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Monte Carlo approach to multiparticle production in a quark-parton model

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We present a simple Monte Carlo quark-parton model based on the following intuitive picture: During a hadron-hadron collision gluons are first converted to $Q\bar{Q}$ pairs. Then a compound system is formed in which quarks and antiquarks are distributed according to longitudinal phase space modified by Kuti-Weisskopf weight factors, pressing valence quarks to higher values of momentum fractions x. Quark-antiquark pairs and QQQ and $\bar{Q}\bar{Q}\bar{Q}$ triplets (whose members are nearby in rapidity) form hadrons in the SU(6) 35-plet of mesons and 56-plets of baryons and antibaryons. Stable hadrons observed in the final state are to a large extent decay products of resonances. Results on multiplicities and inclusive spectra are compared with the data.

I. INTRODUCTION

The present picture of the nucleon is to a large extent based on the data from deep-inelastic lepton-nucleon scattering and on the quark-parton-model interpretation[1] of this data. The dynamics of multiparticle production is more complicated and less known. Still, if a coherent picture of the nucleon and its interactions is to be obtained, it will be necessary to understand also multiparticle production from the quark-parton-model point of view. This idea has already been followed in papers by Satz,[2] by Anisovich and Shekhter and their collaborators,[3,4,5] by Bjorken and Farrar,[6] by Montvay and collaborators,[7] and by Van Hove, Pokorski, and Białkowski.[8] Recently the version of Anisovich and Shekhter was corroborated by results of Morrison's group at CERN.[9] The situation was summarized in Chliapnikov's talk at the Tbilisi Conference.[10]

A great amount of data on multiparticle production has already been accumulated, ranging from detailed studies of some exclusive channels to inclusive characteristics like rapidity spectra, charge distributions and fluctuations, rapidity gaps, etc. It seems, however, that the dynamics of multiparticle production is so complicated that no part of the data, if taken separately, can lead to a deeper understanding of the process. In this situation it is, in our opinion, unavoidable to construct Monte Carlo models[11] which permit us to compare the results of a particular model with all the available experimental information.

The purpose of this paper is to describe a Monte Carlo model for multiparticle production based on the quark-parton picture of hadronic collisions, and to compare the model predictions with the simplest features of the data, like particle multiplicities and rapidity distributions. In the near future we are going to study correlations, rapidity gaps, energy dependence of multiplicities, and other subjects. In this way we hope to learn something about the mechanism of multiparticle production and in particular about the role which enigmatic gluons play in this process.

We assume that in the near future Monte Carlo approaches to multiparticle production will be frequently used and that the description of a particular Monte Carlo method may be of some interest. Because of that we shall show below in some detail how our model works.

The paper is organized as follows: Basic ideas and motivations are presented in the next section. A detailed description of the model is given in Sec. III with some technicalities shifted to the Appendix. Presently available results are compared with data in Sec. IV. Comments and conclusions are given in Sec. V.

II. BASIC IDEAS AND MOTIVATIONS

According to the parton-model picture[1] the two colliding hadrons are represented as coherent superpositions of valence quarks, $Q\bar{Q}$ pairs (the "sea"), and gluons. A collision initiated by the interaction of "wee" partons destroys at least partly the coherence and leads to the formation of a compound state. It is well known that the recombination of valence and sea quarks to hadrons cannot give the observed number of outgoing particles. In fact, using the data on deep-inelastic e^-p scattering, in particular

$$\nu W_2(x) = e_u^2 x[u(x)+\bar{u}(x)] + e_d^2 x[d(x)+\bar{d}(x)] + e_s^2 x[s(x)+\bar{s}(x)] \approx 0.3$$

for $x\to 0$, and assuming that near $x=0$ all the probability densities $u(x), \ldots, \bar{s}(x)$ are about the same, one finds that in the proton there are about 0.55 $Q\bar{Q}$ pairs per rapidity unit (RU). The "soft" interaction of wee partons can hardly cause a substantial change in the density of $Q\bar{Q}$ pairs the original sea. However, 0.55 $Q\overline{Q}$ pairs per RU are clearly insufficient to produce by recombination the observed number of final-state particles in a hadronic collision (about 3/RU).

