

Extended partially conserved axial-vector current hypothesis and model-dependent results*

C. A. Dominguez

Departamento de Física, Centro de Investigación, y de Estudios Avanzados del Instituto Politécnico Nacional, Apartado Postal 14-740, México 14, D. F.

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The corrections to Goldberger-Treiman relations for $\Delta S = 0$ and $|\Delta S| = 1$ β decays (Δ_π and Δ_K , respectively) are estimated from a Veneziano-type model for three-point functions. The effect of unitarizing the model is also discussed, and it turns out that Δ_π and Δ_K are almost insensitive to a variation in the widths of the pseudoscalar-meson daughters. Moreover, the predictions for Δ_π and Δ_K are in close agreement with experiment. Finally, on-mass-shell extrapolation factors for chiral anomalies in $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\gamma$ are also derived, and agreement with experiment is found without the need for invoking η - η' mixing. In summary, the model discussed here seems to be a suitable implementation of the recently proposed extended partially conserved axial-vector current hypothesis.

I. INTRODUCTION

Some time ago it was suggested¹ that there might exist a heavy pion and a heavy kaon which could account for the rather large magnitude of the corrections to Goldberger-Treiman relations² (GTR's) for $\Delta S = 0$ and $|\Delta S| = 1$ β decays (Δ_π and Δ_K , respectively). This idea was then incorporated³ into a Veneziano-type model for the πNN vertex function predicting a value for Δ_π in close agreement with experiment. Later on, a two-component partially conserved axial-vector current (PCAC) hypothesis was discussed by other authors.⁴

Recently, we proposed^{5,6} a generalization of the standard "strong" PCAC hypothesis that accommodates, in a model-independent way, a family of heavy bosons. This extended PCAC (EPCAC) hypothesis reads

$$\partial^\mu A_\mu^\alpha = \sum_{n=0}^N m_{a_n}^2 f_{a_n} \phi_{a_n}^\alpha \quad (a = \pi, K, \eta), \quad (1)$$

where N ($N > 1$), m_{a_n} , f_{a_n} , and $\phi_{a_n}^\alpha$ are left unspecified. Equation (1) coupled with an assumption about dominance of certain diagrams in the limit of zero-mass bosons predicts a universality among Δ_π on the one hand and among Δ_K on the other.⁵ This universality may also be obtained within specific models as has been shown recently by Fuchs⁷ in the context of the quark model. Another virtue of EPCAC is that it links several chiral-symmetry-breaking problems together in a unified fashion and improves the numerical predictions of the soft-meson theorems of PCAC and current algebra. As it turns out, on-mass-shell extrapolation factors are determined entirely by the chiral-symmetry-breaking universal parameters Δ_π and Δ_K which can be taken from experiment. However, if one wishes to obtain predictions for Δ_π and Δ_K it is obviously necessary to go beyond the general hy-

pothesis, Eq. (1), and study specific models for m_{a_n} , f_{a_n} , etc.

To this end, we reexamine in this paper the three-point-function Veneziano model⁷ that has already been used³ to predict Δ_π . An advantage of such an approach⁸ is that one has a single free-parameter formula for the form factor with a transparent physical interpretation. Successful applications of this ansatz include,⁷ besides Δ_π , the electromagnetic form factors of the nucleon, pion, kaon, and $\Delta(1236)$ as well as the nucleon axial-vector form factor. In every instance this model gives better predictions (in the spacelike region) than, e.g., dipole fits.

The Veneziano-type electromagnetic form factors might seem to rely upon a more credible basis due to the real existence of at least two vector-meson daughters.⁹ However, a kaon daughter with a mass in good agreement with the prediction of the Veneziano mass spectrum has already been detected at SLAC.¹⁰ There is yet no evidence, though, for a heavy pion and a plausible cause could be a very large width. For this reason we study here how the width affects the prediction for Δ_π . This is done following the unitarization scheme proposed by Urrutia¹¹ for the Veneziano-type electromagnetic pion form factor. The result is that the value of Δ_π is almost insensitive to a variation in the width of the pion daughters. In fact, it turns out that if these daughters have all infinite widths the prediction for Δ_π is just one half of the value obtained in the zero-width approximation.

