Total cross sections of nucleon diffractive excitations*

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We discuss theoretical implications and resolutions of the recent observation that the total cross section of (diffractively produced) nucleon excitations off nucleon targets falls with the excitation mass. Both multichannel eikonal models and the scattering of deformed objects are studied.

In this paper we explore the theoretical implications of the recent work[1] of Edelstein *et al.*, who used their measurements to infer the total cross-sections of diffractively produced nucleon excitations (DPNE's) on nucleon targets as a function of the mass of the DPNE's. They find a systematic decrease of this cross section with the mass, as shown in Fig. 1. We believe that any final model of hadrons must incorporate one or both of the ingredients we discuss.

We find two types of models in which this result can be understood. The first type, a class of multichannel eikonal models,[2,3] accommodates the result in a conventional fashion. In this model the incorporation of s-channel unitarity is crucial to the decrease in the cross sections. The second model is less conventional, and is based on a geometric picture in which intrinsically deformed nucleons and DPNE's scatter from each other. By allowing the shapes of the nucleon and its excitations to vary systematically with mass (as might be required in a quark confinement picture)[4] we can obtain a cross section which behaves as in Fig. 1. This purely geometrical model is interesting in this context because, in contrast with the

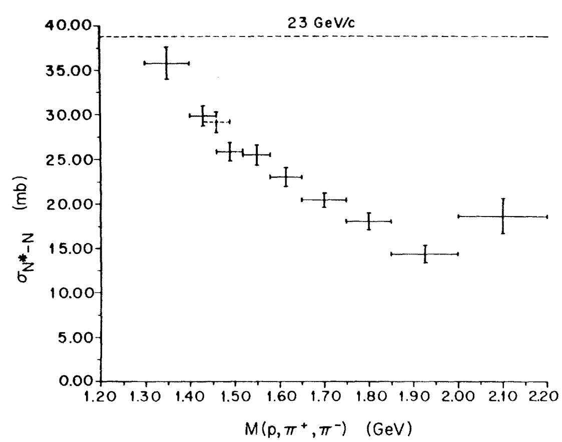

FIG. 1. The N^*N total cross section at 23 GeV/c as a function of the N^* mass M_{N^*}. M_{N^*} is the three-body mass $M(p, \pi^+, \pi^-)$ in the reaction $p + A \rightarrow A + (p, \pi^+, \pi^-)$ (Ref. 1). The horizontal dashed line is the NN total cross section.

first model, the effect is not a result of s-channel unitarity. We treat these two models in turn.

In the multichannel eikonal models[2,3] we employ[3] ("hopping" models) diffractive channels are treated explicitly and elementary transitions are allowed only between neighboring states. The transitions are triggered by an input eikonal or equivalently a profile function. All nondiffractive inelastic channels are treated through the absorptivity of the input eikonal, while the DPNE's are explicitly handled in unitary fashion by multiple exchange of the elementary object. This class of models qualitatively reproduces[2,3] many of the observed features[1] of diffractive excitation, such as the mass dependence of slopes and dips of production cross sections. Indeed, the possible influence of multichannel transitions on these features has been known for some time.[2] In this note, we employ a hopping model in which all DPNE's (labeled by a channel index j, the ground-state nucleon has $j=0$ while j increases as the mass of the DPNE increases) have equal elementary elastic transition strength. We discuss the derivation of the results of this model elsewhere.[5] (This model differs from that in Ref. 3 by its omission of an allowed elementary transition between the highest and lowest states. In other words, the model of Ref. 3 is a "ring" and is easier to treat mathematically than the model we use here; it is uninteresting, however, in relation to the data we discuss, because in such a ring model the symmetry of the problem makes all total cross sections identical.)

The cross sections are described by profile functions $\Gamma_{ij}(b)$, where b is the impact parameter and where i and j respectively label the initial and final two-particle states (one of the two particles is a nucleon, the other is the DPNE labeled i or j). Then

$$\sigma_i^{\text{tot}} = 4\pi \operatorname{Re} \int_0^\infty b\, db\, \Gamma_{ii}(b) \,, \tag{1}$$

$$\sigma_{i\rightarrow j} = 2\pi \int_0^\infty b\, db\, |\Gamma_{ij}(b)|^2 \,. \tag{2}$$

