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Retention of quantum numbers by quark and multiquark jets*

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Quantum-number retention in the fragmentation region of a jet is investigated as a means to identify and classify quark and multiquark systems in lepton- and hadron-induced reactions. We show that a simple "inside-outside" cascade model with a causal space-time structure predicts that the mean charge of hadrons in the jet fragmentation region equals the charge of the parent quark system modulo a universal constant η_Q (the mean quark charge of the sea) confirming previous analyses. Specific tests are discussed, including deep-inelastic neutrino scattering, e^+e^- annihilation with a strongly leading hadron, and massive-lepton-pair production. We emphasize that the Drell-Yan process could play a crucial role in testing these ideas for multiquark jets and identifying a possible "hole" fragmentation region. We also discuss the utility of using a charge-momentum vector as a discriminant of jet structure. Finally we emphasize the utility of charge-retention tests in large-p_T hadronic reactions in order to discriminate between various contributing subprocesses.

I. INTRODUCTION

All the evidence for the quark structure of hadrons has been necessarily indirect, since quarks have never been experimentally observed. However, one of the more direct indications of underlying quark dynamics has been the discovery of a jet structure in the final-state hadrons in e^+e^- annihilation at SPEAR.[1] Hadrons are emitted preferentially with a limited transverse momentum relative to a jet axis which has a $1+\cos^2\theta_{\text{c.m.}}$ distribution characteristic of a virtual photon producing two spin-$\frac{1}{2}$ particles. It is thus natural to identify the parents of this jet with spin-$\frac{1}{2}$ quarks.

Jets are seen in many other reactions ranging from deep-inelastic lepton scattering to the production of high-transverse-momentum hadrons in hadronic collisions.[2] These jets can be identified, with various degrees of certitude, as systems of quark (or multiquark) parentage. Ideally, the hadrons emitted by such jets have essentially limited transverse momentum relative to the jet axis and the inclusive single-hadron distributions scale. This means that the inclusive cross section

$$\frac{dN}{dx}=\frac{1}{\sigma_{\text{inel}}}\int\frac{d^3\sigma}{d^2k_\perp dx}\,d^2k_\perp \tag{1}$$

with

$$x=\frac{p_{\text{hadron}\,\parallel}+E_{\text{hadron}}}{p_{\text{jet}}+E_{\text{jet}}}$$

is independent of jet momentum, p_{jet}, for large p_{jet}. As implied by the Feynman distribution, $dN/dx\sim 1/x$ as $x\to 0$, hadron multiplicities grow logarithmically with energy and, where the energy is sufficiently high, they are distributed uniformly in rapidity (y) for finite y. Here $E_{\text{hadron}}=m_\perp\cosh y$, $m_\perp=(m_{\text{hadron}}{}^2+k_\perp{}^2)^{1/2}$, so that

$$y=\ln[(E_{\text{hadron}}+p_{\text{hadron}\,\parallel})/m_\perp]$$
$$=\ln x+\ln[(p_{\text{jet}}+E_{\text{jet}})/m_\perp].$$

Although evidence for jets with these distribution properties is accumulating, further tools and harder evidence are needed to establish their quark parentage.

One test, suggested by Feynman,[3] is to look at the charge of the hadrons in the fragmentation region of the jet ($y_{\text{c.m.}}>0$). He conjectured that the total charge of all the fragments in this region averaged over events should be equal to the (fractional) charge of the parent quark. In fact all quark quantum numbers should be retained on the average. Although this has been shown not to be true in general[4a] in the quark model, a weaker version is still possible and will be discussed in Sec. III.

In this paper we shall be concerned with several new applications of charge retention as a test for the quark parentage of jets. In the next section we construct a simple "inside-outside" cascade model for jet fragmentation which we shall use in our discussion of charge retention. This model has a causal space-time structure. We shall then review the charge retention problem in Sec. III. In Sec. IV we review briefly what has been learned from deep-inelastic $\nu p\to\mu^- h^\pm X$ at Fermilab and discuss in detail what can be learned from experiments on e^+e^- annihilation and on the Drell-Yan process in hadronic collisions ($h_1+h_2\to l^+l^-+X$ with $l=\mu$ or e). Finally in Sec. V we discuss the possibility of using a charge-momentum vector as a tool for studying the quantum numbers of jets.

II. A MODEL FOR JET FRAGMENTATION

The problem of jet fragmentation has been discussed in detail by many authors.[3,4a,4b,4c,5] It has been pointed out[5] that the space-time evolution of the final state limits the possible realistic models. Bjorken has suggested that the space-time evolution must be as in Fig. 1. The initial quark and antiquark (in $e^+e^- \to$ hadrons, for example) are moving almost at the speed of light. The pion emission must (apart from quantum fluctuations) happen near the hyperboloid $t^2 - x^2 = d^2$, where d is some typical hadronic dimension. The total time needed for this development is then proportional to $\sqrt{s}$ since the hyperboloid joins the quark at $t = \gamma d$. The initial quark and antiquark are then free for a time $t \propto \sqrt{s}$, which justifies their being treated as free particles in the calculation of the cross section.[6]

In constructing a model for this process we must keep in mind that the emission of one hadron cannot cause or directly influence the emission of any other hadron since they are at a spacelike separation. The "cause" of the emission must then somehow come from the region $t^2 - x^2 < d^2$. The simplest such mechanism is shown in Fig. 2. At $x = t = 0$ (or in practice at $t^2 - x^2 < \epsilon^2 \ll d^2$) a large number of (virtual) gluons[7] are emitted with all velocities and with a flat distribution in rapidity ($y = \tanh^{-1} v_{\text{gluon}}$). This condition ensures Lorentz invariance. We assume that the gluons live, on the average, a constant characteristic proper time $\tau = 1/d$ and they then produce a $q\bar{q}$ pair. This production will then always happen near the hyperboloid. Hadrons are formed by joining quarks and antiquarks from adjacent gluons into color singlets as is shown in Fig. 2. Production of baryons could be incorporated as in Fig. 3. Note that in this model baryon number would be compensated locally in rapidity.

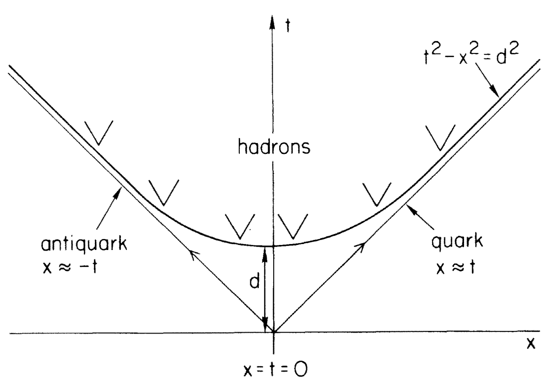

FIG. 1. Space-time evolution of the hadronic final state in e^+e^- annihilation. The initial $q\bar{q}$ pair are produced at $x = t = 0$ and the hadrons are produced near the hyperboloid, $t^2 - x^2 = d^2$. The transverse direction is not shown in the diagram.