This indicates that gluons play an essential role in all processes initiated by hadronic collisions. Various quark-parton models of multiparticle production differ mostly in (tacit or explicit) assumptions about the conversion of gluons to hadrons. The assumption that we are going to accept is inspired by the following consideration:

A simple estimate based for instance on the McElhaney-Tuan distribution functions[12] shows that gluons carry about 3.6 times more momentum than all partons (here and below parton means either Q or $\overline{Q}$) from the sea. If gluons were converted to $Q\overline{Q}$ pairs and if the distribution of these pairs were about the same as that of the original ones, the total density of $Q\overline{Q}$ pairs after the conversion of gluons would be about 2.5/RU. This is about as much as one needs to produce by recombination the observed number of particles. We shall therefore assume that during the first stage of the hadron-hadron collision gluons are converted to $Q\overline{Q}$ pairs and a compound system consisting of valence quarks and of partons coming from (sea and converted) $Q\overline{Q}$ pairs is formed.

In our approach we shall not try to construct a model for this stage of the hadronic collision, but we shall start from the rapidity distribution of partons in the compound state. In order to obtain such a distribution we shall consult the model of Kuti and Weisskopf.[13] In their approach the qualitative features of the nucleon structure functions have been well reproduced by a model where gluons and partons are distributed according to the longitudinal phase space, and where valence quarks receive additional factor $\sqrt{x}$, pushing them to higher values of momentum fraction x. The following assumption is therefore natural to accept:

The partons within the compound system formed during the first stage of a hadronic collision are distributed according to the longitudinal phase space multiplied by the following factors:

(i) G^n, where G is a "coupling constant" adjusting the mean number of partons pairs,

(ii) W_{id}, a factor for identical particles, and

(iii) $\prod_1^6 (|x_i|)^{1/2}$, where x_i is the c.m.-system momentum fraction of the ith valence quark.

The probability of finding six valence quarks with rapidities $y_1, y_2, \ldots, y_6$, n quarks with rapidities $y_7, y_8, \ldots, y_{n+6}$, and n antiquarks with rapidities $y_{n+7}, \ldots, y_N$ $(N=2n+6)$ is then given by the expression

$$dP_N(y_1, \ldots, y_N) = KW_{id}G^n \left[\prod_1^6 (|x_i|)^{1/2}\right] \times \delta\left(\sum_1^N p_i\right)\delta\left(E-\sum_1^N E_i\right)\prod_1^N dy_i, \tag{1}$$

where K is a constant independent of n, p_i is longitudinal momentum and E_i the energy of the ith parton, and E is the total energy of colliding particles.

The tendency of valence quarks to form leading particles is built in through the Kuti-Weisskopf factor $\sqrt{x}$. Distribution (1) reflects also the fact that the collision was initiated by a "soft" process. A hard collision of two quarks would lead to a compound state with characteristics depending on the momenta and quantum numbers of the two quarks. No such dependence is seen in (1). As follows from (1) the converted Q's and $\overline{Q}$'s "do not remember" their parent gluon. This assumption can be checked only by comparing the results with the data, and if necessary one should be prepared to introduce correlations between Q's and $\overline{Q}$'s in (1). We shall consider no such correlations here.

As seen further from (1) in the present version of the model we neglect the transverse momenta of partons in the compound state. This simplifying assumption has to be modified in the further development of the model. In our model we start with generating random configurations of partons satisfying the conservation laws. Each event is then assigned the weight following from (1).

In the next step we let neighboring (in rapidity) $Q\overline{Q}$'s and QQQ's recombine to (valence quarks of) mesons and baryons from SU(6) 35- and 56-plets. Most of them are resonant states. In this way the model naturally gives short-range correlations between particles of opposite charges (such particles frequently appear as decay products of resonances like $\rho^0 \to \pi^+\pi^-$). This origin of short-range correlations between particles of unlike charges may well be consistent with the data.[14]

Some other qualitative features of the data which may be perhaps interpreted as pointing to a quark-parton-model origin are, for instance, the inclusive spectra[15] (in particular a rapid decrease of K^-/π^- ratio with increasing x) and the behavior of charge distribution as function of rapidity in pp collisions, which are reminiscent of charge distributions giving by the parton distribution function.[16]

III. DESCRIPTION OF THE MODEL

In this section we present in some detail our model by describing the program used in the calculations.

