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Class of solutions for the strong-gravity equations

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We solve the Einstein equations for strong gravity in the limit that weak gravity is neglected. The class of solutions we find reduces to the Schwarzschild solution (with the weak-gravity Newtonian constant replaced by a strong-coupling parameter) in the limit $M^2 \to 0$, where M is the mass of the strong-gravity spin-2 meson. These solutions may be of relevance for the problem of defining temperature and confinement in hadronic physics.

I. INTRODUCTION

The proposal that tensor fields may play a fundamental role in strong-interaction physics was introduced some time ago.[1] This idea was given expression in a two-tensor theory of strong and gravitational interactions where the strong tensor fields are governed by equations formally identical to the Einstein equations apart from the coupling parameter $\kappa_f \simeq 1\ \mathrm{GeV}^{-1}$ which replaces the Newtonian $\kappa_g \simeq 10^{-19}\ \mathrm{GeV}^{-1}$. The equations for the strong field $f_{\mu\nu}$ and the gravitational tensor $g_{\mu\nu}$ are derived from the Lagrangian

$$\mathcal{L} = \frac{1}{\kappa_g{}^2}\sqrt{-g}\,R(g) + \frac{1}{\kappa_f{}^2}\sqrt{-f}\,R(f) + \mathcal{L}_{fg} + \mathcal{L}_{\text{matter}}\ , \tag{1.1}$$

where the first term is the usual Einstein Lagrangian and the second is its strong analog (identical in form apart from the replacement $\kappa_g \to \kappa_f$). All other fields are grouped into the term $\mathcal{L}_{\text{matter}}$.

To give the f mesons a mass (as well as their weak gravitational interaction) we need a mixing term between f and g fields. One of the simplest possible covariant mixing terms is given by

$$\mathcal{L}_{fg} = \frac{-M^2}{4\kappa_f{}^2}\sqrt{-g}\,(f^{\mu\nu} - g^{\mu\nu})(f^{\kappa\lambda} - g^{\kappa\lambda}) \times (g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\lambda}g_{\mu\nu})\,, \tag{1.2}$$

where M is a constant with the dimensions of mass.[2]

In Ref. 1 (on the basis of a linearization of these equations, with $f_{\mu\nu} = \eta_{\mu\nu} + \kappa_f\phi_{\mu\nu}$, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa_g h_{\mu\nu}$), it was suggested that the equations resulting from (1.1) and (1.2) describe a massless graviton given by the field combination

$$\left(\frac{1}{\kappa_g{}^2}g_{\mu\nu} + \frac{1}{\kappa_f{}^2}f_{\mu\nu}\right)\left(\frac{1}{\kappa_g{}^2} + \frac{1}{\kappa_f{}^2}\right)^{-1}$$

plus a strongly interacting massive spin-2 field described by the orthogonal combination[3] $(f_{\mu\nu} - g_{\mu\nu})$.

In this note we wish to solve the equations for pure strong gravity, in the limit $\kappa_g \to 0$, without any further approximation. We are particularly interested in that class of solutions which are soft in the limit $M^2 \to 0$ and which—as may be expected from the structure of (1.1)—reduce to Schwarzschild-type solutions (with κ_f replacing κ_g). Our interest in such solutions stems from the possibility offered by the recent work of Hawking—to interpret them as strong-gravity solitons, radiating all species of hadrons thermally, with a temperature which is proportional to strong surface gravity.[4]

II. LAGRANGIAN AND EQUATIONS OF MOTION

From now on we deal with the purely strong-gravity situation. All matter, as well as ordinary gravity, will be ignored. In this situation the Lagrangian (1.1) reduces to the form

$$\mathcal{L} = \frac{1}{\kappa_f{}^2}\sqrt{-f}\,R(f) + \mathcal{L}_{\text{mass}}\ , \tag{2.1}$$

where κ_f ($\sim 1\ \mathrm{GeV}^{-1}$) denotes the strong analog of the Newtonian coupling and the first term here, expressed in terms of the tensor $f_{\mu\nu}$, and its inverse $f^{\mu\nu}$, is identical in form with the Einstein Lagrangian (except for the interchange of κ_f for κ_g). The second term, which gives mass to the tensor meson, takes the form

