Extended partially conserved axial-vector current hypothesis and soft-meson theorems *

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General expressions are derived for soft-meson theorems in the framework of the recently proposed extended partially conserved axial-vector current hypothesis (EPCAC). Applications are made to K_{l3}, $K \rightarrow \pi\pi\pi$, $K \rightarrow 2\pi$, nonleptonic hyperon decays, pseudoscalar-meson photoproduction, and meson-nucleon scattering. In all cases it is found that the EPCAC predictions improve the agreement between soft-meson results and experiment. Moreover, the EPCAC approach allows a unified treatment of several chiral-symmetry-breaking problems. Finally, an extension to SU(4) $\times$ SU(4) is briefly discussed.

I. INTRODUCTION

In a recent paper[1] (hereafter referred to as I) we proposed an extended partially conserved axial-vector current hypothesis (EPCAC) incorporating a family of heavy pseudoscalar mesons (π_n', K_n', η_n') in a model-independent way. This extension of ordinary "strong" partially conserved axial-vector current (PCAC) was mainly motivated by the impossibility of accounting for the corrections to the Goldberger-Treiman relations (GTR),[2] both in SU(2)$\times$SU(2) and SU(3)$\times$SU(3), by means of ordinary dynamical continua.[3,4] The suggestion that there might exist a heavy pion[3,5] which could be responsible for the rather large magnitude of those corrections was incorporated, some time ago, into different kinds of models.[6-9] In I we defined EPCAC by

$$\partial^\mu A_\mu^\alpha = \sum_{n=0}^{N} f_{a_n} m_{a_n}{}^2 \phi_{a_n}{}^\alpha \quad (a = \pi, K, \eta), \tag{1}$$

where $N (N \geq 1)$, $m_{a_n}{}^2$, $\phi_{a_n}{}^\alpha$, and f_{a_n} were left unspecified in order to have a model-independent formulation.[10] The second working hypothesis made in I referred to the strong couplings of the meson daughters, a_n, to hadrons, H and H', viz.,

$$g_{a_n HH'} = g_{a_0 HH'} \quad (n = 0, 1, \ldots). \tag{2}$$

However, this was not strictly necessary and could be replaced by the following alternative assumption: One can assume that for q^2 small the dominant diagrams are those of Figs. 1 and 2 [Fig. 3 defines the $a_n - a_0$ coupling $h_{a_n}(q^2)$]. In this case it was shown in I that the corrections to GTR are given by

$$\Delta_a = -\frac{1}{f_a} \sum_{n=1}^{N} f_{a_n} h_{a_n}(0). \tag{3}$$

Moreover, it turned out that Δ_a is universal, i.e., the corrections to GTR in SU(2)$\times$SU(2), Δ_π, are independent of the hadrons undergoing the $\Delta S = 0$ β decay, and the same situation holds for Δ_K in SU(3)$\times$SU(3). This result is important because it links several chiral-symmetry-breaking problems together in a unified fashion as was already discussed in I. Quite recently, Fuchs[11] made the interesting discovery that the universality among Δ_π on the one hand and among Δ_K on the other may be obtained in the framework of the quark model. This brings the EPCAC approach to chiral-symmetry breaking into close contact with the quark-model approach.[12] The purpose of the present paper is to derive general expressions for soft-meson theorems, in the framework of EPCAC,[13] and discuss additional applications not covered in I.

The paper is organized as follows: Sec. II is devoted to the derivation of soft-meson theorems; in Sec. III we discuss kaon decays (K_{l3}, $K \rightarrow \pi\pi\pi$, and $K \rightarrow \pi\pi$) and in Sec. IV, hyperon decays; Sec. V is devoted to pseudoscalar-meson photoproduction, Sec. VI to meson-nucleon scattering, and Sec. VII to conclusions.

II. SOFT-MESON THEOREMS

Let us start by considering a process involving one soft meson, a^α, in an external "field" H, i.e.,

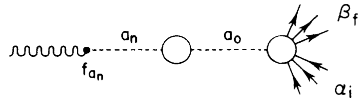

FIG. 1. Off-diagonal contribution to the matrix element of the divergence of the axial-vector current between hadronic states α_i and β_f.

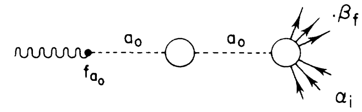

FIG. 2. Diagonal contribution to the matrix element of the divergence of the axial-vector current between hadronic states α_i and β_f.