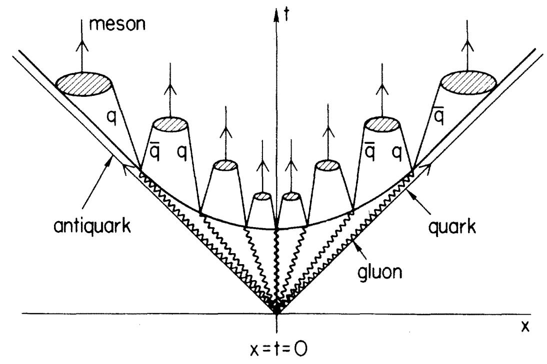

FIG. 2. The mediation of virtual gluons in the formation of hadrons in e^+e^- annihilation.

Let us assume for simplicity that the (virtual) gluon mass is fixed[8]: $m_g \geq 2m_q$ (here m_q is the mass of the u, d quarks assumed equal). In rapidity, the process $e^+e^- \to$ hadrons now looks as in Fig. 4 (in the e^+e^- c.m. frame and neglecting production of strange quarks). In Fig. 4

$$\cosh z = \frac{m_g}{2m_q}, \quad \cosh z_\pi = \frac{m_\pi}{2m_q}. \tag{2}$$

The transverse-momentum fluctuations will only affect the details of the model and not its charge-retention properties. In this simplified version of our model we have assumed the gluons to be evenly placed in rapidity and, when gluon $\to q\bar{q}$, the q and $\bar{q}$ always align themselves to ensure that they make hadrons. The rapidities of all the produced mesons are thus fixed.

If there are $2n+1$ gluons then energy conservation gives

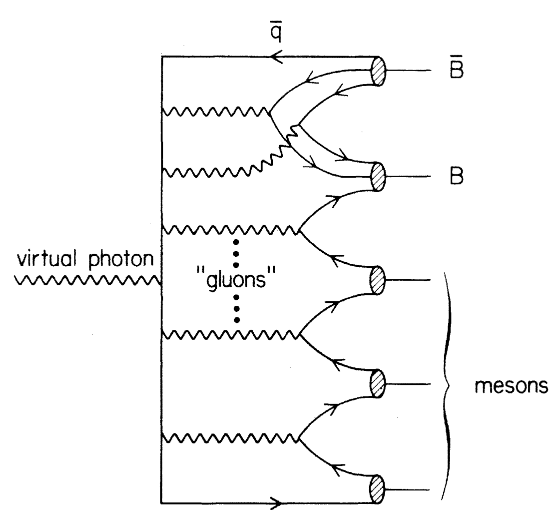

FIG. 3. Baryon production in the model of Sec. II.

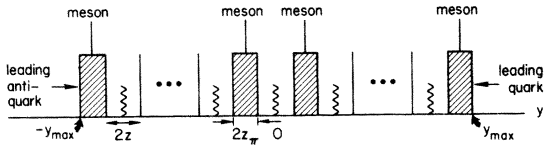

FIG. 4. Rapidity distribution of the gluons and hadrons in the model of Sec. II. Here $\cosh z = m_g/2m_q$ and $\cosh z_\pi = m_\pi/2m_q$.

$$\sqrt{s} = m_g + 2\sum_{\kappa=1}^{n} m_g \cosh(\kappa z_0) + 2m_q \cosh y_{\max}, \quad (3)$$

where

$$z_0 = 2(z+z_\pi), \quad y_{\max} = (n+1)z_0 - z,$$

and $\sqrt{s}$ is the total c.m. energy. The sum in Eq. (3) can be done and for $\cosh y_{\max} \approx e^{y_{\max}}/2$. The result is

$$\sqrt{s} = m_q \exp(y_{\max}) G \quad (4)$$

with

$$G = \frac{1+e^{-2z_\pi}}{1-e^{-z_0}}.$$

Then

$$y_{\max} = \ln\left(\frac{\sqrt{s}}{m_q G}\right) \quad (5)$$

and

$$2(n+1) = \frac{2}{z_0}(y_{\max}+z) = \frac{2}{z_0}\left[\ln\left(\frac{\sqrt{s}}{m_q G}\right) + z\right].$$

The number of produced pions $n_\pi = 2n+2$. Thus the mean multiplicity $\langle n_\pi \rangle$ will be

$$\langle n_\pi \rangle = \frac{2}{z_0} \ln\left(\frac{\sqrt{s}}{m_q}\frac{e^z}{G}\right). \quad (6)$$

The simplified model given here can be extended in a straightforward way to include transverse momentum, rapidly fluctuations, and production of various types of hadrons and resonances. However, we shall not need these for our discussion of charge retention.

III. CHARGE RETENTION

Let us now look at the charge-retention problem in the framework of the model of Sec. II. In $e^+e^- \rightarrow$ hadrons if we could look only at events starting with $e^+e^- \rightarrow u\bar{u}$ and distinguish the u-jet fragmentation region we should, if charge is retained, see a mean charge of $\frac{2}{3}$. It was pointed out by Farrar and Rosner[4a] that this hypothesis need not be correct. In the model of Sec. II it is clear why this happens. Since the average charge in the central rapidity region is zero, in defining the fragmentation region for the purpose of charge retention tests we may stop at any reasonably small value of y. Let us define the fragmentation region by $y > y_0$. The situation is shown in Fig. 5. The meson with smallest $y > y_0$ is composed of $q_2 + \bar{q}_1$. Had we looked at the total charge for $y > y_0$ but *not including* the quark q_2 (i.e., had we cut in the middle of the pion) we would have got the charge of the leading $\bar{q}_L$ *event by event.* In fact the net value of any additive quantum number Λ would, event by event, be the quantum number of $\bar{q}_L$: $\Lambda = \Lambda_{\bar{q}_L}$. However, to this we must add the Λ of q_2. So in this event $\Lambda = \Lambda_{\bar{q}_L} + \Lambda_{q_2}$. Thus the mean quantum number of all hadrons with $y > y_0$ is simply

$$\langle \Lambda \rangle = \Lambda_{\bar{q}_L} + \langle \Lambda_{q_2} \rangle. \quad (7)$$