The Monte Carlo program simulates the assumed mechanism of multiparticle production by random generation of events, and calculates the appropriate weight for each event. The required information is kept in memory for the final handling. The generation of an event proceeds in the following steps:

(i) Generation of the parton configuration. The program generates an equal number of events with a given number of $Q\overline{Q}$ pairs (n) between and $n=0$ and $n=n_{\max}$ ($n_{\max}$ has to be specified beforehand). The multiplicity distribution of partons is thus regulated by the weights of events. In an event containing, for instance, four $Q\overline{Q}$ pairs we start with the set (for the pp collision)

$$u_v u_v d_v\, Q\overline{Q}Q\overline{Q}Q\overline{Q}Q\overline{Q}\, u_v u_v d_v \ ,$$

where valence partons have the index v. Now we select at random quantum numbers of $Q\overline{Q}$ pairs. We have used probabilities $P_u = P_d = 0.439$, $P_s = 0.122$ for a $Q\overline{Q}$ pair to become $u\overline{u}$, $d\overline{d}$, or $s\overline{s}$, respectively. Similar phenomenological suppression factors for producing strange partons have been used in papers by the Leningrad group.[3,4,5] In this way we may get the set

$$u_v u_v d_v\, u\overline{u}\, d\overline{d}\, u\overline{u}\, s\overline{s}\, u_v u_v d_v \ . \tag{2}$$

Now the partons are randomly reordered with the supplementary constraint that the left (right) valence quarks remain in the left (right) half of the sequence. In this way we may get from (2)

$$u_v \overline{u}\, s\, d_v \overline{d}\, u\, u_v\, u\, d\, u_v\, \overline{s}\, \overline{u}\, d_v\, u_v \ . \tag{3}$$

(ii) Generation of parton rapidities. In the next step rapidities of partons are randomly generated with constraints given by the energy and longitudinal momentum conservation. In the present version of the program transverse momenta of partons are set equal to zero. The weight of the event due to the longitudinal phase space (LPS) is calculated. The calculation is based on Eq. (1) and is performed by using a part of Jadach's program GENRAP.[17] For the sake of completeness we give some details about this point in the Appendix.

For this calculation one has to specify masses of partons. We have used the following values:

$$m_u = m_d = 0.3\ \mathrm{GeV}/c^2, \quad m_s = 0.45\ \mathrm{GeV}/c^2 \ .$$

(iii) The factor for identical partons. In the present version of the program, spins of partons were not generated. As a consequence the weight factor due to the identity of partons can be estimated only in a very rough way. For an event containing n $u\overline{u}$ pairs we use the weight factor

$$W_{id}^{(u)} = \left(\frac{1}{n_1!}\right)^2 \frac{\sum_m \binom{n_1}{m}^4}{\sum_m \binom{n_1}{m}^2} \ , \tag{4}$$

and the same expression is used also for $d\overline{d}$ and $s\overline{s}$ pairs. This choice of the weight factor is motivated as follows: Let us suppose that in an event m u-quarks have spins up and (n_1-m) down. We assume that this is compensated by $\overline{u}$-quarks in such a way that m of them have spins down and (n_1-m) up. The weight of such a configuration is $[m!\,(n_1-m)!]^{-2}$. There are $\binom{n_1}{m}^2$ configurations of this type. Averaging over m we get Eq. (4).

