PHYSICAL REVIEW D VOLUME 16, NUMBER 7 1 OCTOBER 1977

Consequences of quark-line (Okubo-Zweig-Iizuka) rule*

Susumu Okubo
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 19 May 1977)

Consequences of the validity of the quark-line (Okubo-Zweig-Iizuka) rule have been studied both theoretically and experimentally. The present experimental data are consistent with the validity of the rule for 1^- and possibly 2^+ nonets. With respect to the 0^- nonet, the rule is in reasonable agreement, if we consider a suitable mixing scheme for the η-η' complex. Theoretical implications of these facts are also discussed. The possible violation of the rule has been estimated to be 6% for the 1^- nonet and 15% for the 0^- nonet.

I. INTRODUCTION

The recent discovery[1] of ψ/J has renewed interest in the study of the quark-line (Okubo-Zweig-Iizuka) rule[2-4] (hereafter referred to as QLR). As we shall explain in Sec. V, the QLR is *not* a symmetry in the usual sense, since it cannot become possibly exact (even in principle) under any circumstances in contrast to the ordinary selection rules. Before explaining about the QLR, it may be worthwhile to briefly sketch its historical outline. As we now know, one important supporting evidence for the eightfold SU(3) scheme[5] of Gell-Mann and Ne'eman is the mass formula for the $J^p=\frac{1}{2}^+$ baryon octet,

$$\tfrac{1}{2}[m(N)+m(\Xi)]=\tfrac{1}{4}[3m(\Lambda)+m(\Sigma)], \tag{1.1}$$

which is experimentally well satisfied. However, it is soon realized that the corresponding quadratic mass formula for the 1^- vector octet

$$m^2(K^*)=\tfrac{1}{4}[3m^2(\omega_8)+m^2(\rho)] \tag{1.2}$$

predicts the mass value $m(\omega_8)\simeq 928$ MeV for the eighth member ω_8 of the octet, which should be compared to experimental values $m(\phi)=1020$ MeV and $m(\omega)=783$ MeV, for the masses of ϕ and ω, respectively. In order to account for the discrepancy, Sakurai[6] in 1962 proposed the so-called ω-ϕ mixing model in analogy to the level mixing of atomic and nuclear physics. In this scheme, the physical ω and ϕ are supposed to be coherent mixtures of ω_8 and ω_0,

$$\begin{aligned}\phi&=\cos\theta\omega_8-\sin\theta\omega_0,\\ \omega&=\sin\theta\omega_8+\cos\theta\omega_0,\end{aligned} \tag{1.3}$$

where ω_8 and ω_0 represent the eighth component of the vector octet V_8 and an SU(3) singlet V_0, respectively. The SU(3) mass formula will now simply determine the value of the mixing angle θ to be[7] either

$$\theta=\pm(40°\pm 1°), \tag{1.4a}$$

or

$$\theta=\pm(37°\pm 1°), \tag{1.4b}$$

depending upon whether we use the quadratic mass formula (1.2) or the corresponding linear formula

$$m(K^*)=\tfrac{1}{4}[3m(\omega_8)+m(\rho)]. \tag{1.2'}$$

We notice that the sign of θ is undetermined.

However, this model caused the following problem. Experimentally, the decay width of $\phi\to\pi^+\pi^-\pi^0$ is very small in comparison to that of $\omega\to\pi^+\pi^-\pi^0$. The currently accepted value[7] is

$$\Gamma(\phi\to\pi^+\pi^-\pi^0)/\Gamma(\omega\to\pi^+\pi^-\pi^0)=0.074. \tag{1.5}$$

If we take into account the larger phase volume available for the $\phi\to\pi^+\pi^-\pi^0$ mode, then the ratio of matrix elements for these decays is roughly estimated to be

$$M(\phi\to\pi^+\pi^-\pi^0)/M(\omega\to\pi^+\pi^-\pi^0)\simeq 0.10. \tag{1.6}$$

More accurately, the decay $\phi\to 3\pi$ is now found[8] to proceed dominantly via $\phi\to\rho\pi$, followed by $\rho\to 2\pi$. Then accepting the SU(6) relation

$$g_{\omega\rho\pi}\simeq 2g_{\rho\pi\pi}\simeq 2g_\rho\simeq 11.4\pm 0.5, \tag{1.7}$$

which gives a good value[9] for $\Gamma(\omega\to\pi^0\gamma)$ as well as $\Gamma(\omega\to 3\pi)$ by ρ dominance, we estimate the ratio of coupling constants $g_{\phi\rho\pi}$ and $g_{\omega\rho\pi}$ to be[10]

$$(g_{\phi\rho\pi})^2/(g_{\omega\rho\pi})^2\simeq 0.007. \tag{1.8}$$

In Eq. (1.7), g_ρ represents the standard ρ_0-γ coupling constant. But Eq. (1.3) demands

$$\frac{g_{\phi\rho\pi}}{g_{\omega\rho\pi}}=\frac{\cos\theta g_8-\sin\theta g_0}{\sin\theta g_8+\cos\theta g_0}, \tag{1.9}$$

where g_8 and g_0 are coupling constants of ω_8 and ω_0 to the ρ-π system, respectively. In order to account for the small ratio for (1.8), a large cancellation in the numerator of the right side in (1.9) must be taking place. This could be, of course, purely accidental. But a more interesting possibility is to postulate a principle underlying the cancellation. If such a principle exists, then the octet ω_8 and the singlet ω_0 cannot act independently of each other in contrast to a purely group-theoretical consideration based upon SU(3) symmetry. Hence, it is better to treat both octet V_8 and singlet V_0 as constituting a single entity which we call the vector nonet V_9 and represent by a nontraceless tensor G^μ_ν. Then the nonet hypothesis is the demand that the trace G^λ_λ should not enter into any physical expression. Under this assumption, it was shown[2] by the present author in 1963 that we indeed obtain

$$M(\phi \to \rho\pi)=0, \quad M(\phi \to \pi^+\pi^-\pi^0)=0 \qquad (1.10)$$

with mixing angle θ being equal to

$$\theta_0 = \tan^{-1}\frac{1}{\sqrt{2}} = 35°16', \qquad (1.11)$$

which is known now as the ideal mixing. Note that (1.11) is very close to the values given in (1.4). Moreover, the same consideration leads to the validity of the nonet mass formulas

$$m(\omega)=m(\rho), \qquad (1.12a)$$

$$m^2(K^*)-m^2(\rho)=m^2(\phi)-m^2(K^*), \qquad (1.12b)$$

which are experimentally fairly well satisfied. If we take into account the correction due to the trace part G^λ_λ, then we have to consider the mass operator of the form

$$M^2 = C_1\,\mathrm{Tr}(GG)+C_2\,\mathrm{Tr}(GG\lambda_8)+C_3(\mathrm{Tr}G)^2, \quad (1.13)$$

which leads to Schwinger's mass formula[11]

$$[m^2(\phi)-m^2(\omega_8)][m^2(\omega)-m^2(\omega_8)] = -\tfrac{8}{9}[m^2(K^*)-m^2(\rho)]^2, \quad (1.13')$$

where

$$m^2(\omega_8)=\tfrac{1}{3}[4m^2(K^*)-m^2(\rho)].$$

If we set $C_3=0$ in (1.13), we obtain, of course, two equations (1.12a) and (1.12b) instead of the single equation (1.13′). Experimentally, (1.13′) is very well satisfied.

In his 1964 papers, Zweig[3] noted that the nonet ansatz can easily be reinterpreted in terms of the quark model as follows. Let $q_1=p$ (or u), $q_2=n$ (or d), and $q_3=\lambda$ (or s) be three SU(3) quarks. Suppose that the nontraceless tensor G^μ_ν can be represented as

$$G^\mu_\nu = q_\nu \bar{q}_\mu, \qquad (1.14)$$

which symbolizes a bound state of q_ν and antiquark $\bar{q}_\mu$. Then ω and ϕ are written as

$$\omega = \frac{1}{\sqrt{2}}(G^1_1+G^2_2) = \frac{1}{\sqrt{2}}(q_1\bar{q}_1+q_2\bar{q}_2),$$
$$\phi = -G^3_3 = -q_3\bar{q}_3. \qquad (1.14')$$

Let us call any hadron nonstrange if its quark constituents (or valence quarks in more modern terminology) are purely of q_1 and q_2 and/or of their antiquarks $\bar{q}_1$ and $\bar{q}_2$. Then ω, π, ρ, and the nucleon N are nonstrange, while ϕ and K are not. This is because[11a] the former do not contain q_3 and $\bar{q}_3$ quarks, while ϕ and K do. Then, in terms of the quark lines, these decays, $\omega\to 3\pi$ and $\phi\to 3\pi$, are graphically depicted as in Figs. 1 and 2, respectively. Comparing both, we see that $\omega\to 3\pi$ can be depicted as a graph involving only continuous quark lines, while for $\phi\to 3\pi$, the quark lines for q_3 and $\bar{q}_3$ must be disjoint from the quark lines containing q_1 and q_2 constituting pions. The nonet rule which insists on the nonappearance of G^λ_λ terms is then equivalent to a hypothesis that first, ω and ϕ can be represented as in (1.14′) and second, the disjoint process corresponding to Fig. 2 (the so-called hairpin diagram) must somehow be either zero or very small in comparison to the matrix element of Fig. 1. The same interpretation of the nonet ansatz had also been independently found by Iizuka *et al.*[4] in 1966. Hereafter we shall refer to the rule as the quark-line rule (abbreviated as QLR). Note that the decay $\phi\to K^+K^-$ is allowed by the QLR as is seen from Fig. 3 and it is indeed a dominant decay mode of ϕ. We may say that the nonet formulation is the algebraic version of the QLR, while the quark-line diagrammatic approach is its geometrical visualization.

With respect to the nucleon, the quark model implies that the proton p and the neutron n are nonstrange, since they are supposed to be bound states[11a] of three nonstrange quarks as $q_1q_1q_2$ and $q_2q_2q_1$, respectively. Then, using a straightforward generalization of the algebraic formulation of the nonet ansatz, Sugawara and von Hippel[12] in 1966 predicted[12a] that the coupling constant $g_{\phi N\bar{N}}$ of the ϕ meson with the nucleon must be zero in the exact ideal case. This fact can be more easily

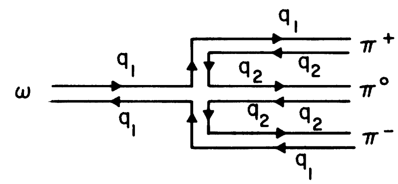

FIG. 1. Quark-line diagram for decay $\omega\to\pi^+\pi^0\pi^-$.

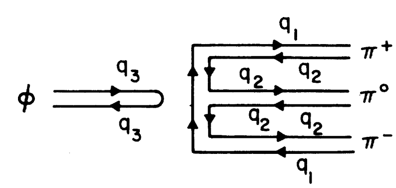

FIG. 2. Quark-line diagram for decay $\phi\to\pi^+\pi^0\pi^-$.

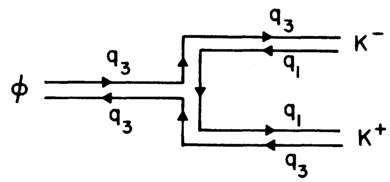

FIG. 3. Quark-line diagram for decay $\phi \to K^- K^+$.

seen from the corresponding graphical version of the QLR. Since the QLR cannot be exact as we will see in Sec. V, this should only imply

$$(g_{\phi N\bar{N}}/g_{\omega N\bar{N}})^2 \ll 1. \tag{1.15}$$

We shall come back to the experimental discussion of this prediction in the next section.