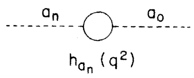

FIG. 3. The $a_n - a_0$ coupling $h_{a_n}(q^2)$.

the reaction[14] $A \to B + a^\alpha$. Defining the amplitude

$$T_\mu^\alpha = i \int d^4x\, e^{iqx} \langle B | T(A_\mu^\alpha(x) H(0)) | A \rangle , \tag{4}$$

and contracting T_μ^α with q^μ and using the EPCAC hypothesis, Eq. (1), one finds

$$q^\mu T_\mu^\alpha = - \sum_{n=0}^{N} m_{a_n}{}^2 f_{a_n} \int d^4x\, e^{iqx} \langle B | T(\phi_{a_n}^\alpha(x) H(0)) | A \rangle - \int d^4x\, e^{iqx} \delta(x_0) \langle B | [A_0^\alpha(x), H(0)] | A \rangle . \tag{5}$$

Extracting the meson poles and assuming, as in I, that for q^2 small the dominant diagrams are those of Figs. 1 and 2, it follows that in the soft-meson limit

$$T^\alpha(q^2 = 0, \nu = 0) \left[f_a + \sum_{n=1}^{N} f_{a_n} h_{a_n}(0) \right] = \langle B | [F_5^\alpha, H(0)] | A \rangle + \lim_{q \to 0} (q^\mu R_\mu^\alpha) , \tag{6}$$

where $T^\alpha(q^2 = 0, \nu = 0)$ stands for the amplitude of $A \to B + a^\alpha$ (soft), $h_{a_n}(q^2)$ is represented in Fig. 3, and R^α contains all other possible singular contributions to T_μ^α, in the soft-meson limit, besides the meson poles. The factorization of T^α in Eq. (6) is a consequence of the assumption that only diagrams of the type illustrated in Figs. 1 and 2 contribute for $q^2 \to 0$. This has already been used in I for the particular case of three-point functions.

Recalling the expression for the corrections to GTR, Eq. (3), one finally has the following EPCAC soft-meson theorem:

$$i f_a (1 - \Delta_a) T^\alpha(q^2 = 0, \nu = 0) = \langle B | [F_5^\alpha, H(0)] | A \rangle + \lim_{q \to 0} (q^\mu R_\mu^\alpha) . \tag{7}$$

In the chiral-symmetry limit Δ_a vanishes and one recovers the standard result.[14] Therefore, the correction to the soft-meson theorem of ordinary PCAC is connected to the breaking of the chiral symmetry as measured by the universal quantity Δ_a.

Let us turn now to processes involving two soft mesons, a^α and a^β, in an external field H, i.e., $A \to B + a^\alpha + a^\beta$. Here, we shall treat both mesons in a symmetrical way and we shall let them become soft at the same time. The relevant amplitude is defined as

$$T_{\mu\nu}^{\alpha\beta} = i \int \int d^4x\, d^4y\, e^{iq_1 x} e^{iq_2 y} \times \langle B | T(A_\mu^\alpha(x) A_\nu^\beta(y) H(0)) | A \rangle . \tag{8}$$

Contracting with $q_1^\mu q_2^\nu$ and symmetrizing, one finds, after some algebra, that

$$q_1^\mu q_2^\nu T_{\mu\nu}^{\alpha\beta} = - T^{\alpha\beta} + \tfrac{1}{2} \lambda^{\alpha\beta\gamma} (q_1 - q_2)^\mu T_\mu^\gamma + \tfrac{1}{2} i \langle B | \Lambda^{\alpha\beta} + \Lambda^{\beta\alpha} | A \rangle + S^{\alpha\beta} + D_1^{\alpha\beta} + D_2^{\beta\alpha} , \tag{9}$$

where

$$T^{\alpha\beta} = i \int \int d^4x\, d^4y\, e^{iq_1 x} e^{iq_2 y} \times \langle B | T(\partial^\mu A_\mu^\alpha(x) \partial^\nu A_\nu^\beta(y) H(0)) | A \rangle , \tag{10}$$

$$D_i^{\alpha\beta} = \int d^4x\, e^{iq_i x} \langle B | T(\partial^\mu A_\mu^\alpha(x) H^\beta(0)) | A \rangle , \tag{11}$$

$$S^{\alpha\beta} = \int d^4x\, e^{i(q_1 + q_2)x} \langle B | T(\sigma^{\alpha\beta}(x) H(0)) | A \rangle , \tag{12}$$

$$T_\mu^\gamma = \int d^4x\, e^{i(q_1 + q_2)x} \langle B | T(V_\mu^\gamma(x) H(0)) | A \rangle , \tag{13}$$

$$i H^\beta(x) \delta(x) = \delta(x_0) [A_0^\beta(x), H(0)] , \tag{14}$$

$$i \sigma^{\alpha\beta}(x) \delta(x - y) = [A_0^\alpha(x), \partial^\mu A_\mu^\beta(y)] \delta(x_0 - y_0) , \tag{15}$$

$$i \lambda^{\alpha\beta\gamma} V_\nu^\gamma(x) \delta(x - y) = [A_0^\alpha(x), A_\nu^\beta(y)] \delta(x_0 - y_0) , \tag{16}$$

