the V_c^- potentials, where below 1 F the relativistic terms are very negative and oppose the Coulomb repulsion. Whenever the relativistic correction to the kinetic energy becomes large, we replace Eq. (15) by

$$\mathrm{KE} = 2mc^2\left[\left(1+\frac{\langle p_1^2+p_2^2\rangle}{2m^2c^2}\right)^{1/2}-1\right] \tag{21}$$

after minimizing with respect to α.

In Fig. 1 we have adjusted the origin so that $V_c(\infty)=0$. The value of $V_c(\infty)$ tells us the binding energy of a quark and a diquark to form a nucleon. The binding energies obtained are (negative means bound)

-33 MeV for $r_0=\infty$,

-19 MeV for $r_0=2.7$ F,

850 MeV for $r_0=0.27$ F.

The overall qualitative features, the spin and parity dependences, are correctly represented. Many improvements are needed: better wave functions, a relativistic treatment, the contribution of diquark exchange, corrections for diquark motion, and overlap of extended diquarks. We defer these formidable undertakings to a future paper.

III. CONCLUSION

Our crude calculations have produced qualitatively correct results, and some fresh insights. For example, the repulsive core at short distances, which has traditionally been credited to vector-meson exchanges, can now be interpreted as a manifestation of diquark-diquark and quark-quark repulsion. Although much better calculations are needed to decide on the details of the theory, there appears to be considerable merit in the idea that nuclear forces are chemical forces.

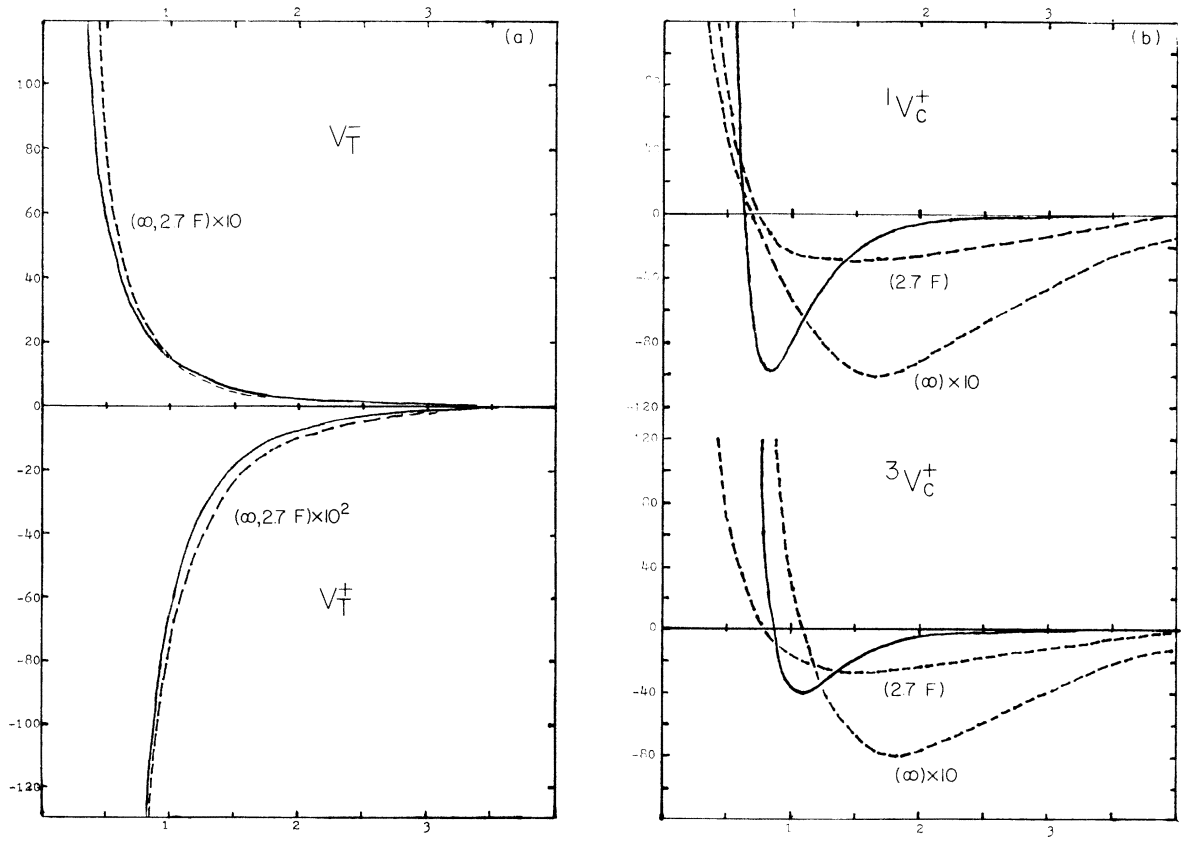

FIG. 1. (*Continued on following page*)