The analog of the nonet structure for the 1^- vector mesons also appears to be present for the 2^+ tensor nonet T_9 consisting of $f(1270\text{ MeV})$, $f'(1514\text{ MeV})$, $A_2(1310\text{ MeV})$, and $K^{**}(1420\text{ MeV})$, which correspond to ω, ϕ, ρ, and K^*, respectively. As noted by Glashow and Socolow[13] in 1965, we can readily understand the smallness of the decay width for $f' \to \pi^+\pi^-$ since the QLR will forbid it just as for $\phi \to 3\pi$. The present experimental data[14,15] indicate

$$\Gamma(f' \to \pi^+\pi^-)/\Gamma(f \to \pi^+\pi^-) = (0.33 \pm 0.20)\times 10^{-2}, \tag{1.16}$$

$$\Gamma(f' \to \pi^+\pi^-)/\Gamma(f' \to K^+K^-) = (1.6 \pm 0.6)\times 10^{-2}. \tag{1.17}$$

In terms of the coupling constants $g_{f'\pi\pi}$, g'_{fKK}, and $g_{f\pi\pi}$, these give[14,15] in fact small values of

$$(g_{f'\pi\pi}/g_{f\pi\pi})^2 \simeq 0.0021 \pm 0.0015, \tag{1.18}$$

$$(g_{f'\pi\pi}/g_{f'K\bar{K}})^2 \simeq 0.0043 \pm 0.0016. \tag{1.19}$$

Note that the suppression in (1.18) is smaller than (1.8) for the 1^- nonet. Also, the Schwinger mass formula (1.13′) is quite well satisfied for the 2^+ nonet as well as for the 1^- case.

We shall review further experimental evidence of the QLR for both 1^- and 2^+ nonets in the next section, for the 0^- nonet in Sec. III, and for the ψ particle in Sec. IV. Theoretical implications as well as the limitation of the QLR will be discussed in the last section.

II. EVIDENCE OF THE QLR FOR 1^- AND 2^+ NONETS

In the preceding section, we have seen some evidence of the validity of the QLR. We shall now examine it in more detail. Let us recall the definition that any hadron is nonstrange, if it does not contain the strange quark q_3. Also, if we assume the ideal mixing angle

$$\theta_0 = \tan^{-1}\frac{1}{\sqrt{2}} = 35°16' \tag{2.1}$$

as in (1.11), then ϕ consists of purely $q_3\bar{q}_3$, while the ω meson is nonstrange. However, we shall consider here a slightly more general case where the mixing angle θ in (1.3) is not exactly equal to θ_0, but differs slightly from it as in (1.4). Noting the fact

$$\omega_8 = \frac{1}{\sqrt{6}}(q_1\bar{q}_1 + q_2\bar{q}_2 - 2q_3\bar{q}_3),$$
$$\omega_0 = \frac{1}{\sqrt{3}}(q_1\bar{q}_1 + q_2\bar{q}_2 + q_3\bar{q}_3), \tag{2.2}$$

then the ϕ meson can now contain a small portion of nonstrange component in its constituent.

Next, let us consider an exclusive reaction

$$A + B \to C_1 + C_2 + \cdots + C_n + (q_3\bar{q}_3) \tag{2.3}$$

and suppose that all particles, $A, B, C_1, C_2, \ldots, C_n$ in the reaction (2.3) are nonstrange, i.e., they contain only q_1, q_2, $\bar{q}_1$, and $\bar{q}_2$ but *not* q_3 and $\bar{q}_3$ quarks. The reaction (2.3) must then proceed only via a disconnected hairpin diagram analogous to Fig. 2 for the q_3 quark line. Therefore, the QLR demands that its matrix element should ideally be zero, i.e.,

$$M[A + B \to C_1 + C_2 + \cdots + C_n + (q_3\bar{q}_3)] = 0. \tag{2.4}$$

More accurately, we imply by (2.4) that the magnitude of ratio

$$Z = \frac{\sqrt{2}M[A + B \to C_1 + \cdots + C_n + (q_3\bar{q}_3)]}{M[A + B \to C_1 + \cdots + C_n + (q_1\bar{q}_1)] + M[A + B \to C_1 + \cdots + C_n + (q_2\bar{q}_2)]} \tag{2.5}$$

be small in comparison to one, i.e.,

$$|Z| \ll 1. \tag{2.6}$$

Rewriting $(q_3\bar{q}_3)$ in terms of physical ϕ and ω by means of (1.3) and (2.2), we find

$$\frac{M(A + B \to C_1 + C_2 + \cdots + C_n + \phi)}{M(A + B \to C_1 + C_2 + \cdots + C_n + \omega)} = -\frac{Z + \tan(\theta - \theta_0)}{1 - Z\tan(\theta - \theta_0)} \tag{2.7}$$

for the ratio of production matrix elements of ϕ

and ω. So far, (2.7) is exact. Since we have assumed $\theta \approx \theta_0$, together with (2.6), i.e., $|Z| \ll 1$, this leads to[12a]

$$\frac{\sigma(A+B \to C_1+C_2+\cdots+C_n+\phi)}{\sigma(A+B \to C_1+C_2+\cdots+C_n+\omega)} \ll 1 \qquad (2.8)$$

for the ratio of production cross sections for ϕ and ω.

We now experimentally check the validity of (2.8).

(1) $p\bar{p} \to \pi^+\pi^- V$ (see Refs. 16 and 16a):

$$\frac{\sigma(p\bar{p} \to \pi^+\pi^-\phi)}{\sigma(p\bar{p} \to \pi^+\pi^-\omega)} = \begin{cases} 0.011^{+0.003}_{-0.004} \text{ at } p_L = 1.2 \text{ GeV}/c, \\ 0.009^{+0.004}_{-0.007} \text{ at } p_L = 3.6 \text{ GeV}/c, \\ 0.020 \pm 0.003 \text{ at } p_L = 2.32 \text{ GeV}/c; \end{cases} \qquad (2.9)$$

(2) $\pi^- p \to nV$ (Ref. 17) at $p_L = 5-6$ GeV/c:

$$\frac{\sigma(\pi^- p \to \phi n)}{\sigma(\pi^- p \to \omega n)} = 0.0035 \pm 0.0015; \qquad (2.10)$$

(3) $\pi^+ n \to pV$:

(a) $p_L = 1.54-2.60$ GeV/c (Ref. 18)

$$\frac{\sigma(\pi^+ n \to \phi p)}{\sigma(\pi^+ n \to \omega p)} = 0.021 \pm 0.011, \qquad (2.11a)$$

(b) $p_L = 5.1$ GeV/c (Ref. 19)

$$\frac{\sigma(\pi^+ n \to \phi p)}{\sigma(\pi^+ n \to \omega p)} < 0.020 \pm 0.004, \qquad (2.11b)$$

(c) $p_L = 5.4$ GeV/c (Ref. 20)

$$\frac{\sigma(\pi^+ n \to \phi p)}{\sigma(\pi^+ n \to \omega p)} < 0.06 \pm 0.02; \qquad (2.11c)$$

(4) $\pi^+ p \to \Delta^{++} V$:

(a) $p_L = 3.70$ GeV/c (Ref. 21)

$$\frac{\sigma(\pi^+ p \to \phi\Delta^{++})}{\sigma(\pi^+ p \to \omega\Delta^{++})} < 0.0033, \qquad (2.12a)$$

(b) $p_L = 8$ GeV/c (Ref. 22)

$$\frac{\sigma(\pi^+ p \to \phi\Delta^{++})}{\sigma(\pi^+ p \to \omega\Delta^{++})} \simeq 0.024; \qquad (2.12b)$$

(5) $\pi^+ p \to \pi^+ pV$:

(a) $p_L = 3.54$ GeV/c (Ref. 23)

$$\frac{\sigma(\pi^+ p \to \phi\pi^+ p)}{\sigma(\pi^+ p \to \omega\pi^+ p)} = 0.019 \pm 0.011, \qquad (2.13a)$$

(b) $p_L = 8$ GeV/c (Ref. 22)

$$\frac{\sigma(\pi^+ p \to \phi\pi^+ p)}{\sigma(\pi^+ p \to \omega\pi^+ p)} < 0.015 \pm 0.003 \qquad (2.13b)$$

(6) $\pi^- p \to \pi^-\pi^-\pi^+ pV$ at $p_L = 19$ GeV/c (Ref. 24):

$$\frac{\sigma(\pi^- p \to \phi\pi^-\pi^-\pi^+ p)}{\sigma(\pi^- p \to \omega\pi^-\pi^-\pi^+ p)} = 0.005^{+0.005}_{-0.002}; \qquad (2.14)$$

(7) $pp \to ppV$, etc. at $p_L = 24$ GeV/c (Ref. 25):

(a) $$\frac{\sigma(pp \to \phi pp)}{\sigma(pp \to \omega pp)} = 0.0265 \pm 0.0188, \qquad (2.15a)$$

(b) $$\frac{\sigma(pp \to \phi\pi^+\pi^- pp)}{\sigma(pp \to \omega\pi^+\pi^- pp)} = 0.001\,15 \pm 0.0008, \qquad (2.15b)$$

(c) $$\frac{\sigma[pp \to \phi(n\pi^+)(n\pi^-)pp]}{\sigma[pp \to \omega(n\pi^+)(n\pi^-)pp]} = 0.018 \pm 0.009. \qquad (2.15c)$$

In (2.15c), the cross sections are averages of those for n-pion pair productions with $n = 0, 1, 2,$ and 3.

Comparing these with (2.8), we can say that they are of the same order as in (1.8), and that the QLR is quite reasonable. However, one possible exception is the unpublished result of the Syracuse group,[26]

$$\frac{\sigma(\bar{p}n \to \pi^-\phi)}{\sigma(\bar{p}n \to \pi^-\omega)} = 0.253 \pm 0.059, \qquad (2.16)$$

at rest $p_L = 0$. Such a large violation of the QLR is rather difficult to reconcile with other experimental data given so far. Further experimental verification of (2.16) is desirable. For analysis which is to be discussed below, we omit taking into account the value in (2.16) because of its anomaly.

To determine the most likely value of the mixing angle θ from these data, we observe first that the value of Z depends upon individual production channels under consideration, while the mixing angle θ does not. We assume that the phase and magnitude of the QLR-violating term Z will vary its values at random with varying reaction channels. In other words, we regard Z to be a random variable with its mean value being zero. If we take the average of the right side of (2.7) over all possible reaction channels, then its dependence upon Z will cancel out in the lowest order of Z. In this way we can approximately determine

$$\theta - \theta_0 = \pm 5°43' \qquad (2.17)$$

if we assign the average value 0.01 to the ratio of cross sections in (2.9)–(2.15).

Alternately, we can proceed as follows. Let us set

$$\beta = \frac{Z + \tan(\theta - \theta_0)}{1 - Z\tan(\theta - \theta_0)}. \qquad (2.18)$$

Then, as we see from (2.9) to (2.15), the experimental range of $|\beta|^2$ varies in an interval (we neglect spin complications)

$$0.0012 \le |\beta|^2 \le 0.026. \qquad (2.19)$$

Since Z is assumed to be a random variable with its average value zero, we may determine the most likely value of $\tan(\theta - \theta_0)$ by requiring the maximum dispersion of $|Z|$ to be minimum when

β changes in the range allowed by (2.19). Then we determine the most likely value of θ to be

$$\theta - \theta_0 = \pm 5°34', \qquad (2.17')$$

which is not so different from the previous rough estimate (2.17). Note that (2.17′) implies θ to be either 39°50′ or 30°42′. The former value of $\theta = 39°50'$ is very close to the value (1.4a) based upon the quadratic mass formula, although (1.4b) could be still reasonable. This excludes a negative θ solution such as $\theta = -40°$ in (1.4a). Also, this dispersion (or variation) of $|Z|$ in this case is calculated to lie in the interval (we neglect complications due to spin)

$$|Z| \leq 0.062. \qquad (2.20)$$

In other words, the QLR violations are less than 6% for all reactions under consideration, i.e., the QLR is satisfied within 6%. As we shall see in Sec. V, the QLR cannot possibly be exact even in principle. Therefore, its validity at better than a level of 6% is quite remarkable. We may remark that the small deviation of the mixing angle θ from the ideal value of $\theta_0 = \tan^{-1} 1/\sqrt{2}$ is probably another reflection of the small violation of the QLR too. However, its precise mechanism depends upon dynamical consideration.[27]

Validity of the QLR for the 1^- nonet can be seen also from various decay rates. One example is (1.5) or (1.8), and the second one is

$$\frac{\Gamma(B \to \pi\phi)}{\Gamma(B \to \pi\omega)} \ll 1, \qquad (2.21)$$

where B is the B meson with mass 1228 MeV, and $J^P = 1^+$. Experimentally,[7] we know[28] that the left side of (2.21) is less than 0.01. In addition, if we assume the validity of exact SU(3) symmetry, then we compute

$$\frac{\Gamma(\phi \to K^+K^-)}{\Gamma(\rho \to \pi\pi)} = \tfrac{3}{4}(\cos\theta)^2 \left[\frac{m(\rho)}{m(\phi)}\right]^2 \left(\frac{k'}{k}\right)^3, \qquad (2.22)$$

where k' and k are magnitudes of the momenta of K and pion in the respective rest frames of ϕ and ρ^0. For three different values of θ, which are equal to θ_0, 40° and 37°, the right side of (2.22) gives 0.013, 0.0114, and 0.012 35, respectively, which should be compared to the experimental value of 0.0126. When we note that a possible SU(3) violation could amount to as much as 10% in this case,[29] the agreement is excellent for all cases.