and $\Lambda^{\alpha\beta}$, in the soft-meson limits, is given by

$$\Lambda^{\alpha\beta} = - [F_5^\alpha, [F_5^\beta, H(0)]] . \tag{17}$$

Some care has to be exercised here in extracting the meson poles when using EPCAC. For instance, the double-pole contributions to $T^{\alpha\beta}$ are

$$T^{\alpha\beta} = \frac{f_a{}^2 m_a{}^4 A_{00}^{\alpha\beta}}{(m_a{}^2 - q_1{}^2)(m_a{}^2 - q_2{}^2)} + \frac{f_a m_a{}^2}{(m_a{}^2 - q_1{}^2)} \sum_{n=1}^{N} \frac{f_{a_n} m_{a_n}{}^2}{(m_{a_n}{}^2 - q_2{}^2)} A_{0n}^{\alpha\beta} + \frac{f_a m_a{}^2}{(m_a{}^2 - q_2{}^2)} \sum_{n=1}^{N} \frac{f_{a_n} m_{a_n}{}^2}{(m_{a_n}{}^2 - q_1{}^2)} A_{n0}^{\alpha\beta} + \sum_{n=1}^{N} \sum_{k=1}^{N} \frac{f_{a_n} m_{a_n}{}^2}{(m_{a_n}{}^2 - q_1{}^2)} \frac{f_{a_k} m_{a_k}{}^2}{(m_{a_k}{}^2 - q_2{}^2)} A_{nk}^{\alpha\beta} , \tag{18}$$

where $A_{nk}^{\alpha\beta}$ stand for the reduced amplitudes. Assuming once again that for $q^2 \to 0$ the dominant diagrams are of the type illustrated in Figs. 1 and 2; $T^{\alpha\beta}$ in the soft-meson limit reduces to

$$\lim_{\substack{q_1\to 0\\ q_2\to 0}} T^{\alpha\beta} = \left[f_a{}^2 + 2f_a \sum_{n=1}^{N} f_{a_n} h_{a_n}(0) + \sum_{n=1}^{N} f_{a_n} h_{a_n}(0) \sum_{k=1}^{N} f_{a_k} h_{a_k}(0)\right] A^{\alpha\beta}$$
$$= f_a{}^2(1-\Delta_a)^2 A^{\alpha\beta} , \tag{19}$$

where $A^{\alpha\beta}$ stands for $A^{\alpha\beta}_{00}$ and Eq. (3) has been used. Using this procedure the soft-meson limit of Eq. (9) becomes

$$f_a{}^2(1-\Delta_a)^2 A^{\alpha\beta}(A \to B + a^\alpha(\text{soft}) + a^\beta(\text{soft}))$$
$$= \tfrac{1}{2} i \langle B | \Lambda^{\alpha\beta} + \Lambda^{\beta\alpha} | A\rangle + \lim_{\substack{q_1\to 0\\ q_2\to 0}} \tfrac{1}{2}\lambda^{\alpha\beta\gamma}(q_1-q_2)^\mu T^\gamma_\mu + \lim_{\substack{q_1\to 0\\ q_2\to 0}} S^{\alpha\beta} - \lim_{\substack{q_1\to 0\\ q_2\to 0}} (q_1^\mu q_2^\nu R^{\alpha\beta}_{\mu\nu}) . \tag{20}$$

where $R^{\alpha\beta}_{\mu\nu}$ contains all singular contributions in the soft limit except the meson poles. Once more, in the chiral-symmetry limit ($\Delta_a = 0$) one recovers the standard result.[11]

The generalization of these soft-meson theorems to any number of mesons is straightforward and can be performed along the lines first discussed by Weinberg[15] in the context of ordinary PCAC. As a rule of thumb, wherever f_a appears in the PCAC version it should be replaced by $f_a(1-\Delta_a)$ in the EPCAC formulation. Therefore, the larger the number of soft mesons emitted, the larger will be the correction to PCAC results.

In the next sections we shall discuss several applications of Eqs. (7) and (20) and see how they improve the agreement between soft-meson predictions and experiment.

III. KAON DECAYS

A. K_{l3} decay

The corrections to the soft-pion[13] and soft-kaon[17] theorems of K_{l3} have been already derived in I following a somewhat different approach. The same results may be obtained from the master equation (7) by choosing $\langle B| = \langle 0|$, $|A\rangle = |K^+\rangle$, and $H(0) = J_\mu^{4-i5}(0)$ for the soft-pion case, and $\langle B| = \langle \pi|$, $|A\rangle = |0\rangle$, and $H(0) = J_\mu^{1+i2}(0)$ for the soft-kaon case. The results are

$$\left|\frac{f_K}{f_\pi}\right| = [f_+(m_K{}^2) + f_-(m_K{}^2)](1-\Delta_\pi) , \tag{21}$$

and

$$\left|\frac{f_\pi}{f_K}\right| = [f_+(\mu_\pi{}^2) - f_-(\mu_\pi{}^2)](1-\Delta_K) , \tag{22}$$

where the experimental values of Δ_π and Δ_K are[18]

$$\Delta_\pi = 1 - \frac{(m_p + m_n) g_A}{\sqrt{2}\, g_{np\pi} + f_\pi} = 0.06 \pm 0.02 \tag{23}$$

and

$$\Delta_K = 1 - \frac{(m_\Lambda + m_p) g^A_\Lambda}{\sqrt{2}\, g_{\Lambda p K} f_K} = 0.30 \pm 0.15 . \tag{24}$$