For example, if only u and d quarks are produced from the gluons and they are produced with equal probability, then

$$\langle Q \rangle = Q_{\bar{q}_L} + \tfrac{1}{2} Q_u + \tfrac{1}{2} Q_d = Q_{\bar{q}_L} + \tfrac{1}{6},$$

where Q is charge. If $\bar{q}_L = \bar{u}$ then $\langle Q \rangle = -\frac{1}{2}$ and charge is not retained. However, for an SU(3)-symmetric sea

$$\langle Q \rangle = Q_{\bar{q}_L} + \tfrac{1}{3}(Q_u + Q_d + Q_s) = Q_{\bar{q}_L},$$

and thus charge is retained due to the fact that the *sum of all quark charges is zero.*[9]

In general let P_u, P_d, P_s, and P_c (etc.) be the respective probabilities for producing a u, d, s, and c quark in the sea (central rapidity region). Then for any additive quantum number Λ,

$$\langle \Lambda \rangle = \Lambda_L \pm \eta_\Lambda, \quad (8)$$

where η_Λ is the universal energy-independent constant

$$\eta_\Lambda = P_u \Lambda_u + P_d \Lambda_d + P_s \Lambda_s + P_c \Lambda_c.$$

Here Λ_L is the quantum number of the parent quark *or multiquark system;* $\Lambda_{u,d,s,c}$ are the quantum numbers of the u, d, s, and c quarks and the sign is + (−) if the leading parent needs a quark (antiquark) to neutralize it. Note that η_Λ *is independent of the process and of the jet type* but depends only

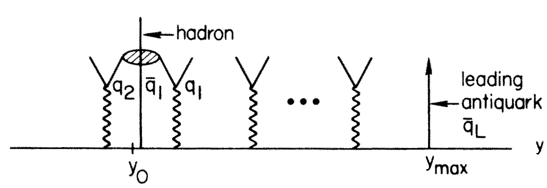

FIG. 5. Quark and antiquark constituents of mesons in the fragmentation region of an antiquark jet. The total hadronic charge for $y > y_0$ is $Q_{\bar{q}_2} + Q_{q_2}$.

on the quantum number Λ. Thus quantum-number retention is true to within a universal additive constant. This important result was first proved in a fragmentation model by Chan and Colglazier.[4b]

As an example consider the case of only an SU(2) sea. Then with $P_u = P_d = \frac{1}{2}$, $P_s = P_c = 0$,

$$\begin{aligned}\eta_{\text{charge}} &= \tfrac{1}{2}(\tfrac{2}{3} - \tfrac{1}{3}) = \tfrac{1}{6} \text{ not retained},\\ \eta_{\text{strangeness}} &= \tfrac{1}{2}(0+0) = 0 \text{ retained},\\ \eta_{\text{baryon number}} &= \tfrac{1}{2}(\tfrac{1}{3} + \tfrac{1}{3}) = \tfrac{1}{3} \text{ not retained},\\ \eta_{I_z} &= \tfrac{1}{2}(\tfrac{1}{2} - \tfrac{1}{2}) = 0 \text{ retained}.\end{aligned} \tag{9}$$

Here I_z is the z component of isospin. Note that baryon number is not retained. In fact for a quark (q) jet, $\langle B\rangle = B_q - \eta_q = \frac{1}{3} - \frac{1}{3} = 0$ as expected. I_z is always retained since we expect $P_u = P_d$ and $I_{zu} = -I_{zd}$, $I_{z_s} = I_{z_c} = 0$. In fact in a model with only SU(2) symmetry (no K production) charge retention is incompatible with I_z retention since if only pions are emitted $\langle Q\rangle = \langle I_z\rangle$ whereas $I_z \neq Q$ on the quark level. From our derivation of the charge retention results it is clear that as long as there are only short-range correlations in rapidity, Eq. (8) is quite model-independent. This is because any effect such as resonance decay, baryon production, or a deviation from the prefect ordering of the hadrons relative to the gluons will lead equally to a positive and negative contribution to $\langle\Lambda\rangle$. It will thus have no effect at all on $\langle\Lambda\rangle$.

The results of this section are consistent with the work of Refs. 4a and 4b in the context of the causal model of Sec. II. We have emphasized that the constant η is universal and is independent of the jet type and thus charge retention is a viable tool for identifying the quantum numbers of the parent. In fact if experimental determination of η_Λ, for all quantum numbers Λ, were possible one could deduce the strange- and charmed-quark content of the sea. (This might or might not be the same as the quark sea of the hadrons as probed in deep-inelastic scattering.) Such experiments are thus of great value in learning about quark fragmentation and possibly about the quark composition of hadrons. In the next section we shall apply these ideas to a study of certain definite experimental tests of charge retention and thus of the quark nature of hadrons.

IV. SPECIFIC TESTS FOR CHARGE RETENTION

The most familiar tests of charge retention are e^+e^- annihilation and deep-inelastic lepton (particularly charged-current) reactions. We shall review the practicality of these tests and then point out that the Drell-Yan[10] reaction ($h_1 + h_2 \to l^+l^- + X$, $l = \mu$ or e) can lead to additional important information on quantum-number retention for quark and multiquark jets.

A. e^+e^- annihilation and deep-inelastic scattering

In e^+e^- annihilation the final hadron jets will be a combination of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ jets. Thus the average charge (above the charm threshold) on the quark-jet side will be

$$\left[\sum_q Q_q^2(Q_q - \eta_Q)\right] \Big/ \sum_q Q_q^2 = \tfrac{7}{15} - \eta_Q . \tag{10}$$

The trouble with this is that we cannot separate the quark jet from the antiquark jet in e^+e^-. One useful test suggested by Bjorken and Miettinen[11] is to look for a strong back-to-back charge correlation in the leading hadrons. In other words, if one triggers on a leading π^+ the probability is large that we are seeing a fragment of either a u jet or a $\bar{d}$ jet. In either case there is a large probability that the leading hadron on the opposite side will be a π^-.

Extending this idea we consider the mean charge in a jet with a strongly leading π^+ (or K^+). Let $D_{h/q}(x)$ be the fragmentation function[3] for a quark q, fragmenting into a hadron, h, with a fraction x of its momentum. Define $\langle Q\rangle_{\pi^+(x)}(\langle Q\rangle_{K^+(x)})$ to be the total charge averaged over events on the side of a leading π^+ (K^+) with a fraction x of momentum. Then

$$\begin{aligned}\langle Q\rangle_{\pi^+(x),\,K^+(x)} &= \frac{(\frac{2}{3})^2(\frac{2}{3} - \eta_Q) + (\frac{1}{3})^2(\frac{1}{3} + \eta_Q)}{(\frac{2}{3})^2 + (\frac{1}{3})^2} + C_{\pi^+(x),\,K^+(x)}\\ &= \tfrac{3}{5}(1 - \eta_Q) + C_{\pi^+(x),\,K^+(x)},\end{aligned} \tag{11}$$

where C_h is a correction due to the possible fragmentation of a nonvalence quark into a π^+ or K^+, and

$$C_{\pi^+(x),\,K^+(x)} \to 0 \text{ as } x \to 1 .$$

The general formula for the charge in the hemisphere of a detected hadron, h, is then

$$\langle Q\rangle_{h(x)} = \frac{\sum_{q=\text{quark, antiquark}} Q_q^2 D_{h/q}(x)(Q_q \mp \eta_Q)}{\sum_q Q_q^2 D_{h/q}(x)} , \tag{12}$$

where $-$ (+) refers to quark (antiquark). Another test of charge retention in e^+e^- annihilation is given by Newmeyer and Sivers.[4c]