Next we calculate Δ_K and find good agreement with experiment. The conditions under which Δ_π and Δ_K might become universal, within this model, are also discussed. Finally, a prediction is made for the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi\pi\gamma$ decay rates which agrees with experiment without the need for invoking η - η' mixing. Although there are many ap-

plications related to chiral anomalies, we have chosen the η decays as a representative example of how misleading might be the usual assumption of ignoring on-mass-shell extrapolations from the soft-meson points.

II. CORRECTIONS TO GOLDBERGER-TREIMAN RELATIONS

Let us consider the β decay $H(p_f) \rightarrow H'(p_i) + \bar{l}\bar{\nu}$, and write the matrix element of the axial-vector current between initial and final hadrons as

$$\langle H'(p_f) | A_\mu^+(0) | H(p_i) \rangle = \bar{u}(p_f) \tau^+ [\gamma_5 \gamma_\mu g_H^A(q^2) + \gamma_5 q_\mu h_H^A(q^2)] u(p_i), \quad (2)$$

where $q = p_f - p_i$. Taking the divergence on both sides of Eq. (2) one has

$$\langle H'(p_f) | i \partial^\mu A_\mu^+(0) | H(p_i) \rangle = \bar{u}(p_f) \gamma_5 \tau^+ D_H(q^2) u(p_i), \quad (3)$$

where

$$D_H(q^2) = (m_H + m_{H'}) g_H^A(q^2) + q^2 h_H^A(q^2). \quad (4)$$

According to the EPCAC hypothesis, Eq. (1), $D_H(q^2)$ can be written as

$$D_H(q^2) = \sum_{n=0}^N \frac{g_{H'Ha_n} f_{a_n} m_{a_n}^2}{m_{a_n}^2 - q^2}. \quad (5)$$

In the framework of the Veneziano model the mass spectrum is given by

$$m_{a_n}^2 = m_0^2 + \frac{1}{\alpha'} n, \quad (6)$$

where m_0 is the mass of the lowest-lying boson and $\alpha' = 1/2m_\rho^2 \cong 1 \text{ GeV}^{-2}$ is the slope of the (linear) Regge trajectory,

$$\alpha_a(q^2) = \alpha'(q^2 - m_0^2) \quad (a = \pi, K, \eta). \quad (7)$$

Furthermore, the vertex function is written as the ratio of two Γ functions, i.e.,

$$D_H(q^2) = C \frac{\Gamma(-\alpha_a(q^2))}{\Gamma(\beta - \alpha_a(q^2))}, \quad (8)$$

where C is a constant fixed by the normalization condition on the lowest-lying boson pole and β is a free parameter related to the asymptotic behavior of the form factor,

$$\lim_{q^2 \rightarrow -\infty} D_H(q^2) = (-q^2)^{-\beta}. \quad (9)$$

If β is an integer then Eq. (8) reduces to a sum of a finite number of poles, while for noninteger values of β , $D_H(q^2)$ is built up from the contribution of an infinite number of poles located on the real axis (zero-width approximation).

The ansatz, Eq. (8), fixes the product $g_{H'Ha_n} f_{a_n}$ to be

$$g_{H'Ha_n} f_{a_n} = \frac{m_0^2}{m_{a_n}^2} f_a \frac{\Gamma(\beta)}{\Gamma(\beta - n)} \frac{(-)^n}{n!} g_{H'Ha}, \quad (10)$$

TABLE I. Predictions for Δ_π .

| β | Δ_π |
|---------|--------------|
| 1 | 0 |
| 1.6 | 0.014 |
| 2.0 | 0.019 |
| 2.6 | 0.025 |
| 3.0 | 0.029 |
| 4.0 | 0.035 |
| 5.0 | 0.040 |

where the normalization condition for D_H has already been used. The expression for $D_H(0)$ becomes

$$D_H(0) = f_a \sqrt{2} g_{H'Ha} \Gamma(\beta) \frac{\Gamma(1 + \alpha' m_0^2)}{\Gamma(\beta + \alpha' m_0^2)} = (m_H + m_{H'}) g_H^A. \quad (11)$$

Recalling the definition of the correction to the GTR's, i.e.,

$$\Delta_a = 1 - \frac{(m_H + m_{H'}) g_H^A}{\sqrt{2} g_{H'Ha} f_a}, \quad (12)$$

one finally has

$$\Delta_a = 1 - \Gamma(\beta) \frac{\Gamma(1 + \alpha' m_0^2)}{\Gamma(\beta + \alpha' m_0^2)}. \quad (13)$$

In the chiral-symmetry limit ($m_0^2 \rightarrow 0$) Δ_a vanishes as expected. The same result is obtained by setting $\beta = 1$, which corresponds to pseudoscalar-meson dominance.

