Conditions for natural suppression of $\mathbb{C}P$ violation*

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We study the conditions for the natural suppression of CP violation at the orders G_F and $G_F\alpha$ of quark–gauge-boson couplings in the $SU(2) \times U(1)$ gauge-theory framework. For the natural suppression of CP at the order G_F , the necessary and sufficient condition is that the quarks of the charges q and $q-1$ do not belong to the same isomultiplets for at least one chirality. However, it is not generally possible to naturally suppress CP at the order $G_F\alpha$ except for the limited class of models such as the standard fourquark model.

L INTRODUCTION

A few extremely small numbers have appeared in elementary-particle physics. They are the K_{L} - K_s mass difference, the $K_L \rightarrow \mu \overline{\mu}$ branching ratio, upper bounds for the branching ratios of μ – $e\gamma$ and $\mu + ee^+e^-$ decays, and CP-violating parameters in $K_L \rightarrow 2\pi$ decay. Do we understand why all these parameters are so small? The small or vanishing experimental numbers can be understood in two ways: by an artificial mechanism or by a natural mechanism.¹ The artificial mechanism implies that there is no alternative choice except one particular set of parameters in the theory to produce experimentally vanishing numbers. On the other hand, the natural mechanism implies vanishing experimental numbers however one may choose the parameters in the theory. Since the artificial mechanism is trivial, we pose the question to the *natural* scheme.

For the natural conservation of strangeness and muon number, the Glashow-Weinberg theorem should be satisfied.¹ But there does not exist an equally useful study for a naturally small CP violation. Ever since the discovery of CP nonconserequally useful study for a naturally small CP vio-
lation. Ever since the discovery of CP nonconser-
vation,² the mystery has not been why it is there,^{3,4} but why it is so weakly violated.⁵ The old idea for the feeble CP violation with an imaginary coupling constant⁶ cannot be incorporated within a gauge theory. If we stay in the standard four-quark scheme of Glashow, Iliopoulos, and Maiani' (GIM), we can introduce ingenious schemes through the spontaneous CP violation⁸ or CP -violating Higgs Lagrangian.⁵ However, they do not solve the basic problem of the smallness of the violation in extended quark models. For example, the six-quark scheme of Kobayashi and Maskawa' introduces a complex parameter in W -boson-quark couplings, but there is no α *priori* reason that this parameter should be small.

At present, there is evidence that we need more quarks than the four quarks of CIM. The most serious phenomenological need arises when we try to fit the heavy lepton τ^- in the scheme. In this possible circumstance of extending quark and lepton models from the minimal four leptons and four quarks, we ask the following; Is it possible to naturally suppress CP violation in gauge-boson-quark couplings? We study this problem essentially in the perturbative treatment. Suppose that in a theory we have the maximum Suppose that in a theory we have the maximum
number of CP -violating parameters.¹⁰ This is equivalent to the maximum number of unremovable imaginary phases through phase redefinition of quark fields. For example, the charged current

$$
J_{\mu}^{\dagger} = \overline{\mathcal{U}}_{L} \gamma_{\mu} U d_{L} \tag{1}
$$

defines a unitary matrix U in the flavor basis (in the flavor basis, the mass matrix is diagonal). Here U is $n \times n$ and u and d are $n \times 1$ column vectors of quarks of charge $\frac{2}{3}$ and $-\frac{1}{3}$, respectively As usual, we identify $d_1 = d$, $d_2 = s$, $u_1 = u$, and u_2 $=c$. Then we study the following problem: How many columns of U can be made real while maintaining the maximum number of independent parameters in U by the phase transformation $V^{\dagger}UW$, where V and W are diagonal unitary? In this specific example (1) , we can make only one column of U be real for $n \geq 3$, but cannot make two or more columns real. (For $n=2$ we can make both of the columns real.) This means we can remove any $\mathcal{C}P$ violation from the order G_F but should include it at the order $G_F\alpha$. Suppose we could make two columns of U be real. Then we can remove CP violation up to the order $G_F\alpha$ since we can redefine the quark field phases such that any desired two columns (or two rows) are real. In this case, the K_L-K_S mass difference which occurs at the one- loop level does not have any imaginary com-

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ponent and CP violation in $K_L+2\pi$ through gaugeboson coupling is expected to occur at the level of ($\alpha m_q^4 / \pi M_w^4$)G_F α , i.e., η_r is order of $\alpha m_q^2 / M_w^2$
We assume an SU(2) × U(1) gauge theory.¹¹ In