$$\mathcal{L}_{\text{mass}} = \frac{-M^2}{4\kappa_f{}^2}\sqrt{-\eta}\,(f^{\kappa\lambda} - \eta^{\kappa\lambda})(f^{\mu\nu} - \eta^{\mu\nu}) \times (\eta_{\kappa\mu}\eta_{\lambda\nu} - \eta_{\kappa\lambda}\eta_{\mu\nu})\,, \tag{2.2}$$

where $\eta_{\mu\nu}$ denotes the flat space-time metric. In the usual rectangular coordinates it equals

$$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)\,.$$

The expression (2.2) is to be interpreted as the relic of a generally covariant form (1.2) in the limit in which ordinary gravitational effects are ignored (i.e., $\kappa_g \to 0$ and $g_{\mu\nu} \to \eta_{\mu\nu}$). It is a strictly phenomenological expression whose origin in vacuum polarization effects we shall not attempt to justify here.[3]

On varying $f^{\mu\nu}$ one obtains the equations of strong gravity,

$$R_{\mu\nu} - \tfrac{1}{2} f_{\mu\nu} R = \kappa_f{}^2 T_{\mu\nu}\,, \tag{2.3}$$

where the left-hand side is the usual Einstein tensor and the right-hand side is simply

$$\kappa^2 T_{\mu\nu} = \tfrac{1}{2} M^2 (f^{\kappa\lambda} - \eta^{\kappa\lambda})(\eta_{\kappa\mu}\eta_{\lambda\nu} - \eta_{\kappa\lambda}\eta_{\mu\nu}) \times (\sqrt{-\eta}/\sqrt{-f})\,. \tag{2.4}$$

$T_{\mu\nu}$ is not a prescribed source; it depends on $f_{\mu\nu}$. We also emphasize that this expression is not generally covariant: It is the flat-space approximation to a generally covariant term (1.2). This means that, although the left-hand side of (2.3) is a tensor, the right-hand side is not. One is not able to remove any components from $f_{\mu\nu}$ by way of coordinate conditions as one would do with a covariant system. Thus, there are altogether 10 independent equations in (2.3) although 4 of them take the form of constraints on $T_{\mu\nu}$,

$$0 = f^{\kappa\mu} \nabla_\kappa T_{\mu\nu}\,, \tag{2.5}$$

where ∇_κ denotes the strong-gravity analog of the covariant derivative.

Without the softening (expected, for example, from the Yang-Mills ansatz motivated in Ref. 5) one might expect the solutions of (2.3) to behave badly in the limit $M^2 \to 0$, since in this limit the equations (becoming generally covariant) decrease in number from ten to six since the four constraints (2.5) are removed. (An analogous situation is encountered in the weak-field approximation[6] where the five degrees of freedom associated with a massive spin-2 field are reduced to two in the massless limit. As is well known, this phenomenon is heralded by the presence of singular factors M^{-2}, M^{-4} in the massive tensor propagator.) Notwithstanding these general considerations, however, the message of this note is that not all solutions of (2.3) are singular in the limit $M^2 \to 0$, and we shall exhibit a class of smooth ones in the following.

III. A STATIC SPHERICALLY SYMMETRIC SOLUTION

When spherical symmetry is assumed, the number of independent components in $f_{\mu\nu}$ is reduced from ten to four. For convenience we use spherical polar coordinates and define the independent components by

$$f_{\mu\nu} dx^\mu dx^\nu = C dt^2 - 2D dt\, dr - A dr^2 - B(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{3.1}$$

where the components $A, \ldots, D$ depend only on r. The inverse is given by

$$f^{\mu\nu}\partial_\mu\partial_\nu = \frac{A}{\Delta}\partial_t{}^2 - \frac{2D}{\Delta}\partial_t\partial_r - \frac{C}{\Delta}\partial_r{}^2 - \frac{1}{B}\left(\partial_\theta{}^2 + \frac{1}{\sin^2\theta}\partial_\varphi{}^2\right), \tag{3.2}$$

where the convenient combination

$$\Delta = AC + D^2 \tag{3.3}$$

is used. In the following we shall exchange D for Δ as the variable of choice.

