Spin test of $\eta'(958)$ from its collinear production and collinear decay

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(Received 7 June 1977)

Motivated by the spin assignment of the $\eta'(958)$, we propose a general spin-parity test for particles produced in a two-body reaction and decaying into three spinless particles. The test which analyzes the events at the limit of collinear production and collinear decay is crucial. Applied to the Amsterdam-CERN-Nijmegen-Oxford Collaboration data, it favors the 0^- assignment. Nevertheless, better statistics obtainable from existing data is highly desired.

A clean-cut test of the η' (958) spin and parity is of the utmost importance for theoretical schemes like mixing inside the SU(3) nonet or SU(4) decasextuplet, and determination of Regge trajectories—especially when other resonances, e.g., $E(1420)$, could be candidates for such schemes. The observation of the decay mode $\eta' \to \gamma\gamma$ excludes the spin-1 assignment.[1] Several analyses[2-4] of the Dalitz plot of the three-body decay modes $\eta' \to \pi^+\pi^-\eta$ and $\eta' \to \pi^+\pi^-\gamma$ favor the 0^- assignment, but they do not exclude the 2^- assignment and higher ones.[5] The angular distribution of the normal to the decay plane seemed to show anisotropies[6,7] when the events produced in the very forward direction are analyzed. But these anisotropies have not been confirmed.[8] This problematic situation was reflected at the Tbilisi conference.[9,10] Recently, the Amsterdam-CERN-Nijmegen-Oxford Collaboration[11] has performed a detailed multidimensional analysis taking into account the $\Lambda\eta'$ joint decay and concluded that "the data unambiguously perfer the 0^- assignment." In this paper we propose a simple crucial test which decides between the 0^- assignment and the 2^- assignment and higher ones. We apply the test to the same Amsterdam-CERN-Nijmegen-Oxford data[11] and arrive at a similar conclusion. We believe that the application of the test to all the existent data would solve the problem definitively.

For a particle state such as η' (958) produced in a two-body reaction

$$K^- p \to \eta' \Lambda \tag{1}$$

and undergoing a three-body decay,

$$\eta' \to \pi^+\pi^-\eta , \tag{2}$$

an especially interesting configuration arises at the limit of collinear production [forward direction in the reaction (1)] and collinear decay [boundary region in the Dalitz plot of the decay (2)]. At this limit the only significant variable[12] is the angle θ between the direction of the decay-three momenta and the direction of the production three-momenta as seen in the η' rest frame. As we shall see, the angular distribution in θ supplies a crucial test (a generalized Adair's[13] test) for the η' spin assignment.

When the reaction (1) is collinear, angular momentum and parity conservation implies that only two transition amplitudes are nonvanishing (one that does not flip and one that flips the helicity of the baryons). Since the initial state is unpolarized, the density matrix of η' can be written in any helicity quantization frame and for any spin assignment j in the following way:

$$\rho = \frac{1+\lambda}{2}\,\rho_0 + \frac{1-\lambda}{2}\,\rho_1 , \tag{3}$$

where ρ_0 and ρ_1 are the density matrices corresponding to the nonflip and flip helicity amplitudes

$$(\rho_0)_{mm'} = \delta_{0m}\,\delta_{0m'} , \tag{3a}$$

$$(\rho_1)_{mm'} = \tfrac{1}{2}\,(\delta_{1m}\delta_{1m'} + \delta_{-1m}\delta_{-1m'}) , \tag{3b}$$

and λ is the normalized difference of the squared moduli of the amplitudes,

$$-1 \le \lambda \le 1 . \tag{3c}$$

On the other hand, when the decay (2) is collinear, angular momentum conservation implies that only one invariant amplitude is nonvanishing, namely the amplitude with η' polarization zero in the decay direction. Furthermore, parity conservation implies that this amplitude vanishes unless the η' spin and the intrinsic parities of the involved particles satisfy

$$(-1)^j = \epsilon_{\eta'}\,\epsilon_\eta\,\epsilon_{\pi^+}\epsilon_{\pi^-} = -\epsilon_{\eta'} . \tag{4}$$