We assume an $\text{SU}(2) \times \text{U}(1)$ gauge theory. 11 In Sec. II, we show that the quarks of charge q and $q-1$ do not belong to the same weak isomultiplets for at least one chirality for the natural CP conservation at the order G_F . This is equivalent to the condition for the microweak CP violation of Lee.¹⁰ condition for the microweak CP violation of Lee.¹⁰ Then we show that it is not generally possible to have natural CP conservation at the order $G_{\mu}\alpha$. In Sec. III, we present a few models which can have natural CP conservation in the K_L-K_S mass matrix up to the one-loop level. None of these models can successfully interpret the high-
anomaly.¹² anomaly.

II. CONDITION FOR NATURAL SUPPRESSION OF CP VIOLATION

In this section we assume an $SU(2) \times U(1)$ gauge In this section we assume an $SU(2) \times U(1)$ gauge theory,¹¹ and define the quark fields in the weakinteraction basis

$$
\xi_L, \quad \xi_R \tag{2a}
$$

and the flavor basis

$$
q_L, \quad q_R \tag{2b}
$$

for each chirality. If necessary, q is decomposed to charge eigenstates

$$
q = \{ \ldots, f, u, d, v, \ldots \}, \qquad (3)
$$

where f, u, d, v , have $Q = \frac{5}{3}, \frac{2}{3}, -\frac{1}{3}$, and $-\frac{4}{3}$, respectively. In the weak-interaction basis gaugeboson coupling is diagonal and in the flavor basis the mass matrix is diagonal. The weak-interaction basis and the flavor basis are related by unitary transformations,

$$
q_L = V_L \xi_L, q_R = V_R \xi_R.
$$
 (4)

The mass matrix

$$
\overline{\xi}_{L}M\xi_{R} + \overline{\xi}_{R}M^{\dagger}\xi_{L}
$$
 (5)

is diagonalized by V_L and V_R ,

$$
\overline{q}M_{D}q=\overline{q}_{L}M_{D}q_{R}+\overline{q}_{R}M_{D}q_{L}\;, \eqno{(6)}
$$

where

$$
M_{D} = V_{L} M V_{R}^{\dagger} \tag{7}
$$

Equation (6) can be decomposed to

$$
\overline{q}M_{D}q = \overline{u}M_{u}u + \overline{d}M_{d}d + \cdots , \qquad (8)
$$

where M_u and M_d are diagonal. The gauge-boson coupling is defined in the weak-interaction basis as

$$
\mathcal{L} = g J_{W}^{\mu} W_{\mu}^{*} + (g^{2} + g'^{2})^{1/2} J_{Z}^{\mu} Z_{\mu} , \qquad (9)
$$

where

$$
J_{\mu}^{W} = \overline{\xi}_{L} \gamma_{\mu} T_{+}^{L} \xi_{L} + \overline{\xi}_{R} \gamma_{\mu} T_{+}^{R} \xi_{R}
$$
 (10)

and

$$
J_{\mu}^{Z} = \overline{\xi}_{L} \gamma_{\mu} (T_{3}^{L} - \sin^{2} \theta_{W} Q^{L}) \xi_{L}
$$

$$
+ \overline{\xi}_{R} \gamma_{\mu} (T_{3}^{R} - \sin^{2} \theta_{W} Q^{R}) \xi_{R} , \qquad (11)
$$

where

$$
Q = Q^L \frac{1 - \gamma_5}{2} + Q^R \frac{1 + \gamma_5}{2} . \tag{12}
$$

Since $[Q, M] = 0$, we have¹³

$$
[Q^L, V_L] = [Q^R, V_R] = 0 , \qquad (13)
$$

which guarantees the flavor conservation in $Q^{L,R}$ terms in J_{μ}^{Z} . For the natural conservation of flavors in neutral currents, $T_3^{L,R}$ should be the multiple of identity for the equally charged quarks, and Z_{μ} coupling to quarks conserves CP . If we require flavor conservation only between d quarks, the Z_{μ} coupling can violate CP but the K_L-K_S mass difference picks up an imaginary component only at the two-loop level of Feynman diagrams. Hence, the contribution to apossibly large CP violation in K_L + 2π can come only from the charged-current coupling (10). This charged current (10) can be decomposed into components,

$$
J_{\mu}^{W} = \overline{u}_{L} \gamma_{\mu} U^{L} d_{L} + \overline{u}_{R} \gamma_{\mu} U^{R} d_{R} + \overline{d}_{L} \gamma_{\mu} X^{L} v_{L}
$$