As discussed in I, Eq. (22) gives a value for $f_K/f_\pi f_+(0)$ in good agreement with experiment if $f_+(0)$ is taken to be $f_+(0) = 0.96$, according to Pagels.[12] The only additional point we wish to make here is that Eq. (22) is in fact predicting the value of $f_+(0)$ since it provides us with $f_\pi/f_K f_+(0)$ while experiment furnishes $f_K/f_\pi f_+(0)$. Using the linear parametrization for $f_\pm(0)$ together with the data from Ref. 19, we find

$$f_+(0) = 0.98 \pm 0.08 . \tag{25}$$

B. $K \to \pi\pi\pi$

Since we are only interested in finding the corrections to PCAC results we shall avoid here any discussion about the $|\Delta I| = \frac{1}{2}$ rule as well as other related problems.[20] The master equation (7) can be used with the following identifications: $H = \mathcal{H}^{\text{pc}}$, $|B\rangle = |\pi_1\pi_2\rangle$, $|A\rangle = |K_i\rangle$, and

$$T = \lim_{q_3\to 0} \langle \pi_1\pi_2\pi_3^\alpha | \mathcal{H}^{\text{pc}} | K_i\rangle ,$$

where $\mathcal{H}^{\text{pc}}$ is the parity-conserving part of the weak Hamiltonian (the parity-violating part $\mathcal{H}^{\text{pv}}$ not contributing to $K\to 3\pi$) and and α, i are isospin indices. Here we shall let the pion π_3 become soft, thus leaving the $\pi_1\pi_2$ system in an $I=0$ state. We then find

$$\lim_{q_3\to 0} i f_\pi (1-\Delta_\pi) \langle \pi_1\pi_2\pi_3^\alpha | \mathcal{H}^{\text{pc}} | K_i\rangle$$
$$= -\tfrac{1}{2}(\tau^\alpha)_{il} \langle \pi_1\pi_2 | \mathcal{H}^{\text{pv}} | K_l\rangle , \tag{26}$$

where we have used

$$[F_5^\alpha, \mathcal{H}^{\text{pc}}] = -[Q^\alpha, \mathcal{H}^{\text{pv}}] \tag{27}$$

and

$$Q^\alpha |K_i\rangle = \tfrac{1}{2}(\tau^\alpha)_{il} |K_l\rangle . \tag{28}$$

The general expression for the isovector amplitude $\vec{A}$ that describes all $K\to 3\pi$ modes can be written as[20]

$$\vec{A}(s_1, s_2, s_3) = \sum_{\text{cycl}} (\vec{\pi}_1 \cdot \vec{\pi}_2)\vec{\pi}_3 f(s_1, s_2, s_3) , \tag{29}$$

where $\sum_{\text{cycl}}$ stands for all cyclic permutations of

123, $\tilde{\pi}_i$ are isospin functions for the pions, $f(s_i, s_j, s_k)$ is symmetric in S_i, S_j to comply with Bose statistics, and $s_i = (p_k - p_{\pi_i})^2$. The experimental information is contained in $f(s_i, s_j, s_k)$ which is usually assumed to have the following linear form[20]:

$$f(s_i, s_j, s_k) = \bar{A}\left[1 - \frac{\sigma}{\mu_\pi^{\,2}}(s_k - s_0)\right], \qquad (30)$$

where $s_0 = \frac{1}{3}m_K^{\,2} + \frac{2}{3}\mu_\pi^{\,2}$. It can be easily seen that the ordinary PCAC predictions for the slopes are unchanged by EPCAC; the only modification coming from Eq. (26) is that $\bar{A}$ is now given by

$$\bar{A} = \frac{|A(K_S^0 \to \pi^+\pi^-)|}{6f_\pi(1-\Delta_\pi)}\,. \qquad (31)$$

Equation (31) gives

$$\bar{A} = (0.78 \pm 0.02)\times 10^{-6}$$

to be compared with the experimental value

$$|\bar{A}(K_L^0 \to \pi^+\pi^-\pi^0)| = (0.82 \pm 0.03)\times 10^{-6}.$$

For $\Delta_\pi = 0$ one obtains instead

$$\bar{A} = 0.73\times 10^{-6},$$

which is off by three standard deviations.

