$\pi^+\pi^- \to \pi^0\pi^0$ in the $M_{\pi\pi} \simeq 1$ GeV/c^2 region

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Using the even-l, even-I $\pi\pi$ scattering phase shifts and elasticities from reactions with charged-pion final states, we calculate the yield in an old observation of the reaction $\pi^- p \to \pi^0\pi^0 n$ at $p_{\pi^-} = 10$ GeV/c. In the region under discussion, 0.75 GeV/c^2 < $M_{\pi\pi}$ < 1.4 GeV/c^2, the fit is bad. We present, only as examples, two modified sets of phase shifts and elasticities which give reasonably good fits to our data.

Several experiments[1-4] have been reported on the $\pi^0\pi^0$ mass distribution in the $M_{\pi\pi} \simeq 1$ GeV/c^2 region, using high-energy incident production of pions from target nucleons. In those experiments[1,2,4] studying $\pi^- p \to \pi^0\pi^0 n$ two bumps in the $\pi^0\pi^0$ mass spectrum are observed. One of these is broad, extending from threshold to $\simeq 1$ GeV/c^2. The second bump is the f^0 meson, centered at $M_{\pi\pi} \simeq 1.27$ GeV/c^2. In our data[1,5,6] (taken with $p_{\pi^-} = 10$ GeV/c bombarding momentum) a deep minimum is observed at $M_{\pi\pi} \simeq 1.05$ GeV/c^2. Our mass distribution is shown in Fig. 1 with a low four-momentum transfer cut [$|t - t_{\min}| < 0.1$ (GeV/c)^2] and with finer binning (50 MeV/c^2) than we have published heretofore. Further subdivision of the data into decay angle bins or into finer mass bins would be useless because of the low numbers per bin.

High-statistics experiments studying the reactions $\pi^+ p \to \pi^+\pi^-\Delta^{++}$ (Ref. 7) and $\pi^- p \to \pi^+\pi^- n$ (Ref. 8) have yielded a set of $\pi\pi$ scattering phase shifts and elasticities. In principle, a study of the reaction $\pi^- p \to \pi^0\pi^0$ should yield $I = 0, 2$ even-l $\pi\pi$ δ's and η's which are consistent with that set.[7,8] However, the $\pi^0\pi^0$ experiments give so few events that it would be hopeless to attempt analyses using them alone. But we can take a published set[8] of phase shifts and elasticities and calculate the $\pi^0\pi^0$ mass distribution; this calculation for our 10-GeV/c experiment is shown as the solid curve in Fig. 1. It is an absolute prediction from the one-pion-exchange model, and relies on (a) the Chew-Low treatment of the $\pi\pi$ vertex with a form factor $e^{7(t-\mu^2)}$ (Refs. 5 and 6), (b) the even-l even-I $\pi\pi$ scattering phase shifts ($\delta_{I=0,2}^{l=0,2}$) and elasticities ($\eta_{I=0,2}^{l=0,2}$) (solid curves in Fig. 2) derived[8] from the charged-pion production reactions, and (c) Monte Carlo generation and selection of $\pi^0\pi^0$ events to model our experiment.

There is a maximum error of about ±25% in this comparison, caused by ±20% uncertainty in the absolute normalization of the data and by ±15% in the Chew-Low form factor. Nevertheless it is clear from Fig. 1 that there are the following disagreements between the data and our calculations, and that these disagreements are systematic rather than statistical: (1) The calculated yield below 1 GeV/c^2 is off by a factor of about 1.5 and is at a mass too high by about 25 MeV/c^2. (2) The calculated minimum is off by a factor of about 2.5 and is at