For the four independent functions we have, of course, four equations to solve. [Two of these will be of the constraint type (2.5), but here the distinction is not a very useful one.] The four nonvanishing components of $T_{\mu\nu}$ are given by

$$\begin{aligned} T_{tt} &= \tfrac{1}{2} M^2 \frac{r^2}{B\sqrt{\Delta}}\left(3 - \frac{2r^2}{B} - \frac{C}{\Delta}\right),\\ T_{tr} &= \tfrac{1}{2} M^2 \frac{r^2}{B\sqrt{\Delta}}\left(\frac{D}{\Delta}\right),\\ T_{rr} &= \tfrac{1}{2} M^2 \frac{r^2}{B\sqrt{\Delta}}\left(-3 + \frac{2r^2}{B} + \frac{A}{\Delta}\right),\\ T_{\theta\theta} &= \tfrac{1}{2} M^2 \frac{r^4}{B\sqrt{\Delta}}\left(-3 + \frac{r^2}{B} + \frac{A+C}{\Delta}\right) \end{aligned} \tag{3.4}$$

in spherical coordinates. The Ricci tensor is given by

$$\begin{aligned} R_{tt} &= -\frac{C}{D} R_{tr} = -\frac{C}{2\Delta}\left(C'' + \frac{B'C'}{B} - \frac{C'\Delta'}{2\Delta}\right),\\ R_{rr} &= \frac{B''}{B} - \frac{B'^2}{2B^2} + \frac{A}{2\Delta}\left(C'' + \frac{B'C'}{B} - \frac{C'\Delta'}{2\Delta} - \frac{B'\Delta'}{AB}\right),\\ R_{\theta\theta} &= -1 + \frac{C}{2\Delta}\left(B'' + \frac{B'C'}{C} - \frac{B'\Delta'}{2\Delta}\right). \end{aligned} \tag{3.5}$$

The identity $DR_{tt} + CR_{tr} = 0$ implies the purely algebraic constraint

$$0 = DT_{tt} + CT_{tr} = \frac{M^2}{2}\frac{r^2}{B\sqrt{\Delta}}\left(3 - \frac{2r^2}{B}\right) D \tag{3.6}$$

when the expressions (3.4) are used. There are, therefore, two categories of solution:

(1) $B = \tfrac{2}{3} r^2$,

(2) $D = 0$ (or $\Delta = AC$).

These values are solutions, it should be noted—not coordinate conditions. Type-2 solutions have been considered by Aragone and Chela Flores,[7] who have shown in a semilinear approximation that these solutions exhibit a Yukawa-type behavior, $(1/M_f r)\exp[-(M_f r)]$ for large r. Our concern is with solutions of type 1. For these, the remaining three equations are easily dealt with and one finds, altogether,

$$\begin{aligned} &B=\tfrac{2}{3}r^2\,,\\ &\Delta=\Delta_0\,,\\ &A+C=\tfrac{2}{3}+\tfrac{3}{2}\Delta_0\,,\\ &C=\tfrac{3}{2}\Delta_0\left(1-\frac{2\mu}{r}-\frac{1}{6}\,\frac{M^2r^2}{\Delta_0^{3/2}}\right), \end{aligned} \tag{3.7}$$

where μ and Δ_0 are arbitrary constants. The crucial step in obtaining this solution was the replacement of D by Δ as a dependent variable. The Ricci component R_{rr} is very complicated when expressed in terms of D and, correspondingly, $D=(\Delta-AC)^{1/2}$ is a nontrivial function of r. The remarkably simple result that Δ is independent of r comes from the combination

$$CR_{rr}+AR_{tt}=\kappa^2(CT_{tt}+AT_{tt})\,.$$

The right-hand side vanishes and the left reduces to

$$C\left(\frac{B''}{B}-\frac{B'^2}{2B^2}\right)-\frac{CB'}{2B}\,\frac{\Delta'}{\Delta}=-\frac{C}{r}\,\frac{\Delta'}{\Delta}\,,$$

when the solution for B is inserted.