If we assume standard Regge-pole analysis together with SU(3) symmetry, then we can test the QLR in the ideal mixing case $\theta = \theta_0$. In this way, Lipkin[30] has found the following relations:

$$\sigma(K^-p \to \omega\Lambda) = \sigma(K^-p \to \rho^0\Lambda), \qquad (2.23)$$

$$\sigma(K^-p \to \phi\Lambda) = \sigma(\pi^-p \to K^{0*}\Lambda). \qquad (2.24)$$

These relations are experimentally well satisfied at $p_L = 3.9$ GeV/c as has been already noted by Lipkin. In passing, we remark that the relations (2.23) and (2.24) are also known as the quark-model predictions of Alexander *et al.*,[31] who originally derived these relations by means of a simple quark model.

Moreover, the QLR together with SU(3) symmetry leads to

$$\frac{\Gamma(K^{**} \to K\omega)}{\Gamma(K^{**} \to K\rho)} = \tfrac{1}{3}\{\cos(\theta - \theta_0) + \sqrt{2}\sin(\theta - \theta_0)\}^2 \times (k'/k)^5. \qquad (2.25)$$

Actually, for the ideal mixing case $\theta = \theta_0$, the validity of this relation depends only upon the QLR involving q_1 and q_2 quarks (rather than q_3 quarks) but not upon SU(3). At any rate, the right side of (2.25) is calculated to be 0.279 for $\theta = \theta_0$, 0.304 for $\theta = 37°$, and 0.346 for $\theta = 40°$. The present experimental value is 0.682 ± 0.432. Considering the experimental error together with a possible SU(3)-violating effect, the agreement is not unreasonable. In comparison, the SU(3) symmetry for other $2^+ \to 1^-0^-$ reactions will predict (without QLR)

$$\frac{\Gamma(K^{**} \to K^*\pi)}{\Gamma(K^{**} \to \rho K)} = \left(\frac{k'}{k}\right)^5 \simeq 3.906,$$
$$\frac{\Gamma(K^{**} \to K^*\pi)}{\Gamma(A_2 \to \rho\pi)} = \frac{3}{8}\left(\frac{k'}{k}\right)^5 \simeq 0.394, \qquad (2.26)$$

which should be compared to experimental values of 4.68 ± 1.52 and 0.461 ± 0.108, respectively.

We have noted in the preceding section that the coupling constant $g_{\phi N\bar{N}}$ should be small as in (1.15). This fact is consistent with the small production cross section of the ϕ meson with the nucleon target as we see from (2.9)–(2.15). However, from a pole analysis of the nucleon, Genz and Höhler[32] find

$$g_1(\phi N\bar{N})/g_1(\omega N\bar{N}) \simeq -0.40 \qquad (2.27)$$

in contrast to (1.15). From (2.5), we can estimate Z in this case to be a very large value,

$$Z \simeq 0.30, \qquad (2.27')$$

which is at least five times larger than the bound in (2.20). We may, however, keep in mind the following facts. First, the pole analysis of the electromagnetic form factor may not be reliable, since we do not really understand the dipole nature of the form factor. Second, an alternative interpretation is possible,[33] if the radially excited state ω' of the ω with mass around 1200 MeV exists and couples strongly with the nucleon. Then the isoscalar contribution, discussed by Genz and Höhler can result from the ω' contribution rather than that from the ϕ. In that case, $g_1(\phi N\overline{N})$ in (2.27) should be replaced by $g_1(\omega' N\overline{N})$ so that the large QLR violation simply disappears. Another possible advantage of the existence of the ω' is the fact[33] that it may help to explain the smallness of the $\rho^\pm \to \pi^\pm\gamma$ width in comparison to $\omega \to \pi^0\gamma$, since the ω' pole in addition to the ω pole could now contribute to the $\rho^\pm \to \pi^\pm\gamma$ decay width. We will come back to the problem of $\rho^\pm \to \pi^\pm\gamma$ in Sec. III. In spite of these attractive features, the possible existence of ω' will give rise to the vexing question of why ω' then does not appreciably mix with ω and ϕ. All present data are in general consistent with the simplest ω-ϕ mixing theory we have been dealing with.

Summarizing what we have found so far, we may say that the QLR is in general excellently satisfied for the 1^- nonet. We could test it in the future, for the $e\overline{e} \to V_9 P_8 P_8$ reactions. In the ideal mixing case $\theta = \theta_0$, the QLR together with SU(3) will predict the following relations[34]:

$$\sigma(e\overline{e} \to \phi\pi^+\pi^-) = 0, \tag{2.28}$$

$$\sigma(e\overline{e} \to \rho^0 K^+ K^-) = \sigma(e\overline{e} \to \omega K^+ K^-), \tag{2.29}$$

$$\begin{aligned}\sigma(e\overline{e} \to \rho^0 K^0 \overline{K}^0) &= \sigma(e\overline{e} \to \omega K^0 \overline{K}^0) \\ &= \sigma(e\overline{e} \to \omega\pi^+\pi^-) \\ &= \sigma(e\overline{e} \to K^{0*}\pi^0\overline{K}^0) \\ &= \tfrac{1}{2}\sigma(e\overline{e} \to \phi K^0\overline{K}^0) \\ &= 3\sigma(e\overline{e} \to K^{0*}\eta_8\overline{K}^0),\end{aligned} \tag{2.30}$$

$$\sigma(e\overline{e} \to \phi K^+ K^-) = \sigma(e\overline{e} \to K^{0*}\pi^+ K^-). \tag{2.31}$$

Also, a possible test of the QLR for ϕ-meson production in extreme high-energy pp collisions is discussed by Frautschi *et al.*[35]

Until now, we have investigated consequences of the QLR for the 1^- nonet. For the 2^+ nonet, we have already seen the smallness of (1.16) and (1.18). The inequality (2.8) should also be valid, if we replace ϕ and ω there by f' and f, respectively. For example, we find at $p_L = 8$ GeV/c (Ref. 22)

$$\frac{\sigma(\pi^+ p \to f'\Delta^{++})}{\sigma(\pi^+ p \to f\Delta^{++})} < 0.037 \pm 0.003, \tag{2.32}$$

which should be compared to (2.12) and (1.16). This is compatible with the validity of the QLR. Also at $p_L = 2.32$ GeV/c, we have[16a]

$$\frac{\sigma(p\overline{p} \to f'\pi^+\pi^-)}{\sigma(p\overline{p} \to f\pi^+\pi^-)} = 0.028 \pm 0.009. \tag{2.32$'$}$$

Another important fact is the f-f' interference.[30] Utilizing this fact on $\pi N \to fN$ and $f'N$ reactions, Pawlicki *et al.*[14] appear to find a similar value to the cross-section ratio, although the exact value is not stated. However, some other data may indicate rather large violations. At $p_L = 1.5$ GeV/c, Picciarelli *et al.*[36] observe

$$\frac{\Gamma(f \to \pi^+\pi^+\pi^-\pi^-)}{\Gamma(f \to \text{all})}\,\sigma(\pi^+ n \to fp) = 9 \pm 1\ \mu b,$$

while Mettel[19] at the same energy finds

$$\frac{\Gamma(f' \to K^*\overline{K}) + \Gamma(f' \to \overline{K}^* K)}{\Gamma(f' \to \text{all})}\,\sigma(\pi^+ n \to f'p) = 6.4 \pm 2.4\ \mu\text{b}.$$

Since we know[7] that

$$\Gamma(f \to \pi^+\pi^+\pi^-\pi^-)/\Gamma(f \to \text{all}) = 0.028,$$

$$\frac{\Gamma(f' \to K^*\overline{K}) + \Gamma(f' \to \overline{K}^* K)}{\Gamma(f' \to \text{all})} \le 0.35,$$

these imply a rather large ratio,

$$\frac{\sigma(\pi^+ n \to f'p)}{\sigma(\pi^+ n \to fp)} \ge 0.080 \pm 0.038. \tag{2.33}$$

This is actually an underestimate, since $\Gamma(f' \to K^*\overline{K})$ is expected to be far smaller [see (2.42)]. However, a possibility exists that the measured data might *not* really be the decay of f'(1514 MeV) with $I = 0$, but rather of F_1(1540 MeV) with $I = 1$ into $K^*\overline{K}$ and $\overline{K}^* K$ modes. Note that $\Gamma(f' \to \text{all}) = 40 \pm 10$ MeV and $\Gamma(F_1 \to \text{all}) = 40 \pm 15$ MeV so that they could overlap each other for the decay channel $K^*\overline{K}$. Also, a possibly rather large cross section for the QLR-violating reaction $p\overline{p} \to \pi^0 f'$ at $p_L = 0.7$ GeV/c has been reported,[37] although by the same complication due to the presence of the F_1 meson and of the f-f' interference, caution is perhaps warranted. In view of the scarcity of data relevant to the f-f' mixing problem, more experimental data on these cross sections are definitely desirable.

If we assume the validity[38] of SU(3) symmetry, then we can test the QLR for the 2^+ nonet. Let θ_T be the mixing angle for the 2^+ nonet just as in (1.3), i.e.,

$$\begin{aligned} f' &= \cos\theta_T f_8 - \sin\theta_T f_0, \\ f &= \sin\theta_T f_8 + \cos\theta_T f_0. \end{aligned} \tag{2.34}$$

Then, the SU(3) mass formula gives[7]

$$\theta_T = \pm(29° \pm 2°),$$

or (2.35)

$$\theta_T = \pm(31° \pm 2°),$$

depending upon whether we use the linear or quadratic mass formula. But we will not consider the case of negative θ_T as in the 1^- case. SU(3) symmetry together with the QLR predicts

$$R_1 = \frac{\Gamma(f\to K\bar{K})}{\Gamma(f\to\pi\pi)} = \tfrac{1}{3}[1+\sqrt{2}\tan(\theta_0-\theta_T)]^2\left(\frac{k'}{k}\right)^5, \quad (2.36)$$

$$R_2 = \frac{\Gamma(f'\to K\bar{K})}{\Gamma(f\to\pi\pi)} = \tfrac{1}{3}[\sqrt{2}-\tan(\theta_0-\theta_T)]^2\left[\frac{m(f)}{m(f')}\right]^2\left(\frac{k'}{k}\right)^5, \quad (2.37)$$

$$R_3 = \frac{\Gamma(f\to K\bar{K})}{\Gamma(A_2\to K\bar{K})} = [\cos(\theta_0-\theta_T)+\sqrt{2}\sin(\theta_0-\theta_T)]^2\left[\frac{m(A_2)}{m(f)}\right]^2\left(\frac{k'}{k}\right)^5, \quad (2.38)$$

where k' and k are magnitudes of momenta of daughter particles in the rest frames of the respective parent particles. Also, we remark that (2.38) is valid without assuming SU(3) symmetry for the case of the ideal mixing $\theta_T=\theta_0$, i.e., that it is a consequence of the QLR (for the q_1 and q_2 quarks rather than the q_3 quark) alone in that case. The numerical values of R_1, R_2, and R_3 are computed to be

$$R_1 = \begin{cases} 0.035, & \theta_T=\theta_0=35°16', \\ 0.043, & \theta_T=31°, \\ 0.047, & \theta_T=29°, \end{cases} \quad (2.36')$$

$$R_2 = \begin{cases} 0.314, & \theta_T=\theta_0=35°16' \\ 0.282, & \theta_T=31°, \\ 0.267, & \theta_T=29°, \end{cases} \quad (2.37')$$

$$R_3 = \begin{cases} 0.711, & \theta_T=\theta_0=35°16', \\ 0.864, & \theta_T=31°, \\ 0.937, & \theta_T=29°. \end{cases} \quad (2.38)$$

These values should be compared[39] to experimental values of

$$R_1 = 0.033\pm0.008,$$
$$R_2 = 0.274\pm0.097, \quad (2.39)$$
$$R_3 = 1.014\pm0.490.$$

The general agreement is reasonable for all cases by the following reasons. First of all, we did not take into account corrections due to finite widths of f, f', and A_2. Second, the SU(3) violation may be considerable. As a matter of fact, the SU(3) symmetry gives us

$$\frac{\Gamma(A_2\to K\bar{K})}{\Gamma(K^{**}\to K\pi)} = \frac{2}{3}\left[\frac{m(K^{**})}{m(A_2)}\right]^2\left(\frac{k'}{k}\right)^5 \simeq 0.127, \quad (2.40)$$

which should be compared to the experimental value of 0.079 ± 0.023.