Thus, the transition amplitude $T_m(\theta,\varphi)$ which describes a collinear decay in the (θ,φ) direction of an η' state with polarization m is (for any point at the boundary of the Dalitz plot and related to any production frame) given by a spherical harmonic

$$T_m(\theta,\varphi) = Y_m^j(\theta,\varphi) , \tag{5}$$

which gives the convenient normalization

$$\int d(\cos\theta)d\varphi\ |\ T_m(\theta,\varphi)\ |^2 = 1\,. \qquad (6)$$

Therefore, the angular distribution for the mixed state described in (3) by the density matrix ρ will be

$$I(\theta,\varphi) = \mathrm{Tr}(T\otimes T^\dagger)\rho$$
$$= \frac{1+\lambda}{2}\ |Y_0^j(\theta,\varphi)|^2 + \frac{1-\lambda}{2}\ |\ Y_1^j(\theta,\varphi)\ |^2\,. \qquad (7)$$

In fact, it becomes independent of the azimuth φ. The corresponding nonvanishing momenta are those with even L and $M=0$, and they are given by

$$y^L \equiv \langle \sqrt{4\pi}\ Y_0^L\rangle$$
$$\equiv \int d(\cos\theta)\,d\varphi\, I(\theta,\varphi)\sqrt{4\pi}\ Y_0^L(\theta,\varphi)$$
$$= \frac{1+\lambda}{2}(2L+1)^{1/2}(2j+1)\begin{pmatrix} j & j & L \\ 0 & 0 & 0\end{pmatrix}^2$$
$$-\frac{1-\lambda}{2}(2L+1)^{1/2}(2j+1)\begin{pmatrix} j & j & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} j & j & L \\ 1 & -1 & 0\end{pmatrix}\,. \qquad (8)$$

For $L=2,4$ we obtain[14]

$$y^2 = \frac{1+\lambda}{2}\sqrt{5}\ \frac{(j+1)j}{(2j+3)(2j-1)} + \frac{1-\lambda}{2}\ \sqrt{5}\ \frac{(j+1)j-3}{(2j+3)(2j-1)}\,, \qquad (8a)$$

$$y^4 = \frac{1+\lambda}{2}\ \frac{27}{4}\ \frac{(j+2)(j+1)j(j-1)}{(2j+5)(2j+3)(2j-1)(2j-3)} + \frac{1-\lambda}{2}\ \frac{27}{4}\ \frac{(j+2)(j-1)[j(j+1)-10]}{(2j+5)(2j+3)(2j-1)(2j-3)}\,. \qquad (8b)$$

Following an anlogous spin test previously proposed,[15] we have drawn in Fig. 1 the plane (y^2, y^4) indicating the line segments $Tj^P - Tj^P$ theoretically allowed for spin-parity assignments $j^P = 2^-, 3^+, 4^-, \ldots, \infty$. The test is manifestly crucial, and it decides between these assignments[16] and the assignment $j^P = 0^-$ (here, of course, the two moments must vanish). The comparison shall be done inside the parachute-shaped boundary imposed by the positivity of the angular distribution in all directions when the higher moments are vanishing.

In order to estimate phenomenologically the limit of double collinearity, some criteria must be used. We propose the Gram determinants as collinearity parameters for production and decay, or, equivalently, the parameters

$$\Delta_p = -\Delta(\vec{p}_{K^-}{}^2, \vec{p}_p{}^2, \vec{p}_\Lambda{}^2)\,,\quad \Delta_d = -\Delta(\vec{p}_\eta{}^2, \vec{p}_{\pi^+}{}^2, \vec{p}_{\pi^-}{}^2)\,, \qquad (9)$$