Probably the most straightforward test of charge retention is the reaction $\nu p \to \mu^- h^\pm X$. This has been discussed in some detail in Refs. 2, 3, 4, and 5. It is most easily visualized in the W^+p c.m. system[2] as in Fig. 6 with $s = (q+p)^2$. Let $x_{bj} = -q^2/2p\cdot q$, where p is the four-momentum of the incident proton and q is the four-momentum of the W^+. Then for $x_{bj} > 0.2$ predominantly the d quark inter-

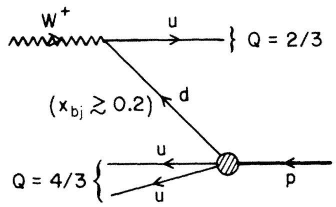

FIG. 6. Parton-model diagram for $\nu p \to \mu^- X$ as viewed in the W^+p c.m. system for the valence region ($x_{bj} \gtrsim 0.2$).

acts. We thus can study the charge retention properties of both a u quark and a uu diquark system. An asymmetry is expected in the charge distribution as shown in Fig. 7. For an SU(3)-symmetric sea ($\eta_Q = 0$) twice as much charge is expected in the proton fragmentation region as in the current (W^+) fragmentation region. For an SU(2) sea $\eta_Q = \frac{1}{6}$ and the charge ratios should be $\frac{3}{2} : \frac{1}{2}$ so that the expected asymmetry is increased. For $x_{bj} < 0.2$ the presence of sea quarks which can be hit by the W^+ tends to further increase the asymmetry between the two hemispheres. Figure 8 shows this clear asymmetry in the Fermilab 15-ft bubble-chamber data.[12] The main problem here is that detailed quantitative results cannot be deduced from these data since they are summed over many values of x_{bj} and q^2 due to small statistics.

It is also interesting to look at the probability that a given charged pion is a π^+ vs a π^-. To do this we take the data in Fig. 8 and plot

$$\left(\frac{dN^+}{dy} - \frac{dN^-}{dy}\right) \Big/ \left(\frac{dN^+}{dy} + \frac{dN^-}{dy}\right)$$

in Fig. 9. Even though we have not reached a region where the rapidity distribution is flat, we still see that hadrons in the fragmentation regions have a larger probability for being positive than those in the central region.

In electroproduction experiments the virtual photon can scatter off either a u or a d quark in the valence region and thus the identity of the jet

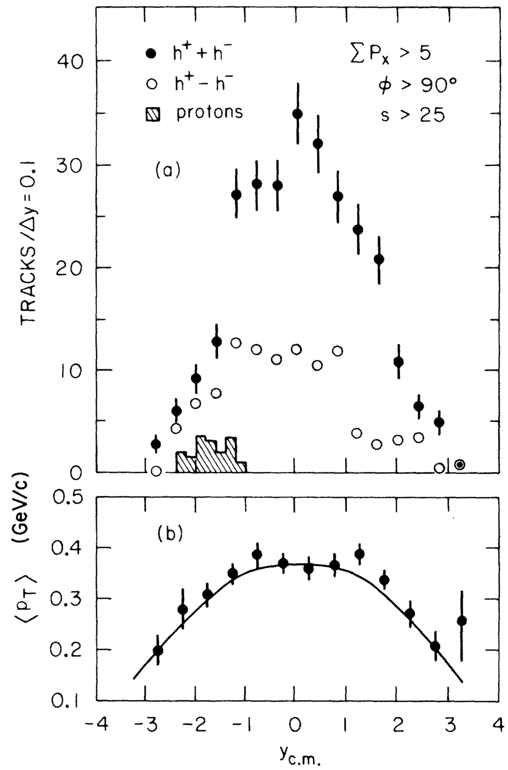

FIG. 8. Data from the Fermilab 15-ft bubble chamber for $\nu p \to \mu^- X$ (Ref. 12). The rapidity distribution and the charge structure $dN^+/dy - dN^-/dy$ of the final state are shown, as well as the transverse-momentum distribution of the emitted hadrons. The curve for $\langle p_T(y)\rangle$ is from pp reactions.

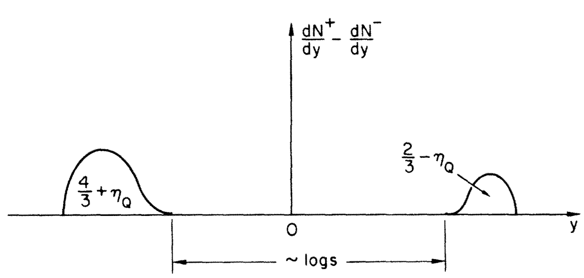

FIG. 7. The idealized distribution of charge in rapidity expected for the process of Fig. 6 as $s \to \infty$.

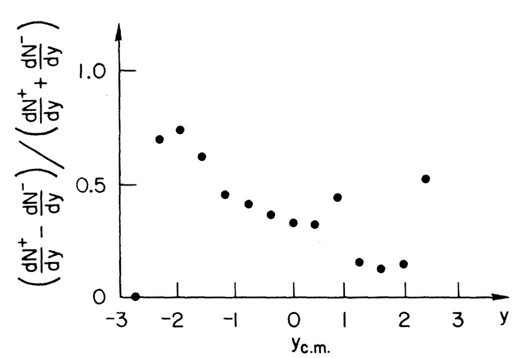

FIG. 9. Fractional excess of positive over negative hadrons for the data of Fig. 8. Error bars are not shown.

is again not determined. However, as we discussed for e^+e^- annihilation, one could look at the charge associated with a leading π^+ taking into account its u and $\bar{d}$ content and obtain definite predictions which could be compared with the experimental results.[13]

B. The Drell-Yan process

We believe that one of the most interesting reactions in which to test charge retention would be the Drell-Yan[10] process $h_1 + h_2 \to l^+l^- + X$, where all the final charged hadrons are observed. We assume that we are in the scaling region where the Drell-Yan mechanism dominates as in Fig. 10. Evidently in the final state we expect two multiquark jets.