C. $K \to 2\pi$

In this example one has to let both pions become soft at the same time in order to be consistent with kinematics. Therefore, we shall use the master equation (20) with the following identifications:

$$H(0) = \mathcal{H}^{\mathrm{pv}}, \quad |B\rangle = |0\rangle, \quad |A\rangle = |K_i\rangle,$$

and (32)

$$A_i^{\alpha\beta} = \lim_{\substack{q_1\to 0\\ q_2\to 0}} \langle \pi^\alpha(q_1)\pi^\beta(q_2)|\mathcal{H}^{\mathrm{pv}}|K_i(p)\rangle\,.$$

The soft-pion theorem then reduces to

$$\lim_{\substack{q_1\to 0\\ q_2\to 0}} \langle \pi^\alpha(q_1)\pi^\beta(q_2)|\mathcal{H}^{\mathrm{pv}}|K_i(p)\rangle$$
$$= -\frac{i}{f_\pi^{\,2}}\frac{1}{(1-\Delta_\pi)^2}\langle 0[Q^\beta,[Q^\alpha,\mathcal{H}^{\mathrm{pv}}]] + [Q^\alpha,[Q^\beta,\mathcal{H}^{\mathrm{pv}}]]\,|K(p_i)\rangle + \frac{1}{f_\pi^{\,2}}\frac{1}{(1-\Delta_\pi)^2}\lim_{\substack{q_1\to 0\\ q_2\to 0}} S^{\alpha\beta}, \qquad (33)$$

where all other terms give no contribution,[14] and the term $S^{\alpha\beta}$ might be absent if the $|\Delta I| = \frac{1}{2}$ rule is exact (this, however, does not seem to be the case[20]). In any case since numerical predictions are dependent on the particular model one chooses for the nonleptonic weak Hamiltonian we shall not proceed any further. However, note that the EPCAC corrections in this decay are larger than for $K \to 3\pi$. In fact, from Eq. (33) one reads a 12% correction due to chiral-symmetry breaking.

IV. NONLEPTONIC HYPERON DECAYS

In this section we point out how nonleptonic hyperon decays may be used as a probe to study the interrelation between chiral- and SU(3)-symmetry breaking. Examples of this situation have already been discussed in I in connection with sets of sum rules which become identities in the chiral- and SU(3)-symmetry limits but retain their forms even if both symmetries are broken.

The general form of the amplitude for $Y \to Y' + \pi$ may be written as

$$i\langle Y'\pi|\mathcal{H}_{\mathrm{weak}}|Y\rangle = \bar{u}'(A + \gamma_5 B)u\,, \qquad (34)$$

where A and B describe parity-violating (pv) s-wave and parity-conserving (pc) p-wave decays. Let us concentrate on pv s-wave decays[21] and use the master equation (7) with $|A\rangle = |Y\rangle$, $|B\rangle = |Y'\rangle$, and $H(0) = \mathcal{H}^{\mathrm{pv}}$, to derive the following EPCAC soft-pion theorm:

$$f_\pi(1-\Delta_\pi)T^{\mathrm{pv}}(Y \to Y' + \pi^\alpha(\mathrm{soft}))$$
$$= \langle Y'|[F_5^\alpha, \mathcal{H}^{\mathrm{pv}}]\,|Y\rangle + \lim_{q\to 0}(q^\mu R_\mu^\alpha)\,. \qquad (35)$$

The soft-pion limit of $q^\mu R_\mu^\alpha$ is finite and of the order of SU(3) breaking since it is proportional to $(m_{Y'} - m_Y)$. However, since we are dealing with s-wave decays we have mass degeneracy and therefore no contribution from such terms to the soft-pion theorem (35). In this case one obtains the following relations:

$$\begin{aligned}
&f_\pi(1-\Delta_\pi)A(\Sigma^+ \to p\pi^0) = \tfrac{1}{2}\langle p|\mathcal{H}^{\mathrm{pc}}|\Sigma^+\rangle\,,\\
&f_\pi(1-\Delta_\pi)A(\Lambda \to n\pi^0) = \tfrac{1}{2}\langle n|\mathcal{H}^{\mathrm{pc}}|\Lambda\rangle\,,\\
&f_\pi(1-\Delta_\pi)A(\Xi^0 \to \Lambda\pi^0) = \tfrac{1}{2}\langle \Lambda|\mathcal{H}^{\mathrm{pc}}|\Xi^0\rangle\,,\\
&f_\pi(1-\Delta_\pi)A(\Sigma^- \to n\pi^-) = \langle n|\mathcal{H}^{\mathrm{pc}}|\Sigma^0\rangle\,,\\
&\sqrt{2}f_\pi(1-\Delta_\pi)A(\Lambda \to p\pi^-) = -\langle n|\mathcal{H}^{\mathrm{pc}}|\Lambda\rangle\,,\\
&\sqrt{2}f_\pi(1-\Delta_\pi)A(\Xi^- \to \Lambda\pi^-) = \langle \Lambda|\mathcal{H}^{\mathrm{pc}}|\Xi^0\rangle\,,\\
&\sqrt{2}f_\pi(1-\Delta_\pi)A(\Sigma^+ \to n\pi^+) = \langle p|\mathcal{H}^{\mathrm{pc}}|\Sigma^+\rangle\\
&\qquad\qquad - \sqrt{2}\,\langle n|\mathcal{H}^{\mathrm{pc}}|\Sigma^0\rangle\,.
\end{aligned} \qquad (36)$$

Clearly, the Lee-Sugawara relation[22] is not modified at all by the presence of Δ_π.