(iv) The weight of an event. To obtain a complete weight of an event the LPS weight $W_{\rm LPS}$ (see Appendix) is multiplied by G^n, by the weight factor for identical partons, and by the Kuti-Weisskopf factor $\sqrt{x}$ for each valence quark [see Eq. (1)]. It should be noted that our procedure of assigning rapidities to partons in the sequence (3) does not guarantee the positivity (negativity) of the rapidities of the beam (target) valence quarks. In the present version of the program we simply throw out such "wrong-sign" events (about 20%), although clearly a more adequate procedure should be contrived. The complete weight of an event reads

$$W = W_{\rm LPS}\, G^n \left[\prod_1^6 (|x_i|)^{1/2}\right] W_{id}^{(u)} W_{id}^{(d)} W_{id}^{(s)} \ . \tag{5}$$

(v) The recombination of partons into hadrons. At this stage the program knows the sequence of partons like (3) and knows their rapidities (consequently also their momenta and energies). The interaction between partons is supposed to be of short range in rapidity. The program now recombines the neighboring partons to $Q\overline{Q}$ and to QQQ and $\overline{Q}\overline{Q}\overline{Q}$ triplets. The program proceeds from left to right in the sequence like (3) always considering three partons (leftovers from the previous recombination plus the nearest right neighbors) and forms $Q\overline{Q}$ pairs and QQQ and $\overline{Q}\overline{Q}\overline{Q}$ triplets. Figure 1 gives rules for forming such combinations. The recombination of the sequence (3) then looks as shown in Fig. 2. These rules were devised so as to avoid a recombination of partons separated by large rapidity gaps. It is quite possible that in the future they will have to be replaced by better motivated dynamical rules.

In the next step the program decides, on the basis of another random-number generator, which

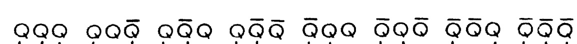

FIG. 1. The rules for forming mesons and baryons from triplets of partons during the process of recombination.

$$u\ \bar{u}\ s\ d\ \bar{d}\ u\ u\ u\ d\ u\ \bar{s}\ \bar{u}\ d\ u$$

FIG. 2. The recombination of the sequence (3) to mesons and baryons according to the rules shown in Fig. 1.

resonance arises from a particular $Q\overline{Q}$, QQQ, or $\overline{Q}\overline{Q}\overline{Q}$ combination. Relative weights of various possibilities are given by squares of the coefficients in wave functions of hadrons within the SU(6) scheme, averaged over spins of initial partons and summed over spins of hadrons being formed. The series (3) can in this way turn to

$$\rho^0 \overline{K}{}^0 p \Delta^+ K^0 \pi^0 \ . \tag{6}$$

The momentum of each resonance is simply given as the sum of momenta of its constituents.

(vi) Resonance decay. In the next step resonances and unstable particles decay to observed hadrons. Branching ratios are taken directly from Particle Data Group tables and an isotropic decay distribution is assumed for each decay. The decay angle is chosen by a random number generator. The sequence (6) turns with some probability to

$$\pi^+\pi^-\pi^+\pi^- p p \pi^0 K_L^0 \pi^0 \ . \tag{7}$$

If desired, any particle may be declared as stable, its decays are forbidden and it appears in the final state in the generated event. This is important for comparison with the data since in an electronic experiment one cannot see, for instance, K_s^0, while in a bubble-chamber experiment this is visible.

The final result of a generation of a particular event is the sequence of hadrons like (7) with specified rapidities. To the whole event there is ascribed the weight as given above which in our model expresses the probability for producing a given final state in the pp collision at high energy.

IV. COMPARISON WITH DATA

We shall now compare results obtained in our model for pp collisions at 300 GeV/c with multiparticle production data. At this energy one can use both Fermilab and CERN ISR data. Fermilab results were obtained in the bubble chamber and the ISR data come from electronic experiments.

We made, therefore, two computer runs at this energy. In the run corresponding to the electronic experiment we had only $p, \overline{p}, n, \overline{n}, \pi^\pm, K^\pm, K_L^0, \pi^0$ in the final states. In the "bubble-chamber" run we declared as stable also $\Lambda, \Sigma^\pm, \Xi$ hyperons together with their antiparticles and K_s^0 (these particles appeared in the final state). In each run we have generated about 33 000 events. We have put $G = 0.615$ (coupling constant adjusted to give a correct charged multiplicity). Results on average multiplicities are summarized and compared to the data[18-20] in Table I. In general the comparison seems to be favorable, although it has to be noted

TABLE I. Comparison of calculated averaged multiplicities with the data.