The simplest example of this is $K^-p \to l^+l^- + X$ with x_K (x_p) the (light-cone) fraction of momentum of the $K^-(p)$ carried by the annihilating quarks. Since the invariant mass $\mathfrak{M}^2$ and the longitudinal-momentum fraction x_L of the lepton pair can be measured we can deduce both x_K and x_p. In fact assuming transverse effects to be small we have

$$\mathfrak{M}^2 = x_K x_p s,\quad x_L = x_K - x_p. \tag{13}$$

If x_L is not too large and $\mathfrak{M}^2$ is large so that both x_K and x_p are >0.2 then only the valence quarks annihilate. We then expect two clearly distinguishable jets as shown in Fig. 11. In the K^- fragmentation region a strange jet is expected with charge $-\frac{1}{3}$ and in the proton fragmentation region we expect a diquark jet with charge $+\frac{1}{3}$. Thus the charge structure expected is shown in Fig. 12. It is an interesting separate question how broad the peaks should be, but it is usually expected that they should be of a constant rapidity width as the neutral plateau increases logarithmically with the beam energy. The mean strangeness in the K^- fragmentation region should be $-1-\eta_s$ [-1 for SU(2) sea, $-\frac{2}{3}$ for SU(3) sea] and $0+\eta_s$ on the proton side [0 for SU(2) sea, $-\frac{1}{3}$ for SU(3) sea]. It will be interesting to compare these results with the charge distribution obtained in normal had-

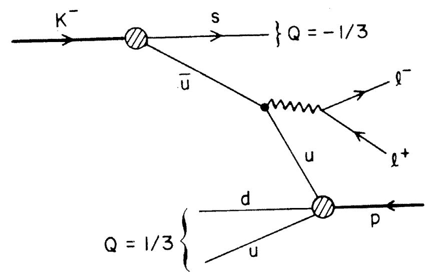

FIG. 11. The Drell-Yan mechanism for $K^-p \to l^+l^- + X$ in the valence-valence region ($x_K, x_p \gtrsim 0.2$).

ronic collisions.[14] The crucial observation will be to watch the decrease in the area under the $dN_+/dy - dN_-/dy$ curves in each hemisphere as one goes from normal events to wee-quark annihilation to valence-quark annihilation.

In order to decide whether experiments in the foreseeable future will be able to see such effects, we must estimate the width of the charge peaks. From the result of the $\nu p \to \mu^- X$ experiment shown in Fig. 3 we might expect a width of about 3 units in rapidity. However, those results are summed over many values of s and include sea contributions which modify the picture as we shall see. There are several factors to consider:

(1) Production and decay of leading resonances.
(2) Fluctuations in the rapidity of the leading hadrons.
(3) Transverse-momentum fluctuations.
(4) Background contributions to the Drell-Yan process.

If a ρ meson decays into $\pi\pi$ the mean longitudinal momentum of the pion will be $\frac{1}{2}$ its momentum (in the ρ rest frame). The resulting maximum and mean rapidities are shown in Fig. 13. We thus expect fluctuations of at least $\sim\frac{3}{2}$ units in rapidity. The effects (2), (3), and (4) are more difficult to estimate. However, in experiments where a plateau is seen, it tends to set in near $x \approx 0.2$.[15] This corresponds to ≈ 1.6 units of rapidity. We can then expect that in any experiment with $\gtrsim 4$ or 5 units of rapidity available end to end and where it is clear

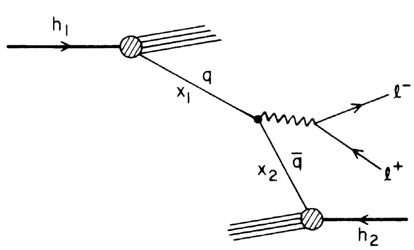

FIG. 10. Parton-model diagram for the Drell-Yan mechanism $h_1 + h_2 \to l^+l^- + X$.

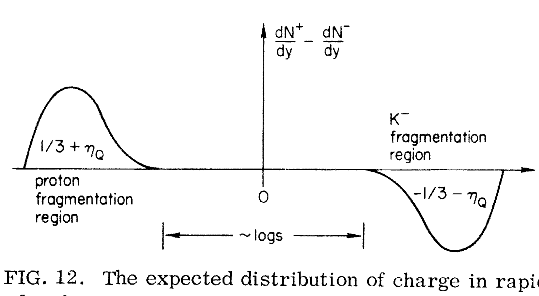

FIG. 12. The expected distribution of charge in rapidity for the process of Fig. 11.

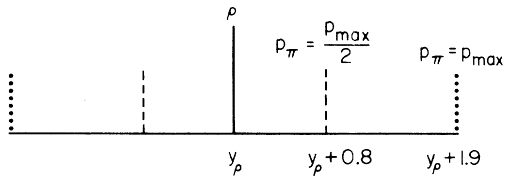

FIG. 13. The effect of ρ decay on the charge distribution near the fragmentation region. Here $p_{max}^2 = E_{max}^2 - m_\pi^2$ and $E_{max} = m_{\rho/2}$ in the rest frame of the decaying ρ meson.

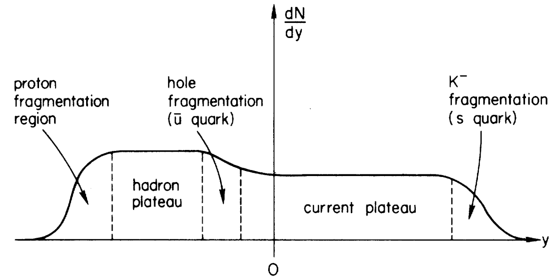

FIG. 15. The expected rapidity distribution for the process of Fig. 14 in the "hole fragmentation" model. In principle, the height of the hadron plateau need not be greater than that of the current plateau. We sketched it thus only to distinguish between the two regions.

that there is no hole fragmentation (i.e., where we have the simple situation of Fig. 11 with no annihilation of wee quarks) the charges should substantially be separated into respective hemispheres. More detailed information on the structure of the charge plateaus and the charge-flow distributions at finite energies can be estimated from the analysis of the pp data in Ref. 15.

Let us now look at $K^-p \to l^+l^-X$ but with x_L large and $\mathfrak{M}^2$ small so that $x_K \gtrsim 0.2$ but $x_p \lesssim 0.2$. Then to the contribution of Fig. 11 we must add the diagrams of Fig. 14 where wee quarks from the proton are struck. Both the rapidity distribution $dN^+/dy + dN^-/dy$ and the mean charge distribution $dN^+/dy - dN^-/dy$ would be extremely interesting quantities to study experimentally, especially since there is theoretical disagreement on their expected form and a good experiment could distinguish between these. We shall now study these various possibilities. For simplicity we consider the process of Fig. 14(a) which dominates over that of 14(b) by $\frac{4}{9} : \frac{1}{9}$ due to the charge values of the annihilating quarks.

