PHYSICAL REVIEW D VOLUME 16, NUMBER 7 1 OCTOBER 1977

How many leptons and quarks?

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(Received 17 March 1977)

Based on the recently derived relation between the fine-structure constant and the Newtonian gravitational constant in a unified model of the Nambu–Jona-Lasinio type for all elementary-particle forces including gravity, I predict that there exist a dozen leptons (six neutrinos and six charged leptons) and a dozen flavors and three colors of quarks (6×3 up quarks and 6×3 down quarks) and that the arithmetic- and geometric-like-average masses of the charged leptons and quarks are about 35.2 GeV and 23.7 GeV, respectively.

Recently, Akama, Chikashige, Matsuki, and the present author have proposed a unified model of the Nambu–Jona-Lasinio type[1] for all elementary-particle forces including gravity.[2-5] Starting with a nonlinear fermion Lagrangian of the Heisenberg type and imposing the massless conditions of Bjorken[6] on vector auxiliary fields, we have constructed an effective Lagrangian which combines the unified gauge theory of Weinberg and Salam[7] for the weak and electromagnetic interactions of leptons and quarks and the asymptotically free gauge theory of Gross, Wilczek, and Politzer[8] for the strong interaction of quarks. The photon γ, the weak vector bosons $W^\pm$ and Z, and the physical Higgs scalar η appear as collective excitations of lepton-antilepton or quark-antiquark pairs while the color-octet gluons G^a ($a=1,2,3,\ldots,8$) appear as those of quark-antiquark pairs.

The most important results of our unified model of the Nambu–Jona-Lasinio type for strong, weak, and electromagnetic interactions[2,3] are the following: the Weinberg angle θ_W is determined to be[2,3]

$$\sin^2\theta_W = (\sum I_3^{\,2})/(\sum Q^2) = \tfrac{3}{8} \qquad (1)$$

for fractionally charged quarks where $\vec{I}$ and Q are the isospin and charge of leptons and quarks. The gluon coupling constant is also determined to be $\frac{8}{3}$ times the fine-structure constant α. These results coincide with those of Georgi and Glashow in their unified SU(5) gauge model.[9] However, our results are due not to such an assumed higher symmetry as SU(5) but to the Nambu–Jona-Lasinio dynamics in our model with only $\mathrm{SU}(3)_{\mathrm{color}}\times[\mathrm{SU}(2)\times \mathrm{U}(1)]_{\text{Weinberg-Salam}}$. In any case, the masses of the weak vector bosons are predicted in our model to be[3]

$$m_{W^\pm} = (\pi\alpha/\sqrt{2}\,G_F\sin^2\theta_W)^{1/2} = 60.9\ \mathrm{GeV}$$

and

$$m_Z = m_{W^\pm}/\cos\theta_W = 77.0\ \mathrm{GeV}, \qquad (2)$$

where G_F is the Fermi coupling constant ($G_F m_p^{\,2} = 1.026\times 10^{-5}$).

Entirely new and proper to our model are the following relations between the masses of the physical Higgs scalar and weak vector boson and those of leptons (ν_i and l_i) and quarks (u_i and d_i)[3]:

$$m_\eta = 2\left[\frac{\sum_{i=1}^{N}(m_{l_i}^{\,4}+3m_{u_i}^{\,4}+3m_{d_i}^{\,4})}{\sum_{i=1}^{N}(m_{l_1}^{\,2}+3m_{u_i}^{\,2}+3m_{d_i}^{\,2})}\right]^{1/2}$$

and

$$m_{W^\pm} = \left[\frac{3}{8N}\sum_{i=1}^{N}(m_{l_i}^{\,2}+3m_{u_i}^{\,2}+3m_{d_i}^{\,2})\right]^{1/2}. \qquad (3)$$

These relations strongly suggest that there exist much heavier leptons and/or quarks whose masses reach or go beyond the weak-vector-boson masses and that the physical Higgs scalar is roughly as heavy as the weak vector boson. Another important result is the relation between the fine-structure constant and the sum of the charge squared of leptons and quarks[4]:

$$\alpha = 3\pi/(\sum Q^2)\ln(\Lambda^2/m^2), \qquad (4)$$

where Λ is the cutoff momentum and m is the geometric-like-average mass of charged leptons and quarks defined by[10]

$$m = \prod_i m_i^{\,Q_i^2/(\Sigma Q^2)}. \qquad (5)$$

The relation (4) is essentially a result of Gell-Mann and Low in 1954.[11]

In our model of the Nambu–Jona-Lasinio type for gravity,[4,5] the graviton is also a collective excitation of a fermion-antifermion pair. A similar idea was first proposed by Phillips in 1966.[12] We have started with a very simple nonlinear fermion Lagrangian and again imposed the massless condition of Bjorken[6] on a tensor field, the gravitational field. The effective Lagrangian derived, then, reproduces the familiar Newtonian gravitational potential if the gravitational constant G is related with the total number N_0 of leptons and quarks (where neutrinos be counted as $\frac{1}{2}$ for each)[4,5]:

$$G = 4\pi/\kappa_0 N_0\Lambda^2, \qquad (6)$$

where $\kappa_0 = \frac{2}{3}$ or $\frac{5}{9}$ depending on the straight or invariant cutoff procedure. Very recently,[5] we have also found that, in a certain coordinate condition, the equation of motion derived from the effective Lagrangian can be reduced to the Einstein equation of motion in the weak-field approximation. We have further unified the unified model of all elementary-particle forces[2,3] and the model of gravity[4,5] into a truly unified model of all elementary-particle forces including gravity.[4]