Also, SU(3) symmetry predicts

$$\frac{\Gamma(f'\to K^*\bar{K})+\Gamma(f'\to\bar{K}^*K)}{\Gamma(A_2\to\rho\pi)} = \tfrac{3}{2}(\cos\theta_T)^2\left(\frac{k'}{k}\right)^5, \quad (2.41)$$

which gives

$$\frac{\Gamma(f'\to K^*\bar{K})+\Gamma(f'\to\bar{K}^*K)}{\Gamma(f'\to \text{all})} = \begin{cases} 0.010, & \theta_T=\theta_0=35°16', \\ 0.011, & \theta_T=31°, \\ 0.012, & \theta_T=29°. \end{cases} \quad (2.42)$$

III. QLR FOR 0⁻ NONET

In preceding sections we have tested the validity of the QLR for the 1^- and 2^+ nonets. The same idea may be applicable to the 0^- nonet. However, there are several possible complications involved for a simple extension to the 0^- case. First, we may have more than two candidates for the nonet partner of η. They are η'(958 MeV) and E(1416 MeV). It is known[40] that the Schwinger mass formula is better satisfied by a choice of $(\pi,K,\bar{K},\eta,E)$ rather than $(\pi,K,\bar{K},\eta,\eta')$ for the 0^- nonet. However, from a study of $\eta'\to\eta\pi^+\pi^-$ and $E\to K\bar{K}\pi$, Ueda[41] has concluded that η' is preferable to E as the nonet partner of the η. Moreover, a recent experiment[42] strongly suggest that $J^P=1^+$ assignment for E, so as to preclude E. In this note, we assume that η' is the nonet partner of η. Second, the 0^- nonet does not satisfy the typical mass formula of the 1^- and 2^+ nonet. Indeed, the analog of Eq. (1.12a) is $m(\eta')=m(\pi)$, which is very badly violated. Third, the 0^- particles are likely to be Nambu-Goldstone particles which are related to spontaneous breakdown of the chiral symmetry $U_R(3)\otimes U_L(3)$. Hence, they may have characteristics entirely different from the 1^- and 2^+ nonets. Indeed, the second and third points mentioned above are connected with the so-called η puzzle[43] in quantum chromodynamics[44] (hereafter referred to as QCD), although the pseudoparticle solution by G. 't Hooft[45] may have resolved the difficulty. Fourth, in QCD, η and η' could couple strongly to two-gluon states. In comparison, ω and ϕ can couple with three gluons. If we accept the asymptotically free QCD model,[46] this will imply that the two-gluon state contained in η and η' may not be negligible.[47] This fact may be related to a relatively large $C_3(\mathrm{Tr}G)^2$ in (1.13) so as to invalidate (1.12) (see Sec. V). The neglect of three-gluon states in ω and ϕ may be justified in view of the smaller coupling constant due to the larger masses of ω and ϕ. Or at least its major effect will be absorbed presumably into the small deviation of the mixing angle θ from its ideal value θ_0.

Regardless of these problems, it is nevertheless worthwhile to consider consequences of the QLR for the 0^- nonet. To maintain the general aspect discussed above, let us, however, suppose that η and η' may mix not only between them but also with additional unspecified particle or particles which we collectively refer to as ξ. Then the physical η and η' would be phenomenologically expressed as

$$\begin{aligned} \eta &= S_1(\cos\theta_1\eta_8 - \sin\theta_1\eta_0) + S_1'\xi, \\ \eta' &= S_2(\sin\theta_2\eta_8 + \cos\theta_2\eta_0) + S_2'\xi, \end{aligned} \tag{3.1}$$

where S_1, S_2, S_1', S_2' are some constants and we introduced two mixing angles θ_1 and θ_2. This generalizes the simple mass-mixing scheme which demands

$$\begin{aligned} &\theta_1 = \theta_2 \equiv \theta_P, \\ &S_1 = S_2 = 1, \quad S_1' = S_2' = 0, \end{aligned} \tag{3.2}$$

where the SU(3) mass formula gives

$$\theta_P = \pm(24° \pm 1°), \tag{3.3a}$$

or

$$\theta_P = \pm(11° \pm 1°), \tag{3.3b}$$

depending upon whether we use the linear or the quadratic formula. Note that the sign of θ cannot be determined in this way. Equation (3.1) contains also the case corresponding to the current mixing scheme[48] as a special case when we have $\xi = 0$. We may interpret ξ in several possible ways. First, ξ may correspond to two-gluon states[47] or bound states[49] of two gluons. Or, ξ may represent other 0^- states such as E or more generally any radially excited state of a $q\bar{q}$ system with $I = Y = J = 0$ and/or daughter trajectories of η and η'. Presumably, the last interpretation corresponds to the model of Inami, Kawarabayashi, and Kitakado,[50] who base their argument on the dual unitary picture of Chew and Rosenzweig.[51] A somewhat similar view has been recently expressed by Lipkin.[52] Although ξ could contain in principle $\eta_c = q_4\bar{q}_4$, where $q_4 \equiv C$ is the charmed quark, its effect for problems involving η and η' is expected[53] to be too small and we can ignore it.

Let us set for simplicity

$$Z = \frac{\sqrt{2}M[A+B\to C_1+\cdots+C_n+(q_3\bar{q}_3)]}{M[A+B\to C_1+\cdots+C_n+(q_1\bar{q}_1)]+M[A+B\to C_1+\cdots+C_n+(q_2\bar{q}_2)]}, \tag{3.4}$$

$$\frac{Z'}{Z} = \frac{M(A+B\to C_1+\cdots+C_n+\xi)}{M[A+B\to C_1+\cdots+C_n+(q_3\bar{q}_3)]}, \tag{3.5}$$

where $q_j\bar{q}_j$ $(j=1,2,3)$ are fictitious ${}^1S_0$ bound states of q_j and $\bar{q}_j$. Then we find

$$\frac{M(A+B\to C_1+\cdots+C_n+\eta')}{M(A+B\to C_1+\cdots+C_n+\eta)} = \frac{S_2[\cos(\theta_0-\theta_2)+Z\sin(\theta_0-\theta_2)]+S_2'Z'}{S_1[\sin(\theta_0-\theta_1)-Z\cos(\theta_0-\theta_1)]+S_1'Z'}, \tag{3.6}$$

where θ_0 is,[54] as before,

$$\theta_0 = \tan^{-1}\frac{1}{\sqrt{2}} = 35°16'. \tag{3.7}$$

So far, there is no approximation. Suppose now that $A, B, C_1, C_2, \ldots, C_n$ are nonstrange hadrons as in the preceding section. Then as a generalization of the QLR, we assume

$$|Z| \ll 1, \quad |S_1'Z'| \ll 1, \quad |S_2'Z'| \ll 1. \tag{3.8}$$

An important difference in comparison to the 1^- case is the fact that both $\cos(\theta_0-\theta_2)$ and $\sin(\theta_0-\theta_1)$ cannot be small theoretically as we shall see shortly. Therefore, from Eqs. (3.6) and (3.8), we find that if we define K by

$$K = \frac{\bar{\sigma}(A+B\to C_1+\cdots+C_n+\eta')}{\bar{\sigma}(A+B\to C_1+\cdots+C_n+\eta)}, \tag{3.9}$$

then the QLR predicts[54]

$$K \approx K_0 \equiv \left[\frac{S_2\cos(\theta_0-\theta_2)}{S_1\sin(\theta_0-\theta_1)}\right]^2. \tag{3.10}$$

In Eq. (3.9), $\bar{\sigma}$ represents the cross section σ divided by the phase volume available in the final state. For the mass-mixing case (3.2), several special cases of (3.9) and (3.10) have been derived also from a simple quark model by Alexander *et al.*[31] Our derivation of (3.10) is obviously independent of SU(3) symmetry and of the Regge-pole analysis, so that it directly tests the QLR.

Especially, we note that K_0 in (3.10) is a constant independent of any energy and angular variables associated with the reaction $A+B\to C_1+\cdots+C_n+\eta(\eta')$. Also, it is independent of any specific reaction channel. Its value can be computed once a theoretical model of η-η' mixing is given. We shall consider several cases of mixing models of practical interest. We consider only the case with $S_1' = S_2' = 0$ hereafter.

(a) Linear mass-mixing model with negative θ_P:

$$\begin{aligned} &\theta_1 = \theta_2 \equiv \theta_P = -24°, \quad S_1 = S_2 = 1, \\ &K_0 = 0.35. \end{aligned} \tag{3.11a}$$

(b) Linear mass-mixing model with positive θ_P:

$$\theta_1=\theta_2\equiv\theta_P=+24°,\quad S_1=S_2=1,\quad K_0=23.9. \qquad (3.11b)$$

(c) Quadratic mass-mixing model with negative θ_P:

$$\theta_1=\theta_2=-10°,\quad S_1=S_2=1,\quad K_0=0.95. \qquad (3.11c)$$

(d) Quadratic mass-mixing model with positive θ_P:

$$\theta_1=\theta_2=+10°,\quad S_1=S_2=1,\quad K_0=4.37. \qquad (3.11d)$$

(e) No mixing at all:

$$\theta_1=\theta_2=0°,\quad S_1=S_2=1,\quad K_0=2. \qquad (3.11e)$$

(f) Rosenzweig model[55]:

$$\theta_1=-10°,\quad \theta_2=-20°,\quad S_1=S_2=1,\quad K_0=0.63; \qquad (3.11f)$$

(g) Model of Inami *et al.*[50]:

$$\theta_1=-6°,\quad \theta_2=-20°,\quad (S_2/S_1)^2=0.69\pm0.19,\quad (S_1)^2=0.64\pm0.11,\quad K_0=0.50\pm0.14. \qquad (3.11g)$$

For the model (3.11g), we ignore hereafter all errors quoted therein and we use only their central values.

Next, let us compare our prediction (3.10) with currently available experimental data:

$$(1)\ \frac{\overline{\sigma}(\pi^+p\to\eta'\Delta^{++})}{\overline{\sigma}(\pi^+p\to\eta\Delta^{++})}=\begin{cases}0.40\pm0.18, & p_L=3.65\ \text{GeV}/c,\ (\text{Ref. 56}), & (3.12a)\\ 0.24\pm0.11, & p_L=5.45\ \text{GeV}/c,\ (\text{Ref. 57}), & (3.12b)\\ 0.70\pm0.40, & p_L=8.0\ \text{GeV}/c,\ (\text{Ref. 58}); & (3.12c)\end{cases}$$

$$(2)\ \frac{\overline{\sigma}(\pi^-p\to\eta'\Delta^0)}{\overline{\sigma}(\pi^-p\to\eta\Delta^0)}=0.25\pm0.025,\quad p_L=7.1\ \text{GeV}/c\ (\text{Ref. 59}); \qquad (3.13)$$

$$(3)\ \frac{\overline{\sigma}(\pi^+n\to\eta'p)}{\overline{\sigma}(\pi^+n\to\eta p)}=\begin{cases}0.27\pm0.06, & p_L=1.66\text{–}2.10\ \text{GeV}/c\ (\text{Ref. 60}),\\ 0.56\pm0.28, & p_L=2.10\text{–}2.22\ \text{GeV}/c\ (\text{Ref. 60}),\end{cases} \qquad (3.14)$$

$$(4)\ \frac{\overline{\sigma}(\pi^-p\to\eta'n)}{\overline{\sigma}(\pi^-p\to\eta n)}=0.50\pm0.14,\quad p_L=3.8\text{–}200\ \text{GeV}/c\ (\text{Refs. 61 and 62}); \qquad (3.15)$$

$$(5)\ \frac{\overline{\sigma}(p\overline{p}\to\eta'\pi^+\pi^-)}{\overline{\sigma}(p\overline{p}\to\eta\pi^+\pi^-)}=0.73\pm0.15,\quad p_L=0\ \text{GeV}/c\ (\text{Ref. 63}). \qquad (3.16)$$

We may remark that the ratio for $\sigma(\pi^-p\to\eta'n)/\sigma(\pi^-p\to\eta n)$ is experimentally found to be approximately constant over the wide energy range[61,62] of $p_L=4\text{–}200$ GeV/c, and over some ranges[64] of the angular variable t. Also, for the reaction $p\overline{p}\to\pi^+\pi^-\eta(\eta')$, the Dalitz plots for η and η' are experimentally found to be essentially identical to each other. These are in accord with our prediction of the constancy of K_0. [See a discussion after Eq. (3.10).]