From Eq. (13) one sees that if the asymptotic behavior of the $H'Ha$ vertex function is the same for all baryons H' and H , then Δ_a is universal. However, it is clear that even if β is the same for the $H'H\pi$ and $H'HK$ form factors, Δ_π and Δ_K are still different.

Tables I and II show the predictions of Eq. (13) for Δ_π and Δ_K , respectively, as a function of β . The axial-vector form factor of the nucleon, $g^A(q^2)$, is presently fitted¹² with $\beta \cong 2.5$; therefore, one might expect something not very different from this behavior for $D_H(q^2)$. The experimental values¹³ of Δ_π and Δ_K are

TABLE II. Predictions for Δ_K .

| β | Δ_K |
|---------|------------|
| 1 | 0 |
| 1.6 | 0.14 |
| 2.0 | 0.20 |
| 2.6 | 0.25 |
| 3.0 | 0.28 |
| 4.0 | 0.34 |
| 5.0 | 0.38 |

$$\Delta_{\pi} = 0.06 \pm 0.02 \quad (14)$$

and

$$\Delta_K = 0.30 \pm 0.15. \quad (15)$$

The agreement for Δ_K is impressive if one takes into account that theoretical estimates based on the saturation of dispersion relations give¹⁴ $\Delta_K \simeq 0.051$. Moreover, assuming the same value of β for Δ_{π} and Δ_K one has (for every β up to $\beta \simeq 4$)

$$\frac{\Delta_{\pi}}{\Delta_K} \simeq 0.1, \quad (16)$$

in remarkable agreement with the model-independent prediction of EPCAC,⁵ $\Delta_{\pi}/\Delta_K = 0.09 \pm 0.01$.

As a final point we note that if only even daughters are considered [$n = \text{even number in Eq. (6)}$], then Δ_a is reduced by approximately a factor of $\frac{1}{2}$, i.e., Δ_a (even daughters) $\simeq \frac{1}{2}\Delta_a$ (all daughters).

III. UNITARIZED MODEL

The zero-width ansatz relevant to Δ_{π} , Eq. (8), may be written as

$$D^0(q^2) = C \sum_{n=0}^{\infty} \frac{(-)^n}{\Gamma(\beta-n)} \frac{1}{n!} \frac{1}{\mu_{\pi_n}^2 - q^2}, \quad (17)$$

where

$$C = f_{\pi} \sqrt{2} g_{\pi NN} \mu_{\pi}^2 \Gamma(\beta). \quad (18)$$

On the other hand, $D(q^2)$ is assumed to satisfy an unsubtracted dispersion relation,¹ i.e.,

$$\text{Im}D(q^2) = (\text{Born term}) + \theta(q^2) C \sum_{n=1}^{\infty} \frac{(-)^n}{\Gamma(\beta-n)} \frac{1}{n!} \frac{\Gamma_n \mu_{\pi_n}}{(\mu_{\pi_n}^2 - q^2)^2 + \Gamma_n^2 \mu_{\pi_n}^2} \frac{q^2}{\mu_{\pi_n}^2}. \quad (23)$$

The only remaining unknown is Γ_n , which we assume behaves as for heavy vector mesons,¹⁵ i.e.,

$$\Gamma_n = \gamma \mu_{\pi_n} \quad (n \geq 1), \quad (24)$$

where γ must be regarded, in the present case, as a second free parameter.