In order that $D=\pm(\Delta-AC)^{1/2}$ is real, we must choose either

$$\mu>0,\quad \tfrac{4}{9}>\Delta_0,\quad \text{and } \sqrt{\Delta_0}>0 \tag{3.8}$$

or

$$\mu<0,\quad \tfrac{4}{9}<\Delta_0,\quad \text{and } \sqrt{\Delta_0}<0\,. \tag{3.9}$$

The choice (3.8) corresponds to the Schwarzschild-de Sitter-type solution in weak-gravity theory. (For $M\to 0$ we recover the pure Schwarzschild case, and for $\mu\to 0$ the pure de Sitter case.) For comparison with the known solutions in weak-gravity theory, it is perhaps instructive to exhibit (3.1) in the form

$$\begin{aligned} f_{\mu\nu}dx^\mu dx^\nu=\tfrac{3}{2}\Delta\Big\{&(1-p)dt^2-(1+p+\alpha)dr^2\\ &-2dt\,dr[p(p+\alpha)]^{1/2}\\ &-\frac{4r^2}{9\Delta}(d\theta^2+\sin^2\theta\,d\phi^2)\Big\}\,, \end{aligned} \tag{3.10}$$

where

$$p(r)=2\mu/r+\tfrac{1}{6}M^2r^2\Delta^{-3/2}\,, \tag{3.11}$$

$$\alpha=4/(9\Delta)-1\,. \tag{3.12}$$

Defining

$$d\tilde{t}=\frac{1}{(1+\alpha)^{1/2}}\left(dt-dr\,\frac{[p(p+\alpha)]^{1/2}}{1-p}\right), \tag{3.13}$$

we can cast (3.10) into the form

$$\tfrac{2}{3}[(1-p)d\tilde{t}^2-(1-p)^{-1}dr^2-r^2d\theta^2-r^2\sin^2\theta\,d\phi^2]\,.$$

This is the Schwarzschild–de Sitter solution and will therefore possess the corresponding Schwarzschild and cosmological horizons.

Now Gibbons and Hawking[8] have recently analyzed the problem of thermal emission from this class of soliton-like solutions and concluded that there are two distinct temperatures which can be associated with the two masses μ and M in these solutions. They have also argued for an observer dependence of the radiation associated with the cosmological horizon. If our basic notion of associating a fundamental spin-2 field of the above variety with strong-interaction physics is correct, it is clear that a direct test of (the partly controversial) conclusions arrived at by Gibbons and Hawking may be sought in experimentation in hadronic physics. A more direct use of the "potential" (3.11) may lie in the possibility it offers for confinement in hadronic physics for the repulsive case (3.9) when $\sqrt{\Delta}<0$.

Appreciation is expressed to Dr. C. J. Isham for stimulating discussions.

ADDENDUM

The mixing term used in the text is by no means the only feasible one. An alternative "cosmological"[1] one is given by

$$\begin{aligned} \mathcal{L}'_{fg}=\lambda\sqrt{-g}+\lambda'\sqrt{-f}&\\ -(\lambda+\lambda')(-f)^\alpha(-g)^\beta&\\ \times\{-\det[xg^{-1}+(1-x)f^{-1}]\}^{\alpha+\beta-1/2}&\,, \end{aligned} \tag{A1}$$

where $g=\det g_{\mu\nu}$ etc. and the contravariant tensors $(g^{-1})^{\mu\nu}$ and $(f^{-1})^{\mu\nu}$ are defined as the matrices inverse to $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively. If the parameters are restricted by the two constraints

$$2[-x\alpha+(1-x)\beta](\lambda+\lambda')=-x\lambda'+(1-x)\lambda\,, \tag{A2}$$

$$(\alpha+\beta-\tfrac{1}{2})x(x-1)(\lambda+\lambda')^2=\tfrac{1}{4}\lambda\lambda'\,, \tag{A3}$$

then the coupled system possesses a stable flat-space-time solution $f_{\mu\nu}=g_{\mu\nu}=$ Minkowski metric. In the linearized version there is, in addition to the graviton, a tensor meson with mass given by

$$M^2=\tfrac{1}{4}(\kappa_f^{\,2}+\kappa_g^{\,2})\lambda\lambda'/(\lambda+\lambda')\,. \tag{A4}$$

The mixing term (A1) was introduced (with $x=\frac{1}{2}$) in the Appendix of Ref. 1.