Suppose that the wee $u\bar{u}$ pair was produced in the target by some neutral object in the sea (such as a gluon). Then the momentum of the u and $\bar{u}$ will be approximately the same.[16] According to Bjorken,[5] Feynman,[3] and others,[4] after the $u\bar{u}$ annihilation into l^+l^-, the $\bar{u}$-s system composed of the leading s and the remnant wee $\bar{u}$ will be a jet pair similar to that in e^+e^-, whereas the proton jet is simply a fragmenting proton which fragments as in usual hadronic collisions. The final rapidity distribution is then divided into the five regions of Fig. 15. According to this picture the charge will be distributed in rapidity as shown in Fig. 16. Thus the charge (-1) of the K^- should be spread over a much broader region than that of the proton.

According to the color model of Ref. 7 there *need not be* a specific hole fragmentation region since no multiplicity is produced until color has separated. Thus the height of the plateau could be constant throughout the central region. Thus there is a possibility that the charge will be smeared between the proton and the $\bar{u}$ quark and it would average to $\frac{1}{3} + \eta_Q$ over the entire region as shown in Fig. 17. This would occur, for example, if the wee quarks were daughters of mesons in the proton cloud; for example, via $p \to n + \pi^+ \to n + u\bar{d}$. In this case the wee u would be accompanied by a leading neutron and a wee $\bar{d}$. Alternatively if the quarks in a multiquark state are in equilibrium before the interaction, the wee u quark will not necessarily be accompanied by a wee $\bar{u}$ quark. It is thus clear that an experiment of this type would be extremely helpful in distinguishing these possibilities and thus in understanding the nature of quark fragmentation.

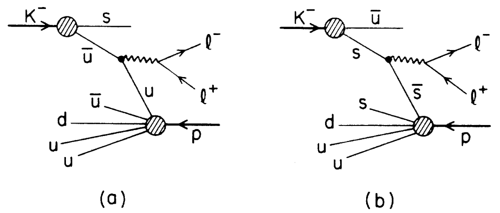

FIG. 14. The Drell-Yan mechanism for $K^-p \to l^+l^-X$ in the "valence-sea" region ($x_K \gtrsim 0.2$, $x_p \lesssim 0.2$).

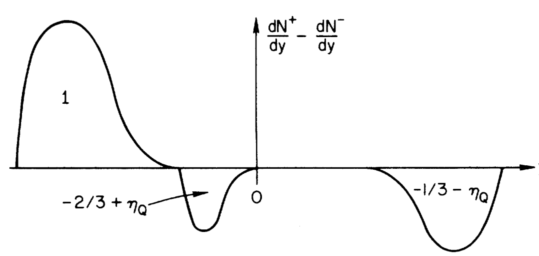

FIG. 16. The expected distribution of charge in rapidity for the process of Fig. 14 in the "hole fragmentation" model.

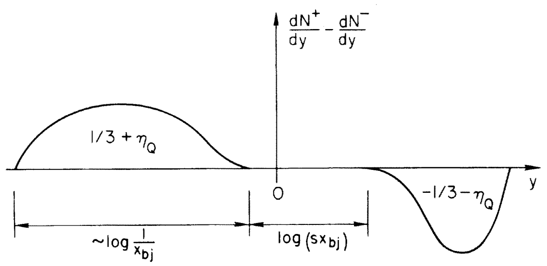

FIG. 17. An alternate possibility for the distribution of charge in rapidity for the process of Fig. 14.

The reaction $K^+p \to l^+l^- + X$ can also lead to interesting tests of these ideas. The dominant interaction for $x_K > 0.2$ will be a $u\bar{u}$ annihilation, leading to a dramatic $(\frac{1}{3}+\eta_Q):(\frac{5}{3}-\eta_Q)$ charge ratio in the $K^+:p$ fragmentation regions. The less frequent $\bar{s}s$ annihilation gives a $(\frac{2}{3}-\eta_Q):(\frac{4}{3}+\eta_Q)$ ratio.

A very important Drell-Yan experiment is $pp \to l^+l^- + X$ at the CERN ISR due to the large available energy. The protons have more than 8 units of rapidity available end to end at $\sqrt{s} = 62$ GeV. After the lepton-pair formation the available energy is reduced by

$$E_{l^+l^-} \approx \frac{\sqrt{s}}{2}\left(x_L{}^2 + \frac{4\mathfrak{M}^2}{s}\right)^{1/2}. \tag{14}$$

The available rapidity for the proton is

$$\Delta y = \Delta y_{\text{initial}} + \ln\left[1 + \frac{\mathfrak{M}^2}{s} - \left(x_L{}^2 + \frac{4\mathfrak{M}^2}{s}\right)^{1/2}\right]. \tag{15}$$

For example, if $\mathfrak{M}^2 \ll s$ and $x_L = \frac{1}{2}$ then only ≈ 0.7 units of rapidity are lost.

The minimal Drell-Yan process for pp is shown in Fig. 18 and allows a study of both a qq and a $qqqq$ jet system. Let us consider first $x_1 > 0.2$ and $x_2 < 0.2$, i.e., $x_L > 0$. Figure 18(b) will dominate since there are two u quarks in the proton and there is an enhancement from the (charge)^2 of the annihilating quarks. According to the hole fragmentation model we should expect the situation in Fig. 19, where the charge structure of Fig. 19(b) dominates. Of course, all the possible alternatives mentioned in connection with the K^-p Drell-Yan process are possible here as well. The rapidity distribution at ultrahigh energies will probably be flat throughout the central region. However, the charge $\frac{5}{3} - \eta_Q$ could be spread out on the left side of the rapidity distribution. In any case a charge asymmetry of $\frac{5}{3} - \eta_Q : \frac{1}{3} + \eta_Q$ between the two hemispheres would be striking since in ordinary pp scattering the proton charges are clearly deposited in their fragmentation regions.[14,15]

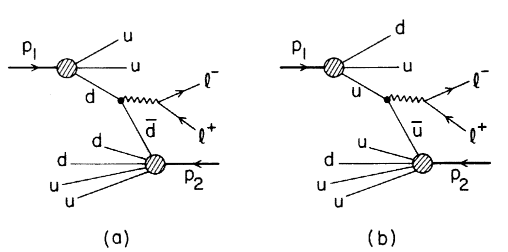

FIG. 18. The Drell-Yan mechanism for the process $pp \to l^+l^- + X$ in the valence-sea region ($x_1 \gtrsim 0.2$, $x_2 \lesssim 0.2$).

The study suggested above is in the valence-sea region (e.g., if $\mathfrak{M}^2 = 0.004\ s$ and $x_L = 0.39$ then $x_1 = 0.01$ and $x_2 = 0.4$). If both quarks are in the sea then the situation is much more complicated. In fact the knowledge of x_L and $\mathfrak{M}^2$ is not sufficient, in any given event, to distinguish the two jets. As $\mathfrak{M}^2/s \to 0$ one would expect the hadron distributions to become very similar to those in ordinary pp scattering.