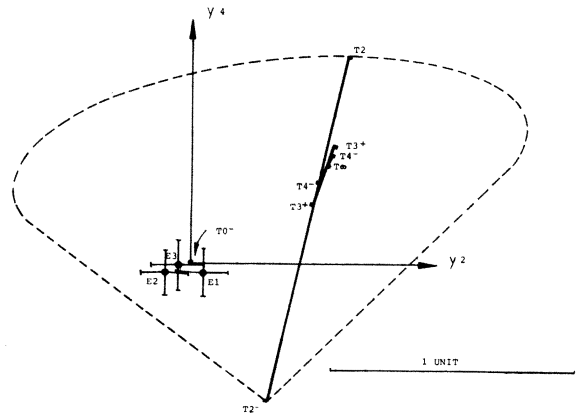

FIG. 1. Plot of the spin-parity test of a particle state collinearly produced in a two-body reaction and collinearly decaying into three pseudoscalars. The coordinates y^2 and y^4 are the moments defined in Eq. (8). The dashed parachute-shaped boundary is imposed by the positivity of the angular distribution. The origin $T0^-$ and the line segments $Tj^P - Tj^P$ correspond to spin-parity assignments $j^P = 0^-, 2^-, 3^+, 4^-, \cdots, \infty$. The points $E3$, $E2$, $E1$ show experimental values of the moments y^2, y^4 for three ensembles of events approach- the limit of double collinearity. The data are borrowed from Amsterdam-CERN-Nijmegen-Oxford Colloboration.

where the three-momenta are defined in the η' rest frame, and the triangularity function

$$-\Delta(a^2, b^2, c^2) = (a+b+c)(a+b-c)(a-b+c)(-a+b+c) \qquad (10)$$

is the square of four times the area of a triangle of sides a, b, c. If we want to relate the collinearity in production with the collinearity in decay for a limited sample of events, we can take a realistic criterion. For instance, we can associate the cuts in Δ_p and Δ_d which exclude separately the same number of events of the sample.

To perform the experimental test, we have used the data of the Amsterdam-CERN-Nijmegen-Oxford Collaboration[11] consisting of 916 events of $\eta'(958)$ resonance produced through reaction (1) at $p_{\rm lab} \sim 4.2$ GeV/c and undergoing the decay (2). Following the indicated criteria, the data have been separated in three mutually exclusive ensembles of ~100 events $E3, E2, E1$ approaching the limit of double collinearity. Using the values of the angle θ, we have computed the moments y^2 and y^4 with their standard errors. The precise definition of these ensembles and the values of the moments are given in Table I. These moments have been also plotted in Fig. 1. The limit of the experimental polarization points $E3, E2, E1$ seems to be compatible with the 0^- assignment and to ex-

TABLE I. Definition of the three ensembles of η' events approaching the limit of double collinearity, and experimental values of the corresponding moments. The collinearity parameters Δ_p and Δ_d and the moments y^L are defined in Eqs. (9) and (8), respectively.

Ensemble	Cut in Δ_p (GeV^4)	Cut in Δ_d ($GeV^4/10^4$)	Excluding	Number of events	y^2	y^4
$E1$	3.84	3.64	...	98	0.052 ± 0.100	-0.039 ± 0.097
$E2$	5.59	5.41	events in $E1$	109	-0.099 ± 0.090	-0.038 ± 0.091
$E3$	7.29	6.58	events in $E1$ and $E2$	94	-0.047 ± 0.010	-0.008 ± 0.101

clude the 2^- assignment and higher ones. Nevertheless, more restrictive cuts in collinearity are desired, and they could be done with reasonable statistics by reassembling existing data of similar experiments.[2,3,6-8] Notice that it does not matter that these experiments have been performed at different p_{lab}.

We are greatly indebted to R. Armenteros, M. Cerrada, and the whole Amsterdam-CERN-Nijmegen-Oxford team for permitting us to use their unpublished data and for their comments. We also acknowledge the discussions with A. Bramón, C. Pajares, R. Pascual, E. de Rafael, and E. Ugaz.

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[5]The observation of events of the boundary of the Dalitz plot excludes a series of spin-parity assignments. In this case, as we shall see [Eq. (4)], the natural series is excluded.

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[12]When the Λ decay is taken into account, another significant variable at the limit of double collinearity is the difference of aximuths of the two decay directions (related to the production direction). Two correlation moments could be measured, but they do not supply new crucial tests.

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[14]*Tables of the Clebsch-Gordan Coefficients* (The Institute of Atomic Energy, China, 1965).

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[16]The 1^+ assignment cannot be excluded because the $T1^+ - T1^+$ line segment includes the origin. In this case, for instance, it is ruled out by the $\gamma\gamma$ decay mode.