The Γ_{ij} are described in terms of the input profile $\Gamma^{(0)}(b)$ (containing the elementary elastic transition strength) and a parameter y, the ratio of the elementary inelastic transition strength to the elementary elastic transition strength, taken to be independent of b. Solutions of this hopping model are

$$\Gamma_{ij}(b)=\delta_{i,j}-e^{-A(b)}\{I_{i-j}[-2yA(b)] - I_{i+j+2}[-2yA(b)]\}, \quad (3)$$

where the input eikonal function is described by $A(b)$:

$$A(b)=-i\chi(b)=-\ln[1-\Gamma^{(0)}(b)]\,. \quad (4)$$

We assume that the input eikonal is purely absorptive, i.e., $A(b)$ is real. Unitarity in the one-dimensional problem (where, for example, only the ground state is considered) in the form $\sigma_{el}^{(0)} \leq \sigma_{tot}^{(0)}$ requires $0\leq\Gamma^{(0)}(b)\leq 1$. Unitarity for the multichannel problem then restricts the allowed value of y for a given $A(b)$ because of the restriction $\sigma_i^{tot}\geq\sum_j\sigma_{i\to j}$; since unitarity is diagonal in partial waves, this can be written

$$2\Gamma_{ii}(b)\geq\sum_j|\Gamma_{ij}(b)|^2\,. \quad (5)$$

All y's we use below will satisfy this restriction. We remark here, for use below, that if the input profile is completely black at some $b=b_0$ $(\Gamma(b_0)=1)$, then $A(b)\to_{b\to b_0}\infty$, and the condition (5) translates into the bound $|y|\leq\frac{1}{2}$. On the other hand, if the input profile is only gray rather than black $[\max(\Gamma(b))<1]$, then y is allowed to increase above $\frac{1}{2}$.

We can see immediately from Eq. (3) and the properties of the Bessel functions that the elastic profile $\Gamma_{ii}(b)$ falls for all b as the channel number i increases; in other words, the cross sections σ_i^{tot} and σ_i^{el} fall with i to a finite asymptotic value calculable from the asymptotic profile function

$$\Gamma_{ii}(b)\xrightarrow[i\to\infty]{} 1-e^{-A(b)}I_0(2yA(b))\,. \quad (6)$$

This establishes the qualitative agreement between the model and experiment; it remains to show that it is possible to find acceptable quantitative agreement.

We have investigated[6] the Gaussian and Fermi distributions for the input profile; together, these can approximate any reasonable profile. We find that the asymptotic ratio

$$R=\lim_{i\to\infty}\sigma_0^{tot}/\sigma_i^{tot}$$

increases with y, but that the maximum value of this ratio when y is majorized by $\frac{1}{2}$ is only ~1.15. Since the experimental value is ~2, we conclude either that the input profile must not be perfectly black at any b and/or that a model where the elementary transition strength y becomes dependent on i is necessary. (If one prefers the former alternative and believes that as energy increases the absorption due to nondiffractive channels also increases, then one should expect R to increase with energy. While the data of Ref. 1 show little energy dependence, within errors, the range of energy is rather small: 15- and 23-GeV/c incident momenta.) In fact, we find, by keeping y independent of i, but increasing the transparency, that we can approximate R. More detailed agreement with the curve of Fig. 1, however, requires a channel spacing of ~500 MeV, probably unrealistically large. Thus a more complicated model may be required.

We close discussion of this class of model with the observation that the slope at $t=0$ of the i-channel elastic differential cross section is given by

$$a^{(i)}=\frac{1}{2}\,\frac{\int_0^\infty b^3\,db\,\Gamma_{ii}(b)}{\int_0^\infty b\,db\,\Gamma_{ii}(b)}\,. \quad (7)$$

Computation of this quantity shows that we would expect an increase (steepening) in this slope with channel number to an asymptotic value; the relative change is similar to that of the cross section. Unfortunately this is a difficult quantity to measure.

We turn now to the geometric model, which is also consistent with the data. Here we treat DPNE's and nucleons as spheroidally deformed objects whose intrinsic deformations change with excitation energy. For simplicity we consider here the collision of such an object with a point projectile; the scattering of two such objects gives the same qualitative results without the pedagogical clarity.[7] The elastic amplitude may be written in the usual eikonal form, except that now the profile function depends on the orientation of the deformed object through the rotation $\mathcal{R}$ which carries the body-fixed coordinate system into the space-fixed frame:

$$f=\frac{ip}{2\pi}\int d^2b\,e^{i\vec{q}\cdot\vec{b}}\,\Gamma(\vec{b};\mathcal{R}); \quad (8)$$

To obtain physical quantities one must project out appropriately defined rotational states of the deformed object. Omitting details,[7] we find that the total cross section for state i on the point projectile has the form

$$\sigma_i^{tot}=2\int\frac{d\mathcal{R}}{8\pi^2}\int d^2b\,\Gamma_{ii}(\vec{b},\mathcal{R}). \quad (9)$$