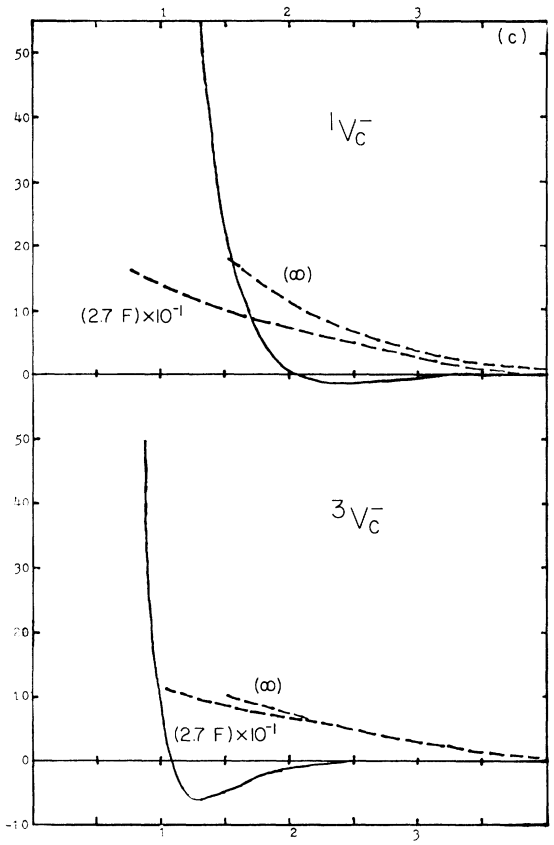

FIG. 1. The static nuclear potentials (in MeV) versus the internucleon separation (in fermis). The solid curves are the Reid soft-core potentials (Ref. 5). The dashed curves are the quark-model results calculated using $r_0=\infty$ and 2.7 F.

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APPENDIX

Listed here are the molecular integrals used in Eq. (16). They were evaluated using methods described in Refs. 6 and 7. Defining $w \equiv \alpha R$, we write

$$J' = \alpha^{-1}\langle ab|r^{-1}|ab\rangle = \int d^3r_1 d^3r_2 \psi^*(r_{1a})\psi^*(r_{2b})(\alpha r)^{-1}\psi(r_{1a})\psi(r_{2b}) = w^{-1} - e^{-2w}(w^{-1}+\tfrac{11}{8}+\tfrac{3}{4}w+\tfrac{1}{6}w^2), \quad \text{(A1)}$$

$$K' = \alpha^{-1}\langle ba|r^{-1}|ab\rangle = \int d^3r_1 d^3r_2 \psi^*(r_{1b})\psi^*(r_{2a})(\alpha r)^{-1}\psi(r_{1a})\psi(r_{2b}) = w^5(\tfrac{1}{18}W_{00}^{00} - \tfrac{1}{3}W_{00}^{20} + \tfrac{1}{2}W_{00}^{22} + \tfrac{2}{45}W_{20}^{00}), \quad \text{(A2)}$$

$$X = (\langle ba|ab\rangle)^{1/2} = e^{-w}(1+w+\tfrac{1}{3}w^2), \quad \text{(A3)}$$

$$J = \alpha^{-1}\langle ab|-r_{1b}^{-1}|ab\rangle = e^{-2w}(w^{-1}+1) - w^{-1}, \quad \text{(A4)}$$

$$K = \alpha^{-1}\langle ba|-r_{1b}^{-1}|ab\rangle = -Xe^{-w}(1+w), \quad \text{(A5)}$$

$$J_L = \alpha\langle ab|r_{1b}|ab\rangle = w^{-1} + w - e^{-2w}(w^{-1}+\tfrac{1}{2}), \quad \text{(A6)}$$

$$K_L = \alpha\langle ba|r_{1b}|ab\rangle = \tfrac{3}{2}Xe^{-w}(1+w+\tfrac{4}{9}w^2+\tfrac{1}{9}w^3), \quad \text{(A7)}$$

$$J_L' = \alpha\langle ab|-r|ab\rangle = e^{-2w}(2/w+\tfrac{29}{16}+\tfrac{5}{8}w+\tfrac{1}{3}w^2) - 2/w - w, \quad \text{(A8)}$$