Comparing Eqs. (3.12)–(3.16) with (3.10) and (3.11) we can safely eliminate the cases corresponding to (3.11b), (3.11d), and (3.11e). In other words, the mixing angle $\theta_P=\theta_1=\theta_2$ for the mass-mixing case cannot be positive nor zero. This fact is gratifying since other studies[65,66] especially on $\eta\to2\gamma$, $\eta'\to2\gamma$, and $\eta\to\pi^+\pi^-\gamma$ lead to the same conclusion.

Similarly for the decay of the A_2 meson, the QLR predicts

$$\frac{\Gamma(A_2\to\pi\eta')}{\Gamma(A_2\to\pi\eta)}=K_0\left(\frac{k'}{k}\right)^5=0.041K_0, \qquad (3.17)$$

where k and k' are magnitudes of pion momenta in the rest frames of these decay modes. From the known upper limit[7] for $\Gamma(A_2\to\pi\eta')$, we estimate

$$K_0<1.62, \qquad (3.18)$$

which still excludes the cases (3.11b), (3.11d), and (3.11e).

From (3.12) to (3.16), we see that the values of K are not exactly a constant K_0 but are scattered over the range $K\simeq0.24\text{–}0.73$. Moreover, as far as the first four reactions are concerned, they are expected to go through the A_2 Regge trajectory at very high energy. Therefore, irrespective of the validity of the QLR, we expect to have

$$K\simeq(g_{A_2\pi\eta'}/g_{A_2\pi\eta})^2\simeq K_0 \qquad (3.19)$$

for these four reactions in the very-high-energy region. If the discrepancy between (3.13) and (3.15) is real, then we have to conclude that a pure Regge-pole model is not working well and that a Regge cut which also will violate the QLR must be contributing considerably to these reactions. However, the energies involved in the reactions (3.14) and (3.12a) are not high enough perhaps to permit us the use of the Regge theory. Nevertheless, the values of K are not significantly different from other cases. Noting that our derivation of (3.10) is independent of the Regge-theory and that the energy variations in these reactions (3.12)–(3.16) range in the wide region of $p_L = 0$–200 GeV/c, the near constancy of K is still remarkable.

We now propose to find the most likely value of K_0. As in Sec. II, we suppose that the QLR-violating terms Z and Z' of (3.4) and (3.5) will change their phases and magnitude randomly with varying reaction channels around the median $\langle Z\rangle = 0$ and $\langle Z'\rangle = 0$. Then if we take the average of K over all available reactions, we expect that the dependence of Z and Z' will cancel each other at least in the lower order to give $\langle K\rangle_{\text{ave}} = K_0$. In this way, we can roughly estimate K_0 to be

$$K_0 \simeq 0.5. \tag{3.20}$$

This is gratifying since the QLR is expected theoretically to be better at higher energy (see Sec. V) so that the most likely value of K_0 should be near the one given by (3.15).

Now the variations of K which are found among (3.12)–(3.16) are regarded as due to QLR-violating effects. However, the magnitude of the violation depends upon the theoretical model of η-η' mixing. If we adopt the model of Inami *et al.*[50] with $S_1' = S_2' = 0$ (or $Z' = 0$), as in (3.11g), then we estimate (we neglect complications due to spin)

$$|Z| \leq 0.143 \tag{3.21}$$

from (3.6) for all cases of $0.24 \leq K \leq 0.73$. The bound $|Z| \leq 0.170$ can be obtained also for the linear mass-mixing model (3.11a) with $\theta_P = -24°$, but the Rosenzweig model (3.11f) leads to a larger variation of

$$|Z| \leq 0.19. \tag{3.21$'$}$$

This fact may imply that the latter model is less favorable in comparison to the former ones. Actually, for the mass-mixing case, we can determine the most likely value of θ_P as in the ω-ϕ case by demanding minimum variations of $|Z|$. Then we find

$$\theta_0 - \theta_P = \pm 56°42', \quad K_0 = 0.43, \quad |Z| \leq 0.126, \tag{3.22}$$

which gives either $\theta_P = -21°30'$ or $+91°50'$. The former value is very close to $\theta_P = -24°$ of the linear mass case. Thus, the linear mass-mixing model is definitely better than the quadratic one. Note that the second solution $\theta_P = -90°50'$ implies that η and η' consist almost of pure SU(3) singlet η_0 and SU(3) octet η_8, respectively. Although this fact is rather suggestive, this solution is *not* compatible with the SU(3) mass formula so that it can be ruled out.

Returning to the discussion of the variation of $|Z|$, Eq. (3.21) implies that the violations of the QLR for models (3.11g) and (3.11a) are at most 15 to 17%, respectively. If we accept the linear mass-mixing model with angle given by (3.22), i.e., $\theta_P = -21°30'$, this is reduced further to 13%. Compared to the analogous value of 6% for the QLR violation of the 1^- nonet, this is not good. But considering the large experimental uncertainties as well as the much larger energy range involved ($p_L = 0$–200 GeV/c), this fact is nevertheless remarkable. It is certainly better than what we would expect theoretically. For example, the QCD gluon model would qualitatively predict a much larger violation of the QLR for the 0^- nonet, although a quantitative calculation is almost impossible. In this context, we should emphasize the fact that a large part of the QLR-violating effects are implicitly hidden in the form of normalization constants S_1 and S_2 as well as of mixing angles θ_1 and θ_2. Hence, we may say more correctly that once these effects are subtracted, then the remaining residual QLR-violating terms Z and Z' are small, and at most of order 13–19%.

We have seen that the model of Inami *et al.* (3.11g) and linear mass-mixing model (3.11a) as well as possibly the Rosenzweig model (3.11f) are reasonably compatible with the QLR. It is very difficult at the moment to select the best candidate from these three. If the mass-mixing model (3.2) is assumed, then Lipkin[30] has shown the validity of the relation

$$\sigma(K^-p \to \eta\Lambda) + \sigma(K^-p \to \eta'\Lambda) = \sigma(K^-p \to \pi^0\Lambda) + \sigma(\pi^-p \to K^0\Lambda) \tag{3.23}$$

on the basis of the QLR and of the SU(3) together with the Regge-pole model. As he remarks, this relation is badly violated at $p_L = 3.9$ GeV/c, where the right side is larger by a factor of 1.6 in comparison to the left side. Recent experimental data[67] at $p_L = 4.2$ GeV/c show not only the same discrepancy at $t \simeq 0$, but also a larger discrepancy of a factor amounting to nearly 3 in a larger t region of $t = 0.5$–0.9 (GeV/c)^2. However, the relation (3.23) is *not* valid for the models of Inami *et al.* and of Rosenzweig. Therefore for the latter two, we have no contradiction. This may suggest that the latter models are better than the linear mass-mixing scheme. However, a word of caution is perhaps advisable. It could be that the Regge-pole model could be bad[68] at these energies especially in larger values of t. Or it would be that the SU(3) symmetry might be badly violated also in the reactions. For example, if SU(3) is violated by an amount of 10% and if the QLR violation is of the order 15%, then the combined 25% violation for the scattering amplitude could easily account for the discrepancy of 1.6 observed near $t=0$.

For the decay modes $2^+ \to 1^-0^-$, SU(3) together with the QLR predicts

$$R_1 = \frac{\Gamma(A_2 \to \pi\eta)}{\Gamma(A_2 \to K\bar{K})} = 2[S_1 \sin(\theta_0 - \theta_1)]^2 \left(\frac{k'}{k}\right)^5, \qquad (3.24)$$

$$R_2 = \frac{\Gamma(K^{**} \to K\eta)}{\Gamma(K^{**} \to K\pi)} = \tfrac{1}{3}(S_1)^2[\sin(\theta_0 - \theta_1) - \sqrt{2}\cos(\theta_0 - \theta_1)]^2 \left(\frac{k'}{k}\right)^5, \qquad (3.25)$$

which give

$$R_1 = \begin{cases} 4.258 \text{ for (3.11a)}, \\ 2.907 \text{ for (3.11c) and (3.11f)}, \\ 1.605 \text{ for (3.11g)}, \end{cases} \qquad (3.24')$$

$$R_2 = \begin{cases} 0.002 \text{ for (3.11a)} \\ 0.008 \text{ for (3.11c) and (3.11f)}, \\ 0.010 \text{ for (3.11g)}. \end{cases} \qquad (3.25')$$

These values should be compared to the experimental values of

$$R_1 = 3.19 \pm 0.59,$$

$$R_2 = 0.036 \pm 0.036.$$

Therefore, the model (3.11a) is better than (3.11g). We may test the QLR also from

$$\frac{\Gamma(f' \to \eta\eta)}{\Gamma(f' \to K\bar{K})} = [S_1 \cos(\theta_0 - \theta_1)]^4 \times \left[\frac{\sqrt{2} - \tan(\theta_0 - \theta_T)\tan^2(\theta_0 - \theta_1)}{\sqrt{2} - \tan(\theta_0 - \theta_T)}\right]^2 \left(\frac{k'}{k}\right)^5 \qquad (3.26)$$

which predicts with $\theta_T \simeq 30°$

$$\frac{\Gamma(f' \to \eta\eta)}{\Gamma(f' \to K\bar{K})} = \begin{cases} 0.033 \text{ for (3.11a)}, \\ 0.156 \text{ for (3.11c) and (3.11f)}, \\ 0.086 \text{ for (3.11g)}. \end{cases} \qquad (3.26')$$

So far, no good experimental value is available to test this.

Next, we find similarly that the QLR gives

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\eta' \to \rho^0\gamma)} = \tfrac{1}{9}[\cos(\theta_0 - \theta)]^2 \times [1 + 2\tan(\theta - \theta_0)\tan(\theta_0 - \theta_2)]^2 \left(\frac{k'}{k}\right)^3. \qquad (3.27)$$

For the ideal mixing case $\theta = \theta_0$ for the 1^- nonet, the right side of (3.27) for all values of θ_2 is simply 0.096, while for $\theta = 39°$ it gives values of 0.143, 0.123, and 0.119, respectively, for (3.11a), (3.11f), and (3.11g). The recent experimental value for this ratio is found[69] to be 0.10 ± 0.02 so that all models are nicely consistent with the experiment. However, the most troublesome problem facing the QLR for $V_9 \to P_8\gamma$ decay is the ratio

$$\frac{\Gamma(\omega \to \pi^0\gamma)}{\Gamma(\rho^\pm \to \pi^\pm\gamma)} = 9[\cos(\theta_0 - \theta)]^2 \left(\frac{k'}{k}\right)^3 \simeq 9.47, \qquad (3.28)$$

which differs from the present experimental value by a factor of 2.5. For the ideal angle $\theta = \theta_0$, (3.28) is a direct consequence of the QLR involving q_1 and q_2 but not q_3 quarks. As is well known,[70] this discrepancy cannot be resolved unless we demand either a large SU(3) violation or a large QLR violation (for q_1 and q_2 quarks but not necessarily for q_3 quarks) or both. With respect to other decay widths such as $\Gamma(\phi \to \eta\gamma)$, Borchardt and Mathur[71] have investigated them in detail, so that we will not repeat them here. The models (3.11g) and possibly (3.11a) appear to give reasonably good values for these widths except for $\Gamma(K^* \to K\gamma)$ and $\Gamma(\rho^\pm \to \pi^\pm\gamma)$.