Interestingly, the total cross section does not depend on the angular momentum of the state i, but only on its intrinsic deformation. (However, the elastic and transition cross sections do depend on angular momenta.) Equation (9) has the form of the average over orientations of the effective projected area of the deformed object. The additional factor of 2 is simply the usual edge-diffraction effect. In the limit of a completely black object, the total cross section of a spheroid is easily evaluated. Taking the surface to be given, in the intrinsic system of coordinates, by

$$\frac{x^2}{\alpha_i^{\,2}}+\frac{y^2}{\alpha_i^{\,2}}+\frac{z^2}{\beta_i^{\,2}}=1, \tag{10}$$

the total cross section becomes

$$\sigma_i^{\text{tot}}=\begin{cases}\pi\alpha_i\beta_i\left[\dfrac{\alpha_i}{\beta_i}+\dfrac{\sin^{-1}(1-\alpha_i^{\,2}/\beta_i^{\,2})^{1/2}}{(1-\alpha_i^{\,2}/\beta_i^{\,2})^{1/2}}\right], & \alpha_i<\beta_i\\[2ex] \pi\alpha_i\beta_i\left[\dfrac{\alpha_i}{\beta_i}+\dfrac{\sinh^{-1}(\alpha_i^{\,2}/\beta_i^{\,2}-1)^{1/2}}{(\alpha_i^{2}/\beta_i^{2}-1)^{1/2}}\right], & \alpha_i<\beta_i\,.\end{cases} \tag{11}$$

Presumably the dynamics of the intrinsic baryon states will provide restrictions which relate α_i and β_i in some manner analogous to volume conservation in spheroidal nuclei. Thus we might envision relationships such as

$$\alpha_i^{\,2}\beta_i=r^3=\text{const (constant volume)} \tag{12}$$

or

$$2\alpha_i^{\,2}+\beta_i^{\,2}=3r^2=\text{const (fixed rms radius)}. \tag{13}$$

In the first case we get (we drop the channel indices henceforth)

$$\sigma^{\text{tot}}=\pi\frac{r^3}{\alpha}\left\{\frac{\alpha^3}{r^3}+\frac{\sin^{-1}[1-(\alpha/r)^6]^{1/2}}{[1-(\alpha/r)^6]^{1/2}}\right\}, \tag{12$'$}$$

whereas in the second case we obtain

$$\sigma^{\text{tot}}=\pi\alpha(3r^2-2\alpha^2)^{1/2}\left\{\frac{\alpha}{(3r^2-2\alpha^2)^{1/2}}+\frac{\sin^{-1}\left[\dfrac{3(r^2-\alpha^2)}{3r^2-2\alpha^2}\right]^{1/2}}{\left[\dfrac{3(r^2-\alpha^2)}{3r^2-2\alpha^2}\right]^{1/2}}\right\}. \tag{13$'$}$$

In either case the curve has an extremum at zero deformation, $\alpha=\beta=r$; however, in (12′) it is a minimum, whereas in (13′) it is a maximum.

The observed decrease of the total DPNE-nucleon cross section with excitation energy can thus be understood either as a prolate→spherical (→oblate?) metamorphosis, as in (12′), or as a spherical→oblate transition as in (13′). Physical intuition suggests that a prolate deformation for a highly excited state is less likely than an oblate one, hence the directions proposed above. Interestingly, if we insist on conservation of $\langle r^n\rangle$, then for $n\to\infty$ the spherical→oblate transition of (13′) yields an asymptotic ratio of $R=2$. On the other hand, if we fit using a curve with a minimum [as in (12′)], the nucleon is required to have an intrinsic deformation in its ground state, a result inconsistent with bag models at their present stage of development.[8]

In conclusion, it is perhaps appropriate to comment on the relationship between the two models discussed above. The multichannel eikonal model permits only transitions between adjacent states, but sums all orders of perturbation theory and in principle includes diffractive transitions in which the intrinsic state of the hadron can change. By contrast, the geometric model, as formulated herein, includes diffractive inelastic excitations which may be represented as rotational states of an intrinsically deformed object, i.e., the intrinsic state of the hadron remains the same. Moreover, in its present stage of development, the geometric model only includes one- and two-step transitions, but not higher orders. (Note, however, that the geometric model can be extended to include multistep and intrinsic excitations, thereby bringing the two models into closer correspondence.)[9] In other words, although the two models are definitely related, we believe that, as currently formulated, they each produce the observed decrease through quite different physical mechanisms, and are for this reason worth contrasting. On the other hand, there remains in our opinion too much freedom and lack of theoretical justification to warrant detailed numerical fits in either case. For the same reason we do not see any really clean and currently feasible experimental way to distinguish the models.

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