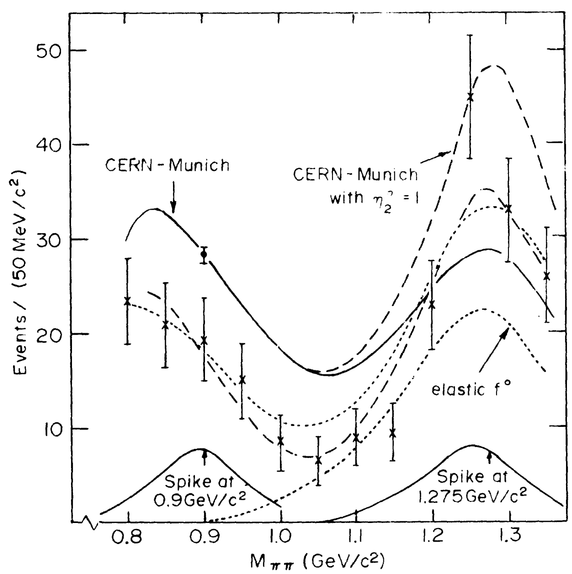

FIG. 1. Experimental data points (in 50-MeV/c^2 bins) and Monte Carlo curves. Reading down, the upper solid curve uses CERN-Munich[8] δ's and η's; a sample statistical error is shown for a 50-MeV/c^2 bin centered on 900 MeV/c^2. The dashed extension of the upper solid curve above 1025 MeV/c^2 uses CERN-Munich δ's and η's except that $\eta_2^0 = 1$. The upper dotted curve uses our first modified set of δ's and η's (see text). The lower dashed curve uses our second set of modified δ's and η's (see text). The lower dotted curve is elastic f^0 using resonance parameters given in text, but yield divided by a factor of 4. The lower solid curves are resolutions for mass spikes generated at 0.9 and 1.275 GeV/c^2 with decay angular distributions given by CERN[8] δ's and η's.

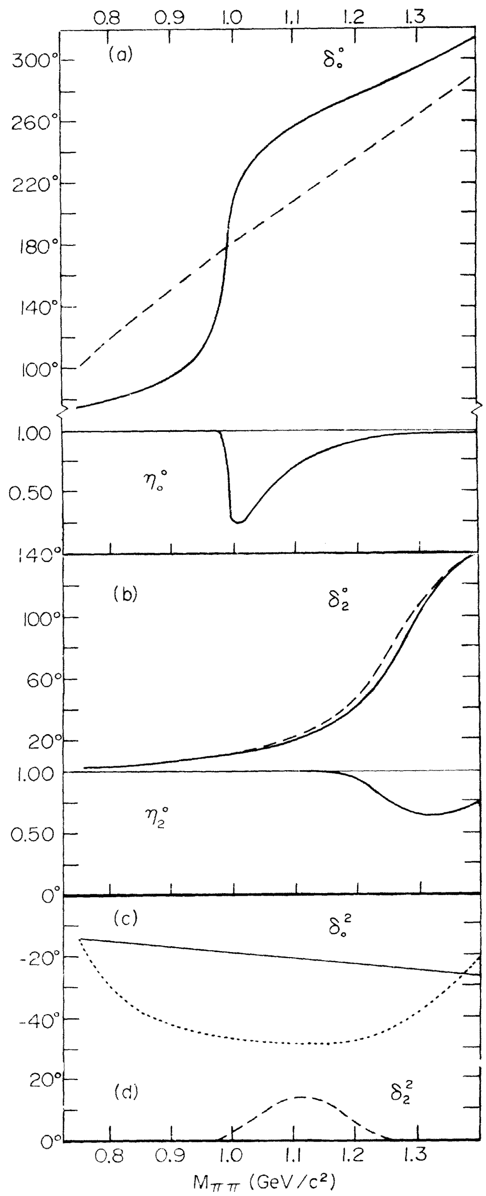

FIG. 2. Phase shifts (in degrees) and elasticities. (a) δ_0^0 and η_0^0; (b) δ_2^0 and η_2^0; (c) δ_0^2; (d) δ_2^2. Note that $\eta_0^2 = \eta_2^2 = 1$ throughout, and that $\delta_2^2 = 0$ in the CERN-Munich analysis. The solid curves are from the CERN-Munich analysis and are retained in the first of our trial modifications, except that δ_0^2 is replaced by the dotted curve. In the second trail modification, the dashed curves are the δ's ($\delta_0^2 = 0$), and all four η's are taken to be unity.

a mass too high by about 25 MeV/c^2. (3) The calculated f^0 peak is off by a factor of about 0.8 and is also too broad.