Inserting Eq. (23) into Eq. (19) and performing the integrations one finds for $D(0)$ the following result:

$$D(0) = D(0) \Big|_{\gamma=0} \left(1 - \frac{\tan^{-1}\gamma}{\pi} \right) + f_{\pi} \sqrt{2} g_{\pi NN} \frac{\tan^{-1}\gamma}{\pi}. \quad (25)$$

while $\Delta_{\pi}(\gamma)$ is given by

$$\Delta_{\pi}(\gamma) = \Delta_{\pi}(\gamma=0) \left(1 - \frac{1}{\pi} \tan^{-1}\gamma \right), \quad (26)$$

and in particular

$$\Delta_{\pi}(\gamma = \infty) = \frac{1}{2} \Delta_{\pi}(\gamma = 0), \quad (27)$$

$$D(q^2) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\text{Im}D(q'^2)}{q'^2 - q^2} dq'^2, \quad (19)$$

from which it follows that

$$\text{Im}D^0(q^2) = (\text{Born term}) + \theta(q^2 - 9\mu_{\pi}^2) C\pi \times \sum_{n=1}^{\infty} \frac{(-)^n}{\Gamma(\beta-n)} \frac{1}{n!} \delta(\mu_{\pi_n}^2 - q^2) \quad (20)$$

and the Born term corresponds to the usual pion.

Following Urrutia,¹¹ we accomplish the unitarization by the following replacement:

$$\delta(\mu_{\pi_n}^2 - q^2) \rightarrow \frac{1}{\pi} \frac{\Gamma_n \mu_{\pi_n}}{(\mu_{\pi_n}^2 - q^2)^2 + \Gamma_n^2 \mu_{\pi_n}^2} \quad (n \geq 1). \quad (21)$$

Next, the threshold behavior of $\text{Im}D(q^2)$ is needed. In the $\text{SU}(2) \times \text{SU}(2)$ symmetry limit one knows that¹

$$\text{Im}D(q^2) = O(q^2), \quad (22)$$

while the physical threshold is located at $q^2 = 9\mu_{\pi}^2$. Since we are treating the ordinary pion as a Born term, the second-sheet poles in the complex q^2 plane start at $\mu_{\pi_1}^2 \simeq 1.02 \text{ GeV}^2$, which is far away from $q^2 = 9\mu_{\pi}^2$. Therefore, as a first approximation one can effectively shift the beginning of the cut to the origin and use the threshold condition at $q^2 = 0$. As we shall see later this approximation does not introduce any significant difference in the final result for Δ_{π} .

In this case one has

which shows how little Δ_{π} depends on the widths of the pion daughters. It is a simple exercise to show that Eq. (27) is still valid in the case in which the threshold is located at $q^2 = 9\mu_{\pi}^2$.

In summary, if a heavy pion of a large width is eventually detected, the predictions of the non-unitarized model shall suffer only minor modifications.

IV. RADIATIVE η DECAYS

The EPCAC predictions for the extrapolation of chiral-anomaly results to the mass shell have been discussed in detail in Ref. 5. Therefore, we shall proceed directly to the application of the Veneziano model, discussed in the previous sections, to the case of $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi\pi\gamma$ decays. The amplitudes for these processes, \mathcal{F}_n and \mathcal{G}_n , respectively, are defined by

$$M(\eta \rightarrow \gamma\gamma) = \epsilon_{\mu\nu\alpha\beta} k_1^\mu k_2^\nu \epsilon_1^\alpha \epsilon_2^\beta \mathfrak{F}_\eta(q^2), \quad (28)$$

$$M(\eta \rightarrow \pi^+ \pi^- \gamma) = \epsilon_{\mu\alpha\beta\gamma} \epsilon^\mu k^\alpha P_+^\beta P_-^\gamma \mathfrak{G}_\eta(p_+ \cdot k, p_- \cdot k, \dots), \quad (29)$$

where ϵ and k are the polarizations and momenta of the photons, $q = k_1 + k_2$ is the η momentum, and P_\pm are the π^\pm momenta.

The low-energy theorems implied by the anomalous Ward identities are¹⁶

$$\mathfrak{F}_\eta(0) = -\frac{1}{\sqrt{3}} \frac{2\alpha}{\pi} \frac{1}{f_\eta} S, \quad (30)$$

and

$$\mathfrak{G}_\eta(0, 0, \dots) = -\frac{1}{\sqrt{3}} \frac{\sqrt{\pi\alpha}}{\pi^2} \frac{1}{f_\pi^2 f_\eta} S, \quad (31)$$

where to any finite order in renormalized perturbation theory,¹⁷ S is fixed by the pointlike constituents that circulate around the triangle and/or square loops. In the three-color triplet quark model¹⁸ $S = \frac{1}{2}$, a value demanded by $\pi^0 \rightarrow \gamma\gamma$ in the framework of ordinary PCAC or even EPCAC.⁵

The on-mass-shell extrapolation factors are defined by¹⁶

$$E_\eta = \frac{\mathfrak{F}_\eta(m_\eta^2)}{\mathfrak{F}_\eta(0)} \quad (32)$$

and

$$\frac{\mathfrak{G}_\eta(m_\eta^2, \mu_\pi^2, \mu_\pi^2, 0, \dots)}{\mathfrak{G}_\eta(0, 0, 0, 0, \dots)} = E_\pi^2 E_\eta, \quad (33)$$

where E_π is the extrapolation factor for $\pi^0 \rightarrow \gamma\gamma$ defined analogously to E_η .