Average multiplicity	Calculation[a] "electronic run"	Calculation[a] "bubble-chamber run"	Data	Ref.
total	14.25	13.48		
charged	$\underline{8.75}$	7.78	8.77	18[b]
π^+	$\underline{3.69}$	3.44	3.7	18[b]
π^-	$\underline{3.10}$	2.69	3.1	18[b]
π^0	3.96	$\underline{3.57}$	3.9	19[c]
K^+	$\underline{0.36}$	0.34	0.37	18[b]
K^-	$\underline{0.18}$	0.19	0.26	18[b]
K_S^0		$\underline{0.27}$	0.22	19
K_L^0	0.27	0.25		
p	$\underline{1.33}$[d]	1.12	1.3	18[b,d]
$\overline{p}$	$\underline{0.09}$[d]	0.04	0.08	18[b,d]
Λ/Σ^0		$\underline{0.31}$	0.11	19
$\overline{\Lambda}/\overline{\Sigma}^0$		$\underline{0.05}$	0.03	19
n	0.83	0.62	0.5	20[e]
$\overline{n}$	0.07	0.05		
Σ^+		0.05		
Σ^-		0.03		
γ[f]	0.39	0.40		
Ξ^0		0.01		
Ξ^-		0.01		

[a] The underlined number is to be compared with the data.
[b] Interpolation of ISR data at 240 GeV/c and 480 GeV/c.
[c] Estimated from the number of γ's observed.
[d] Note that protons from Λ/Σ^0 decays are included in an electronic experiment.
[e] Estimated on the basis of the compilation of the data and baryon-number conservation.
[f] Our γ's come only from decays of resonances, γ's from neutral pion decays are not included here.

TABLE II. Average multiplicities of some particles which are directly produced by the recombination of partons in 300-GeV/c pp collisions.

Particle	$\langle n\rangle$	Particle	$\langle n\rangle$	Particle	$\langle n\rangle$
π^+	0.27	Δ^{++}	0.22	ρ^+	0.79
π^-	0.16	Δ^+	0.63	ρ^0	0.71
π^0	0.22	Δ^0	0.35	ρ^-	0.53
K^+	0.11	Δ^-	0.05	K^{*+}	0.28
K^0	0.07	Y^{*+}	0.08	K^{*0}	0.18
K^-	0.04	Y^{*0}	0.11	K^{*-}	0.09
$\overline{K}{}^0$	0.04	Y^{*-}	0.04	$\overline{K}{}^{*0}$	0.10
p	0.34	Σ^+	0.04	ω	0.72
n	0.16	Σ^0	0.05	ϕ	0.09
$\overline{p}$	0.01	Σ^-	0.02	η	0.11
$\overline{n}$	0.01	Λ	0.06	X^0	0.16

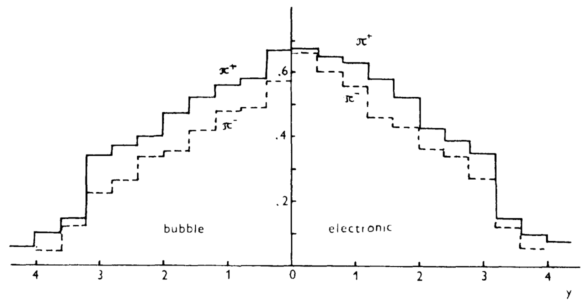

FIG. 3. Rapidity distributions of π^+ in pp collisions at 300 GeV/c as calculated in the present Monte Carlo program.

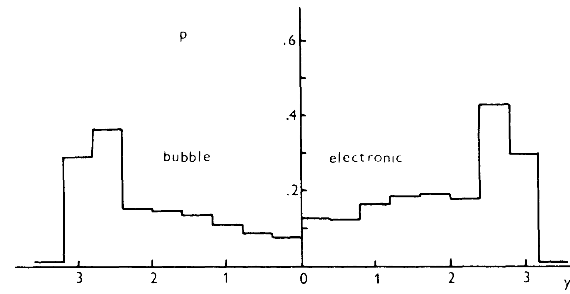

FIG. 5. Proton rapidity distributions.

that the charged multiplicity is regulated by G and the overall suppression of strange particles is given by $P_s/(P_u+P_d)$.