We remark that the QLR predicts

$$\frac{\Gamma(\eta' \to \rho^0\gamma)}{\Gamma(\rho^0 \to \eta\gamma)} = 3K_0\left(\frac{k'}{k}\right)^3 = 1.95K_0 \qquad (3.29)$$

and

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \eta\gamma)} = 3K_0\left(\frac{k'}{k}\right)^3 \simeq 1.52K_0, \qquad (3.30)$$

where we assumed ideal mixing for ω and ϕ in (3.30). From (3.29) and (3.30), we obtain also

$$\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\rho^0 \to \eta\gamma)} = 1.28\,\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\eta' \to \rho^0\gamma)}. \qquad (3.31)$$

A measurement by Andrews *et al.*[72] gives two possible solutions:

$$\Gamma(\rho^0 \to \eta\gamma) = 50 \pm 13 \text{ keV}, \quad \Gamma(\omega \to \eta\gamma) = 3.0^{+2.5}_{-1.8} \text{ keV}, \qquad (3.32a)$$

or

$$\Gamma(\rho^0 \to \eta\gamma) = 76 \pm 15 \text{ keV}, \quad \Gamma(\omega \to \eta\gamma) = 29 \pm 7 \text{ keV}. \qquad (3.32b)$$

However, the second solution (3.32b) is in rather bad agreement with (3.31), if we use the experimental value[69] of $\Gamma(\eta' \to \omega\gamma)/\Gamma(\eta' \to \rho^0\gamma) = 0.10 \pm 0.02$. Accepting the solution (3.32a) and using the known branching ratio[7] of

$$\Gamma(\eta' \to \rho^0\gamma)/\Gamma(\eta' \to \text{all}) = 0.304,$$

then (3.29) predicts

$$\Gamma(\eta' \to \text{all}) \simeq (160 \pm 42)\ \text{keV}, \qquad (3.33)$$

if we use the value $K_0 = 0.5$.

All these decay modes are intimately related to $e\bar{e} \to V_9 P_8$ reactions. Assuming the exact SU(3) together with the QLR, we find

$$\bar{\sigma}(e\bar{e} \to \pi^0\phi) = 0, \qquad (3.34)$$

$$\bar{\sigma}\ (e\bar{e} \to \pi^0\omega) = 9\bar{\sigma}\ (e\bar{e} \to \pi^0\rho^0) = 9\bar{\sigma}(e\bar{e} \to \pi^+\rho^-), \qquad (3.35)$$

$$\bar{\sigma}(e\bar{e} \to \eta\rho^0) = 9\bar{\sigma}(e\bar{e} \to \eta\omega) = \tfrac{9}{4}\tan^2(\theta_0 - \theta_1)\bar{\sigma}\,(e\bar{e} \to \eta\phi) = 9S_1{}^2 \sin^2(\theta_0 - \theta_1)\bar{\sigma}(e\bar{e} \to \pi^0\rho^0), \qquad (3.36)$$

$$\bar{\sigma}(e\bar{e} \to \eta'\rho^0) = 9\bar{\sigma}(e\bar{e} \to \eta'\omega) = \tfrac{9}{4}\cot^2(\theta_0 - \theta_2)\bar{\sigma}(e\bar{e} \to \eta'\phi) = 9S_2{}^2 \cos^2(\theta_0 - \theta_2)\bar{\sigma}(e\bar{e} \to \pi^0\rho^0), \qquad (3.37)$$

where for simplicity we assumed ideal mixing for ω and ϕ. From these, we may hopefully be able to discriminate various models of the η-η' mixing in the future. Also, studies of $\sigma(e\bar{e} \to \pi^{\mp}\rho^{\pm})$ will shed light on the vexing problem of $\Gamma(\rho^{\pm} \to \pi^{\pm}\gamma)$.

Some other predictions of the QLR are given in Ref. 54. We did not discuss here the decay widths of $\eta \to 2\gamma$ and $\eta' \to 2\gamma$ since that involves various other assumptions. Some calculations based upon the mass-mixing models can be found in Ref. 66. Also, it is possible that the QLR involving the q_3 quark is better than the QLR for q_1 and q_2 quarks. Note that the QLR involving the charmed quark q_4 is better satisfied than that for the q_3 quark as we will see in the next section.

IV. QLR FOR ψ/J AND 0^+

The narrow decay width of 67 keV for ψ/J with mass 3100 MeV definitely requires a new quantum number.[1] Ordinarily, we introduce the fourth quark[73] q_4 (or c) which is called the charmed quark. ψ/J is then assumed to be a bound state of $q_4\bar{q}_4$ in the $^3S_1$ state just as $\phi = q_3\bar{q}_3$ in the ideal mixing case. Then the QLR readily forbids the decay of ψ/J into ordinary hadrons involving only q_1, q_2, q_3 and their antiquarks. Therefore, the hadronic decays of ψ are possible only in a weaker QLR-violating mechanism. The narrow width of ψ/J implies that the QLR involving the fourth quark q_4 is much better satisfied than the QLR involving the third quark q_3. Also, the cross section $\sigma(e\bar{e} \to \psi + \text{hadrons})$ at 4.0–5.0 GeV/c is found[74] to be less than 0.1% of total hadronic cross sections. This is also compatible with the validity of the QLR with respect to the fourth quark, q_4. The answer for this may be sought[74a] in the QCD gluon model. At any rate, we refer, hereafter, to the QLR violation only those with respect to the third quark, q_3, but not to q_4.

One outstanding problem is how to explain a relatively large decay rate

$$\frac{\Gamma(\psi \to \phi\pi^+\pi^-)}{\Gamma(\psi \to \omega\pi^+\pi^-)} = 0.20 \pm 0.10. \qquad (4.1)$$

Since the decay $\psi \to \phi\pi^+\pi^-$ violates the q_3 quark QLR, we expect the ratio in (4.1) to be of the order of 0.01 as we judge from the results of Sec. II. However, a recent experiment[75] may have resolved this dilemma. They discovered the two-pion invariant-mass spectra are markedly different[76] between $\psi \to \phi\pi^+\pi^-$ and $\psi \to \omega\pi^+\pi^-$. It appears that the decay $\psi \to \phi\pi^+\pi^-$ is really a two-step decay process in which the ψ first decays into

$$\psi \to \phi\epsilon' \qquad (4.2)$$

and then the resonance ϵ' with mass value around 900 MeV decays into

$$\epsilon' \to \pi^+\pi^-. \qquad (4.3)$$

Here ϵ' could be a new 0^+ resonant state with rather narrow width or it could be identical to S^*(993 MeV), except for its slightly lower mass value. To simplify the argument, let us suppose that ϵ' consists dominantly almost of $q_3\bar{q}_3$. Then the reaction (4.2) is allowed by the QLR, but

$$\psi \to \omega\epsilon' \qquad (4.4)$$

is forbidden by the QLR in conformity with the experiment. Therefore, the experimental difference of the dipion mass distributions for $\psi \to \phi\pi^+\pi^-$ and $\psi \to \omega\pi^+\pi^-$ is readily explained. Indeed, the decay $\psi \to \omega\pi^+\pi^-$ dominantly proceeds via $\psi \to \omega f$, followed by $f \to \pi^+\pi^-$. Now the decay (4.3) is normally forbidden by the QLR just like $\phi \to \rho\pi$. However, because of zero phase space, the QLR-allowed decay

$$\epsilon' \to K\bar{K} \qquad (4.5)$$

is kinematically forbidden. As a result, the normally forbidden mode (4.3) can now be essentially the only dominant decay process possible, thus explaining the large ratio in (4.1) as well as the rather small width of ϵ'.

Moreover, they find[75] other evidence for the validity of the QLR such as

$$\frac{\Gamma(\psi\to\omega f')}{\Gamma(\psi\to\omega f)}<0.084\pm0.035, \quad (4.6a)$$

$$\frac{\Gamma(\psi\to\omega f')}{\Gamma(\psi\to\phi f')}<0.20\pm0.12, \quad (4.6b)$$

$$\frac{\Gamma(\psi\to\phi f)}{\Gamma(\psi\to\phi f')}<0.46\pm0.29, \quad (4.6c)$$

$$\frac{\Gamma(\psi\to\phi f)}{\Gamma(\psi\to\omega f)}<0.19\pm0.18. \quad (4.6d)$$

If the QLR involving the q_3 quarks is exact, then values of the left sides in Eq. (4.6) should be zero in the ideal-mixing limit.

Also, if $C_1, C_2, \ldots, C_n$ are all nonstrange, then we should have[54]

$$\frac{\overline{\Gamma}(\psi\to C_1+C_2+\cdots+C_n+\eta')}{\overline{\Gamma}(\psi\to C_1+C_2+\cdots+C_n+\eta)}=K_0 \quad (4.7)$$

just as (3.10), where $\overline{\Gamma}$ is the decay width divided by the phase volume. This relation may be tested in the future. A similar test of the η-η' mixing theory is suggested by Lipkin.[52] Moreover, if the interaction responsible for hadronic decays of ψ is dominantly SU(3)-singlet, the QLR gives

$$\overline{\Gamma}(\psi\to\pi^0\rho^0):\overline{\Gamma}(\psi\to\eta\omega):\overline{\Gamma}(\psi\to\eta\phi):\overline{\Gamma}(\psi\to\eta'\omega):\overline{\Gamma}(\psi\to\eta'\phi)$$
$$=1:|S_1|^2\sin^2(\theta_0-\theta_1):|S_1|^2\cos^2(\theta_0-\theta_1):|S_2|^2\cos^2(\theta_0-\theta_2):|S_2|^2\sin^2(\theta_0-\theta_2), \quad (4.8)$$

where we assumed ideal mixing for ω and ϕ. So far, the present experimental data[75] are not accurate enough to discriminate among various models of the η-η' mixing discussed in the preceding section, although the data are roughly consistent with (4.8).

In pp high-energy collision, ψ appears to be produced[77] via a QLR-violating process rather than the QLR-preserving reaction

$$pp\to D\overline{D}\psi+\cdots. \quad (4.9)$$

However, a similar situation already exists[24,25] for the ϕ production in high-energy pp and πp collisions, where the ϕ meson is usually more copiously produced via QLR-forbidden processes rather than by the QLR-allowed reactions

$$pp\to K\overline{K}\phi+\cdots. \quad (4.10)$$

These facts are presumably due to smaller unfavorable phase volumes for the QLR-preserving reactions (4.9) and (4.10) at the currently available energy range.

Up to now, we noted that the QLR are quite well satisfied experimentally. However, we may have one theoretical problem of the following nature. Computing the so-called pion σ term on the basis of the chiral $SU_L(3)\otimes SU_R(3)$ model, Cheng[78] has obtained a rather large value for

$$Z=\frac{\sqrt{2}\langle N|\overline{q}_3(0)q_3(0)|N\rangle}{\langle N|[\overline{q}_1(0)q_1(0)+\overline{q}_2(0)q_2(0)]|N\rangle}\simeq 0.35, \quad (4.11)$$

which is significantly far from the value 0 predicted by the QLR. This is essentially two times larger than the maximum value $|Z|\leq 0.17$ for the 0^- nonet. In (4.11), $\overline{q}_j(0)q_j(0)$ is the scalar density operator involving the jth quark. In the pole approximation, this fact implies a large coupling constant $g_{\epsilon' N\overline{N}}$ of ϵ' to the nucleon, suggesting a large violation of the ideal mixing in the nonet structure of the 0^+ meson. However, on the other hand, the decay width of $\psi\to\omega\epsilon'$ appears to be small in comparison to that of $\psi\to\phi\epsilon'$. This fact requires contrarily that the ideal nonet structure for the 0^+ meson must be reasonable. One possible way to resolve this dilemma will be to measure the ratio of cross sections such as

$$\frac{\overline{\sigma}(\pi^-p\to n\epsilon')}{\overline{\sigma}(\pi^-p\to n\epsilon)}=\left|\frac{Z+\tan(\theta_S-\theta_0)}{1-Z\tan(\theta_S-\theta_0)}\right|^2, \quad (4.12)$$

where ϵ with mass 1200 MeV is probably the 0^+ meson corresponding to quark structure $(1/\sqrt{2})(q_1\overline{q}_1+q_2\overline{q}_2)$ (see Sec. II).