Coupling the η family to the bare constituents with γ_5 coupling one has, for $\eta \rightarrow \gamma\gamma$,

$$\mathfrak{F}_\eta(q^2) = A \frac{m_\eta^2 - q^2}{m_\eta^2} \sum_n \frac{f_\eta m_n^2}{m_n^2 - q^2} g_n, \quad (34)$$

where A is a constant and g_n are the strong couplings of the η_n to the constituents. It is important to note that if \mathfrak{F}_η develops eventually a q^2 dependence this comes from the η -daughter propagators and not from quark structure. In analogy with Eq. (10), the Veneziano ansatz gives

$$E_\eta = \frac{\Gamma(\lambda + \alpha' m_\eta^2)}{\Gamma(\lambda) \Gamma(1 + \alpha' m_\eta^2)}, \quad (35)$$

which in the $SU(3) \times SU(3)$ symmetry limit reduces to the expected result $E_\eta = 1$.

The model-independent prediction of EPCAC was⁵ $E_\eta = (1 - \Delta_\eta)^{-1}$, where Δ_η is the correction to a hypothetical GTR. In Ref. 5 it was assumed that Δ_η could be replaced by Δ_K and that $f_\eta \simeq f_\pi$, in which case a value for $\Gamma(\eta \rightarrow \gamma\gamma)$ in agreement with

experiment was obtained without the need for invoking η - η' mixing. Comparing Eq. (35) with Eq. (13) we see that the assumption $E_\eta \simeq (1 - \Delta_K)^{-1}$ is fully justified in the context of the Veneziano model. In fact, the error introduced by that assumption lies between 3% and 7% for values of λ between 1.6 and 3.

Using Eq. (35) and $f_\eta = f_\pi$, the predicted $\eta \rightarrow \gamma\gamma$ rate is $\Gamma(\eta \rightarrow \gamma\gamma) = 0.25 - 0.37$ keV for $\lambda = 1.5 - 2.8$, to be compared with the experimental value¹³ $\Gamma(\eta \rightarrow \gamma\gamma) = (0.323 \pm 0.46)$ keV, and consistent with the EPCAC result.⁵

Finally, regarding $\eta \rightarrow \pi^+ \pi^- \gamma$, instead of computing the rate directly we can find the ratio $\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) / \Gamma(\eta \rightarrow \gamma\gamma)$, which is independent of f_η and therefore free of the $SU(3)$ assumption $f_\eta = f_\pi$. The result is

$$\frac{\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma)}{\Gamma(\eta \rightarrow \gamma\gamma)} = E_\pi^4 \frac{1}{f_\pi^4} \frac{1}{4\pi\alpha} \phi, \quad (36)$$

where

$$\phi = \frac{m_\eta^4}{96\pi^2} (7.48 \times 10^{-3}) \quad (37)$$

is the phase-space factor¹⁶ and

$$E_\pi = \frac{\Gamma(\beta + \alpha' \mu_\pi^2)}{\Gamma(\beta) \Gamma(1 + \alpha' \mu_\pi^2)}. \quad (38)$$

The prediction for the ratio, Eq. (36), is 0.115–0.123, corresponding to values of β between 1.6 and 3, while the experimental number is¹³ $\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) / \Gamma(\eta \rightarrow \gamma\gamma) = 0.13 \pm 0.04$.

V. SUMMARY

From the results obtained we conclude that the notion of heavy pseudoscalar mesons, implemented by the Veneziano model for three-point functions, provides a working explanation for the rather large magnitude of Δ_π and Δ_K . In addition, the results for radiative η decays make unnecessary the assumption of η - η' mixing, at least with a negative mixing angle.

In summary, the model discussed in this paper seems to be a suitable implementation of EPCAC.

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