+
$$
\overline{d}_{R} \gamma_{\mu} X^{R} v_{R} + \overline{f}_{L} \gamma_{\mu} Y^{L} u_{L} + \overline{f}_{R} \gamma_{\mu} Y^{R} u_{R}
$$

+
$$
\cdots , \qquad (14)
$$

where $U^{L,R}$, $X^{L,R}$, $Y^{L,R}$, etc. are unitary matrices and $u_{L,R}$, $d_{L,R}$, etc. are column vectors with quark fields.

Now we investigate systematically the conditions for CP conservation at the order G_{κ} and the order $G_{\mathbf{F}}\alpha$. For this we also assume that the quark mass matrix is completely arbitrary.

A. Natural CP conservation at the order G_F

Any unitary transformation $A^{L,R}$, $B^{L,R}$, etc. which is needed for making a particular element of $U^{L,R}$, $X^{L,R}$, etc. real should leave the diagonal quark mass matrix M_p unchanged,

$$
\overline{q}M_{D}q = \overline{u}_{L}M_{u}u_{R} + \overline{u}_{R}M_{u}u_{L} + \overline{d}_{L}M_{d}d_{R} + \overline{d}_{R}M_{d}d_{L} + \cdots
$$

Namely,

$$
M_u = A^{L^{\dagger}} M_u A^R = A^{R^{\dagger}} M_u A^L ,
$$

\n
$$
M_d = B^{L^{\dagger}} M_d B^R = B^{R^{\dagger}} M_d B^L ,
$$
\n(15)

 $(8')$

²)^{1/2}J^u_Z μ _u, (9) etc., where $A^{L,R}$ and $B^{L,R}$ redefine the quark fields

 $u_{L,R}$ and $d_{L,R}$, respectively. Since M is completel arbitrary, $A^{L,R}$, $B^{L,R}$, etc. are diagonal unitar (i.e., only the phase redefinition of quark fields is allowed) and left- and right-unitary matrices are equal,

$$
A^L = A^R, \quad B^L = B^R, \quad \text{etc.} \tag{16}
$$

As an illustration, let us take U^L and U^R in Eq. (14). By phase transformations $A^{L,R}$ and $B^{L,R}$, we want, without loss of generality, to make the first elements of $U^{L,R}$ be real,

$$
(A^{L^{\dagger}} U^{L} B^{L})_{11}
$$
 = real ,
\n $(A^{R^{\dagger}} U^{R} B^{R})_{11}$ = $(A^{L^{\dagger}} U^{R} B^{L})_{11}$ = real ,

which is equivalent to

$$
A_{11}^{L^*} U_{11}^L B_{11}^L = \text{real} \,,\tag{17a}
$$

$$
A_{11}^{L*} U_{11}^R B_{11}^L = \text{real} \tag{17b}
$$

Equations (17a) and (17b) imply that

phase of
$$
U_{11}^L
$$
 = phase of U_{11}^R = phase of $A_{11}^L B_{11}^{L^*}$, (18)

which *cannot* be generally satisfied for arbitrary U^L and U^R . Hence, one of U^L and U^R is not allowed for natural CP conservation at the order g. The physical implication of Eqs. (17a) and (17b) is that we can redefine quark field phases such that the interaction $\overline{u}\gamma_{\mu}(a+b\gamma_{5})dW^{\mu}$ conserves CP. Therefore, the decay W^+ - $u\bar{d}$ conserves CP. This argument ean be applied to any final state of the quark and antiquark combinations, and Eqs. (17a) and (17b) guarantee the CP conservation at the order g in the decays W^* + $u_i \overline{d}_j$. Further, we note that if one of U^R or U^L were forbidden, we could make any one row (or column) of U^L of U^R real by appropriate phase matrices A^L and B^L , i.e.,

phase of $U_{i_1}^L$ = phase of $A_{i_1}^L B_{i_1}^L$ ^{*}

if U^R were forbidden.