The extrapolation of the standard soft-pion theorem [Eq. (35) with $\Delta_\pi = 0$] to the physical world involves corrections due to both SU(2)×SU(2)- and SU(3)-symmetry breakings. The first correction has already been taken care of by means of EPCAC, as shown by the appearance of the universal quantity Δ_π in Eq. (35). Therefore, one has reduced the problem to the SU(3)-breaking part, and once this is accounted for one can gain insight into the interrelationship between the breakings of both symmetries. In a sense this resembles the situation that holds in connection with the Dashen-Weinstein sum rules[23] for the hadronic corrections to generalized GTR. These corrections can be parametrized in terms of f and d couplings and therefore one derives relations between Δ_π, Δ_K and strong-coupling constants. We have already discussed in I how, from the universality of Δ_π and of Δ_K, one can obtain more information from those sum rules and thus study the interrelationship between chiral- and SU(3)-symmetry breakings. The nonleptonic hyperon decays are just another example of such a situation.

V. PSEUDOSCALAR-MESON PHOTOPRODUCTION

We shall concentrate here on the EPCAC prediction for single π photoproduction. It is a straightforward exercise to extend this result to other single and multiple pseudoscalar-meson photoproduction as well as electroproduction processes.

Using the master equation (7) with $|A\rangle = |N(p_1)\rangle$ and $B = |N(p_2)\rangle$ representing nucleon states, and $H(0) = V_\nu^{\text{em}}(0)$ being the electromagnetic current, one has the following EPCAC soft-pion theorem:

$$f_\pi(1-\Delta_\pi)T_\nu^\alpha(\gamma N \to N\pi^\alpha(\text{soft})) = i\epsilon^{\alpha 3\gamma}\langle N(p_2)|A_\nu^\gamma|N(p_1)\rangle + \lim_{q\to 0}(q^\mu R_{\mu\nu}^\alpha), \qquad (37)$$

where

$$T_\nu^\alpha = i\langle N(p_2)\pi^\alpha(\text{soft})|V_\nu^{\text{em}}|N(p_1)\rangle .$$

Next we define the invariant decomposition

$$\epsilon^\nu T_\nu^\alpha = \gamma_5(\gamma\cdot\epsilon)T_1^\alpha + \gamma_5[\tfrac{1}{2}(p_1+p_2)\cdot\epsilon]T_2^\alpha , \qquad (38)$$

and the isospin decomposition

$$T_i^\alpha = \tfrac{1}{2}[\tau_\alpha, \tau_3]T_i^{(-)} + \delta_{\alpha 3}T_i^{(+)} + \tau_\alpha T_i^{(0)} (i=1,2) . \qquad (39)$$

The charged-pion photoproduction amplitudes can then be written as

$$T(\gamma p \to \pi^+ n) = \sqrt{2}(T^{(-)} + T^{(0)}) , \qquad (40)$$

$$T(\gamma n \to \pi^- p) = \sqrt{2}(T^{(-)} - T^{(0)}) . \qquad (41)$$

Following standard steps[14] one finds that the differential cross section for π^+ photoproduction at threshold is given by

$$\left[\frac{|\vec{k}|}{|\vec{q}|}\frac{d\sigma(\pi^+)}{d\Omega}\right]_{\text{theor}} = \frac{\alpha}{4\pi}\left(\frac{M}{M+\mu_\pi}\right)^2 \frac{g_A^{\,2}}{2f_\pi^{\,2}(1-\Delta_\pi)^2} \times\left(\frac{\mu_\pi}{2M}-1\right)^2 , \qquad (42)$$

where M is the nucleon mass and g_A the β-decay constant. In the chiral-symmetry limit ($\mu_\pi = 0$, $\Delta_\pi = 0$) Eq. (42) becomes the Kroll-Ruderman theorem, while for $\Delta_\pi = 0$, $\mu_\pi \neq 0$ one recovers the standard soft-pion theorem[14] (in our approach, however, this last situation is inconsistent). Numerical results are as follows:

$$\left[\frac{|\vec{k}|}{|\vec{q}|}\frac{d\sigma(\pi^+)}{d\Omega}\right]_{\text{theor}} = \begin{cases} 20.8\ \mu\text{b/sr}, & \mu_\pi = \Delta_\pi = 0 \\ 13.6\ \mu\text{b/sr}, & \text{PCAC } (\Delta_\pi = 0) \\ 15.4\pm 0.4\ \mu\text{b/sr}, & \text{EPCAC, Eq. (42)} \\ 15.6\pm 0.5\ \mu\text{b/sr}, & \text{expt. (Ref. 23)} \end{cases}$$

As one can see, both the Kroll-Ruderman and the standard soft-pion prediction are not in good agreement with experiment, while the EPCAC result has improved the prediction.