It is a simple matter to obtain exact solutions to the coupled f-g equations for the case of the new mixing term (A1) by imposing the constraint

$$0=\det[xg^{-1}+(1-x)f^{-1}], \tag{A5}$$

which decouples the equations for f and g provided $\alpha+\beta>\frac{3}{2}$. At one's disposal there are now eight coordinate conditions, one of which must be used to satisfy (A5). To illustrate, consider the de Sitter solutions

$$g_{\mu\nu}dx^\mu dx^\nu=\left(1+\frac{\lambda}{6}r^2\right)dt^2-\left(1+\frac{\lambda}{6}r^2\right)^{-1}dr^2 - r^2(d\theta^2+\sin^2\theta\, d\varphi^2), \tag{A6}$$

$$f_{\mu\nu}dx^\mu dx^\nu = C\,dt^2-2D\,dtdr-A\,dr^2 - B(d\theta^2+\sin^2\theta\,d\varphi^2) = \left(1+\frac{\lambda'}{6}\bar r^2\right)d\bar t^2-\left(1+\frac{\lambda'}{6}\bar r^2\right)^{-1}d\bar r^2 - \bar r^2(d\theta^2+\sin^2\theta\,d\varphi^2). \tag{A7}$$

The coordinates t, r are fixed by assuming the standard form here for $g_{\mu\nu}$. Likewise, the $\bar t, \bar r$ coordinates correspond to the standard form for $f_{\mu\nu}$. One must now construct a transformation such that (A5) is satisfied, i.e.,

$$0=-\det[xf^{-1}+(1-x)g^{-1}] = (r^4\sin^2\theta)^{-1}\left(x+(1-x)\frac{r^2}{B}\right)^2\left\{x^2+\frac{x(1-x)}{\Delta}\left[\left(1+\frac{\lambda}{6}r^2\right)A+\left(1+\frac{\lambda}{6}r^2\right)^{-1}C\right]+\frac{(1-x)^2}{\Delta}\right\}. \tag{A8}$$

There are many ways to satisfy this; perhaps the simplest is by choosing

$$\sqrt{B}=\bar r=[(x-1)/x]^{1/2}r, \quad \bar t=t, \tag{A9}$$

so that

$$f_{\mu\nu}dx^\mu dx^\nu=\left(1+\frac{\lambda'}{6}\frac{x-1}{x}r^2\right)-\frac{x-1}{x}\left(1+\frac{\lambda'}{6}\frac{x-1}{x}r^2\right)^{-1}dr^2-\frac{x-1}{x}r^2(d\theta^2+\sin^2\theta\,d\varphi^2). \tag{A10}$$

Alternatively, one may take

$$\sqrt{B}=\bar r=\left(\frac{x-1}{x}\right)^{1/2}r, \quad \bar t=\left(\frac{x}{x-1}\Delta\right)^{1/2}[t+f(r)] \tag{A11}$$

and choose $f(r)$ so as to make the last factor in (A8) vanish. One finds

$$f_{\mu\nu}dx^\mu dx^\nu=\frac{x}{x-1}\Delta\left\{\left(1+\frac{r^2}{R^2}\right)dt^2+2\left(1+\frac{r^2}{R^2}\right)f'dtdr-\left[\frac{1}{\Delta}\left(\frac{x-1}{x}\right)^2\left(1+\frac{r^2}{R^2}\right)^{-1}-\left(1+\frac{r^2}{R^2}\right)f'^2\right]dr^2\right\} - \frac{x-1}{x}r^2(d\theta^2+\sin^2\theta\,d\varphi^2), \tag{A12}$$

where $1/R^2=[(x-1)/x]\lambda'/6$ and

$$f'=\left[\left(1+\frac{\lambda}{6}r^2\right)^{-1}-\left(1+\frac{r^2}{R^2}\right)^{-1}\right]^{1/2}\left[\left(1+\frac{\lambda}{6}r^2\right)^{-1}-\left(\frac{x-1}{x}\right)^2\frac{1}{\Delta}\left(1+\frac{r^2}{R^2}\right)^{-1}\right]^{1/2}. \tag{A13}$$

This solution reduces to that in the text (with $\mu=0$) if one takes $x=3$ and $\lambda=0$. (If $\lambda\neq 0$, then it coincides with the exact solution recently obtained by Isham and Storey[9] for the old mixing term.)