A copious production of resonances is a typical feature of our model. To get some feeling about this point, we present multiplicities of directly produced (i.e., not via decay of resonances) particles in Table II.

Rapidity distributions of charged pions are shown in Fig. 3. The comparison with the data is rather difficult, since bubble-chamber data are available at lower energies (see Figs. 61, 62 in Ref. 20) and ISR data should be integrated over p_T with uncertainties at low p_T. Figure 4 contains rapidity distributions for charged kaons. In this case there is no difference between the "bubble-chamber" and "electronic" runs. The shape of the histogram for K^- indicates that fluctuations are still present. In order to obtain more smooth curves one would need to increase considerably the number of generated events.

In Fig. 5 we present the rapidity spectra of the proton, both for the "electronic" and for the "bubble-chamber" runs. Antiproton distributions are shown in Fig. 6. Here, as expected the difference between both runs is rather large.

The bubble-chamber data of Sheng *et al.*[19] on K_s^0, Λ, and $\bar{\Lambda}$ production are compared with our results in Fig. 7. The agreement is quite satisfactory for K_s^0, debatable for $\bar{\Lambda}$, and really bad for Λ. The case of Λ is no surprise since the average multiplicity of Λ's (see Table I) disagrees with the data. The Λ production is also notoriously known to be the source of troubles for quark models of multiparticle production. The problem has to be studied in more detail in the future.

The charge distribution dQ/dy obtained in our model is shown in Fig. 8. In a qualitative way the shape of the curve corresponds to that given by Sivers[21] and based on the compilation of ISR data. The enhancements at $|y|\approx 3$ is caused by the tendency of valence quarks to stay at large values of momentum fractions. In our model this was built in by the Kuti-Weisskopf factors $\sqrt{x_i}$. The same holds true for the rapidity distribution of protons (see Fig. 5).

Finally, in Fig. 9 we present probabilities for charge transfer ΔQ across $y=0$ calculated from our model. The dispersion of the curve $\langle \Delta Q^2\rangle = 1.03$, in agreement with experimental values[22] $\langle \Delta Q^2\rangle = 0.99 \pm 0.03$ at 205 GeV/c and $\langle \Delta Q^2\rangle = 1.11 \pm 0.03$ at 405 GeV/c.

V. COMMENTS AND CONCLUSIONS

In a Monte Carlo model one can calculate any quantity measured in the experiment. In some cases, however, the amount of computer time necessary for the calculation becomes prohibitively large. For instance, in order to calculate correla-

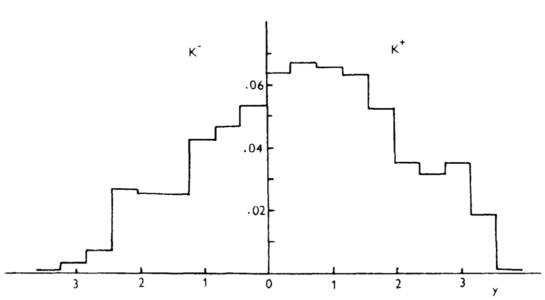

FIG. 4. Rapidity distributions of charged kaons.

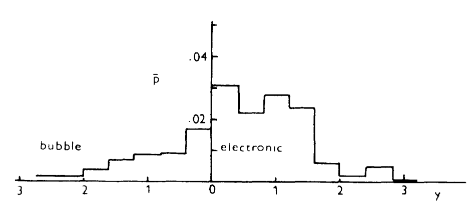

FIG. 6. Rapidity distributions of antiprotons.

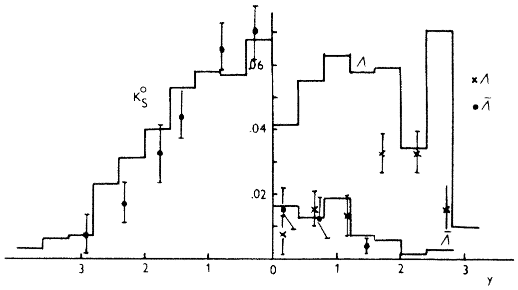

FIG. 7. Λ, $\bar{\Lambda}$, and K_S^0 rapidity distributions. Histogram shows our results from the "bubble-chamber run", data points come from the Fermilab bubble-chamber experiment (Ref. 19).