Therefore, we obtain a necessary condition for the natural CP conservation at the order G_F : The quarks of charge q and $q-1$ do not belong to the same weak isomultiplets for at least one chirality. This is also a *sufficient* condition since we can make any one element real for any one unitary matrix by appropriate phase transformations similar to Eq. (17). Further, if the above condition is satisfied, the one- W -boson-exchange diagrams, e.g.,

$$
u_i\overline{d}_j + W^+ + u_k\overline{d}_l
$$
 or $f_k\overline{u}_l$ or $d_k\overline{v}_l$, etc.,

can contain at most the overall phase factors which cannot be tested experimentally. Namely, the above condition is necessary and sufficient for the natural conservation of CP at the

order g and $G_{\boldsymbol{F}}$.

If this condition for natural CP conservation at the order $G_{\mathbf{r}}$ is satisfied, it is easy to see that the eleetrie dipole moment of quarks cannot arise in the second order of g since the electromagnetic coupling of fermions conserve CP and flavor. The electric dipole moment of the quark in this theory is estimated by Lee¹⁰ to be of the order of 10^{-27} e cm for $m_w \approx 60$ GeV and $m_q \approx 3$ GeV. (Here we have not considered the factor $\epsilon \approx 10^{-3}$ of Ref. 10.)

We note that the vector models¹⁴ do not have natural G_F -order CP conservation. But the Kobayashi and Maskawa six-quark model⁹ conserves CP at the order G_F .

B. Condition for natural CP conservation at the order $G_F\alpha$

The feeble CP violation observed in $K_L \rightarrow 2\pi$ decay, viz., $\eta_{12} \approx \eta_{00} \approx 2 \times 10^{-3}$, suggests that CP is also suppressed at the one-loop level of the $s+s-d+d$ amplitude due to the fact that the CP-conserving K_t-K_s mass difference is consistent with the oneloop calculation.¹⁵ Since we can use the freedom of redefining quark field phases, the necessary and sufficient condition for the natural CP conservation at the order $G_{\vec{r}}\alpha$ is the existence of two unitary diagonal matrices, say A and B , which removes all the complex numbers from two columns (or two rows) from an arbitrary unitary matrix U in addition to the condition for the natural $G_{\mathbf{r}}$ -order CP conservation. (This kind of argument can be extended to the natural CP conservation at higher orders.) Note that if this condition on the order $G_F\alpha$ is satisfied, the K_L-K_S mass difference does not get an imaginary part up to $G_F \alpha (m_a^2/M_w^2)$ due to the GIM suppression since the dominant contribution to the mass difference comes from the oneloop calculation which is known to be suppressed by quark mass terms. Without loss of generality, let us transform U^L by phase matrices A^L and B^L such that the first two columns are real, i.e.,

$$
A_{jj}^{L^*} U_{j1}^L B_{11}^L = \text{real} \,,\tag{19a}
$$

$$
A_{jj}^{L^*} U_{j2}^L B_{22}^L = \text{real} \tag{19b}
$$

 $(j$ is not summed) namely,

phase of
$$
U_{j1}^L B_{11}^L
$$
 = phase of $U_{j2}^L B_{22}^L$ (20)

and the phase of A_{jj}^L is determined from (19a) or (19b).

For an arbitrary $n \times n$ unitary matrix U^L , there are *n* equations corresponding to (20), i e., $j=1$, \ldots , *n*, to solve for two parameters B_{11}^L and B_{22}^L . Therefore, for $n \geq 3$, there do not exist phase matrices A^L and B^L such that (19a) and (19b) are true. On the other hand, if $n = 1$ or 2, CP is conserved to all orders in gauge-boson-quark couplings. This is the original observation of Kobayashi and Maskawa.

Since it is hard to observe CP violation outside the neutral K system, we may relax the condition on natural CP conservation at the one-loop level in other channels, which will be the case for the second and third models in Sec. III.

At this point, it is worthwhile to examine the consequences of the flavor-changing neutral currents. In this case the matrix U^L in Eqs. (19a) and (19b) is $m \times n$, where $m \neq n$. Corresponding to Eqs. (19a) and (19b), there are $2m$ equations for the phase, and we have *m* adjustable phases in A_{ij}^L and two adjustable phases in B_{11}^L and B_{22}^L . Therefore, the case $m \leq 2$ admits solutions without restricting the value of n . However, the case m =2< n which corresponds to two Q = $\frac{2}{3}$ quarks and n Q = $-\frac{1}{3}$ quarks presents other disastrous problem such as the ones in $Q = -\frac{1}{3}$ neutral currents which violate both flavors and CP.

llL MODELS

In this section we shall give several examples of natural $G_{\mathbf{r}}\alpha$ -order CP conservation in d-quarkgauge-boson couplings. This ensures the reality of the K_L-K_S mass matrix up to the one-loop level.