As a final example we consider $\pi^\pm p \to l^+l^-X$ in the valence-valence region. The expected processes and their charge distributions are shown in Fig. 20. These experiments with sufficiently high energy would also be useful in studying the quark nature of these jets.

V. THE CHARGE-MOMENTUM VECTOR

In this section we shall consider how the "charge momentum" might be used to study jet structure. The idea of charge-momentum flow was first introduced in studying charge-exchange reactions in hadronic collisions.[17] We shall study its applicability to jet structure. Consider, for example, $e^+e^- \to$ hadrons. Define the "charge-momentum" vector[2] for any additive quantum number Λ by

$$J^\mu_\Lambda = \sum_{\substack{\text{hadrons}\\(i)}} P^\mu_i \Lambda_i . \tag{16}$$

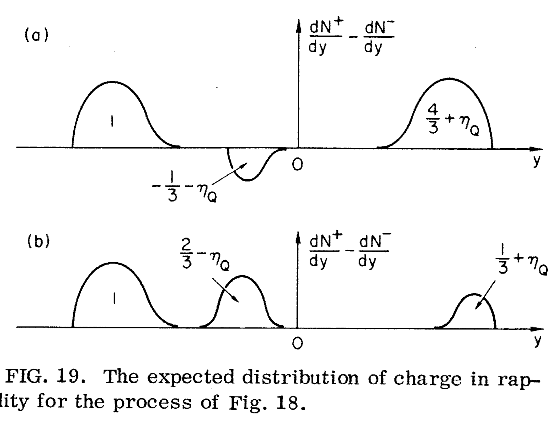

FIG. 19. The expected distribution of charge in rapidity for the process of Fig. 18.

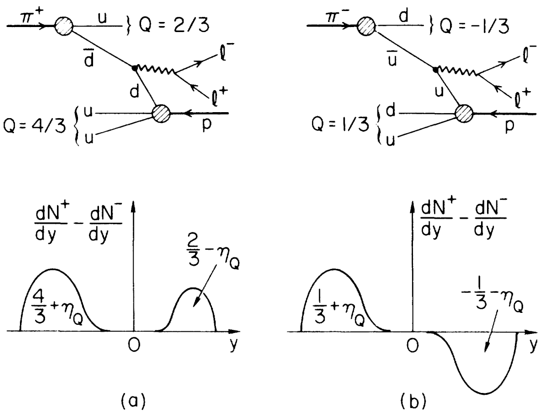

FIG. 20. The Drell-Yan mechanism and the expected charge distribution for $\pi^{\pm}p \rightarrow l^+l^- + X$ in the valence-valence region ($x_\pi, x_p \gtrsim 0.2$).

Here Λ_i is the quantum number of hadron i and P_i^μ is its four-momentum. We could also consider the "convection current"

$$J^{\mu}_{(V)\Lambda} = \sum_{\substack{\text{hadrons}\\(i)}} \Lambda_i P_i^\mu / m_i = \sum_i V_i^\mu \Lambda_i . \quad (17)$$

Here m_i and V_i^μ are the mass and four-velocity of the hadron i. Besides being covariant, an essential advantage of J^μ is that it emphasizes leading hadrons over those in the central rapidity region.

In the reaction $e^+e^- \rightarrow$ hadrons, on the quark level we have $e^+e^- \rightarrow q\bar{q}$ and thus in the e^+e^- c.m. system

$$\begin{aligned} J^0_\Lambda &= \Lambda_q \frac{\sqrt{s}}{2} - \Lambda_q \frac{\sqrt{s}}{2} = 0 , \\ \vec{J}_\Lambda &\approx \left(\Lambda_q \frac{\sqrt{s}}{2} + \Lambda_q \frac{\sqrt{s}}{2}\right)\hat{k} = \Lambda_q \sqrt{s}\, \hat{k} , \end{aligned} \quad (18)$$

where Λ_q is the quantum number of the quark and $\hat{k}$ is a unit vector in the jet direction.

For $e^+e^- \rightarrow$ hadrons in the one-photon approximation $\langle J^\mu_Q \rangle_{\text{events}} = 0$ due to charge-conjugation invariance. Instead it is interesting to look at $\langle (J^0)^2 \rangle$ in the c.m. system and $\langle J^\mu J_\mu \rangle$. If J^μ on the hadron level were similar to that on the quark level we would expect $\langle J^\mu J_\mu \rangle \propto s$ and $\langle (J^0)^2 \rangle \ll \langle J^\mu J_\mu \rangle$. In all jet models which we have considered we find $\langle J^2 \rangle \propto s$ but $\langle J^2 \rangle / s$ is very model-dependent as is any general conclusion about $\langle (J^0)^2 \rangle$. In fact we found $\langle (J^0)^2 \rangle \propto s$ as well. This points out very clearly that the charge-momentum vector is not, in any sense, conserved. As an example let us calculate $e^+e^- \rightarrow \mu^+\mu^- \rightarrow e^+e^-\bar{\nu}_e\nu_e\bar{\nu}_\mu\nu_\mu$. For the reaction $e^+e^- \rightarrow \mu^+\mu^-$ we find as for the quark case that $\langle (J^0)^2 \rangle = 0$ and $\langle \vec{J}^2 \rangle \approx s$ (for Q = charge in units of the proton charge). From the decay distribution of $\mu^\pm \rightarrow e^\pm \nu\bar{\nu}$ we can evaluate $\langle (J^0)^2 \rangle$ and $\langle \vec{J}^2 \rangle$ for the final state and find

$$\begin{aligned} \langle (J^0)^2 \rangle &\approx \tfrac{1}{48} s , \\ \langle \vec{J}^2 \rangle &\approx \tfrac{3}{20} s , \\ \langle J^\mu J_\mu \rangle &\approx \tfrac{31}{240} s , \end{aligned} \quad (19)$$

so that

$$\frac{\langle (J^0)^2 \rangle}{\langle J^\mu J_\mu \rangle} \approx \tfrac{5}{31} .$$

We have calculated $\langle (J^0)^2 \rangle$ and $\langle \vec{J}^2 \rangle$ in several models including the model of Sec. II and find that the ratio $\langle (J^0)^2 \rangle / \langle \vec{J}^2 \rangle$ is very model-dependent.