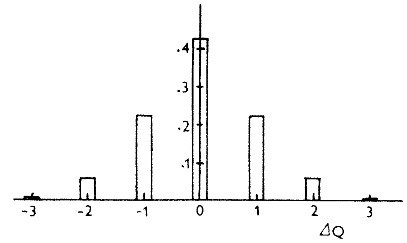

FIG. 9. The distribution of charge transfer ΔQ across $y=0$ as calculated in our model.

tions we would need to increase the statistics by an order of magnitude. For the present we have concentrated mainly on integrated quantities postponing more computer-time-consuming calculation until the limitations of the model are better understood on the level of single-particle distributions.

We shall now mention briefly some uncertain points of the present model:

Diffractive dissociation. We have assumed a complete incoherence of the initial parton distribution. Because of this we do not have events due to the diffractive dissociation. As a consequence we have depletion of low-multiplicity events and our multiplicity distributions are much closer to the Poisson distribution than the data are (we have $f_2^{cc} \approx 0$).

Correlations. We have not yet performed a detailed study of correlations. It seems, however, that we shall have about a right amount of short-range correlations between particles of opposite charges. It is also clear that this model, as it stands, will not be able to describe correlations between like particles, which are most likely due to Bose-Einstein effects.[23]

The recombination process. Our prescriptions for forming QQ pairs and QQQ triplets are probably oversimplified. More global and more dynamically motivated criteria should probably be contrived. Moreover, at present we do not take into account any possible relationships between masses of resonances being formed and effective masses of respective pairs or triplets.

The identity of partons. Since we are working with probabilities and not with wave functions of partons we can in an approximate way include the effects due to identity of particles into the phase space, but we are unable to include these effects in the wave functions. Simple arguments based on counting the number of permutations indicate that this enhances *uds* combinations relative to *uud* or *udd*. This is perhaps the reason for producing too many Λ's in our model.

Resonance production. Our model predicts the ρ^0 average multiplicity at 300 GeV/c of about 0.7 while the data[24] suggest something like 0.3. This indicates that the recombination of a $Q\bar{Q}$ pair into a vector meson is suppressed relative to the exact-SU(6) case. This may be a dynamical effect due to higher masses of vector mesons.

Transverse momenta. For a detailed comparison with single-particle spectra it seems to be necessary to build in transverse momenta of partons which we have neglected in the present version of the model. This will somewhat narrow the single-particle spectra since a part of the total energy will be consumed by the transverse motion. The full inclusion of transverse momenta is also necessary in calculating the short-range correlations in rapidity. It is also impossible to adjust quark masses before building in the transverse momenta of partons.

The role of gluons. In our model gluons are converted into $Q\bar{Q}$ pairs prior to generation of rapidities. The model by Van Hove *et al.*[8] ascribes quite a different role to gluons: The central region is populated mostly by gluons and one would not expect a significant baryon production there. Deeper understanding of the role of gluons requires apparently a lot of further work.

To conclude, it seems to us that the present Monte Carlo quark-parton model is able to describe the general features of the multiparticle production and that a more detailed comparison with

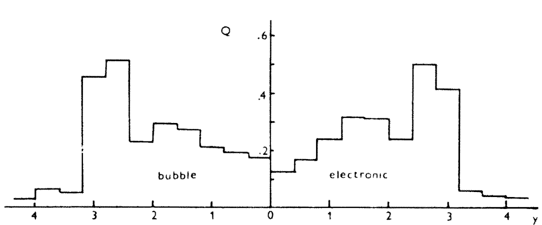

FIG. 8. Charge distribution dQ/dy in pp collisions at 300 GeV/c.

data may bring information about the dynamics of the hadronic collision. Further work along these lines might clarify the uncertain points listed above and throw, finally, some light on the role of gluons.