VI. MESON-NUCLEON SCATTERING

Let us start by considering π-nucleon scattering and use the master equation (20) to obtain the EPCAC version of the soft-pion theorm[14] for the forward amplitude

$$T_{\pi N}^{\alpha\beta} = T^{\alpha\beta}(\pi^\alpha(q) + N(p) \to \pi^\beta(q) + N(p)) = T_{\pi N}^{\alpha\beta}(\nu, t, q^2, q^2) ,$$

where $\nu = p\cdot q$. Writing the standard isospin decomposition

$$T_{\pi N}^{\alpha\beta} = \delta_{\alpha\beta}T^{(+)} + \tfrac{1}{2}[\tau^\alpha, \tau^\beta]T^{(-)} , \qquad (43)$$

it follows that, in the soft-pion limit,

$$\lim_{\nu\to 0}\frac{T^{(-)}(\nu,0,0,0)}{\nu} = \frac{1-g_A^{\,2}}{f_\pi^{\,2}(1-\Delta_\pi)^2} . \qquad (44)$$

Therefore, the EPCAC correction amounts to a 12% over the standard result. Substituting the experimental numbers[18] in Eq. (44) one finds

$$\lim_{\nu\to 0}\frac{T^{(-)}(\nu,0,0,0)}{\nu} = \begin{cases} -(1.29\pm 0.05)\mu_\pi^{\,-2}, & \Delta_\pi = 0 \\ -(1.46\pm 0.09)\mu_\pi^{\,-2}, & \text{EPCAC, Eq. (44)} \end{cases}$$

to be compared with on-shell values[24] ranging from $-0.88\mu_\pi^{\,-2}$ to $-1.1\mu_\pi^{\,-2}$. Clearly, with such errors it is not possible to distinguish unambiguously between the PCAC and EPCAC predictions. A more sensitive test is provided by the Tomozawa-Weinberg relation,[25] holding at the physical threshold, which in the EPCAC version reads

$$a_1 - a_3 = \frac{3\mu_\pi M}{8\pi(M+\mu_\pi)}\frac{1}{f_\pi^{\,2}(1-\Delta_\pi)^2} , \qquad (45)$$

where a_1 and a_3 are the s-wave π-N scattering lengths for isospin $\frac{1}{2}$ and $\frac{3}{2}$ in the s-channel and M is the nucleon mass. Numerical results are as follows:

$$a_1 - a_3 = \begin{cases} 0.239\ \mu_\pi^{-1}, & \Delta_\pi = 0 \\ (0.27 \pm 0.01)\ \mu_\pi^{-1}, & \text{EPCAC, Eq. (45)} \\ (0.259 \pm 0.006)\ \mu_\pi^{-1}, & \text{expt. (Ref. 26)}, \end{cases}$$

showing that the PCAC result is off by more than three standard deviations while the prediction of EPCAC is in good agreement with experiment.

Concerning the even amplitude, it is well known that its off-mass-shell value is given by $T^{(+)}(0,0,0,0) = -\sigma_{\pi N}/f_\pi^{\,2}$, where $\sigma_{\pi N}$ is the σ term.[27] The EPCAC expression is

$$T^{(+)}(0,0,0,0) = -\frac{\sigma_{\pi N}}{f_\pi^{\,2}(1-\Delta_\pi)^2}\,. \tag{46}$$

As with the off-mass-shell odd amplitude, Eq. (44), one finds a 12% correction which is the approximate size of the ambiguities in $\sigma_{\pi N}$ and therefore we cannot draw any definite conclusion.

A similar situation prevails in connection with the Adler-Weisberger relation,[28] which now reads

$$1 = (g_A)^2 + f_\pi^{\,2}(1-\Delta_\pi)^2\,\frac{2}{\pi} \times \int_{\nu_0}^{\infty} \frac{d\nu}{\nu}\,[\bar\sigma_{\text{tot}}(\pi^- p) - \bar\sigma_{\text{tot}}(\pi^+ p)]\,, \tag{47}$$

where $\bar\sigma_{\text{tot}}$ stands for the off-mass-shell total cross section. In fact, the 6% correction to $|g_A|$ obtained from Eq. (47) is presently undetectable. However, this is no longer true for the soft-kaon sum rule, i.e.,

$$2 = (g_\Lambda^A)^2 + (g_\Sigma^A)^2 + f_K^{\,2}(1-\Delta_K)^2\,\frac{2M}{\pi} \times \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2}\,k(\nu)[\bar\sigma_{\text{tot}}(K^- p) - \bar\sigma_{\text{tot}}(K^+ p)]\,, \tag{48}$$