In ending this section, we simply remark that there may exist a further hierarchy of importance among QLR-preserving diagrams. One example is the dominance[79] of the so-called quark-rearrangement diagram over the other diagrams in $p\overline{p}\to V_9P_8P_8$ reactions.

V. LIMITATION OF QLR AND THEORETICAL MODELS

In spite of its reasonable successes, the QLR fundamentally differs from the usual selection rules, since it is in general incompatible with the unitarity condition

$$\mathrm{Im}T(i\to f)=\sum_n T^*(f\to n)T(i\to n)\delta(E-E_n). \quad (5.1)$$

As is well known,[80,81] the unitarity correction can lead to violation of the initial QLR rule. A simple example is the case with $i=\phi$, $f=\pi^-\rho^+$, and $n=K^+K^-$, as we may see from Figs. 3, 4, 5, and 6. Since the real part $\mathrm{Re}T(i\to f)$ can be computed from the imaginary part $\mathrm{Im}T(i\to f)$ on the basis of the dispersion relation, the validity of the QLR suggests that the summation over all intermediate states n in Eq. (5.1) must be canceling[82] greatly,

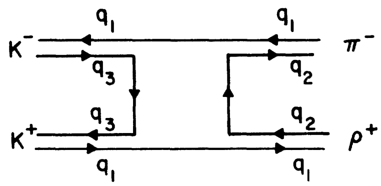

FIG. 4. Quark-line diagram for reaction $K^-K^+ \rightarrow \pi^-\rho^+$.

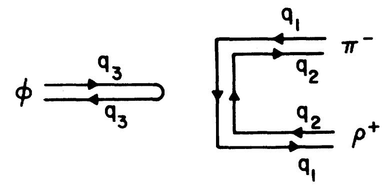

FIG. 6. Quark-line diagram for $\phi \rightarrow \pi^+\rho$, which is topologically equivalent to Fig. 5.

leaving a small residual QLR-violating term. Such a behavior may be partially understandable, if we assume an ansatz analogous to the so-called local-compensation hypothesis[83] on the multiple-particle production reactions. Suppose that the initial state "i" differs from the final one "f", i.e., $i \neq f$. Then in high energy, $T(i \rightarrow n)$ and $T(f \rightarrow n)$ would randomly and independently change its magnitude and phase, as the intermediate state "n" varies. Because of this mismatch between phases of $T(i\rightarrow n)$ and $T(f \rightarrow n)$ the cancellation would result.[83] If this argument is valid, then we expect that the QLR is better satisfied in higher energies, since many intermediate channels "n" will be open in high energy so that the cancellation will be better.[83] However, this argument fails for $i = f$, since the sum in (5.1) is then coherently additive as we see:

$$\mathrm{Im}T(i \rightarrow i) = \sum_n |T(i \rightarrow n)|^2 \delta(E - E_n). \tag{5.2}$$

Therefore, the QLR is *not* expected to be good for the reaction $i \rightarrow i$. Indeed, the cross section $\sigma(\phi p \rightarrow \phi p)$ with $i = \phi p$ is known to be large via the Pomeron exchange, which symbolizes a large QLR violation.

The unitarity correction is likely related to the presence of the sea quarks[84] inside any hadrons. Since the sea quark is expected to be an SU(3) scalar, this implies that the physical proton and pion can contain strange-quark pairs $q_3\bar{q}_3$ inside them. This will be another possible mechanism for violations of the QLR.

So far, the best possible explanation of the QLR is the asymptotically free color-SU(3) gluon model. Since this fact is well known, we need not go into detail. However, one interesting consequence of the gluon model is an interpretation of the term $C_3(\mathrm{Tr}G)^2$ in (1.13). It can be due to contributions of three- or two-gluon intermediate states to the mass operator of 1^- or 0^- and 2^+ nonets, as $q\bar{q} \rightarrow$ (3 or 2 gluons) $\rightarrow q\bar{q}$. Since we expect that this contribution would be small for the 1^- nonet, we can set $C_3 = 0$ so as to obtain a reasonable validity of the two nonet mass formulas (1.12a) and (1.12b). On the other hand, the 2^+ nonet requires two gluon exchanges and hence the coefficient C_3 may not be so small as compared to the 1^- case. This may account for a slightly poorer validity of (1.12a) and (1.12b) for the 2^+ case. With respect to 0^-, the C_3 term could be very large because of the smaller masses of the 0^- particles. Therefore, the nonet formulas (1.12) will be very bad. Also, because of this the Schwinger mass formula (1.13′) itself is not well satisfied for the 0^- nonet in comparison to both 1^- and 2^+ nonets since we may require the additional presence of $C_4\mathrm{Tr}(G\lambda_8)\mathrm{Tr}G$ term.

The QLR-violating scattering $\phi p \rightarrow \phi p$ mentioned already can proceed via gluon exchanges of possibly infinite numbers. Then the Pomeron may be identifiable with exchanges of possibly infinite number of soft gluons, as has been suggested by some authors. The QLR-forbidden reactions

$$p\bar{p} \rightarrow \Omega^-\bar{\Omega}^-, \phi\phi, f'f', \tag{5.3}$$

$$p\bar{p} \rightarrow f'\phi \tag{5.4}$$

are interesting in the gluon model. The reactions (5.3) can proceed via a minimum of two gluon exchanges, while the reaction (5.4) requires at least three gluon exchanges because of the charge conjugation. Therefore, we expect a much smaller cross section for (5.4) in comparison to those of reaction (5.3) in the very-high-energy region. Note also that $p\bar{p} \rightarrow \phi\phi$ is a crossed reaction of $\phi p \rightarrow \phi p$.

Last, we simply remark that the QLR may be somehow related to the asymptotic chiral theory of Oneda. Indeed, Oneda and his collaborators[85]

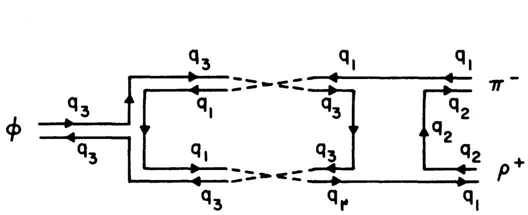

FIG. 5. Quark-line diagram for combinations of Figs. 3 and 4, representing the two-step mechanism $\phi \rightarrow K^-K^+ \rightarrow \pi^-\rho^+$.

have derived $\Gamma(\phi \to \pi\rho) = \Gamma(f' \to 2\pi) = 0$ as well as the Schwinger mass formula by his method without explicit uses of the QLR. A possible reason for this fact is that the asymptotic freedom of the color-gluon theory may automatically imply the validity of the asymptotic $SU_R(3)\times SU_L(3)$ in Oneda's sense.

Note added. (i) After this work was completed, it came to my attention that R. Baldi *et al.*, Phys. Lett. 68B, 381 (1977), found

$$\sigma(\pi^- p \to \phi\pi^- p)/\sigma(\pi^- p \to \omega\pi^- p) = 0.006 \pm 0.003\,,$$

$$\sigma(pp \to \phi pp)/\sigma(pp \to \omega pp) = 0.020 \pm 0.005\,,$$

by 10-GeV/c pion and proton beams. (ii) The quark-line rule could be exact in the sense of the usual symmetry. Suppose that we have an additional symmetry group G_0 which may be finite, and that all low-lying hadrons belong to nontrivial representations of G_0, while the colored gluons are assumed to be singlets of G_0. Then the ϕ and ψ/J mesons can never couple with any numbers of gluons because of the new symmetry G_0. This implies that the quark-line rule for hairpin diagrams can be exact in some theories.

Note added. (i) After this work was completed, it came to my attention that R. Baldi *et al.*, Phys. Lett. 68B, 381 (1977), found

$$\sigma(\pi^- p \to \phi\pi^- p)/\sigma(\pi^- p \to \omega\pi^- p) = 0.006 \pm 0.003\,,$$

$$\sigma(pp \to \phi pp)/\sigma(pp \to \omega pp) = 0.020 \pm 0.005\,,$$

by 10-GeV/c pion and proton beams. (ii) The quark-line rule could be exact in the sense of the usual symmetry. Suppose that we have an additional symmetry group G_0 which may be finite, and that all low-lying hadrons belong to nontrivial representations of G_0, while the colored gluons are assumed to be singlets of G_0. Then the ϕ and ψ/J mesons can never couple with any numbers of gluons because of the new symmetry G_0. This implies that the quark-line rule for hairpin diagrams can be exact in some theories.

ACKNOWLEDGMENTS

The author would like to express his gratitude to K. Jagannathan for collecting many experimental data used in this work, and to Professor T. Ferbel and Professor C. Rosenzweig for informing him of some experimental data.

---

*Work supported in part by the U. S. Energy Research and Development Administration under Contract No. E(11-1)-3065.

[1]E.g., see *Proceedings of the 1975 International Symposium on Lepton and Photon Internations at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976).

[2]S. Okubo, Phys. Lett. 5, 165 (1963).

[3]G. Zweig, CERN Report Nos. TH-401 and TH-412, 1964 (unpublished).

[4]J. Iizuka, Prog. Theor. Phys. Suppl. 37–38, 21 (1966); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. 35, 1061 (1966).

[5]E.g., M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).

[6]J. J. Sakurai, Phys. Rev. Lett. 9, 472 (1962).

[7]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[8]G. Parrour *et al.*, Phys. Lett. 63B, 362 (1976).

[9]E.g., see R. L. Heimann, Nuovo Cimento 39A, 461 (1977).

[10]D. Cohen *et al.*, Phys. Rev. Lett. 38, 269 (1977), obtained smaller values $(g_{\phi\rho\pi}/g_{\omega\rho\pi})^2 = 0.0022$ or 0.0036 from values of $g_{\omega\rho\pi}$ determined respectively by R. Dashen and D. Sharp, Phys. Rev. 133, B1585 (1964), and by J. Yellin, Phys. Rev. 147, 1080 (1966).

[11]J. Schwinger, Phys. Rev. Lett. 12, 237 (1964). This formula has since been rederived by many other techniques. See e.g. E. Takasugi and S. Oneda, Phys. Rev. D 12, 198 (1975) for references. The physical meaning of the $C_3(\mathrm{Tr}\,G)^2$ in Eq. (1.13) may be that we are taking into account three-gluon intermediate states in mass matrix as we will discuss in Sec. V. The exact QLR demands $C_3 = 0$.

[11a]For a simple review of the quark model, see H. J. Lipkin, Phys. Rep. 8C, 173 (1973).

[12]H. Sugawara and F. von Hippel, Phys. Rev. 145, 1331 (1966).

[12a]Such a prediction had been made earlier on the basis of the nonchiral $SU(3)\times SU(3)$ group by J. Schwinger, Phys. Rev. 135, B816 (1964), and F. Gürsey, T. D. Lee, and M. Nauenberg, *ibid.* 135, B467 (1964).

[13]S. L. Glashow and R. H. Socolow, Phys. Rev. Lett. 15, 329 (1965).

[14]A. J. Pawlicki *et al.*, Phys. Rev. Lett. 37, 971 (1976); Phys. Rev. D 12, 631 (1975); 15, 3196 (1977).

[15]W. Beusch *et al.*, Phys. Lett. 60B, 101 (1975), give a somewhat smaller value of $(g_{f'\pi\pi}/g_{f\pi\pi})^2 \leq \frac{1}{800}$.

[16]R. A. Donald *et al.*, Phys. Lett. 61B, 210 (1976) and earlier references quoted therein.

[16a]C. K. Chen, T. Fields, D. Rhines, and J. Whitmore, Report No. ANL-HEP-PR-77-22 (unpublished).

[17]D. S. Ayres *et al.*, Phys. Rev. Lett. 32, 1463 (1974); D. Cohen *et al.*, *ibid.* 38, 269 (1977).

[18]D. W. Davies *et al.*, Phys. Rev. D 2, 506 (1970) for $\sigma(\pi^+ n \to \phi p)$ and J. S. Danburg *et al.*, *ibid.* 2, 2564 (1970) for $\sigma(\pi^+ n \to \omega p)$. The result quoted is the average value over the energy range $E_{\mathrm{c.m.}} = 1.95$–2.40 GeV.