$$K_L' = \alpha\langle ba|-r|ab\rangle = w^7[\tfrac{1}{45}W_{00}^{00} - \tfrac{7}{30}W_{00}^{20} + \tfrac{1}{6}W_{00}^{40} - \tfrac{3}{5}W_{00}^{31} + \tfrac{1}{2}W_{00}^{22} - \tfrac{1}{2}W_{00}^{42} + \tfrac{3}{50}W_{10}^{11} - \tfrac{1}{5}W_{10}^{31} + \tfrac{1}{6}W_{10}^{33} + \tfrac{2}{315}W_{20}^{00} + \tfrac{2}{175}W_{30}^{11} + \tfrac{1}{300}W_{11}^{00} - \tfrac{1}{30}W_{11}^{20} + \tfrac{1}{12}W_{11}^{22} + \tfrac{1}{1575}W_{31}^{00}], \quad \text{(A9)}$$

$$K_2 = \alpha^{-2}\langle ba|r_{1b}^{-2}|ab\rangle = 2X(2wf_{01} - e^{-w}), \tag{A10}$$

$$J_D = \alpha^{-3}\langle ab|\pi\delta(\vec{r}_{1b})|ab\rangle = e^{-2w}, \tag{A11}$$

$$K_D = \alpha^{-3}\langle ba|\pi\delta(\vec{r}_{1b})|ab\rangle = Xe^{-w}, \tag{A12}$$

$$J'_D = \alpha^{-3}\langle ab|-\pi\delta(\vec{r})|ab\rangle = -e^{-2w}(\tfrac{1}{8}+\tfrac{1}{4}w+\tfrac{1}{6}w^2), \tag{A13}$$

$$K'_D = \alpha^{-3}\langle ba|-\pi\delta(\vec{r})|ab\rangle = J'_D, \tag{A14}$$

$$J_T = \alpha^{-3}\langle ab|\tfrac{1}{2}[3(\hat{r}_{1b}\cdot\hat{R})^2-1]r_{1b}^{-3}|ab\rangle = w^{-3}[1-e^{-2w}(1+2w+2w^2)], \tag{A15}$$

$$K_T = \alpha^{-3}\langle ba|\tfrac{1}{2}[3(\hat{r}_{1b}\cdot\hat{R})^2-1]r_{1b}^{-3}|ab\rangle = 6X[e^{-w}(w^{-2}+w^{-1})-f_{02}+\tfrac{1}{3}f_{00}], \tag{A16}$$

$$J'_T = \alpha^{-3}\langle ab|-\tfrac{1}{2}[3(\hat{r}\cdot\hat{R})^2-1]r^{-3}|ab\rangle \approx -J_T, \tag{A17}$$

$$K'_T = \alpha^{-3}\langle ba|-\tfrac{1}{2}[3(\hat{r}\cdot\hat{R})^2-1]r^{-3}|ab\rangle \approx -K_T, \tag{A18}$$

$$J'_B = \alpha^{-3}\langle ab|-(2r)^{-1}(\vec{p}_1\cdot\vec{p}_2+\vec{p}_1\cdot\hat{r}\vec{p}_2\cdot\hat{r})|ab\rangle \approx 0, \tag{A19}$$

$$K'_B = \alpha^{-3}\langle ba|-(2r)^{-1}(\vec{p}_1\cdot\vec{p}_2+\vec{p}_1\cdot\hat{r}\vec{p}_2\cdot\hat{r})|ab\rangle \approx 0. \tag{A20}$$

The functions

$$W_{lm}^{n\bar{n}}(w) = \int_1^\infty d\xi_1 \int_1^\infty d\xi_2 \exp[-w(\xi_1+\xi_2)]\xi_1^n\xi_2^{\bar{n}} \times (\xi_1^2-1)^{m/2}(\xi_2^2-1)^{m/2} \times Q_l^m(\xi_>)P_l^m(\xi_<), \tag{A21}$$

$$f_{ln}(w) = \int_1^\infty d\xi\, e^{-w\xi}Q_l(\xi)\xi^n \tag{A22}$$

are tabulated in Ref. 7.

The Breit term has been evaluated for the hydrogen molecule,[8] where it was found to be much smaller than the other relativistic terms. The tensor integrals J'_T and K'_T are very difficult to evaluate in closed form, so we have approximated them. In the future we will evaluate these integrals numerically.

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