The most exciting result of this grand unification is a simple relation between the fine-structure constant and the Newtonian gravitational constant:

$$\alpha = 3\pi/(\sum Q^2)\ln(4\pi/\kappa_0 N_0 G m^2)\,. \tag{7}$$

This relation (let us call it the G-α relation) can be easily derived from combining the two relations (4) and (6). Thus, we have succeeded in calculating the fine-structure constant in the model. Historically, a relation of this type was conjectured in an implicit form by Landau in 1955,[13] based on the idea that the effects of gravitational interaction may exceed the electromagnetic effects at the cutoff energy Λ. Together with the weak-interaction (or superficial) cutoff in quantum electrodynamics by the weak-vector-boson mass $m_{W^\pm}$ ($\sim 10^2$ GeV), which the present author suggested in 1969,[14] this gravitational (or genuine) cutoff approximately by the Planck mass $G^{-1/2}$ ($\sim 10^{19}$ GeV) or at the extremely short distance r_0 ($= G^{1/2} \sim 10^{-33}$ cm) would eliminate all existing infinities in quantum field theories.

What are left as the most fundamental problems for future investigations in our unified model are the following: (1) How many leptons and quarks? (2) Why the lepton and quark masses? (3) Why the Cabibbo angle? and (4) Why CP violation? In this short note, I shall present an answer to the first question and a possible clue to the second problem: (1) There exist a dozen leptons (six neutrinos and six charged leptons) and a dozen flavors and three colors of quarks (6×3 up quarks and 6×3 down quarks); and (2) the arithmetic- and geometric-like-average masses of the charged leptons and quarks are about 35.2 GeV and 23.7 GeV, respectively. The reason for these will be given in what follows.

Let us transform the G-α relation into the form of

$$m = (4\pi/\kappa_0 N_0 G)^{1/2}\exp[-3\pi/2\alpha(\sum Q^2)]\,. \tag{8}$$

Next, notice that $\sum Q^2 = \frac{8}{3}N$ and $N_0 = \frac{15}{2}N$ for N Weinberg-Salam multiplets of leptons and quarks. Furthermore, take $\kappa_0 = \frac{2}{3}$ for the straight cutoff procedure, which is more favorable to reproduce Einstein's gravity.[5] Then, the transformed G-α relation becomes

$$m = (4\pi/5NG)^{1/2}\exp(-9\pi/16\alpha N)\,. \tag{9}$$

This shows that the geometric-like-average mass of the charged leptons and quarks is given as a known function of the integer N when both the fine-structure constant and the Newtonian gravitational constant are fixed. Given the experimental value of α and G ($\alpha^{-1} = 137.036$ and $G^{-1/2} = 1.221\times 10^{19}$ GeV), the G-α relation finally becomes

$$m = (1.936\times 10^{19}/\sqrt{N})\exp(-242.163/N)\text{ GeV}. \tag{10}$$

The numerical results are, for example, $m = 8.11$ MeV for $N=5$, $m = 23.7$ GeV for $N=6$, and $m = 7.02$ TeV for $N=7$.

Since 8.11 MeV is even smaller than the muon mass, the case of $N=5$ must be excluded. Also, as 7.02 TeV seems too large to be the average mass of leptons and quarks, the case of $N=7$ should be physically excluded. What is left is the case where $N=6$ and $m = 23.7$ GeV. The geometric-like average of 23.7 GeV seems to be quite natural for the six charged leptons and a dozen flavors and three colors of quarks since the (kinetic) mass of the fourth lightest quark, the charmed quark, seems to lie between 1.5 GeV and 2.0 GeV and since the mass of the possible third lightest charged lepton, the heavy lepton τ, has been reported to be around 2 GeV.[15] Furthermore, I would like to present the following additional circumstantial evidence for the case of $m = 23.7$ GeV: As stated in (3), in our unified model, $\sqrt{3}$ times the arithmetic-like-average mass of leptons and quarks is equal to the mass of the charged weak vector boson. Remember also that, in the same model, the latter mass is predicted to be 60.9 GeV. Therefore, the arithmetic-like-average mass of leptons and quarks is predicted to be 35.2 GeV, which is of the same order of magnitude as 23.7 GeV.