The stability of our solution against changes in the details of the mixing term encourages us to believe that it may have a more basic significance than the explicit derivation would appear to suggest. This stability is perhaps relevant also to the discussion of Boulware and Deser[3] who showed (in the approximation $g=\eta$) that a sixth (scalar) degree of freedom is excited in the interacting f system. For the particular mixing term used in the text of our comment, they show that the Hamiltonian is unbounded below. No such statement is possible, on the face of it, for the term (A1), nor is there any statement for the complete f-g theory in either version of the mixing term. In fact, with the constraint (A5), the f-g Lagrangian essentially reduces, for this solution, to a sum of two "independent" cosmological Lagrangians. Thus, presumably, the problem of the boundedness of the Hamiltonian is reduced to the familiar problem of a pure gravitational Lagrangian containing a cosmological term. We believe that the question of such extra degrees of freedom in f-g theories is a deep problem and will perhaps be resolved when tensor masses are generated by a spontaneous mechanism.[5]

[1]C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. D 3, 867 (1971).

[2]This term is a scalar density with respect to general coordinate transformations. It must contain either a factor $\sqrt{-g}$ or $\sqrt{-f}$ or any other combination of these of total weight unity. In this note we have chosen the factor as $\sqrt{-g}$.

[3]According to D. G. Boulware and S. Deser [Phys. Rev. D 6, 3368 (1972)], the addition of a Pauli-Fierz type of mass term leads to the appearance of an additional spin-zero degree of freedom of f-g theory. To obviate this, we have suggested [Abdus Salam and J. Strathdee, Phys. Rev. D 14, 2830 (1976)] the use of the f-g Lagrangian

$$\mathcal{L}=\sqrt{-g}\,\frac{1}{\kappa_g{}^2}R(g)+\sqrt{-f}\,\frac{1}{\kappa_f{}^2}R(f)+\sqrt{-f}\,\tfrac{1}{4}\mu^2(f^{\mu\lambda}-g^{\mu\lambda})(f^{\nu\lambda}-g^{\nu\lambda})\,(\vec{F}_{\mu\nu}\cdot\vec{F}_{\kappa\lambda})\,,$$

where $\vec{F}_{\mu\nu}$ are Yang-Mills field strengths for a non-Abelian gauge theory. A soft mass term for the f field is generated as a result of quantum effects, with the operator products $A^i_\mu A^j_\nu$ and $f_{\kappa\lambda}f_{\mu\nu}$ (spontaneously and self-consistently) developing c-number parts. In the present note we have not used this model but have presented solutions for the original f-g equations resulting from (1.1) and (1.2) with the Newtonian constant $\kappa_g=0$.

[4]S. W. Hawking, in *Quantum Gravity*, edited by C. J. Isham, R. Penrose, and D. W. Sciama (Clarendon, Oxford, 1974), p. 219; Abdus Salam and J. Strathdee, Phys. Lett. (to be published).

[5]Abdus Salam and J. Strathdee, Ref. 3.

[6]The weak-field approximation is obtained by substituting the form $f_{\mu\nu}=\eta_{\mu\nu}+\kappa_f\phi_{\mu\nu}$ into (2.3) and expanding in powers of κ_f. The linear terms read

$$\eta^{\kappa\lambda}\,(\phi_{\mu\nu,\kappa\lambda}-\phi_{\mu\kappa,\lambda\nu}-\phi_{\nu\kappa,\lambda\mu}+\phi_{\kappa\lambda,\mu\nu})-\eta_{\mu\nu}\eta^{\kappa\lambda}\eta^{\sigma\tau}(\phi_{\kappa\lambda,\sigma\tau}-\phi_{\kappa\sigma,\lambda\tau})=-M^2\,(\phi_{\mu\nu}-\eta_{\mu\nu}\eta^{\kappa\lambda}\phi_{\kappa\lambda})\,,$$

corresponding to the propagation of a particle of spin 2 and mass M.

[7]C. Aragone and J. Chela Flores, Nuovo Cimento 10A, 818 (1972); J. Chela Flores, Int. J. Theor. Phys. 10, 103 (1974).

[8]G. W. Gibbons and S. Hawking, Phys. Rev. D 15, 2738 (1977).

[9]C. J. Isham and D. Storey, Phys. Rev. D (to be published).