where $k(\nu) = (1/M)(\nu^2 - M^2 m_K^{\,2})^{1/2}$. Here we expect a sizable correction to g_Λ^A. Some time ago, Lôpez[29] made a model-independent evaluation of Eq. (48) (with $\Delta_K = 0$) finding very good agreement between g_Λ^A and its experimental value. This result, however, does not contradict our previous statement because Lôpez had explicitly assumed that the GTR for SU(3)×SU(3) were exact, and therefore evaluated the following sum rule:

$$\frac{2}{(g_\Lambda^A)^2} = 1 + \left(\frac{g_\Sigma^A}{g_\Lambda^A}\right)^2 + \frac{(M+M_\Lambda)^2}{g_{\Lambda p K}^{\;2}}\,\frac{M}{\pi} \times \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2}\,k(\nu)[\bar\sigma_{\text{tot}}(K^- p) - \bar\sigma_{\text{tot}}(K^+ p)]\,, \tag{49}$$

where f_K had been replaced by its GTR value ($\Delta_K = 0$). Recalling the definition of Δ_K, Eq. (24), we can see that Eq. (49) reduces to the EPCAC sum rule Eq. (48). Therefore, the numerical results of Lôpez are actually the predictions of EPCAC,[30] Eq. (48), which give $|g_\Lambda^A| = 0.67 \pm 0.06$ to be compared with the experimental value[18] $|g_\Lambda^A| = 0.66 \pm 0.05$.

As a final point let us consider the EPCAC version of the analog of the Adler-Weisberger relation for πK scattering,[31] i.e.,

$$f_\pi^{\,2}(1-\Delta_\pi)^2 \int_{\nu_0}^{\infty} \frac{d\nu}{\nu}\,[\bar\sigma_{\text{tot}}(K^+\pi^-) - \bar\sigma_{\text{tot}}(K^+\pi^+)] = f_K^{\,2}(1-\Delta_K)^2 \int_{\nu_0}^{\infty} \frac{d\nu}{\nu}\,[\bar\sigma_{\text{tot}}(K^-\pi^+) - \bar\sigma_{\text{tot}}(K^+\pi^+)]\,. \tag{50}$$

Assuming the equality of the integrals one obtains

$$\left|\frac{f_K}{f_\pi}\right| = \frac{1-\Delta_\pi}{1-\Delta_K} = 1.3 \pm 0.3\,, \tag{51}$$

where the number has been derived from Eqs. (23) and (24). If one takes the value $f_+(0) = 0.97$, then from Eq. (51) one has $f_K/f_\pi f_+(0) = 1.3 \pm 0.3$ to be compared with the experimental value of 1.26 ± 0.02. On the other hand, combining Eq. (51) with the soft-kaon theorem, Eq. (22), one has a prediction for $f_+(0)$, i.e.,

$$f_+(\mu_\pi^{\,2}) - f_-(\mu_\pi^{\,2}) = \frac{1}{1-\Delta_\pi}\,. \tag{52}$$

Using the linear parametrization for $f_\pm(0)$ and the K_{l3} data from Ref. 19 one finds

$$f_+(0) = 0.93 \pm 0.02\,, \tag{53}$$

which is consistent with Eq. (25), although the prediction (53) is less uncertain since Δ_π is known more accurately than Δ_K.

VIII. CONCLUDING REMARKS

We have seen in this paper how the EPCAC hypothesis improves the soft-pion and soft-kaon predictions of PCAC and current algebra. However, a more important point is, perhaps, the unification of many chiral-symmetry-breaking problems achieved through the universality of the corrections to GTR in SU(2)×SU(2) and SU(3) ×SU(3). Moreover, this universality allows one to obtain more information about the interrelationship between chiral- and SU(3)-symmetry breakings.

As a final remark we would like to suggest an extension of the EPCAC approach to the case of SU(4)×SU(4). Here one would be in principle reluctant to derive soft-charmed-meson predictions due to the large chiral-symmetry breaking. In other words, one would expect soft-meson theorems and sum rules to be in worse shape than the corresponding ones for SU(3)×SU(3). However, as we have seen in I and in the present paper one can satisfactorily account for the corrections to kaon and η PCAC in the framework of EPCAC. Therefore, if an extrapolation to SU(4)×SU(4) is tried one could presumably handle the chiral-symmetry-breaking effects in the same fashion as for SU(3)×SU(3) and SU(2)×SU(2). Besides, from the universality of the corrections to GTR for charm-changing baryon β decays, one could gain insight into the interrelationship between SU(4)×SU(4) and SU(4) breakings through the use of generalized Dashen-Weinstein sum rules and charmed-baryon nonleptonic decays. Clearly, much more experimental information would be needed before attempting to develop such a program.

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$$f_{a_n} = \frac{m_0^2}{m_{a_n}^2}\tilde{f}_{a_n},$$

where m_0 is the mass of the ground-state meson (π, K, or η).

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