(1) Since the calculated yield below 1 GeV/c^2 is determined almost exclusively by ($\delta_0^0 - \delta_0^2$), with $\eta_0^0 = \eta_0^2 = 1$, we have to modify the accepted S-wave phase shifts to get agreement. $\delta_0^0 - \delta_0^2$ must be farther beyond 90° at a lower mass than in the CERN-Munich[8] analysis in order that the yield after spreading by the Monte Carlo match the data in the 0.85-GeV/c^2 region.

(2) The location and shape of the minimum comes primarily from the passage of ($\delta_0^0 - \delta_0^2$) through 180°. It is also affected by interference with δ_2^0, given by the tail of the f^0. We find that ($\delta_0^0 - \delta_0^2$) must climb through 180° more slowly than in the CERN-Munich[8] analysis if the minimum is not to be filled in by our experimental resolution width [140 MeV/c^2 full width at half maximum (FWHM) at 1 GeV/c^2; see Fig. 1 for resolutions at 0.9 and 1.275 GeV/c^2].

(3) The sensitivity to the elasticity η_2^0 of the $I = 0$ D wave is shown by setting $\eta_2^0 = 1$, with the CERN-Munich δ's and other η's unchanged (upper dashed curve, Fig. 1). The sensitivity to interference between the f^0 and the other partial waves is shown by the yield for an isolated purely elastic f^0 (lower dotted curve, Fig. 1, plotted at $\frac{1}{4}$ scale). We have taken resonance parameters $M_{f^0} = 1270$ MeV/c^2, $\Gamma_{f^0} = 170$ MeV/c^2 and with an $l = 2$ square-well penetration factor[9] with $r_{f^0} = 5$ GeV^{-1}. This differs only slightly from the treatment of the f^0 in the CERN-Munich analysis [compared in Fig. 2(b)]. To scale, the elastic f^0 peak is a factor of 2.5 higher than the experimental peak. We believe that the disagreement between the full CERN-Munich calculation and our data in the f^0 region lies not in the f^0 but in the S waves or the $I = 2$ D wave.

To give a better match to our experimental mass distribution over the whole mass range, we have tried modifying the S waves and $I = 2$ D wave. This search for a good fit has by no means been exhaustive, and we have only two illustrations to offer:

(1) Leave the $I = 0$ δ's and η's of CERN-Munich as given and vary δ_0^2 and δ_2^2, assuming these latter to be purely elastic. The δ_0^2 of the dotted curve in Fig. 2(c) predicts the mass distribution of the upper dotted curve in Fig. 1. This gives a good fit except in the 0.975 to 1.2 GeV/c^2 mass range, where the calculated yield is too high. This disagreement cannot be removed by the addition of an $I = 2$ D wave.

(2) Assume all waves to be purely elastic and the $I = 0$ D wave to be given by the f^0 resonance parameters above [dashed curve, Fig. 2(b)]. (This is greatly oversimplified, since the f^0 inelasticity is particularly well established.) We then alter the CERN-Munich δ_0^0 drastically to that of the dashed curve [Fig. 2(a)]. Adding a small δ_2^2 [Fig. 2(d)] to suppress further the yield on the low-mass side of the f^0 peak gives a very good fit to the data (lower dashed curve, Fig. 1).

We believe the disagreement does not come from the data. There were two points that required special attention:

(1) The geometric efficiencies are strongly angle-dependent, so the Monte Carlo calculation of the total yield in each mass bin depends on the production angle distribution and particularly strongly on the decay angle distribution. However, we found no indication of any special sensitivity to the geometry of the experiment that might help explain the mass dependence of the yield.

(2) Background events must be calculated, generated by Monte Carlo, and finally subtracted without characteristic identification. By our best estimates,[5,6] the background is far too small to alter the shape significantly, and in any case larger background subtractions would almost certainly make the disagreement worse.

We gain confidence for these data from the subsequent confirmation of several related results[10-12] obtained using 2-γ and 3-γ events from the same encoding. We believe that better 4-γ experiments will confirm the disagreements exhibited here between our data on the one hand and the calculations based on currently available $\pi^+\pi^- \to \pi^+\pi^-$ amplitudes on the other.

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