The first example is the four-quark model of $GIM₁$

$$
\begin{pmatrix} u' \\ d \end{pmatrix}_{L}, \begin{pmatrix} c' \\ s \end{pmatrix}_{L}, \begin{pmatrix} u_R & c_R \\ d_R & s_R \end{pmatrix}, \tag{21}
$$

where

$$
\begin{pmatrix} u' \\ c' \end{pmatrix}_L = U^{L^{\dagger}} \begin{pmatrix} u \\ c \end{pmatrix} \tag{22}
$$

Any complex parameter in U^L can be removed by redefining quark field phases, and CP is conserved to all orders of gauge-boson couplings.

The second example is^{16}

$$
f_{1L}, f_{2L}, {f_1 \choose u''}_R, {f_2 \choose c''}_R,
$$

$$
{u' \choose d}_L, {c' \choose s}_L, d_R, s_R,
$$
 (23)

where

$$
\binom{u'}{c'}_L = U^L \binom{u}{c}_L \tag{24}
$$

and

$$
\binom{u''}{c''}_R = Y^R \binom{u}{c}_R \tag{25}
$$

In this model the $G_{\vec{r}}\alpha$ -order CP problem in the neutral K system depends only on the matrix U^L which can be made real following the arguments of Sec. II. The same conclusion can be drawn for Y^R . But we cannot make both U^L and Y^R real, simultaneously. The reason is that the phase freedom is restricted for the u quark field due to Eq. (16). For example, if we made Y^R real, the freedom for A^L is exhausted and only two parameters in U^L can be removed by B^L [refer to Eq. (19)], leaving one complex parameter. Hence, CP is conserved in $K_L+2\pi$ up to the order $G_F\alpha$ of the gauge-boson couplings and η_{\uparrow} from this source is expected to be of order of $(\alpha/\pi) m_a^2/M_{\rm w}^2$. (In this model other sources of CP violation such as the ones discussed in Refs. 5 and 8 play a more important role in $K_L \rightarrow 2\pi$ decay.) The same degree of CP conservation is expected for mesons, $(\overline{f}_1 f_2 \pm f_1 \overline{f}_2)/\sqrt{2}$. However, $D_{1,2} \equiv (u\overline{c} \pm \overline{u}c)/\sqrt{2}$ is expected to substantially violate CP unless it is suppressed by real or imaginary (or both) Cabibbo angles.

As the third example, we consider

$$
\begin{pmatrix} f_1' \\ u' \\ d' \end{pmatrix}, \begin{pmatrix} f_2' \\ c' \\ s \end{pmatrix}, \begin{matrix} f_{1R}, f_{2R}, \\ u_R, c_R, \\ f & g_R, s_R, \end{matrix}
$$
 (26)

The phenomenological implication of this model is similar to that of the second model. The three models have the same unfortunate fate for the models have the same unfortunate fate for the
high-y anomaly.¹² But this model is the simples
one to include the τ^* lepton in the scheme,¹⁷ one to include the τ^* lepton in the scheme,¹⁷

$$
\begin{pmatrix} \nu_e \\ e^- \\ E^+ \end{pmatrix} \begin{pmatrix} \nu_{\mu} \\ \mu^- \\ M^+ \end{pmatrix} \begin{pmatrix} \nu_{\tau} \\ \tau^- \\ T^+ \end{pmatrix} \begin{pmatrix} \nu_l \\ l^- \\ L^+ \end{pmatrix}, \text{ and } V + A \text{ singlets. (27)}
$$

We note that (26) and (27) do not produce the Adler
Bell-Jackiw anomaly.¹⁸ Bell-Jackiw anomaly.

IV. CONCLUSION

The natural suppression of CP violation in gaugeboson-quark couplings in the $SU(2) \times U(1)$ theory has been studied. We have obtained a necessary and sufficient condition for the order- G_F suppression of CP: The quarks of charge q and $q-1$ should not belong to the same weak isomultiplets for at least one chirality. However, we could not remove the CP phase at the order $G_{\mu} \alpha$ except for the special cases of Sec. III.

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