In reactions other than $e^+e^- \rightarrow$ hadrons the situation is even more complicated. In the case of $\nu p \rightarrow \mu^- X$ for $x_{bj} > 0.2$ we have, in the W^+p c.m. system the situation of Fig. 6. On the quark level

$$\begin{aligned} J^0 &= \tfrac{4}{3} E_{(uu)} + \tfrac{2}{3} E_u \propto 2\sqrt{s} , \\ |\vec{J}| &= \tfrac{4}{3} P_{(uu)} - \tfrac{2}{3} P_\mu \propto \tfrac{2}{3}\sqrt{s} , \end{aligned} \quad (20)$$

where $E_{(uu)}, P_{(uu)}, E_u, P_u$ are the energy and momenta of the uu diquark system and the u quark. Here $\sqrt{s}$ is the total c.m. energy of the W^+p system. Observe that the situation is reversed and $\langle \vec{J}^2 \rangle < \langle (J^0)^2 \rangle$ might be expected.

In general, the evaluation of $\langle J^\mu J^\nu \rangle$ can be a convenient and covariant method by which to label the properties of a jet. However, specific predictions depend critically on the fragmentation model, and one can thus possibly differentiate between different mechanisms. One might expect $\vec{J}$ in any event to point in the jet direction. However, it might tend to overemphasize transverse-momentum fluctuations. For example, if a π^+ is emitted with transverse momentum $p_\perp$ relative to the jet axis and a π^- compensates this $p_\perp$, then $\vec{J}$ will acquire a transverse component of $2p_\perp$ rather than canceling.

Another use of the charge-momentum vector would be in identifying the onset of weak effects in $e^+e^- \rightarrow$ hadrons in high-energy storage rings. Owing to the predicted interference between $e^+e^- \rightarrow \gamma \rightarrow$ hadrons and $e^+e^- \rightarrow Z^0 \rightarrow$ hadrons, we expect $\langle J^\mu \rangle \neq 0$. This is also the case for the interference between the 2γ and 1γ process in e^+e^- annihilation.[18]

VI. DISCUSSION AND CONCLUSIONS

In this paper we have emphasized that quantum-number retention in the fragmentation region can be a viable method for verifying the quark nature of hadrons and for identifying specific quark and multiquark systems. We have shown that a simple "inside-outside" cascade model with a causal space-time structure predicts that the mean charge of hadrons in the jet fragmentation region equals the charge of the parent quark system modulo a universal constant η_Q (the mean quark charge of the sea). This result appears to be quite general and agrees with earlier work[4] utilizing fragmentation models. Although it is well known that the hypothesis of exact quantum-number retention is in general incorrect, it has not been sufficiently stressed that the correction terms η_Λ are universal numbers which can be established empirically.

There is already some evidence from analyses of experiment for a nonzero value of η_Q. Field and Feynman[19] find from their parametrizations of the quark fragmentation distributions $D_{h/q}(x)$ that

$$\langle Q\rangle_q \simeq \sum_{h=\pi^\pm K^\pm} Q_h \int_0^1 D_{h/q}(x)dx = 0.59, -0.40, -0.39$$

for $q=u,d,s$ respectively. Their analysis uses e^+e^- and electroproduction single-particle inclusive data and neglects baryon production. The results are consistent with $\eta_Q = 0.07 \pm 0.01$ (although the uncertainty in the analysis is probably larger than this error estimate).

As we have discussed in Sec. IV B, one of the most interesting areas of application of charge retention will be massive-lepton-pair production where the Drell-Yan mechanism is expected to dominate. Charge-retention studies should allow a specific identification of the quark and multi-quark jet system expected in the beam and target fragmentation regions. In addition one can study the distribution of the hadronic charge when sea quarks annihilate and thus test the ideas of hole fragmentation[5] to see if there is local compensation of charge. It will also be interesting to compare events with high-mass lepton pairs on and off the J/ψ and other resonances as a check of the production mechanism (e.g., production by gluons). It is also interesting to compare the charge distribution in the processes discussed above with that in normal inelastic hadronic events.

In the case of e^+e^- several tests of charge retention are possible as discussed in Sec. IV A. It will be interesting to compare jet charges above and below the charm threshold. Another test, suggested in Ref. 20, is to measure hadron charge asymmetries associated with photons produced at large transverse momentum relative to the jet axis. The ratio of hadron to muon asymmetries

$$R_h^{(3)}(x) \equiv \frac{\dfrac{d\sigma(e^+e^-\to\gamma h^+X)}{d^3k/k_0 d\Omega dx} - \dfrac{d\sigma(e^+e^-\to\gamma h^-X)}{d^3k/k_0 d\Omega dx}}{\dfrac{d\sigma(e^+e^-\to\gamma\mu^+\mu^-)}{d^3k/k_0 d\Omega_{\mu^+}} - \dfrac{d\sigma(e^+e^-\to\gamma\mu^-\mu^+)}{d^3k/k_0 d\Omega_{\mu^-}}}$$

is given in the quark-parton model as $(Q_{\mu^+}=1)$

$$R_h^{(3)}(x) = \sum_q Q_q^3 [D_{h/q}(x) - D_{\bar h/q}(x)]\,.$$

As we have shown here, the mean charge in the quark jet is

$$\langle Q\rangle_{\rm jet} = \sum_{\substack{h=\text{positive}\\ \text{hadrons}}} Q_h \int_0^1 dx [D_{h/q}(x) - D_{\bar h/q}(x)]$$
$$= Q_q - \eta_Q\,.$$

Thus $R_h^{(3)}(x)$ satisfies the sum rule

$$\sum_{\substack{h=\text{positive}\\ \text{hadrons}}} Q_h \int_0^1 dx R_h^{(3)}(x) = \sum_q (Q_q^4 - \eta_Q Q_q^3)\,.$$

The testing of this sum rule as well as the scaling behavior of $R_h^{(3)}(x)$ would be an important test of the quark ideas.

Once one confirms the quantum-number-retention analysis in experiments discussed here, the technique can become an effective tool for analyzing subprocesses which control large-p_T hadronic reactions. Thus in a model based exclusively on quark-quark scattering[21,19] ($qq\to qq$) one expects an equal average charge for each of the large-p_T jets (toward and away), both for events triggered by single particles or by jets. On the other hand, in the constituent-interchange model,[22] in regions where the subprocess $Mq\to Mq$ is expected to dominate, the jet on the trigger side (usually the side of the meson, M) should have a charge structure different from the away side jet. For $pp\to K^+X$ or $pp\to\pi^+X$, the dominant recoil jet will correspond to a u quark, giving $\langle Q\rangle = \frac{2}{3} - \eta_Q$. Care will be needed to separate the large-p_T jets from background contributions. The beam fragmentation regions will also have a different quantum-number structure for the different subprocess models. Thus quantum-number retention in charge, strangeness, and baryon number may turn out to be a useful method for discriminating underlying hadronic mechanisms at short distances.

Clues to the structure of jets in each of the processes discussed here can also be obtained by utilizing the $x\to 1$ behavior of the leading particles in the jet. This is discussed in Ref. 23 and Ref. 2.

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