[19]See N. Armenise *et al.*, Nuovo Cimento 65A, 637 (1970), for $\sigma(\pi^+ n \to \omega p)$, and D. Mettel, Ph.D. thesis, 1970, Institut de Physique Nucleaire, Paris [quoted in Particle Data Group, Lawrence Berkeley Laboratory Report No. LBL-53, 1973 (unpublished), p. II-186] for $\sigma(\pi^+ n \to \phi p)$.

[20]M. S. Farber *et al.*, Nucl. Phys. B29, 237 (1971).

[21]W. R. Butler *et al.*, Phys. Rev. D 7, 3177 (1973).

[22]M. Aderholtz *et al.*, Nucl. Phys. B8, 45 (1968); B11,

259 (1969); B14, 255 (1969).

[23]M. Abolins *et al.*, Phys. Rev. Lett. 11, 381 (1961).

[24]P. L. Woodworth *et al.*, Phys. Lett. 65B, 89 (1976).

[25]V. Blobel *et al.*, Phys. Lett. 59B, 88 (1975).

[26]T. E. Kalogerpoulos (private communication); J. Roy, Ph.D. thesis, Syracuse University, 1974 (unpublished); L. Gray *et al.*, Phys. Rev. Lett. 17, 501 (1966). The present author would like to express his thanks to Professor Kalogeropoulos for calling his attention to these data.

[27]E.g., see Chan H.-M., K. Konishi, K. Kwiecinski, and P. G. Roberts, Phys. Lett. 58B, 469 (1976); K. Akama and S. Wada, *ibid.* 61B, 279 (1976). Also, see the comment made in Ref. 11.

[28]For some applications of the QLR involving 1^+ and 0^+ particles, see G. C. Joshi, Lett. Nuovo Cimento 17, 269 (1976).

[29]SU(3) symmetry predicts

$$\Gamma(K^*\to K\pi)/\Gamma(\rho\to\pi\pi)=\frac{3}{4}\left[\frac{m(\rho)}{m(K^*)}\right]^2\left(\frac{k'}{k}\right)^3=0.289,$$

while the experimental value is 0.325.

[30]H. J. Lipkin, Phys. Lett. 60B, 371 (1976). See also, A. D. Martin, C. Michael, and R. J. N. Phillips, Nucl. Phys. B43, 13 (1972).

[31]G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Lett. 17, 412 (1966).

[32]H. Genz and G. Höhler, Phys. Lett. 61B, 389 (1976).

[33]K. Fujikawa and M. Kuroda, Lett. Nuovo Cimento 18, 539 (1977); see also H. Genz and C. B. Lang, Nuovo Cimento 40A, 313 (1977).

[34]S. Okubo, Phys. Rev. D 14, 108 (1976). These are consequences based upon the standard QLR discussed so far. The other alternative based upon the modified QLR is not given here, since a new datum on $\psi\to\phi\pi^+\pi^-$ decay definitely rules out its validity as we will discuss in Sec. V.

[35]S. C. Frautschi, S. Pakvasa, and S. F. Tuan, Phys. Lett. 66B, 47 (1977); Nucl. Phys. B121, 141 (1977).

[36]V. Picciarelli *et al.*, paper presented at the 1970 International Conference on High Energy Physics, Kiev, 1970, quoted in Particle Data Group, Lawrence Berkeley Laboratory Report No. LBL-53, 1973 (unpublished), p. II-208.

[37]B. Lörstad *et al.*, Nucl. Phys. B14, 63 (1969).

[38]For a general test of the SU(3) symmetry, see N. P. Samios, M. Goldberg, and B. T. Meadows, Rev. Mod. Phys. 46, 49 (1974); J. L. Rosner, Phys. Rep. 11C, 190 (1974).

[39]We assumed that $f'\to K\bar{K}$ is essentially 100% decay modes of f'. From Eqs. (2.42) and (3.26), we expect that $f'\to K^*\bar{K}$ and $f'\to\eta\eta$ are at most of 15%.

[40]E.g., M. D. Slaughter and S. Oneda, Phys. Rev. D 15, 879 (1977).

[41]Y. Ueda, Phys. Rev. D 15, 870 (1977).

[42]V. Vuillenin *et al.*, Lett. Nuovo Cimento 14, 165 (1975).

[43]S. Weinberg, Phys. Rev. D 11, 3583 (1975).

[44]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); Phys. Rev. D 8, 4492 (1973); H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).

[45]G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976).

[46]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975).

[47]N. Fuchs, Phys. Rev. D 14, 1912 (1976); G. Karl, Nuovo Cimento 38A, 315 (1977).

[48]S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964); N. M. Kroll, T. D. Lee, and D. Zumino, *ibid.* 157, B1376 (1967); J. J. Sakurai, *Currents and Mesons* (Univ. of Chicago Press, Chicago, 1969).

[49]P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975); Univ. of Chicago Report No. EFI 76/71, 1976 (unpublished).

[50]T. Inami, K. Kawarabayashi, and S. Kitakado, Prog. Theor. Phys. 56, 1570 (1976).

[51]C. Rosenzweig and G. F. Chew, Phys. Lett. 58B, 93 (1975); G. F. Chew and C. Rosenzweig, Nucl. Phys. 104, B290 (1976).

[52]H. J. Lipkin, Phys. Lett. 67B, 65 (1977).

[53]E.g., V. S. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D 11, 2572 (1975); H. Harari, Phys. Lett. 60B, 172 (1976).

[54]S. Okubo and K. Jaganathan, Phys. Rev. D 15, 177 (1977). However, the definition of θ_0 there corresponds to $\pi/2-\theta_0$ in the present paper.

[55]C. Rosenzweig, Phys. Rev. D 13, 3080 (1976).

[56]G. H. Trilling *et al.*, Phys. Lett. 19, 427 (1965).

[57]I. J. Bloodworth *et al.*, Nucl. Phys. B39, 525 (1972).

[58]M. Bardadin-Otwinoska *et al.*, Phys. Rev. D 4, 2711 (1971).

[59]K. W. Lai *et al.*, in a paper presented at the XVI International Conference on High Energy Physics, Fermilab, 1972, quoted by F. D. Gault *et al.*, Nuovo Cimento 24A, 259 (1974).

[60]For $\sigma(\pi^+n\to\eta'p)$, see R. K. Rader *et al.*, Phys. Rev. D 6, 3059 (1972) and for $\sigma(\pi^+n\to\eta p)$, J. S. Danburg *et al.*, *ibid.* 2, 2564 (1970). We have divided for convenience the energy regions into two parts: 2.00 GeV $\leq E_{\text{c.m.}}\leq$ 2.20 GeV and 2.20 GeV $\leq E_{\text{c.m.}}\leq$ 2.35 GeV and have taken average values of cross sections over these energy ranges.

[61]W. D. Appel *et al.*, Phys. Lett. 46B, 459 (1973); V. N. Boltov *et al.*, *ibid.* 48B, 280 (1974); W. D. Appel *et al.*, HIEP Report No. 74-118 (unpublished).

[62]The data for $\sigma(\pi^-p\to\eta n)$ for p_L = 20–200 GeV/c can be found in O. I. Dahl *et al.*, Phys. Rev. Lett. 37, 80 (1976). The experimental values for $\sigma(\pi^-p\to\eta' n)$ are preliminary data by the same group (private communication from A. Barnes).

[63]M. Foster *et al.*, Nucl. Phys. B8, 174 (1968).

[64]The exception is the data at p_L = 8.4 GeV/c by K. W. Edwards *et al.*, in a paper submitted to the XVIII International Conference on High Energy Physics, Tibilisi, 1976 (unpublished) and University of Toronto Report No. COO-1545-193, 1976 (unpublished). However, the average of K over t is again around 0.5, so that it will not affect our argument to determine the best value of K_0.

[65]E.g., F. D. Gault, H. F. Jones, M. D. Scadrons, and R. L. Thews, Nuovo Cimento 24A, 259 (1974); H. Nagai and A. Nakamura, Prog. Theor. Phys. 53, 523 (1975); A. Bramon, Phys. Lett. 51B, 87 (1974).

[66]M. S. Chanowitz, Phys. Rev. Lett. 35, 977 (1975); K. Ohnishi, T. Teshina, and I. Umemura, Prog. Theor. Phys. 53, 1145 (1975).

[67]F. Marzano *et al.*, Nucl. Phys. B123, 203 (1977).

[68]We note that the reaction $\pi^-p\to\omega n$ at 6 GeV/c requires the Regge cut. See A. J. Pawlicki *et al.*, Argonne National Lab report, 1976 (unpublished), for the experiment and N. N. Achasov *et al.*, Institute of Mathe-

matics Report, USSR Academy of Science, Novosibirsk, 1976 (unpublished) for a theoretical explanation of the experiment.

[69]C. J. Zanfino *et al*., Phys. Rev. Lett. 38, 930 (1977).

[70]See B. J. Edwards and A. N. Kamal, Phys. Rev. D 15, 2019 (1977); P. J. O'Donnell, Phys. Rev. Lett. 36, 177 (1976); D. H. Boal, R. H. Graham, and J. W. Moffat, *ibid*. 36, 714 (1976).

[71]S. Borchardt and V. S. Mathur, Rochester report, 1977 (unpublished).

[72]D. E. Andrews *et al*., Phys. Rev. Lett. 38, 198 (1977).

[73]D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Phys. Lett. 11, 190 (1964); J. D. Bjorken, and S. L. Glashow, *ibid*. 11, 255 (1964); Y. Hara, Phys. Rev. 134, B701 (1964); Z. Maki, Prog. Theor. Phys. 31, 725 (1964); P. Tarjanne and V. L. Teplitz, Phys. Rev. Lett. 11, 447 (1963).

[74]J. Burmester *et al*., Phys. Lett. 68B, 283 (1977).

[74a]E.g., see H. Harari, Ref. 1, p. 317.

[75]F. Vannucci *et al*., Phys. Rev. D 15, 1814 (1977). The present author would like to express his thanks to Professor C. Rosenzweig for calling his attention to this paper.

[76]This experimental fact immediately rules out the conjecture of the modified nonet ansatz postulated in S. Okubo, Phys. Rev. Lett. 36, 117 (1976); Phys. Rev. D 13, 1994 (1976); 14, 108 (1976); see also the comment in Ref. 34.

[77]J. G. Branson *et al*., Phys. Rev. Lett. 38, 580 (1977).

[78]T. P. Cheng, Phys. Rev. D 13, 2161 (1976); Univ. of Missouri report, 1976 (unpublished).

[79]E. g., R. K. Logan, S. Konitz, and S. Tanaka, in *Antinucleon-Nucleon Interactions*, edited by G. Ekspong and S. Nilsson (Pergamon, New York, 1977); Vol. 29; T. Fields, Phys. Rev. Lett. 36, 489 (1976).

[80]C. Schmid, D. M. Webber, and C. Sorensen, Nucl. Phys. B111, 317 (1976).

[81]H. J. Lipkin, in *New Fields in Hadronic Physics*, proceedings of the XI Rencontre de Moriand, edited by J. Tran Thanh Van (CNRS, Paris, 1976).

[82]E.g., see N. A. Törnquist, Phys. Lett. 64B, 348 (1976); 69B, 193 (1977); V. Ruuskanen and N. A. Törnquist, Univ. of Helsinki Report No. RITP-3-77 (unpublished). See also J. Pasupathy, Phys. Rev. D 12, 2929 (1975); Phys. Lett. 58B, 71 (1975).

[83]D. Weingarten, Phys. Rev. D 13, 1474 (1976); D. Weingarten *et al*., Phys. Rev. Lett. 37, 1717 (1976) and references quoted therein.

[84]E.g., R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, N.Y., 1972); J. Kogut and L. Susskind, Phys. Rep. 8C, 76 (1973). According to V. Barger and R. J. N. Phillips, Nucl. Phys. B73, 269 (1974), the λ quark in the proton sea is of order 5%.

[85]S. Oneda and S. Matsuda, in *Fundamental Interactions in Physics*, 1973 Coral Gables Conference, edited by B. Kurşunoğlu and A. Perlmutter (Plenum, N.Y., 1973), p. 175; S. Oneda and E. Takasugi, Phys. Rev. D 12, 198 (1975), and in Proceedings of the International Symposium on Mathematical Physics, Mexico City, Mexico, 1976, edited by A. Böhm, Vol. 2, p. 585.