A Monte Carlo model like this one may also be useful from a practical point of view since it permits a rough estimate of what one can expect in a particular experiment on multiparticle production.

Within the framework of our model the recombination of partons to mesons and baryons is made possible by the destruction of coherence relations between partons during the first part of the hadronic collision. It is then natural to assume that $Q\overline{Q}$ pairs can either recombine or annihilate via virtual photons to e^+e^- or $\mu^+\mu^-$ pairs.

Recently, we have performed the corresponding calculations and obtained a qualitative description of the production of dimuons with low dimuon masses[25] and the production of electrons[26] with low p_T. These results give support to the idea, first advocated by Bjorken and Weisberg,[27] that the annihilation of Q's and $\overline{Q}$'s produced during the collision provides a substantial contribution to the direct lepton production. If this picture turns out to be qualitatively correct, the dilepton production and the multiparticle processes will be two complementary sources of information about the behavior of hadronic constituents during the hadronic collision. There are at least two points in which the dilepton production can provide essential information: First, the effective mass of hadronic constituents might be reflected in the shape of the dimuon mass spectrum at low masses and second, the quantum number of constituents which annihilated to a dilepton may be inferred from the comparison of multiparticle distributions in events with and without a dilepton pair. If such information were available it would be a considerable help in attempts to construct models for multiparticle production.

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APPENDIX

We use a part of Jadach's program GENRAP for assigning rapidities to partons in the sequence (3) and for calculating the LPS weight of the given event. In the present version of our model we put all transverse momenta of partons equal to zero. For the sake of completeness we shall reproduce here briefly the main points of the relevant part of Jadach's Monte Carlo rapidity generator.

The LPS of an n-particle configuration (without a factor for identical particles) is given by the expression

$$L_n=\int\left(\prod_1^n \frac{dp_i}{2E_i}\right)\delta\left(\sum p_i\right)\delta\left(E-\sum E_i\right). \qquad \text{(A1)}$$

After some simple algebra the formula can be rewritten (using rapidity variables) as

$$L_n=\frac{E}{2^{n-1}}\int\left(\prod_1^n dy_i\right)\delta\left(E^2-\left(\sum m_i e^{y_i}\right)\left(\sum m_i e^{-y_i}\right)\right)\delta\left(E-\sum m_i e^{y_i}\right) \qquad \text{(A2)}$$

In order to get rid of the δ functions, Jadach makes the change of variables

$$\begin{aligned} y_1&=Z\,,\\ y_2&=Z+Y\xi_2\,,\\ &\vdots\\ y_{n-1}&=Z+Y\xi_{n-1}\,,\\ y_n&=Z+Y\,. \end{aligned} \qquad \text{(A3)}$$

Equation (A2) then becomes

$$L_n=\frac{n(n-1)}{2^{n-1}E^2}\int\left(\prod_{i=2}^{n-1}d\xi_i\right)\frac{Y^{n-2}}{D}\,, \qquad \text{(A4)}$$

where Y and D are implicit functions of $(n-2)$ independent variables $\xi_2,\ldots,\xi_{n-1}$ given by the equations

$$E^2-\left(\sum_{i=1}^n m_i e^{-\xi_i Y}\right)\left(\sum_{i=1}^n m_i e^{\xi_i Y}\right)=0\,, \qquad \text{(A5)}$$

$$D=\left|\frac{\partial}{\partial Y}\ln\left(\sum_1^n m_i e^{\xi_i Y}\right)\left(\sum_1^n m_i e^{-\xi_i Y}\right)\right|, \qquad \text{(A6)}$$

and $\xi_0=0$, $\xi_n=1$ is understood in summing over ξ's. In the computer program one selects random numbers ξ_i, $0<\xi_i<1$, $i=2,\ldots,n-1$, calculates numerically Y from (A5) and Z from the condition

$$E-\sum_{i=1}^n m_i e^{Z+\xi_i Y}=0\,. \qquad \text{(A7)}$$

Rapidities are then calculated from (A3) and, according to (A4), the appropriate weight reads

$$W_{\text{LPS}}=\frac{n(n-1)}{2^{n-1}E^2}\,\frac{Y^{n-1}}{D}\,.$$

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