For the above reasons, I reach the following conclusion: The case of $N=6$ is the most plausible and all the other cases are very unlikely. I, therefore, predict with confidence that there exist a dozen leptons (six neutrinos and six charged leptons) and a dozen flavors and three colors of quarks (6×3 up quarks and 6×3 down quarks) and also that the arithmetic- and geometric-like-average masses of the charged leptons and quarks are about 35.2 GeV and 23.7 GeV, respectively. Let me call these leptons and quarks as follows, paying respect to Perl and Harari:

$$\begin{gathered}\begin{pmatrix}\nu_e\\ e\end{pmatrix},\quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix},\begin{pmatrix}\nu_{\tau_n}\\ \tau\end{pmatrix},\\ \begin{pmatrix}p\\ n'\end{pmatrix},\quad \begin{pmatrix}p'\\ \lambda'\end{pmatrix},\quad \begin{pmatrix}t_n\\ b_n\end{pmatrix},\end{gathered} \tag{12}$$

for $n = 1, 2, 3, 4$.

Some physicists may now ask themselves why there exist so many leptons and quarks. In the remaining part of this short note, I shall present a possible answer by Akama and the present author to this question.[16] It is a "subquark" model of leptons and quarks in which leptons and quarks are made of three "subquarks" of spin $\frac{1}{2}$, w_i ($i = 1, 2$), h_i ($i = 1, 2, \ldots, N$), and C_i ($i = 0, 1, 2, 3$). The left-handed w_L and the right-handed w_{1R} and w_{2R} are a doublet and singlets of the Weinberg-Salam SU(2), respectively. The h_i's form an N-plet of the unknown H symmetry. Also, the C_0 and C_i's ($i = 1, 2, 3$) are singlet and triplet under the SU(3) color symmetry. A dozen leptons and a dozen flavors and three colors of quarks are expressed in terms of these subquarks as follows:

$$\nu_e = (w_1 h_1 C_0) \quad \nu_\mu = (w_1 h_2 C_0) \quad \nu_{\tau_n} = (w_1 h_{n+2} C_0)$$

$$e = (w_2 h_1 C_0) \quad \mu = (w_2 h_2 C_0) \quad \tau_n = (w_2 h_{n+2} C_0)$$

$$p_i = (w_1 h_1 C_i) \quad p_i' = (w_1 h_2 C_i) \quad t_{ni} = (w_1 h_{n+2} C_i)$$

$$n_i = (w_2 h_1 C_i) \quad \lambda_i = (w_2 h_2 C_i) \quad b_{ni} = (w_2 h_{n+2} C_i)$$

for $i = 1, 2, 3$ and $n = 1, 2, 3, 4$.

In the unified subquark model of all elementary-particle forces,[3] which is an alternative to the unified lepton-quark model, the gauge bosons γ, $W^\pm$, and Z appear as collective excitations of a w-$\bar{w}$ pair which behave as $(\bar{w}Qw)$, $(\bar{w}\tau^\pm w)$, and $(\bar{w}Rw)$ (where R is orthogonal to Q), respectively, while the color-octet gluons G^a appear as those of a C-$\bar{C}$ pair $(\bar{C}\lambda^a C)$. As a result, we have derived the following relations between the masses of the physical Higgs scalar and weak vector bosons and those of the w's:

$$m_\eta \simeq 2 m_w, \quad m_{W^\pm} \simeq \sqrt{3}\, m_w,$$

and (14)

$$m_Z \simeq (\sqrt{3}/\cos\theta_W) m_w .$$

These relations suggest that the masses of the physical Higgs scalar and weak vector bosons may be very close to the threshold of w-pair production, if any. This possible situation is very much similar to the one in which the masses of J/ψ and ψ', the new vector mesons, are very close to the threshold of the recently reported charmed-meson-pair production in the e^+e^- colliding-beam experiments.[17] I, therefore, strongly urge experimentalists to be still alert for producing possible w pairs even after the anticipated exciting discovery of the weak vector bosons in the 1980's. In any case, "subquark diagrams" would become relevant and useful in discussing the strong, weak, and electromagnetic interactions of leptons and quarks as quark diagrams in discussing those of hadrons.[16]

In conclusion, let me show the results for integrally charged quarks of the Han-Nambu type.[18] Since $\sum Q^2 = 4N$ (N even) in this case, the G-α relation becomes $m = (4\pi/5NG)^{1/2}\exp(-3\pi/8\alpha N)$. The numerical results are, for example, $m = 1.22 \times 10^{-7}$ eV for $N = 2$, $m = 29.1$ GeV for $N = 4$, and $m = 823$ TeV for $N = 6$. This strongly indicates that the case of $N = 4$ is the right answer. Namely, there exist eight leptons (four neutrinos and four charged leptons) and eight flavors and three colors of quarks (4×3 up quarks and 4×3 down quarks) and the geometric-like-average mass of the charged leptons and quarks is 29.1 GeV if quarks are integrally charged.

I would like to thank Professor A. Pais for discussions and the warm hospitality at Rockefeller University where this manuscript has been prepared, Professor S. D. Drell for discussions at Stanford Linear Accelerator Center, and Professor M. Gell-Mann and others for discussions and the warm hospitality at California Institute of Technology where this manuscript has been completed.

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*This work is supported in part by the Yoshida Foundation for Science and Technology.

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